On the decay mode $Z \to Hgg$

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We present analytic results for the matrix elements relevant for a process with one Z boson, one Higgs boson, and two gluons as external particles to lowest order in perturbation theory. These results are used to study Higgs-boson production associated with two gluon jets through Z decay. We find $\Gamma(Z \to Hgg)/\Gamma(Z \to \text{all}) < 5\times10^{-9}$ for $M_H \geq 24$ GeV and $m_t \geq 77$ GeV. $Z \to Hgg$ turns out to be more than 4 orders of magnitude suppressed as compared to the Born process $Z \to Hq\bar{q}$ independently of possible angular cuts on the two jets.

I. INTRODUCTION

The existence of a neutral Higgs boson H is a fundamental assumption within the minimal standard model of electroweak interactions, and its discovery would provide crucial evidence in favor of this theory. The standard model predicts the vacuum expectation value of the Higgs particle to be equal to $2^{-1/4}G_F^{-1/2} \approx 246$ GeV, but leaves its mass practically undetermined, which renders it difficult to look for it.

One of the central objectives of Z factories, such as the Large Electron-Positron Collider (LEP) at CERN or the SLAC Linear Collider (SLC) consists in the search for the Higgs boson in the window $M_H \leq M_Z$. Recently, the ALEPH Collaboration at CERN has obtained the experimental bound $M_H \geq 24$ GeV (Ref. 1) by looking for Higgs-boson production in Z-boson decay through the Bjorken process $Z \to Hf\bar{f}$,[2] where f denotes a lepton or a quark. This reaction can be realized already on the Born level and is, therefore, expected to be a copious source of Higgs bosons.

After fragmentation, the parton-level process $Z \to Hgg$ leads to the same class of final states as $Z \to Hq\bar{q}$: one leptonically or hadronically decaying Higgs boson and two jets. To lowest order it proceeds through the set of triangle and box diagrams depicted in Fig. 1. Intuitively, because of the strong suppression factor $(\alpha_s/\pi)^2$, one would expect the channel $Z \to Hgg$ not to be competitive with $Z \to Hq\bar{q}$. In a recent publication,[3] however, it has been claimed that the mechanism $Z \to Hgg$ significantly contributes to $Z \to H+\text{jet}+\text{jet}$ for $M_H \geq 40$ GeV and is even dominant for $M_H \geq 50$ GeV.

In this paper we repeat the calculation and find, in disagreement with Ref. 3, that $\Gamma(Z \to Hgg)/\Gamma(Z \to Hq\bar{q}) < 10^{-4}$ for $M_H \geq 24$ GeV and $m_t \geq 77$ GeV. For a fixed value of the Higgs-boson energy E_H, the respective differential decay rates $d^2\Gamma(Z \to Hjj)/(dE_H d\cos\theta_{jj})$ turn out to have almost the same dependence on the angle between the two jets θ_{jj}, so that the ratio cannot be appreciably improved by the application of kinematical cuts. As Ref. 3 does not provide an analytic representation of the solutions, we are not in the position to determine the source of this discrepancy.

The outline of this paper is as follows. In Sec. II and the appendixes, we list closed analytic expressions for the $ZHgg$ vertex to lowest order and work out the leading behavior for both light and heavy virtual quarks. In Sec. III we apply these results to the process $Z \to Hgg$. We derive a simple approximation formula for the integrated decay width $\Gamma(Z \to Hgg)$, assuming idealized quark masses, i.e., $m_t \to \infty$ and $m_q = 0$ $(q \neq t)$, which is correct within 25%, that is, less than the uncertainty introduced by the strong coupling constant $\delta\alpha_s^2/\alpha_s^2 \approx 50\%$. Quantitatively, the bulk of $Z \to H+\text{jet}+\text{jet}$ events is seen to originate from the parton-level process $Z \to Hq\bar{q}$, while $Z \to Hgg$ plays the role of a small second-order QCD correction.

II. $ZHgg$ VERTEX

To lowest order the $ZHgg$ coupling is generated by one triangle diagram in connection with a ZZH vertex and three box diagrams as illustrated in Fig. 1. From color conservation it is obvious that the two gluons form a color singlet, if they are both in the initial or final state, or that they carry the same color, if one is incoming and the other is outgoing, so that the SU(3) group structure reduces to a Kronecker symbol δ^{ab}. As a consequence of charge-conjugation invariance, the Z boson couples only axially to the internal quark, so that the contribution from a mass-degenerate weak isodoublet of quarks vanishes. Applying the reduction algorithm developed in Ref. 4 and repeatedly using Shouten's identity

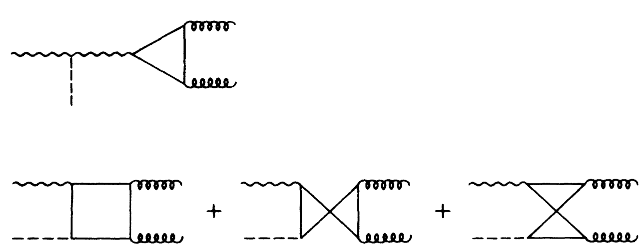

FIG. 1. Feynman diagrams pertinent to the $ZHgg$ vertex in lowest order.

