Radiative corrections and electroweak observables

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Different electroweak observables probe different combinations of the parameters $\bar{\theta}$ (the weak mixing angle as defined in terms of couplings at the Z pole) and $\rho \equiv M_W^2/(M_Z^2\cos^2\bar{\theta})$. A simple, approximate method is given for relating these parameters to the angle θ defined in terms of gauge-boson masses by $\cos^2\theta \equiv M_W^2/M_Z^2$, and it is shown how these various observables shed light on θ and quantities such as the top-quark mass that directly affect ρ.

I. INTRODUCTION

The recent precise measurements of the Z mass in hadronic[1] and electron-positron[2,3] collisions have provided important knowledge about electroweak parameters. Measurements of the W mass[4,5] also are improving in accuracy. When combined with other information on neutral-current interactions,[6–9] these data can shed light on quantities such as the top-quark mass that affect radiative corrections[6–30] to these processes.

In recent comprehensive discussions of radiative corrections affecting the properties of the Z and W (see, e.g., Ref. 18) the most important effects are the running of the fine-structure constant and heavy top-quark contributions to W and Z self-energies. It is the purpose of the present paper to apply a simplified discussion based on these effects to specific electroweak observables measured in various reactions. We shall give an approximate prescription which shows what parameters are specified by any given reaction. The method is not meant to substitute for exact calculations, which should be used in determining the exact values of these parameters. However, it is simple enough to yield the statistical impact of any given experiment with just a few lines of calculation. A preliminary account of this work has already appeared.[20] (The present paper corrects some algebraic errors in the parametrizations of electroweak observables given in Ref. 20). The spirit of the approach is quite similar to that advocated, for example, in Refs. 9, 11, 15, 16, and 21.

More general methods appear to be needed to parametrize electroweak radiative corrections in the presence of large numbers of degenerate doublets of heavy fermions or pseudo-Nambu-Goldstone bosons of technicolor theories.[22,23,30] Analyses based on such extended parametrizations are now starting to appear.[22,30] The simplified treatment presented here is equivalent to setting the parameter S, defined in the second of Refs. 22, equal to zero.

Different electroweak observables probe different combinations of the parameters $\bar{\theta}$ (the weak mixing angle as defined in terms of coupling at the Z pole) and

$$\rho \equiv M_W^2/(M_Z^2\cos^2\bar{\theta}) \ . \tag{1.1}$$

The parameter ρ is related to one called T in the second of Refs. 22 by $\rho = 1 + \alpha T$, where α is the fine-structure constant. A simple, approximate method is given in Sec. II for relating ρ and $\bar{\theta}$ to the angle θ defined in terms of gauge-boson masses by

$$\cos^2\theta \equiv M_W^2/M_Z^2 \ . \tag{1.2}$$

The way in which ρ is affected by quantities such as the top-quark and Higgs-boson masses is given in Sec. III. In Sec. IV it is shown how various electroweak observables shed light on θ and ρ (and, consequently, on such interesting quantities as the top-quark mass). Section V concludes.

II. RELATIONS AMONG ELECTROWEAK PARAMETERS

A. Assumptions

We shall define an electroweak mixing angle $\bar{\theta}$ in terms of coupling constants at the Z-boson pole. Thus, the effective electroweak interaction between fermions and a physical Z boson will be written

$$\mathcal{L}_{Zf\bar{f}} = -(g^2+g'^2)^{1/2}\bar{\psi}\gamma^\mu Z_\mu(I_{3L}-Q\bar{x})\psi \ , \tag{2.1}$$

where g and g' are the respective $SU(2)_L$ and $U(1)_{Y_W}$ coupling constants, and $\bar{x} \equiv \sin^2\bar{\theta}$. Here

$$g = e/\sin\bar{\theta}, \quad g' = e/\cos\bar{\theta} \ , \tag{2.2}$$

with $e^2/4\pi \equiv \alpha$ and[31]

$$\alpha^{-1} \equiv \alpha^{-1}(M_Z^2) = 128.02 \pm 0.12 \ . \tag{2.3}$$

The value (2.3) is based on the relation $\alpha^{-1}(M_Z^2) = \alpha^{-1}(m_e^2)(1-\Delta r_{\rm lept} - \Delta r_{\rm had})$, where[31] $\Delta r_{\rm lept} = 0.0330$, $\Delta r_{\rm had} = 0.0328 \pm 0.0009$ for five flavors of light quarks, and $\alpha^{-1}(m_e^2) = 137.036$. We shall adopt the value on the right-hand side of (2.3) for both $\alpha^{-1}(M_W^2)$ and $\alpha^{-1}(M_Z^2)$. Quantities written without an explicit momentum dependence are assumed to be evaluated at M_Z^2, and variations between M_W^2 and M_Z^2 are neglected. The quantity $\sin^2\bar{\theta}$ is essentially that defined as $\sin^2\hat{\theta}(M_Z^2)$ in the modified-minimal-subtraction ($\overline{\rm MS}$) scheme by

Fanchiotti and Sirlin in Ref. 12, as $\sin^2\theta^* \equiv s_*$ by Kennedy and Lynn (last of Refs. 13), and as $\sin^2\bar{\theta}$ by Hollik (first of Refs. 18) and by Consoli, Hollik, and Jegerlehner (last of Refs. 28). Small differences among these quantities are neglected in our simplified approach.

We shall further assume that the exchange of virtual W and virtual Z bosons at small momentum transfers is governed by propagators which behave as $[q^2-M_W^2(q^2)]^{-1}$ or $[q^2-M_Z^2(q^2)]^{-1}$. Thus, a W-exchange process at low q^2 such as muon decay will be governed by

$$G_F(q^2)/\sqrt{2}=g^2(q^2)/8M_W^2(q^2)\ , \tag{2.4}$$

while a corresponding low-q^2 Z-exchange process will have an amplitude proportional to

$$[g^2(q^2)+g'^2(q^2)]/8M_Z^2(q^2)\ .$$

Now, since $g^2(q^2)+g'^2(q^2)=g^2(q^2)/\cos^2\bar{\theta}(q^2)$, if we use the definition (1.1), we have

$$\rho(q^2)G_F(q^2)/\sqrt{2}=[g^2(q^2)+g'^2(q^2)]/8M_Z^2(q^2)\ , \tag{2.5}$$

so that *neutral-current processes appear with an amplitude proportional to ρ if expressed in terms of the angle $\bar{\theta}$.* It is this extremely simple relation upon which we wish to capitalize.

It has been shown by explicit calculation in Ref. 21 that the ratios on the right-hand sides of Eqs. (2.4) and (2.5) are very slowly varying as functions of q^2. Thus, we are justified in taking these equations to the particle poles, where they define couplings in terms of masses and the Fermi constant $G_F=1.166\,37\times10^{-5}$ GeV^{-2}. The fact that both (2.4) and (2.5) turn out to be slowly varying in q^2 means that the constant ρ can also be regarded as slowly varying in q^2. This becomes of some use when analyzing neutral-current processes at low q^2, as will be shown in more detail in Sec. II D.

The effective constancy of Eq. (2.5) (and hence ρ) as a function of q^2 is not a feature of certain minimal extensions of the standard model, such as those involving degenerate doublets of heavy fermions or pseudo-Nambu-Goldstone bosons of technicolor theories.[22,23,30] Our parametrization will make use of the known quantities G_F, $\alpha(m_e)$, and M_Z, and the still-to-be-specified quantity $\rho\equiv1+\Delta\rho$ [where it will turn out that $\Delta\rho\le0.012$ (95% C.L.)] to describe electroweak phenomena. It may turn out that a more general description,[22,23,30] making use of five parameters rather than four, becomes necessary as data become more precise. We shall see that present experiments do not require such a more general approach. As the precision of experiments improves, one might worry also about other effects we have neglected, such as vertex corrections and box diagrams.

B. Relation among θ, $\bar{\theta}$, and ρ

We may combine the definitions (1.1) and (1.2) to obtain the relation $\rho=\cos^2\theta/\cos^2\bar{\theta}$. Writing now

$$\rho=1+\Delta\rho,\quad \cos^2\theta=1-x,\quad \cos^2\bar{\theta}=1-\bar{x}\ , \tag{2.6}$$

we have (to first order in $\Delta\rho$)

$$\bar{x}\approx x+(1-x)\Delta\rho\ , \tag{2.7}$$

where we recall that the quantities $\bar{x}$ and ρ without explicit arguments refer to their values at M_Z^2. We shall substitute the expression (2.7) into several electroweak quantities $\bar{f}(\bar{x},\Delta\rho)$ to obtain corresponding quantities in terms of x and $\Delta\rho$:

$$\bar{f}(\bar{x},\Delta\rho)=f(x,\Delta\rho)\ . \tag{2.8}$$

The expressions $\bar{f}(\bar{x},\Delta\rho)$ can usually be written down more or less by inspection with the help of the Lagrangian (2.1). The expressions $f(x,\Delta\rho)$, on the other hand, contain valuable information about what sorts of constraints are provided by each kind of electroweak processes.

C. Constraint due to Z mass measurement

It was pointed out previously[17] that a precise Z mass measurement would provide valuable information on electroweak parameters. Such a measurement is now available. A compilation of recent data[32] gives

$$M_Z=91.165\pm0.031\ \text{GeV}/c^2\ . \tag{2.9}$$

We shall now combine (2.7) with other information to obtain a second constraint[21] relating x and $\bar{x}$.

From Eqs. (2.2) and (2.4) we have

$$M_W^2=\frac{g^2}{4\sqrt{2}G_F}=\frac{\pi\alpha}{\sqrt{2}G_F\bar{x}}\ . \tag{2.10}$$

Then

$$x=1-\frac{M_W^2}{M_Z^2}=1-\frac{\pi\alpha}{\sqrt{2}G_FM_Z^2\bar{x}}\equiv1-\frac{c_0}{\bar{x}}\ , \tag{2.11}$$

where

$$c_0\equiv\frac{\pi\alpha}{\sqrt{2}G_FM_Z^2}=0.1790\pm0.0002 \tag{2.12}$$

for $M_Z=91.165\pm0.031$ GeV/c^2, $\alpha^{-1}=128.02\pm0.12$.

Equations (2.12) and (2.7) can be combined to give a relation between x and $\Delta\rho$ (or $\bar{x}$ and $\Delta\rho$) stemming from the precise measurement of the Z mass. This relation is shown for $x\equiv\sin^2\theta$ and $\Delta\rho$ by the solid lines in Fig. 1, corresponding to the central value and error bars of the coefficient in Eq. (2.12). For $\Delta\rho=0$, a specific value of $x=\bar{x}$ then follows which we shall call x_0. Numerically, $x_0=0.2335\pm0.0004$. Contours of fixed $\bar{x}$ are shown by the diagonal dashed lines.

Approximate linear relations hold between $\Delta\rho$ and x or $\bar{x}$ once the Z mass is known. Combining (2.7) and (2.12), we have

$$\Delta\rho=a(x-x_0)=b(\bar{x}-x_0)\ , \tag{2.13}$$

where

$$a=\frac{2x_0-1}{(1-x_0)^2}=-0.9072\pm0.0004\ ,\qquad b=\frac{2x_0-1}{x_0(1-x_0)}=-2.978\pm0.008\ , \tag{2.14}$$

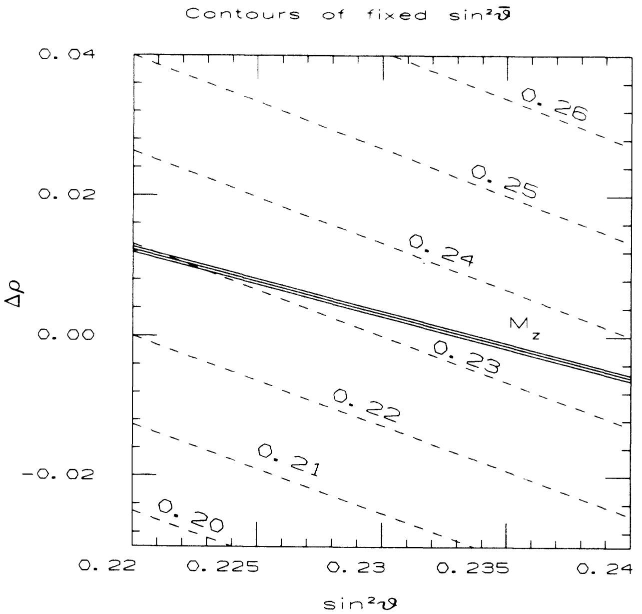

FIG. 1. Contours of fixed $\sin^2\bar{\theta}$ as functions of $x \equiv \sin^2\theta$ and $\Delta\rho$ (dashed lines, labeled by values of $\sin^2\bar{\theta}$). The solid lines correspond to the relation (2.13) (central value and error bars) between x and $\Delta\rho$ arising from the measured Z mass, $M_Z = 91.165 \pm 0.031$ GeV/c^2, with $\alpha^{-1} = 128.02 \pm 0.12$.

for the values of M_Z and α taken above. The value of x_0 is the smaller root of $x_0 = 1 - (c_0/x_0)$ and is given explicitly by $x_0 = (1 - \sqrt{1 - 4c_0})/2$. Another useful relation is

$$\bar{x} - x_0 = \frac{x_0}{1 - x_0}(x - x_0)$$
$$= (0.3046 \pm 0.0007)(x - x_0) \ . \qquad (2.15)$$

One sees immediately from Fig. 1 and Eqs. (2.13)–(2.15) that a given range of $\Delta\rho$ (associated, for example, with a given range of top-quark masses) is correlated with a much wider variation in the angle θ than in $\bar{\theta}$. For $0 \le \Delta\rho \le 0.012$, for example (which we shall see corresponds roughly to $0 \le m_t \le 200$ GeV/c^2), $\sin^2\theta$ ranges from x_0 to about 0.220, while $\sin^2\bar{\theta}$ ranges only from x_0 to about 0.230. Thus, processes which probe a given range of $\sin^2\theta$ are a much better source of information about $\Delta\rho$ than those which probe the same range of $\sin^2\bar{\theta}$. The limited range of a parameter very similar to $\bar{x}$ has been noted recently, for example, in Refs. 8, 18, and 21, and by Fanchiotti and Sirlin in Ref. 12.

We shall present several figures showing the degree to which any given experiment provides constraints on x and $\Delta\rho$. One might argue (see, e.g., Ref. 8) that because the Z mass has been measured so precisely, it is no longer appropriate to depict such two-dimensional constraints. We choose to retain the two-dimensional plots because they illustrate to what degree a given measurement provides information which has already been provided by the Z mass measurement, or provides "orthogonal" (i.e., new) information. If one chose, one could use the relations (2.13) to show how any given experiment constrains a single parameter, be it x, $\bar{x}$, or $\Delta\rho$. We shall do so when discussing most individual experiments and in our conclusions.

D. Neutral-current observables at $q^2 \neq M_Z^2$

The variation of coupling constants between M_Z^2 and lower values of momentum transfer leads to a dependence on q^2 of the effective value of $\bar{x}$ when (2.1) is used to second order to calculate processes involving virtual Z exchange. We estimate the q^2 dependence of the quantity

$$\sin^2\bar{\theta}(q^2) = \bar{x}(q^2)$$
$$\equiv g'^2(q^2)/[g^2(q^2) + g'^2(q^2)] \qquad (2.16)$$

by comparing the q^2 evolution of the fine-structure constant α and of the corresponding SU(2) constant $\alpha_2 \equiv g^2/4\pi$. The q^2 dependences are given by the Feynman parametric integrals[19]

$$\alpha^{-1}(q^2) = \alpha^{-1}(M_Z^2) + \frac{1}{\pi} \sum_f Q_f^2 N_c(f) \int_0^1 2x(1-x)dx \ln \left| \frac{m_f^2 - x(1-x)M_Z^2}{m_f^2 - x(1-x)q^2} \right| , \qquad (2.17)$$

$$\alpha_2^{-1}(q^2) = \alpha_2^{-1}(M_Z^2) + \frac{1}{4\pi} \sum_f N_c(f) \int_0^1 2x(1-x)dx \ln \left| \frac{x m_{f^-}^2 + (1-x)m_{f^+}^2 - x(1-x)M_Z^2}{x m_{f^-}^2 + (1-x)m_{f^+}^2 - x(1-x)q^2} \right| , \qquad (2.18)$$

where $f^{\pm}$ denote fermions which are $I_3 = \pm\frac{1}{2}$ members of weak isodoublets, and $N_c(f)$ is 1 for leptons, 3 for quarks. An excellent approximation to the integrals in Eqs. (2.17) and (2.18) for $|q^2|, M_Z^2 \gg m_f^2$ is provided by $\frac{1}{3}\ln(M_Z^2/q^2)$, while for $q^2 = 0$ and $M_Z^2 \gg m_f^2$ the integral in (2.17) is close to $\frac{1}{3}[\ln(M_Z^2/m_f^2) - \frac{5}{3}]$. We evaluate the integrals using the observed lepton masses and the quark masses[8] $(m_u, m_d, m_s, m_c, m_b) = (5.5, 8, 150, 1200, 5000)$ MeV/c^2, respectively.

The values of q^2 important for our purposes range from zero (applicable to parity violation in atoms) to M_Z^2 (applicable to processes involving real Z bosons). We find, for example, that

$$\delta\bar{x}(0) \equiv \bar{x}(0) - \bar{x}(M_Z^2) = 0.0076 \pm 0.0002 \qquad (2.19)$$

for $m_t = 150 \pm 50$ GeV/c^2. Corresponding estimates in Refs. 8 and 21 for this quantity are about 0.009 and 0.007, respectively. There is also some subtlety, alluded to in Ref. 19, associated with whether the evolution of g_2 is estimated from the W self-energy [as we have done in Eq. (2.18)] or via the Z self-energy; the difference in $\delta\bar{x}(0)$ amounts to about 0.001. In what follows we shall assume

$\delta\bar{x}(0)=0.0076$, recognizing that this quantity may be uncertain by about 0.001 unit. We may also estimate $\delta\bar{x}(q^2)\equiv\bar{x}(q^2)-\bar{x}(M_Z^2)$ for other momentum transfers in the same manner.

Writing

$$\begin{aligned}\bar{x}(q^2)&=\bar{x}(M_Z^2)+\delta\bar{x}(q^2)\\&\approx x+\Delta\rho(1-x)+\delta\bar{x}(q^2)\ ,\end{aligned}\tag{2.20}$$

we see that a neutral-current process at $q^2\neq M_Z^2$ yields a value of x which is *also shifted by* $\delta\bar{x}(q^2)$, to first order in small quantities. For example, we have to subtract 0.0076 from a $q^2\approx0$ measurement of $\sin^2\bar{\theta}$ to compare it with the quantity $\bar{x}\equiv\bar{x}(M_Z^2)$, and a corresponding shift of the same amount must be made for x. This correction is germane to parity violation in atoms, for example. The corresponding shifts for processes at spacelike values of $q^2\neq0$ are found to be smaller: 0.0035 for deep-inelastic neutrino scattering, 0.0048 for polarized electron-deuteron scattering, and 0.006 for neutrino elastic scattering on electrons. We shall discuss these shifts further in the context of specific processes.

Henceforth we shall use the notation

$$\bar{x}'=\bar{x}+\delta\bar{x}(q^2)\ ,\tag{2.21}$$

$$x'=x+\delta\bar{x}(q^2)\ ,\tag{2.22}$$

to refer to quantities at some $q^2\neq M_Z^2$.

Note added. It has been pointed out (see, e.g., Refs. 6 and 12) that vertex corrections neglected here compensate to a large extent the q^2 variation of $\bar{x}$, so that a more self-consistent treatment within the present context would be to neglect the shift $\delta\bar{x}(q^2)$ altogether. This is relevant to neutrino scattering processes, where the effects of a neutrino charge radius induced by higher-order corrections lead to almost precise compensation of the change in $\bar{x}$ from $q^2=M_Z^2$ to $q^2=0$. A similar comment applies to parity violation in electron-quark interactions. However, within the context of considering gauge-boson self-energy effects but ignoring vertex corrections, we must retain the q^2 variation of $\bar{x}$.

E. Extended parametrization

Note added. Our approach can be easily extended to ones[22,23,30] containing a new degree of freedom, corresponding to the difference between wave-function renormalizations of the Z and W. Ignoring q^2 variations, we may write the effective charged- and neutral-current interaction strengths, keeping quantities to first order in radiative corrections, as

$$\frac{G_F}{\sqrt{2}}=[1+(1-x_0)\Delta Z_*]\frac{g^2}{8M_W^2}\ ,\tag{2.23}$$

$$\frac{G_F\rho}{\sqrt{2}}=(1+\Delta Z_*)\frac{g^2+g'^2}{8M_Z^2}\ ,\tag{2.24}$$

in place of (2.4) and (2.5), respectively. Here $Z_*\equiv1+\Delta Z_*$ denotes the wave-function renormalization of the Z, in the notation of the second of Refs. 22. The ratio of W and Z wave-function renormalizations is a consequence of custodial SU(2) symmetry and may not be as shown above if that symmetry is violated.[22,23] Writing

$$x_0(1-x_0)\Delta Z_*\equiv\alpha S/4,\quad \Delta\rho=\rho-1\equiv\alpha T\tag{2.25}$$

and linearizing Eqs. (2.23) and (2.24) in S and T with the help of the relation $x_0(1-x_0)=\pi\alpha/(\sqrt{2}G_FM_Z^2)$ discussed in Sec. II C, we find

$$\bar{x}-x_0=\frac{\alpha}{1-2x_0}[\tfrac{1}{4}S-x_0(1-x_0)T]\ ;\tag{2.26}$$

$$x-x_0=\frac{\alpha}{1-2x_0}[\tfrac{1}{2}(1-x_0)S-(1-x_0)^2T]\ ;\tag{2.27}$$

$$x-\bar{x}=\alpha\left[(1-x_0)T-\frac{S}{4}\right]\ .\tag{2.28}$$

Equations (2.26) and (2.27) are the analogs of Eq. (2.13), while (2.28) corresponds to (2.7), for the case in which $S\neq0$. All the methods applied in this paper to electroweak observables can be extended to the case $S\neq0$ if Eqs. (2.26)–(2.28) are used in substituting for $\bar{x}$ or x.

III. SOURCES OF $\rho\neq1$

A. Quark-mass differences

Gauge-boson self-energy graphs involving fermion loops lead to a finite value[10,11] for $\delta M_W^2-(1-\bar{x})\delta M_Z^2$ in the presence of a weak isodoublet of quarks such as (t,b). Counting a factor of 3 for color, we have[21]

$$\delta M_W^2-(1-\bar{x})\delta M_Z^2\approx\frac{3g^2}{64\pi^2}\left[m_t^2+m_b^2-\frac{2m_t^2m_b^2}{m_t^2-m_b^2}\ln\frac{m_t^2}{m_b^2}\right]\ .\tag{3.1}$$

The quadratic dependence on m_t in Eq. (3.1) is crucial; it stems from the nonconservation of the axial-vector current in the presence of a heavy-fermion mass. If we write

$$\frac{M_W^2}{(1-\bar{x})M_Z^2}=\rho=1+\Delta\rho\tag{3.2a}$$

or

$$M_W^2\approx(1-\bar{x})M_Z^2+M_W^2\Delta\rho\ ,\tag{3.2b}$$

and compare Eqs. (3.2b) and (3.1), we find

$$\Delta\rho_{\text{top}}\approx\frac{3\alpha}{16\pi\bar{x}}\left[\frac{m_t}{m_W}\right]^2\tag{3.3}$$

in the limit $m_t\gg m_b$. Here we have used $4\pi\alpha=e^2=g^2/\bar{x}$. For $m_t=100$ GeV/c^2, and $\bar{x}=0.23$, $\Delta\rho_{\text{top}}\approx0.0032$.

B. Higgs bosons

The presence of virtual Higgs-boson loops in W and Z self-energies leads to a contribution[21,33]

$$\Delta\rho_{\mathrm{Higgs}} = \frac{3\alpha}{16\pi\bar{x}}\left[\frac{M_H^2}{M_H^2-M_W^2}\ln\frac{M_H^2}{M_W^2} - \frac{1}{1-\bar{x}}\frac{M_H^2}{M_H^2-M_Z^2}\ln\frac{M_H^2}{M_Z^2}\right] . \qquad (3.4)$$

This expression behaves for $M_H \gg M_W$ as

$$\Delta\rho_{\mathrm{Higgs}} \approx \frac{-3\alpha}{8\pi(1-\bar{x})}\ln\frac{M_H}{M_W} . \qquad (3.5)$$

Here, in contrast with the correction from the top quark, the effect is only logarithmic in the Higgs-boson mass. For $M_H = 1$ TeV/c^2 Eq. (3.4) implies $\Delta\rho_{\mathrm{Higgs}} = -0.0024$, comparable in magnitude but opposite in sign to the contribution from a top quark of mass $\approx M_W$.

Note added. There is a small additional contribution of Higgs bosons to the parameter S defined in Sec. II E, which has been neglected in the present treatment.

C. Triplet Higgs bosons

Vacuum expectation values of weak isotriplet Higgs bosons can lead to $\rho \neq 1$. Specifically, the deviations are

$$\Delta\rho = -2V_{1,-1}^2/v^2 + 4V_{1,0}^2/v^2 , \qquad (3.6)$$

where $v = 2^{-1/4}G_F^{-1/2} = 246$ GeV is the standard Higgs-doublet vacuum expectation value, while the other vacuum expectation values V are labeled by (I, I_3). (See, e.g., the discussion in Ref. 34, where normalizations are defined.) In particular, with $V_{1,-1} \neq 0$ it is possible to cancel out the effects on ρ of a very heavy top.[35] (With $2V_{1,0}^2 = V_{1,-1}^2$, the isotriplet Higgs-boson contributions to ρ cancel one another, so independent arguments limiting their size must be brought to bear.[35])

D. Comparison with exact formulas

If one uses the relations (2.7) and (2.12) to connect $x = \sin^2\theta$ or $\bar{x} = \sin^2\bar{\theta}$ and $\Delta\rho$, and the approximate formula (3.3) for $\Delta\rho$, one obtains a relation between x or $\bar{x}$ and m_t shown in Fig. 2, respectively, by the dashed and dotted lines. For $m_t = 0$, $x = \bar{x} = x_0 = 0.2335$. The m_t dependence of $\bar{x}$ is much less appreciable than that of x, so that for $m_t = 200$ GeV, $x \approx 0.22$, while $\bar{x} \approx 0.23$. This is a result of the fact that x is a rather artificial quantity, defined in terms of M_W^2/M_Z^2, while $\bar{x}$ refers to couplings at the Z mass scale, whose values should not be so sensitive to m_t once the Z mass is specified. This point has been particularly emphasized recently by Sirlin in Ref. 12.

For comparison we also show in Fig. 2 the values of x predicted in Ref. 18 for various Higgs-boson masses (the solid lines), and values of $\bar{x}$ obtained via Eq. (2.7) from the values of x (dotted-dashed lines). Especially for values of $\bar{x}$, our simple expressions provide a good approximation to the more exact calculation.

E. Combined m_t and M_H dependence

In Fig. 3 we show contours of fixed $\Delta\rho$ as functions of M_H and m_t, based on the sum of the contributions (3.1)

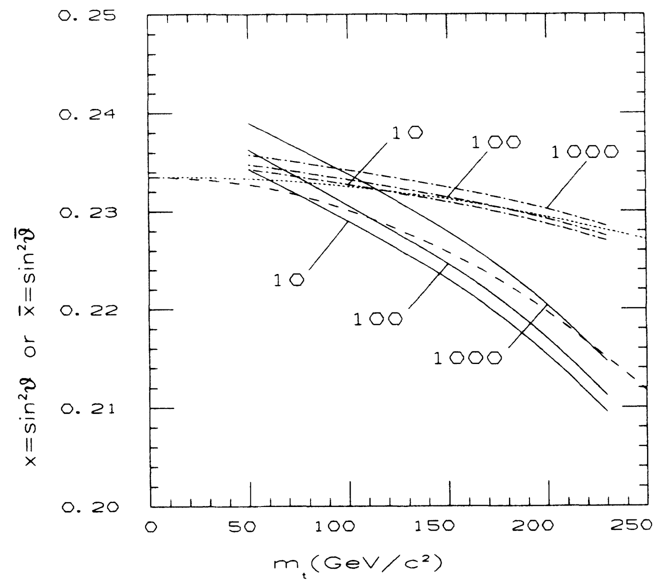

FIG. 2. Values of $x = \sin^2\theta$ (lower four curves) or $\bar{x} = \sin^2\bar{\theta}$ (upper four curves) as functions of top-quark mass. Solid curves, labeled by the Higgs-boson mass in GeV/c^2, correspond to values calculated by Hollik (Ref. 18); dotted-dashed curves correspond to the respective values of $\bar{x}$ obtained from these x values via Eq. (2.7); dashed and dotted curves correspond to our approximate expressions for x and $\bar{x}$, with $\Delta\rho$ calculated from Eq. (3.3).

and (3.4). The dependence on the top-quark mass is much stronger than on the Higgs-boson mass. As M_H ranges from 10 to 1000 GeV/c^2, $\Delta\rho$ ranges from 0.0031 to 0.0006 for $m_t = 100$ GeV/c^2, and from 0.0125 to 0.0101 for $m_t = 200$ GeV/c^2.

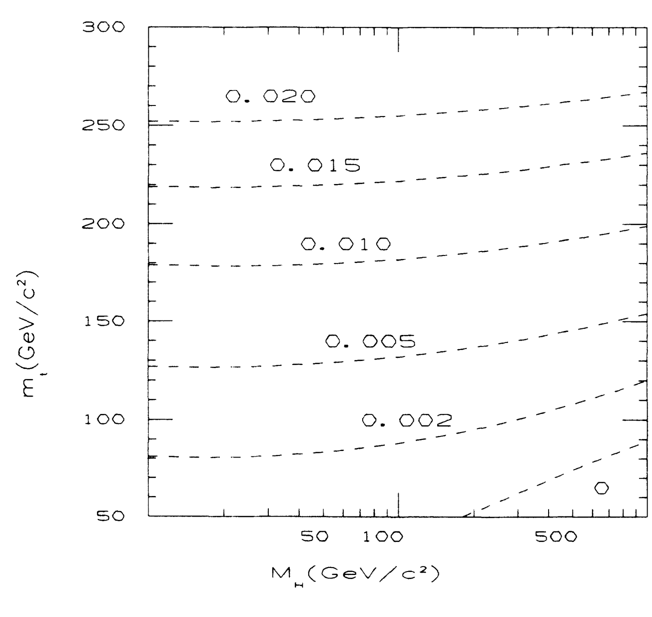

FIG. 3. Contours of fixed $\Delta\rho$ as functions of M_H and m_t (dashed lines, labeled by values of $\Delta\rho$), based on the sum of Eqs. (3.1) and (3.4).

IV. ELECTROWEAK OBSERVABLES

A. *W* mass

Given an exact value of M_Z, the measurement of the W mass[4,5] is a direct source of information about $x=\sin^2\theta$, independent of $\Delta\rho$. (By definition, $\sin^2\theta \equiv 1-M_W^2/M_Z^2$.) The Collider Detector at Fermilab (CDF) Collaboration[4] quotes a value of M_W which, when combined with the CERN LEP measurement of M_Z, leads to

$$\sin^2\theta = 0.2317 \pm 0.0075 \; , \tag{4.1a}$$

while UA2 finds[5]

$$\sin^2\theta = 0.2202 \pm 0.0084 \pm 0.0045 \; . \tag{4.1b}$$

We average the two results to find

$$\sin^2\theta = 0.2273 \pm 0.0059 \; . \tag{4.2}$$

This value is shown in Fig. 4. The corresponding constraint on $\Delta\rho$ when one takes into account the Z mass measurement, as one can see from Fig. 4 or Eq. (2.13), is

$$\Delta\rho = 0.0056 \pm 0.0054 \; . \tag{4.3}$$

On the basis of Fig. 3, this constraint implies $m_t \le 207$ GeV/c^2 at the 1σ limit for $M_H \le 1$ TeV/c^2.

There is a direct relation between M_W and $\bar{x}$ within the context of the assumptions made here, as one sees from Eq. (2.10): $\bar{x}=\pi\alpha/(\sqrt{2}G_F M_W^2)$. We combine Eqs. (4.2) and (2.9) to obtain

$$M_W = (80.14 \pm 0.31)\ \text{GeV}/c^2 \tag{4.4}$$

and hence

$$\bar{x} = 0.2316 \pm 0.0018 \; . \tag{4.5}$$

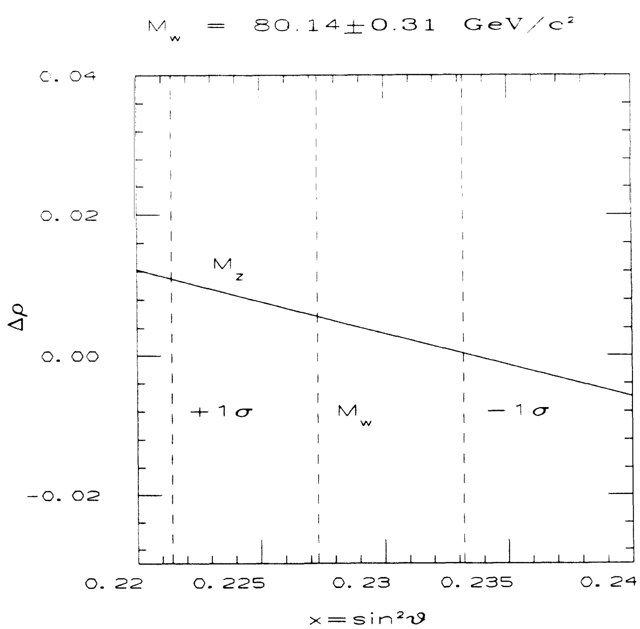

FIG. 4. Constraints on x and $\Delta\rho$ following from the average (4.4) of CDF and UA2 W mass measurements. Solid line: Z mass constraint (central value).

The acceptable range of $\bar{x}$ is quite limited. A similar conclusion was reached by Fanchiotti and Sirlin in Ref. 12.

Our approximate conclusion that the new W mass measurements imply $m_t \le 207$ GeV/c^2 at the 1σ level is supported by the more precise calculations of Ref. 18. In Fig. 5 we show predicted values of M_Z and M_W for various top-quark and Higgs-boson masses, along with the experimental values (2.9) and (4.4). The 1σ upper limit on m_t varies between about 160 and 190 GeV/c^2 as M_H ranges from 10 to 1000 GeV/c^2. [The corresponding upper limit on m_t for a 1-TeV Higgs particle based on (4.4) and the last of Refs. 12 is 196 GeV/c^2.] The 1σ lower limit on m_t is comparable to that ($m_t \ge 89$ GeV/c^2) set by a direct search[36] in $\bar{p}p$ collisions.

B. Deep-inelastic neutrino scattering

Ratios of neutral-to-charged-current cross sections have been measured in several experiments at CERN[37,38] and Fermilab.[39,40] These measurements have been performed both for neutrinos and for antineutrinos. As we shall see, the measurements using neutrinos are one of the best sources of a constraint on both $\Delta\rho$ and x. Antineutrinos provide almost no information on x, and relatively poor information on $\Delta\rho$ as a result of limited statistics.

The value of $\bar{x} \equiv \sin^2\bar{\theta}$ changes between M_Z^2 and the values of q^2 probed in present neutrino deep-inelastic scattering experiments. Here $q^2=-2m_p E_\nu xy$, where E_ν is the neutrino laboratory energy, x is the Bjorken scaling variable, and y is the fraction of the neutrino energy transferred to the target. We choose nominal values $E_\nu = 100$ GeV, $x=0.2$, and $y=0.5$ to estimate an average $q^2=-Q_0^2=-18.8$ GeV^2. Using the parameters described earlier to evaluate (4.17) and (4.18), we then find $\delta\bar{x}(-Q_0^2)\equiv\bar{x}(-Q_0^2)-\bar{x}(M_Z^2)=0.0035\pm0.0002$ for $m_t = 150\pm50$ GeV/c^2. Notice that even though $Q_0^2=(4.3\ \text{GeV})^2$ appears much closer to $q^2=0$ than to M_Z^2, the difference $\delta\bar{x}(-Q_0^2)$ is only about half of $\delta\bar{x}(0)$. The reason is that light leptons and quarks play a major role in vacuum-polarization corrections to both α and α_2.

Note added. As mentioned at the end of Sec. II, there is some question as to whether one should apply the correction $\delta\bar{x}$ at all. A simplified treatment, ignoring it, would yield a higher value of x.

(1) *Neutrinos.* The ratio $R_\nu \equiv \sigma_{\rm NC}(\nu N)/\sigma_{\rm CC}(\nu N)$ for an isoscalar target is given by[41]

$$R_\nu = \rho^2[\tfrac{1}{2} - x' + \tfrac{5}{9}x'^2(1+r)] \; , \tag{4.6}$$

where

$$r = \frac{\sigma_{\rm CC}(\bar{\nu}N)}{\sigma_{\rm CC}(\nu N)} \; . \tag{4.7}$$

We may substitute $\rho^2 \approx 1+2\Delta\rho$ and Eq. (2.7) to obtain an expression for R_ν in terms of x to first order in $\Delta\rho$:

$$R_\nu = \tfrac{1}{2} - x' + \tfrac{5}{9}x'^2(1+r) + \Delta\rho[\tfrac{10}{9}(1+r)-1]x' \; . \tag{4.8}$$

The coefficient of $\Delta\rho$ in Eq. (4.8) is relatively small, as a result of accidental cancellations.[6,9,20] Thus, the ratio R_ν provides an estimate of $x=x'-\delta\bar{x}$ rather independent of $\Delta\rho$.

We shall illustrate the application of Eq. (4.8) for two specific sets of data for which all the relevant quantities have been presented recently.[37,38] Similar analyses are possible for other data sets.[39,40]

(a) CERN-Dortmund-Heidelberg-Saclay-Warsaw (CDHSW) Collaboration data. A correction is necessary for an isoscalar target when the measurement is not performed on one. Furthermore, corrections for the neutrino spectrum must be made. These are applied in Ref. 37, with the result that Eq. (4.8) continues to define x', but with $R_\nu \rightarrow R_\nu^0$ and $r \rightarrow r^0$. The measured, corrected values of these quantities are

$$R_\nu^0 = 0.3122 \pm 0.0034 - 0.009(m_c - 1.5\ \mathrm{GeV}/c^2)\ , \quad (4.9a)$$

$$r^0 = 0.383 \pm 0.014 - 0.004(m_c - 1.5\ \mathrm{GeV}/c^2)\ . \quad (4.9b)$$

The contours in $\Delta\rho$ and x corresponding to the central value and $\pm 1\sigma$ limits of R_ν^0 (errors on r^0 are insignificant by comparison) are shown by the dashed lines in Fig. 6. The corresponding limits on x and ρ, taking into account the constraint from the Z mass, are (for $m_c = 1.5$ GeV/c^2)

$$x = 0.2255 \pm 0.0045; \quad \Delta\rho = 0.0073 \pm 0.0041\ . \quad (4.10)$$

(b) CHARM Collaboration data. The values corresponding to Eqs. (4.9) that we employ for the CHARM data[38] are

$$R_\nu^0 = 0.3091 \pm 0.0031 - 0.009(m_c - 1.5\ \mathrm{GeV}/c^2)\ , \quad (4.11a)$$

$$r^0 = 0.456 \pm 0.014 - 0.004(m_c - 1.5\ \mathrm{GeV}/c^2)\ . \quad (4.11b)$$

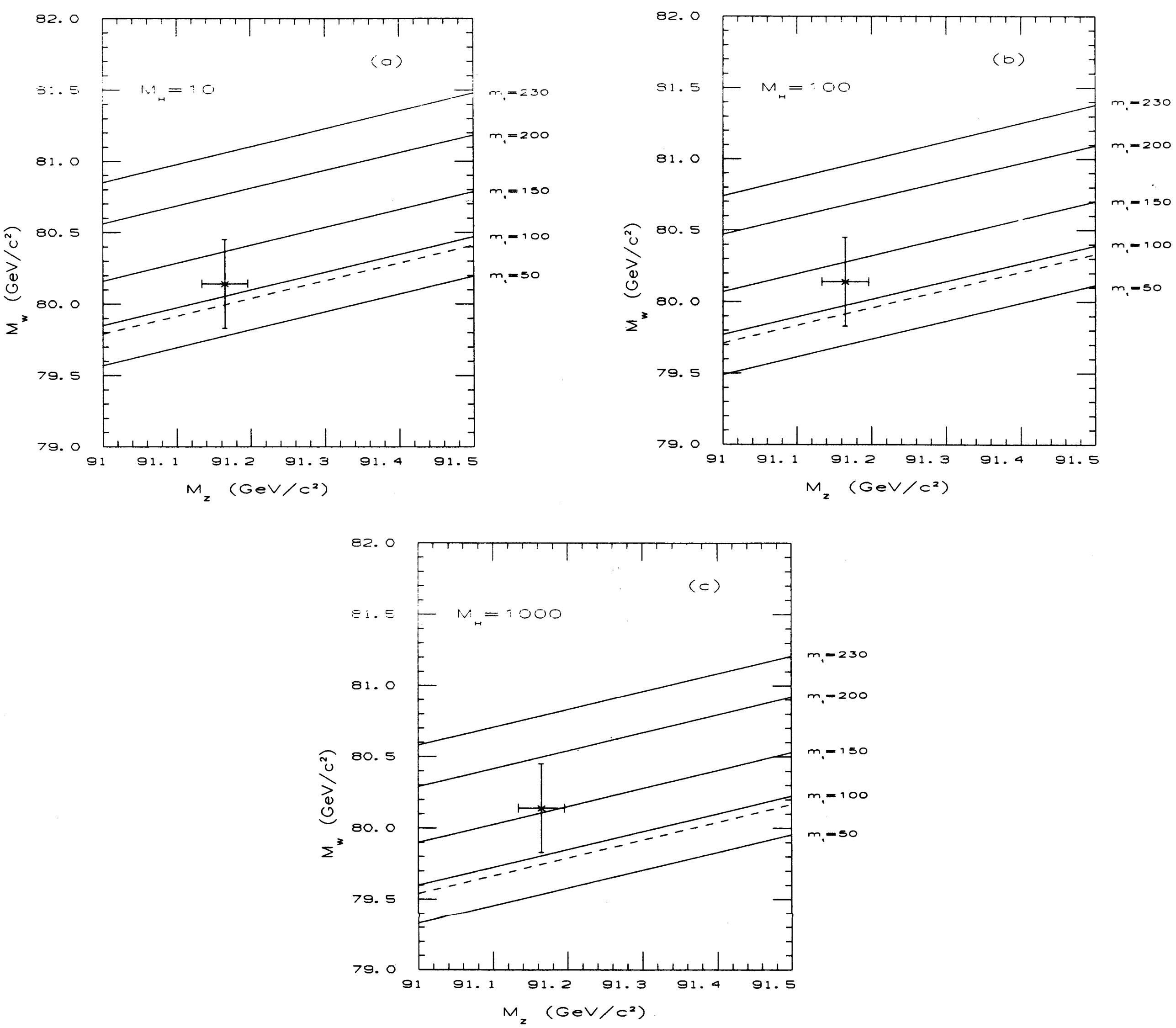

FIG. 5. Comparison of measured Z and W masses [plotted point, Eqs. (2.9) and (4.4)] with predictions of Ref. 18 for various top-quark masses (right-hand labels, in GeV/c^2) and Higgs-boson masses M_H (in GeV/c^2). Values must lie above the dashed line, based on the direct top-quark search of Ref. 36, if the top quark decays conventionally. (a) $M_H = 10$ GeV/c^2; (b) $M_H = 100$ GeV/c^2; (c) $M_H = 1000$ GeV/c^2.

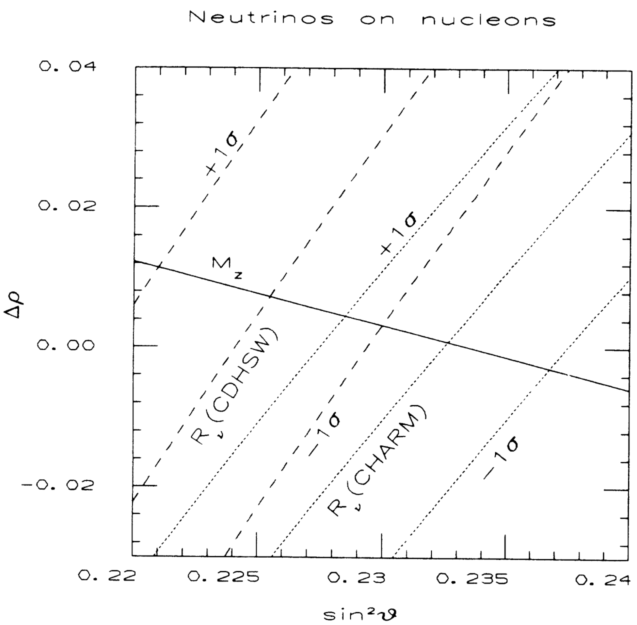

FIG. 6. Constraints on $\sin^2\theta$ and $\Delta\rho$ from measurement of R_ν. Dashed lines correspond to central value and $\pm 1\sigma$ constraints from the CDHSW Collaboration (Ref. 37), while dotted lines correspond to CHARM II results (Ref. 38). Solid line: Z mass constraint (central value).

Here we have applied a net correction equivalent to $\Delta\sin^2\theta = +0.005$ to the value of R_ν presented in Ref. 38 (see Ref. 37), no correction to their value of r, and have assumed the charmed-quark mass dependence to be the same as in Ref. 37. The contours in $\Delta\rho$ and x corresponding to the central value and $\pm 1\sigma$ limits of R_ν^0 are shown by the dotted lines in Fig. 6. The corresponding limits on x and ρ, again taking into account the constraint from the Z mass, are (for $m_c = 1.5$ GeV$/c^2$)

$$x = 0.2326 \pm 0.0041; \quad \Delta\rho = 0.0008 \pm 0.0037 \ . \tag{4.12}$$

(c) Averaged CDHSW and CHARM data. We may average the results of Eqs. (4.10) and (4.12) to obtain

$$x = 0.2294 \pm 0.0030; \quad \Delta\rho = 0.0037 \pm 0.0027 \ . \tag{4.13}$$

Remember that we are taking into account the constraint from the Z mass, so that the error on $\sin^2\theta$ is slightly smaller than if we had kept ρ fixed. However, an additional systematic error in x of $\Delta x = 0.013\Delta(m_c/1$ GeV$/c^2)$ arises from uncertainty in the charmed-quark mass. We shall take the error in x to be ± 0.004, corresponding to a 300-MeV uncertainty in m_c. Additional theoretical uncertainties[37,38] amount to a further error in x of ± 0.003. Adding these in quadrature to the error in (4.14), we then find

$$\begin{aligned} x &= 0.2294 \pm 0.0058 \ ; \\ \bar{x} &= 0.2323 \pm 0.0018 \ ; \\ \Delta\rho &= 0.0037 \pm 0.0053 \ , \end{aligned} \tag{4.14}$$

where we have used Eqs. (2.14) and (2.15) to relate x to $\bar{x}$ and $\Delta\rho$. The value of x is nearly identical to that found by Fanchiotti and Sirlin [Eq. (8) of Ref. 12].

The dashed and dotted lines in Fig. 6 slope gently upward to the right. It is mentioned in Ref. 10 that they slope upward to the *left* if neutrino and antineutrino constraints in analyses of the data[37−40] are combined. We have exhibited the antineutrino constraints separately (see below).

The contributions of vector and axial-vector mesons to the ratio R_ν have been estimated in Ref. 42. There is a possibility, depending on how these contributions fall with increasing Q^2, that $\sin^2\theta$ could be as much as 0.01 larger than estimated on the basis of (4.8) and (4.9). According to Eqs. (2.14) and (4.14), such an increase in x would reduce the 1σ upper bound on $\Delta\rho$ to about 0, favoring top-quark masses near present lower bounds.[42]

Other, slightly less precise, results for R_ν are 0.309 ± 0.009 [Chicago-Caltech-Fermilab-Rochester-Rockefeller Collaboration[39]] (CCFRR) and 0.307 ± 0.007 [average of three Fermilab-MIT-Michigan State (FMM) Collaboration[40] values]. These are compatible with Eqs. (4.9a) and (4.11a). We do not include their effects in Fig. 6 only for economy of presentation. The results should be incorporated into any comprehensive analysis of world data. The CCFRR value, in particular, already has the theoretical errors added in quadrature, so that it has greater statistical weight than the quoted error would imply.

The uncertainties in $\sin^2\theta$ due to m_c and to vector and axial-vector meson production can both be minimized by using higher-energy neutrino beams, such as could be provided at an upgraded Fermilab Tevatron. Proposals exist (see, e.g., Ref. 43) for such experiments.

(2) *Antineutrinos.* For deep-inelastic scattering of antineutrinos on an isoscalar target, one has[41]

$$R_{\bar\nu} = \rho^2 \left[\frac{1}{2} - \bar{x}' + \frac{5}{9}\bar{x}'^2 \left[1 + \frac{1}{r} \right] \right] . \tag{4.15}$$

Substituting as before, we find

$$R_{\bar\nu} \approx \tfrac{1}{2} - x' + \tfrac{5}{9}x'^2 \left[1 + \frac{1}{r} \right] + \Delta\rho \left[\frac{10}{9} \left[1 + \frac{1}{r} \right] - 1 \right] x' \ . \tag{4.16}$$

In contrast with Eq. (4.8), this expression is almost independent of x' for $x' \approx 0.23$, $r \approx \frac{1}{3}$. The dependence on $\Delta\rho$, on the other hand, turns out to be considerably greater.

The values of $R_{\bar\nu}$ and r, corrected for nonisoscalarity of the target and for various other effects,[37] are denoted by $R_{\bar\nu}^0$ and $\bar{r}^0$. They are found in Ref. 37 to be

$$R_{\bar\nu}^0 = 0.378 \pm 0.016 - 0.019(m_c - 1.5 \text{ GeV}/c^2) \ , \tag{4.17a}$$

$$\bar{r}^0 = 0.371 \pm 0.014 + 0.004(m_c - 1.5 \text{ GeV}/c^2) \ . \tag{4.17b}$$

It is these values which are to be used in Eq. (4.16). The corresponding contours in the $x - \Delta\rho$ plane are shown by the dashed lines in Fig. 7 for $m_c = 1.5$ GeV$/c^2$. The value of $R_{\bar\nu}$ (here, uncorrected) from Ref. 38 is

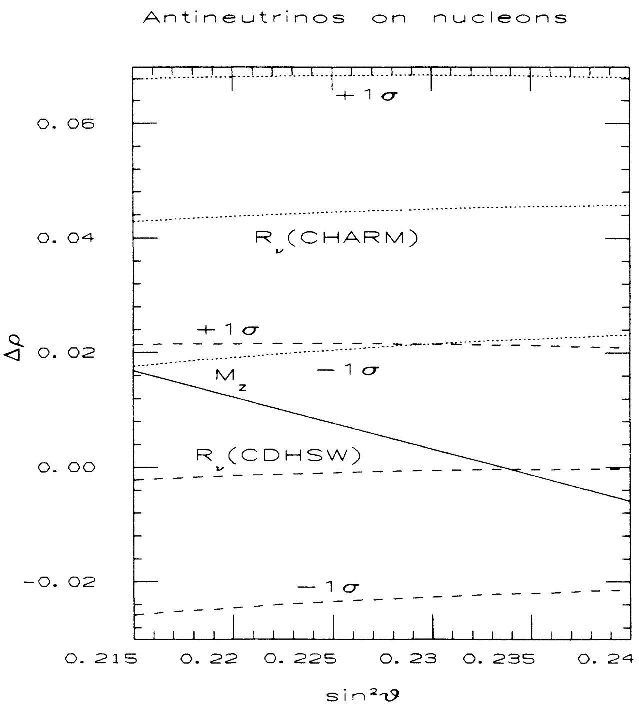

FIG. 7. Constraints on $\sin^2\theta$ and $\Delta\rho$ from measurement of $R_{\bar\nu}$. Lines as in Fig. 6.

$$R_{\bar\nu}=0.390\pm0.014\ , \tag{4.18}$$

and we take r from Eq. (4.11b). The corresponding contours in the $x-\Delta\rho$ plane are shown by the dotted lines in Fig. 7 for $m_c=1.5$ GeV/c^2. The higher values of $\Delta\rho$ in comparison with the CDHSW data arise from the higher values of both r and $R_{\bar\nu}$.

The corresponding values of $R_{\bar\nu}$ from Refs. 39 and 40 are 0.382±0.028 (CCFRR) and 0.384±0.017 (FMM). These lie between Eqs. (4.17a) and (4.18), and provide constraints of comparable quality.

The constraints shown in Fig. 7 provide little additional information beyond that already supplied by the Z mass and the neutrino NC/CC ratio.

(3) *Paschos-Wolfenstein relation.*[44] The ratio

$$R_{\mathrm{PW}}^{-}\equiv\frac{\sigma_{\mathrm{NC}}(\nu_\mu N)-\sigma_{\mathrm{NC}}(\bar\nu_\mu N)}{\sigma_{CC}(\nu_\mu N)-\sigma_{CC}(\bar\nu_\mu N)}=\rho^2(\tfrac{1}{2}-\bar x)$$

is useful in principle because it is independent of sea-quark contributions. In terms of $\Delta\rho$ and x' it is given by[20]

$$R_{\mathrm{PW}}^{-}=\tfrac{1}{2}-x'-x'\Delta\rho\ .$$

Present uncertainties associated with $\sigma_{\mathrm{NC}}(\bar\nu_\mu N)$ prevent this quantity from providing a useful constraint in the x-$\Delta\rho$ plane.

C. Electron-quark interactions

The low-energy effective Lagrangian for electron-quark interactions can be parametrized as[45]

$$\begin{aligned}\mathcal{L}_{\mathrm{eq}}=-\frac{G_F}{\sqrt{2}}\{(\bar e\gamma^\mu\gamma^5 e)[\tfrac{1}{2}\alpha(\bar u\gamma_\mu u-\bar d\gamma_\mu d)\\ +\tfrac{1}{2}\gamma(\bar u\gamma_\mu u+\bar d\gamma_\mu d)]\}\\ +\{(\bar e\gamma^\mu e)[\tfrac{1}{2}\beta(\bar u\gamma_\mu\gamma^5 u-\bar d\gamma_\mu\gamma^5 d)\\ +\tfrac{1}{2}\delta(\bar u\gamma_\mu\gamma^5 u+\bar d\gamma_\mu\gamma^5 d)]\}\ .\end{aligned} \tag{4.19}$$

With our definitions, one can express the parameters α (no relation to the fine-structure constant), β, γ, and δ in terms of ρ and $\bar x'$ as

$$\begin{aligned}&\alpha=\rho(-1+2\bar x'),\quad \beta=\rho(-1+4\bar x')\ ,\\ &\gamma=\rho(2\bar x'/3),\quad \delta=0\ .\end{aligned} \tag{4.20}$$

These couplings are probed in such processes as parity violation in atoms and polarized electron-deuteron scattering.

(1) *Atomic parity violation.* The interference of electromagnetic and neutral-weak-current effects may be seen through parity-violating effects in atoms, such as those which have been measured recently with great precision in atomic cesium.[46] The coherent effect of the neutral current arises from the vector coupling of the virtual Z to the nucleus and its axial-vector coupling to the electron. The weak neutral-current amplitude is proportional to a single power of ρ. One thus measures a quantity linear in ρ and in the vector coupling of the Z to the nucleus:

$$Q_W=-[\alpha(Z-N)+3\gamma(Z+N)]=\rho(Z-N-4Z\bar x')\ . \tag{4.21}$$

Radiative corrections[47,48] for low m_t alter this expression to

$$Q_W=\rho[0.9793(Z-N)-3.8968Z\bar x']\ , \tag{4.22}$$

or, in terms of $\Delta\rho$ and $x'\equiv x+\delta\bar x$,

$$\begin{aligned}Q_W\approx&-0.98(N-Z)-3.9Zx'\\ &-\Delta\rho[0.98(N-Z)+3.9Z]\ .\end{aligned} \tag{4.23}$$

Here we use $\delta\bar x=0.0076$, in accord with the estimate of Sec. II D, to relate x' to x. (*Note added.* Vertex corrections, ignored here, nearly cancel the effect of $\delta\bar x$, as mentioned at the end of Sec. II.) For Cs, with $Z=55$, $N=78$, one finds

$$Q_W(\mathrm{Cs})=-22.5-214.3x'-236.8\Delta\rho\ . \tag{4.24}$$

Application of recent atomic physics corrections[47] to the measurements of Ref. 46 yields a result

$$\frac{-Q_W}{N}=0.909\pm0.020\pm0.010 \tag{4.25}$$

which, for $N=78$, entails[49]

$$Q_W=-70.9\pm1.6\pm0.8\ . \tag{4.26}$$

Equating (4.24) and (4.26), we obtain the contours in the x-$\Delta\rho$ plane shown in Fig. 8 by the dashed lines. Note that they are almost parallel to those entailed by the Z measurement, and are consistent with it at the 2σ level.

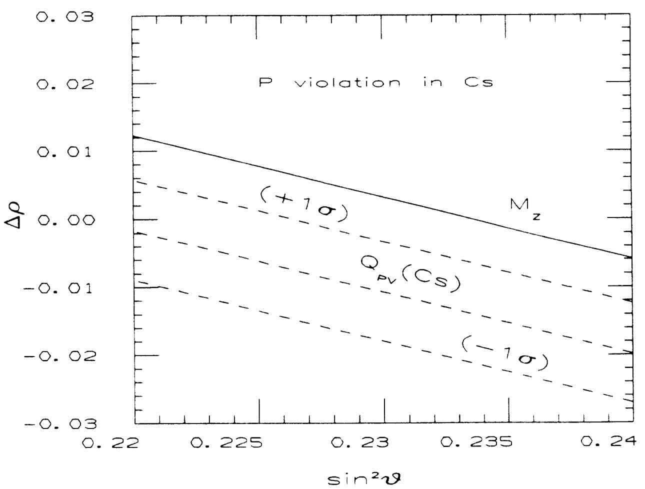

FIG. 8. Constraints on $\sin^2\theta$ and $\Delta\rho$ from parity violation in atomic cesium (dashed lines). Solid line: Z mass constraint (central value).

The measurement of parity violation in atoms does not provide separate information from the Z mass measurement within the context of the radiatively corrected standard model. Such a measurement *can* provide a very powerful constraint on certain unconventional models, such as those involving extra Z's.[6,50–54] We have also found[30] that it already provides one of the best constraints at present on the parameter S discussed in Sec. II E, with the prospect of further substantial improvement.[55]

(2) *Polarized electron-deuteron scattering.* The combinations of parameters measured in an experiment[56] at SLAC are

$$\alpha+\tfrac{1}{3}\gamma=-0.60\pm0.16\ , \tag{4.27}$$

$$\beta+\tfrac{1}{3}\delta=0.30\pm0.50\ . \tag{4.28}$$

Substituting $\bar{x}'=x'+\Delta\rho(1-x')$ into the expressions (4.20), we find

$$\alpha+\tfrac{1}{3}\gamma=-1+\tfrac{20}{9}x'+\tfrac{11}{9}\Delta\rho\ ; \tag{4.29}$$

$$\beta+\tfrac{1}{3}\delta=-1+4x'+3\Delta\rho\ . \tag{4.30}$$

The errors in Eqs. (4.27) and (4.28) are strongly correlated, as one can see from Fig. 2 in the second of Refs. 56. Thus, it is misleading simply to combine these equations with the theoretical predictions (4.29) and (4.30). If we were to do so, we would obtain, respectively,

$$x'+\tfrac{11}{20}\Delta\rho=0.18\pm0.07\ , \tag{4.31}$$

$$x'+\tfrac{3}{4}\Delta\rho=0.33\pm0.13\ . \tag{4.32}$$

Both relations are consistent with present bounds on x and $\Delta\rho$, but a stronger result can be obtained by taking account of correlations.

We have performed a simultaneous fit to the 11 data points quoted in the second of Refs. 56, on the basis of the predictions (4.29) and (4.30). We then obtain regions of x' and $\Delta\rho$ corresponding to $\chi^2\le\chi^2_{\min}+1$. The values of x' must be corrected for the fact that $q^2\neq M_Z^2$. From values presented in the second of Refs. 56, we estimate $\langle q^2\rangle=-1.38$ GeV^2, and $\delta\bar{x}=0.0048\pm0.0002$ for $m_t=150\pm50$ GeV/c^2. (See, however, the note at the end of Sec. II.)

The results are shown in Fig. 9. The central dashed line refers to the χ^2 minimum for each value of $\Delta\rho$, and the $\pm1\sigma$ contours correspond to the range of x leading to $\chi^2\le\chi^2_{\min}+1$ for that value of $\Delta\rho$. (The χ^2 decreases slowly along the central dashed line as $\Delta\rho$ rises.) The error in x is ±0.012, as found in Ref. 56. An additional error in x of ±0.008, stemming from a 5% uncertainty in the electron polarization, is to be added linearly to the other errors in x.[56]

When the constraint due to the Z mass is taken into account, the present error in x of ±0.02 and its correlation with $\Delta\rho$ in the polarized electron-deuteron scattering experiment correspond to an error of about $\Delta\rho=\pm0.04$. Since other experiments which we have been discussing (W mass or neutrino deep-inelastic scattering) constrain $\Delta\rho$ to better than ±0.006 we require an improvement in present accuracy of polarized e-D asymmetries by about a factor of 8 to achieve comparable constraints.

The slopes of the lines in Fig. 9 are rather different from those presented in Ref. 6. We do not understand the source of the discrepancy at present.

Note added. The recent experiment of Ref. 57 provides a constraint on β and δ with smaller errors than Eq. (4.28): $\beta+0.04\delta=0.005\pm0.17$, when values of α and γ are taken from other experiments.

D. Neutrino-electron scattering

(1) *Neutrino-to-antineutrino ratio for neutral currents.* The ratio of muon-neutrino-to-muon-antineutrino cross

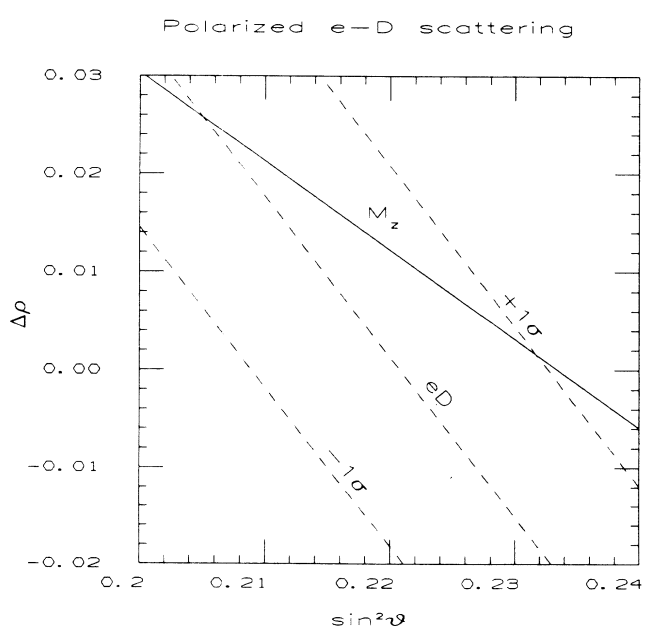

FIG. 9. Constraints on $\sin^2\theta$ and $\Delta\rho$ (dotted lines) from measurement of polarized ed scattering from Ref. 56. Solid line: Z mass constraint.

sections on electrons is

$$\frac{\sigma(\nu_\mu e)}{\sigma(\bar{\nu}_\mu e)} \equiv R_{\nu e} = 3\left[\frac{1-4\bar{x}'+16\bar{x}'^2/3}{1-4\bar{x}'+16\bar{x}'^2}\right] . \tag{4.33}$$

The measured value[58] of this ratio has been interpreted[58,59] in terms of a value

$$\bar{x}'=0.233\pm0.012\pm0.008 . \tag{4.34}$$

To convert this to a value of $\bar{x}=\bar{x}'-\delta\bar{x}$, we estimate $q^2=-2m_e E_\nu y$ in the process as follows.

We take $E_\nu=100$ GeV, and $\langle y\rangle$ from the expected angular distributions in $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering. For $\bar{x}'=\frac{1}{4}$, both processes can be shown to have $\langle y\rangle=\frac{7}{16}$, which is close enough for present purposes. We then find $\langle q^2\rangle=-0.045$ GeV^2, leading to $\delta\bar{x}=0.0060\pm0.0002$ for $m_t=150\pm50$ GeV$/c^2$. We thus interpret (4.34) as implying $\bar{x}=\bar{x}'-0.006=0.227\pm0.014$, consistent with the likely range mentioned earlier (0.23 to 0.234). Considerably greater precision in the measurement of $R_{\nu e}$ is required before an effective challenge of the standard model can be mounted. This and other measurements of $\bar{x}$ already provide useful constraints on models[22,23,30] in which the parameter S (see Sec. II E) is allowed to be nonzero, as pointed out in the second of Refs. 22. As noted earlier, vertex corrections act to nullify the effect of $\delta\bar{x}$.

(2) *Interference between charged and neutral currents in $\nu_e e$ scattering.* The process $\nu_e e^-\to\nu_e e^-$ receives contributions from both charged and neutral currents. A recent experiment[60] has been interpreted in terms of an interference term

$$I=-1.07\pm0.17\pm0.11 , \tag{4.35}$$

which in our notation is expressed as

$$I=\rho(-2+4\bar{x}')\approx-2+4x'+2\Delta\rho . \tag{4.36}$$

The experiment thus constrains the combination

$$x'+\frac{\Delta\rho}{2}=0.23\pm0.05 . \tag{4.37}$$

A useful constraint within the context of the present discussion would begin to emerge when the interference term I is measured about ten times more accurately than at present.

E. Forward-backward asymmetry in $p\bar{p}\to Z+\cdots\to e^+e^-+\cdots$

A value of $\bar{x}$ has been extracted on the basis of the forward-backward asymmetry of the lepton pairs from the Z produced in $\bar{p}p$ collisions:[61]

$$\bar{x}=0.229\pm0.016\pm0.002 . \tag{4.38}$$

This value is consistent with the expected range $0.23\le\bar{x}\le0.234$, but the accuracy must improve considerably for it to have an impact within the context of the standard model. The central value and error are comparable to those obtained from the most precise experiment[58] on neutrino-electron scattering.

F. Partial widths in Z decays

Numerical tables of partial widths for Z decays have been presented in Ref. 18. It is easy to see the qualitative behavior of the various partial widths on the basis of the considerations presented here. A similar discussion has been given in Ref. 14.

The lowest-order expressions for the partial widths of the Z into various fermion species are

$$\Gamma(Z\to\nu\bar{\nu})=\frac{\alpha M_Z}{24\bar{x}(1-\bar{x})}=\frac{G_F M_Z^3}{12\pi\sqrt{2}}\rho\equiv\Gamma_0 , \tag{4.39}$$

$$\Gamma(Z\to l^+l^-)=\frac{\Gamma_0}{2}[1+(1-4\bar{x})^2] , \tag{4.40}$$

$$\Gamma(Z\to q\bar{q})=3\frac{\Gamma_0}{2}[1+(1-4|Q_q|\bar{x})^2] . \tag{4.41}$$

Within the context of the corrections discussed here, Eqs. (4.39)–(4.41) continue to apply to all decays except $Z\to b\bar{b}$, for which loop diagrams involving an internal top quark provide an additional contribution.

Now, $\Gamma_0\equiv\Gamma(Z\to\nu\bar{\nu})$ in Eq. (4.39) is a decreasing function of $\bar{x}$ for $\bar{x}$ in the range of interest. The coefficients of Γ_0 in Eqs. (4.40) and (4.41) are also decreasing functions of $\bar{x}$. In Table I we compare the predictions of Eqs. (4.39)–(4.41) for partial and total Z widths for several values of $\bar{x}$, taking $\Gamma(Z\to b\bar{b})$ separately from Hollik's calculation. The effect of the top-quark loop on $\Gamma(Z\to b\bar{b})$ is actually to suppress the $\bar{x}$ dependence; we have taken an average over very slowly varying values. Experimental values (from Refs. 10, 29, 62, and 63) are shown where available.

The measured leptonic width implies $\bar{x}=0.231\pm0.003$, relatively free of theoretical errors. We expect improvements in the accuracy of this quantity to play a major role in refining parameters of the electroweak theory.

The measured total width favors $\bar{x}=0.2300\pm0.0026$, within the range of values found in other measurements. From Eqs. (2.13)–(2.15) we see that the value of x corresponding to this range of $\bar{x}$ is 0.222 ± 0.009 and that of $\Delta\rho$ is 0.010 ± 0.008.

The ratio $\Gamma_{\rm tot}(Z)/\Gamma(Z\to l^+l^-)$ is predicted to be about 29.6, nearly independent of $\bar{x}$. It is unlikely that the more general parametrization of Refs. 22 and 23 could alter this prediction significantly. The present experimental ratio is compatible with the prediction. Improved measurement of this ratio would be an excellent test for physics beyond that considered here or in Refs. 22, 23, and 30.

The interpretation of the total width measurement is open to some uncertainties, such as QCD corrections and rare decay modes. For example, a change of α_s by ±0.02 (the error assumed in Ref. 29) changes the predicted hadronic and total Z widths by ±11 MeV. The total error quoted on the hadronic width of the Z in Ref. 29 is much larger, amounting to ±25 MeV, but we believe this reflects also uncertainty in $\bar{x}$. A measurement of the total or hadronic Z width to ±10 MeV can provide a potential very useful constraint on standard-model parameters or on new physics, restricting $\bar{x}$ to an accuracy of ±0.0015 when the QCD error is also taken into account.

TABLE I. Partial Z decay widths (MeV).

Channel	$\bar{x}=0.226$	$\bar{x}=0.230$	$\bar{x}=0.234$	Expt.
$\nu\bar{\nu}$	169.6	167.5	165.5	
All $\nu\bar{\nu}$	508.9	502.6	496.6	496 ± 18[a]
l^+l^-	85.6	84.3	83.1	84.0 ± 0.9[b]
All l^+l^-	256.8	252.9	249.3	
$u\bar{u}$[c]	294.6	288.9	283.4	
$u\bar{u}+c\bar{c}$[c]	589.2	577.8	566.8	
$c\bar{c}$[d]	305.9	299.9	294.2	267 ± 109[e]
$d\bar{d}$[c]	378.6	372.1	365.8	
$d\bar{d}+s\bar{s}$[c]	757.3	744.2	731.7	
$b\bar{b}$[c]	356.0[f]	356.0[f]	356.0[f]	377 ± 52[e]
				353 ± 48[g]
All hadrons[d]	1767.5	1742.1	1717.7	1755 ± 21[b]
All	2533.2	2497.6	2463.6	2498 ± 20[b]
All/l^+l^-	29.59	29.63	29.65	29.74 ± 0.40

[a]From Ref. 10 and the second of Refs. 29.
[b]From the second of Refs. 29.
[c]Without QCD corrections.
[d]Including QCD correction of $1+\alpha_s/\pi=1.038$ for $\alpha_s(M_Z^2)=0.12$.
[e]Reference 62.
[f]Figure 12 in second of Refs. 18 implies about 360 MeV.
[g]Reference 63.

G. Asymmetries at the Z

Two major classes of asymmetries will be measured in electron-positron collisions at the Z mass. Both probe the parameter $\bar{x}$ governing intrinsic couplings at the Z pole. Forward-backward asymmetries A_{FB} for production of various fermion species can make use of unpolarized beams, while various types of left-right asymmetries $A_{LR}=(\sigma_L-\sigma_R)/(\sigma_L+\sigma_R)$ measure the asymmetry between cross section for left-handed and right-handed electrons annihilating on positrons. A variant of A_{LR} is measured using the production of polarized τ leptons by unpolarized beams.

(1) *Forward-backward asymmetries.* The differential cross section for the annihilation of e^-e^+ into a fermion f and the corresponding antifermion $\bar{f}$ at a center-of-mass energy $\sqrt{s}$ is (see, e.g., Ref. 64)

$$\frac{d\sigma(e^-e^+\to f\bar{f})}{d(\cos\theta^*)}=\frac{\pi\alpha^2}{8s}[(|f_{LL}|^2+|f_{RR}|^2)(1+\cos\theta^*)^2]+[(|f_{LR}|^2+|f_{RL}|^2)(1-\cos\theta^*)^2]\ , \tag{4.42}$$

where the first subscript on the amplitude refers to the helicity of the fermion f and the second to that of the electron. (We shall neglect all fermion masses.) Here

$$f_{(L,R)L}\equiv -Q+\frac{(I_{3(L,R)}-Q\bar{x})(-\frac{1}{2}+\bar{x})}{\bar{x}(1-\bar{x})}\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}\ , \tag{4.43}$$

$$f_{(L,R)R}\equiv -Q+\frac{I_{3(L,R)}-Q\bar{x}}{1-\bar{x}}\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}\ , \tag{4.44}$$

with Q and I_3 denoting the charge and weak isospin of the fermion f. For ordinary quarks and leptons, $I_{3L}=\pm\frac{1}{2}$, $I_{3R}=0$.

The cross sections for the process corresponding to forward and backward scattering are defined as

$$\sigma_F\equiv\int_0^1 d(\cos\theta^*)\frac{d\sigma(e^-e^+\to f\bar{f})}{d(\cos\theta^*)}\ ;$$
$$\sigma_B\equiv\int_{-1}^0 d(\cos\theta^*)\frac{d\sigma(e^-e^+\to f\bar{f})}{d(\cos\theta^*)}\ . \tag{4.45}$$

The forward-backward asymmetry for $e^-e^+\to f\bar{f}$ is

$$A_{FB}\equiv\frac{\sigma_F-\sigma_B}{\sigma_F+\sigma_B}=\frac{3}{4}\frac{|f_{LL}|^2+|f_{RR}|^2-|f_{LR}|^2-|f_{RL}|^2}{|f_{LL}|^2+|f_{RR}|^2+|f_{LR}|^2+|f_{RL}|^2}\ . \tag{4.46}$$

We show this quantity as a function of $\bar{x}$ for $\sqrt{s}=M_Z=91.165$ GeV/c^2 and for various fermion species in Fig. 10. We have taken $\Gamma_Z=2.54$ GeV, in ac-

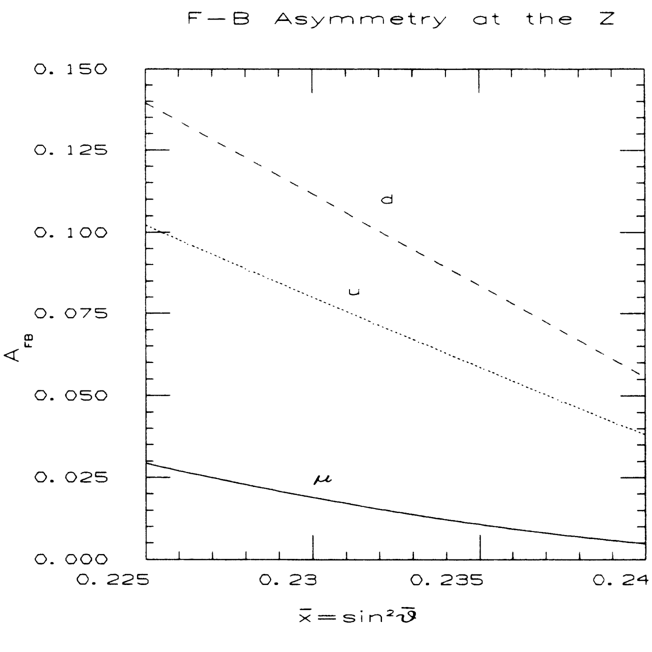

FIG. 10. Forward-backward asymmetries at the Z peak for $e^-e^+\to$ (a) $u\bar{u}$ (dotted line), (b) $d\bar{d}$ (dashed line), and (c) $\mu^-\mu^+$ (solid line), as functions of $\bar{x}$.

cord with the compilation of Ref. 10.

Let us imagine performing a measurement of $\bar{x}$ with an error of $\Delta\bar{x}=0.001$, which would distinguish between the "high-top" ($\bar{x}\approx 0.23$) and "low-top" ($\bar{x}\approx 0.234$) scenarios at the 4σ level. [Recall that since $\Delta\rho=-2.98(\bar{x}-x_0)$, a change in $\bar{x}$ of -0.001 corresponds to a change in ρ of 0.003, while a top quark of mass 200 GeV/c^2 is associated with $\Delta\rho\approx 0.010$ to 0.013.] What measurements of forward-backward asymmetries are capable of yielding such information?

The expected asymmetries for $\bar{x}=0.232\pm 0.001$ are

$$A_{FB}^{(u)}=0.0714\pm 0.0044\ , \tag{4.47a}$$

$$A_{FB}^{(d)}=0.1004\pm 0.0056\ , \tag{4.47b}$$

$$A_{FB}^{(\mu)}=0.0153\pm 0.0016\ . \tag{4.47c}$$

The small value for (4.47c) is a consequence of the quadratic vanishing of $A_{FB}^{(\mu)}$ for $\bar{x}=\frac{1}{4}$. Now, in order to observe a given asymmetry A at the $n\sigma$ level, one needs to observe $N=n/A^2$ events. But, since $n=A/\delta A$, where δA is the error in A, we find $N=(\delta A)^{-2}$. Thus, to measure $\bar{x}$ to ± 0.001 in a forward-backward asymmetry at the Z, we need approximately (5.2×10^4, 3.2×10^4, 3.9×10^5) ($u\bar{u}$, $d\bar{d}$, $\mu^-\mu^+$) events.

Since the branching ratio of Z to $\mu^-\mu^+$ is about 3.3%, one would need more than 10^7 Z's to observe a forward-backward asymmetry in $e^-e^+\to Z\to\mu^-\mu^+$ which probes $\bar{x}$ to ± 0.001. As we have seen from Table I, the same accuracy in $\bar{x}$ would come from measuring the Z total width about three times as accurately as at present, requiring only 10^6 Z's.

In principle one would need significantly fewer quark pairs than muon pairs to probe $\bar{x}$ to the same accuracy in A_{FB}. In practice one has to resort to $c\bar{c}$ pairs for u-type quarks, and $b\bar{b}$ pairs for d-type quarks, tagging the quark by the sign of the lepton emitted in semileptonic decay. For $e^-e^+\to b\bar{b}$, the presence of B^0-$\bar{B}^0$ mixing multiplies the expected asymmetry by about a factor of 0.75 ± 0.10.[63]

The L3 Collaboration[63] has presented a measurement, based on 18 000 hadronic (or about 26 000 total) decays of the Z, of

$$A_{FB}^{(b)}=(13.3\pm 9.9)\%\ , \tag{4.48}$$

compatible with the expected value (4.47b) (even after correction for B^0-$\bar{B}^0$ mixing) but with an error about 18 times as large. We then anticipate that to obtain an error in $\bar{x}$ of ± 0.001 will require a total of nearly 10^7 Z's, almost as many as for $A_{FB}^{(\mu)}$.

(2) *Left-right asymmetries.* Longitudinally polarized electron beams will be used for e^+e^- collisions at the Stanford Linear Collider, and possibly at LEP. The cross section asymmetry A_{LR} is the same for any fermion species in the final state, and depends only on $\bar{x}$ to the approximation considered here:

$$A_{LR}=\frac{\frac{1}{4}-\bar{x}}{\frac{1}{4}-\bar{x}+2\bar{x}^2}\ . \tag{4.49}$$

For $\bar{x}=0.232\pm 0.001$, this expression yields $A_{LR}=0.1433\pm 0.0079$. By the arguments mentioned above, to measure A_{LR} with such an error would require only $(0.0079)^{-2}\approx 1.6\times 10^4$ events, assuming a fully polarized beam. With a beam of polarization P, the numbers of events N_L and N_R obtained with left-handed and right-handed polarization are related to A_{LR} by $(N_L-N_R)/(N_L+N_R)=PA_{LR}$, so one would need $(P\delta A_{LR})^{-2}$ events to measure A_{LR} to an accuracy of δA_{LR}. For $P=40\%$ and δA_{LR} as given above, one would need 10^5 Z events to measure $\bar{x}$ to ± 0.001.

Note added. One should also take into account the error on the polarization in estimating the number of events needed. Since a measurement of A_{LR} with an error of about ± 0.008 corresponds to an error in A_{LR} of about 5%, the polarization must be known at least that accurately. Thus, a beam polarization of $(40\pm 2)\%$ and 10^5 Z events would only yield $\bar{x}$ with an error of about ± 0.0014.

(3) *Polarized τ asymmetry.* An asymmetry described by Eq. (4.49) also applies to the cross sections σ_L and σ_R for production of left-handed or right-handed τ^- in $\tau^-\tau^+$ pairs produced at the Z by unpolarized e^-e^+ annihilations. The polarization of the τ^- may be analyzed by the asymmetry in its decay to $\pi^-\nu$, for example. A detailed discussion of the statistical power of this method is presented in Ref. 65.

V. CONCLUSIONS

As information about electroweak parameters becomes more precise, the question arises of what combination of these parameters is measured in any given experiment. In this paper we have given a simple, approximate prescription for sorting out the main radiative effects in any given process. We work in terms of a weak mixing angle $\bar{\theta}$ defined in terms of intrinsic couplings at the Z pole, and relate it to the angle θ defined in terms of W and Z masses by $\sin^2\theta = 1 - M_W^2/M_Z^2$. The parameter $\rho \equiv M_W^2/M_Z^2\cos^2\bar{\theta}$ deviates from 1 as a result of radiative corrections. For example, a heavy top quark leads to $\Delta\rho = (3\alpha/16\pi\sin^2\bar{\theta})(m_t/M_W)^2$. The precise knowledge of the Z mass leads to a constraint[21] $\cos^2\theta = (0.1790 \pm 0.0002)/\sin^2\bar{\theta}$. When combined with the relation $\sin^2\bar{\theta} \approx \sin^2\theta + \cos^2\theta\Delta\rho$ which follows from our definitions, the Z mass constraint allows us to see what quality of information on θ, $\bar{\theta}$, and $\Delta\rho$ is probed by various electroweak processes.

The measurements of W mass and deep-inelastic neutrino scattering are the most promising near-term sources of information on θ and $\Delta\rho$. By contrast, some other processes provide information on the angle $\bar{\theta}$. If precise enough, this information can be of significant interest, as in the case of the partial widths of the Z. Still other processes, such as parity violation in atomic cesium, provide a combination of information already constrained by the Z mass measurement. A discrepancy would be taken as an indication for deviations from the standard model, but so far no such deviations have been seen.

We summarize in Table II some of the most important constraints on $x \equiv \sin^2\theta$, $\bar{x} \equiv \sin^2\bar{\theta}$, and $\Delta\rho$ provided by the processes discussed here. Many other data points should be added for a proper compilation of world data in the spirit of Refs. 6 and 25. Our purpose has been rather to compare the impact of some recent measurements with that of deep-inelastic neutrino scattering, which is the main source of information from older experiments.

The 95% confidence-level upper limit on $\Delta\rho$ implied by the result of Table II, $\Delta\rho \le 0.012$, corresponds according to Fig. 3 to an upper limit on the top-quark mass of about 220 GeV/c^2 for a Higgs-boson mass below 1 TeV. The lower limit on the top-quark mass stemming from radiative corrections to electroweak processes, in our view, is not yet any stronger than that imposed by direct searches.[36] (Here we are essentially in accord with the analysis of Ref. 66, which appeared as the present work was being prepared for publication.) Although it may appear that little has been gained in comparison with previous discussions (as in Ref. 6), the rapidly increasing accuracy of W mass and Z width measurements will lead to a considerable refinement of our knowledge about the electroweak parameters x, $\bar{x}$, and $\Delta\rho$ in the near future. The present discussion has been intended to show the manner in which this improvement is likely to take place.

We have estimated that our approach is limited in accuracy to specification of $\bar{x}$ to ± 0.001, or x to ± 0.003. For more precise corrections it is important to take into account vertex and box diagrams neglected here. Certain classes of theories (notably ones involving large numbers of additional particles in the spectrum, such as technicolor models[22,23]) also appear not to be amenable to our simplified discussion. One would see effects of such theories by a lack of consistency in our treatment exceeding the inherent limitation of ± 0.001 in $\bar{x}$. Other new physics effects (such as those involving extra Z bosons) could also manifest themselves in this way.

The fine points (and loopholes) of conclusions about the top-quark mass based on the present analysis have been discussed in various places.[9,19–21,67] Here we note only that the conclusion depends (weakly) on the assumed Higgs-boson mass, and much more crucially on the assumed absence of any explicit Higgs-triplet vacuum expectation values. Since the main unknown quantity affecting $\Delta\rho$ in the standard description appears to be the top-quark mass, we expect that a full description of radiative corrections to electroweak processes will have to await the discovery of the elusive sixth quark.

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TABLE II. Some sources of information on $\sin^2\theta$.

	$x \equiv \sin^2\theta$		$\bar{x} \equiv \sin^2\bar{\theta}$		$\Delta\rho$	
Process	Value	Error	Value	Error	Value	Error
R_ν [a]	0.2294	0.0058[b]	0.2323	0.0018	0.0037	0.0053
W mass	0.2273	0.0059	0.2316	0.0018	0.0056	0.0054
$\Gamma(Z \to l^+l^-)$	0.225	0.010	0.231	0.003	0.007	0.009
$\Gamma(Z \to \text{hadrons})$	0.2220	0.0085	0.2300	0.0026[c]	0.0104	0.0077
Average	0.2269	0.0035	0.2315	0.0011	0.0060	0.0032

[a]Average of results from Refs. 37 and 38.
[b]Statistical error of ± 0.003 combined in quadrature with theoretical error of ± 0.003 and assumed error of ± 0.004 due to uncertainty in m_c.
[c]Based on experimental error for $\Gamma(Z \to \text{hadrons})$ of ± 20 MeV combined in quadrature with theoretical error of ± 11 MeV, based on $\Delta\alpha_s = \pm 0.02$ (see first of Refs. 29).

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