$g^{\alpha[\beta}\epsilon^{\gamma\delta\epsilon\varphi]}=0$, where the square brackets symbolize antisymmetrization, we find, for the T matrix element of interest,

$$i\mathcal{T}=\sqrt{2z}\,G_F\frac{\alpha_s}{\pi}\epsilon_\alpha^a(q_1)\epsilon_\beta^b(q_2)\epsilon_\mu(p)\delta^{ab}\times\sum_q I_q T^{\alpha\beta\mu}(q_1,q_2,p,m_q)\ , \tag{2.1}$$

where it is summed over the color indices $a,b=1,\ldots,8$, and the quark flavors q. G_F is the Fermi constant, α_s is the strong coupling constant, and $I_q=\pm\frac{1}{2}$ is the third component of the weak isospin. q_1,q_2,p are the four-momenta of the two gluons and the Z boson, respectively, and $\epsilon^a(q_1),\epsilon^b(q_2),\epsilon(p)$ are their polarization four-vectors. We take all momenta to be ingoing and define

$$s=(q_1+q_2)^2,\quad t=(q_1+p)^2,\quad u=(q_2+p)^2\ ,$$

$$z=p^2,\quad h=(q_1+q_2+p)^2,\quad N=tu-zh\ .$$

The mass-shell conditions read $q_1^2=q_2^2=0$, $z=M_Z^2$, and $h=M_H^2$. Momentum conservation leads to the identity $s+t+u=z+h$. The polarization tensor $T^{\alpha\beta\mu}(q_1,q_2,p,m)$ has the decomposition

$$\begin{aligned}T^{\alpha\beta\mu}(q_1,q_2,p,m)=&\left[\frac{s}{2}\epsilon^{\alpha\beta\mu\rho}q_{2\rho}-q_2^\alpha\epsilon^{\beta\mu\rho\sigma}q_{1\rho}q_{2\sigma}\right]F_1(t,u,z,h,m^2)-\left[\frac{s}{2}\epsilon^{\alpha\beta\mu\rho}q_{1\rho}-q_1^\beta\epsilon^{\alpha\mu\rho\sigma}q_{1\rho}q_{2\sigma}\right]F_1(u,t,z,h,m^2)\\&+\left[p^\alpha+\frac{z-t}{s}q_2^\alpha\right]\epsilon^{\beta\mu\rho\sigma}q_{2\rho}[q_{1\sigma}F_2(t,u,z,h,m^2)+p_\sigma F_3(t,u,z,h,m^2)]\\&+\left[p^\beta+\frac{z-u}{s}q_1^\beta\right]\epsilon^{\alpha\mu\rho\sigma}q_{1\rho}[q_{2\sigma}F_2(u,t,z,h,m^2)+p_\sigma F_3(u,t,z,h,m^2)]\\&+\left[\frac{s}{2}\epsilon^{\alpha\beta\mu\rho}p_\rho-q_2^\alpha\epsilon^{\beta\mu\rho\sigma}q_{1\rho}p_\sigma+q_1^\beta\epsilon^{\alpha\mu\rho\sigma}q_{2\rho}p_\sigma+g^{\alpha\beta}\epsilon^{\mu\rho\sigma\tau}q_{1\rho}q_{2\sigma}p_\tau\right]F_4(t,u,z,h,m^2)\ ,\end{aligned} \tag{2.2}$$

where we use the convention $\epsilon^{0123}=1$. Herein we have already dropped terms proportional to q_1^α, q_2^β, or p^μ, appealing to the transversality conditions for the vector bosons, $q_1\cdot\epsilon^a(q_1)=q_2\cdot\epsilon^b(q_2)=p\cdot\epsilon(p)=0$. The form factors $F_i(t,u,z,h,m^2)$ $(i=1,\ldots,4)$ are listed in Appendix B. Gauge invariance with respect to the gluons manifests itself in

$$q_{1\alpha}T^{\alpha\beta\mu}(q_1,q_2,p,m)=q_{2\beta}T^{\alpha\beta\mu}(q_1,q_2,p,m)=0\ . \tag{2.3}$$

The property $F_4(u,t,z,h,m^2)=-F_4(t,u,z,h,m^2)$ guarantees Bose symmetry:

$$T^{\beta\alpha\mu}(q_2,q_1,p,m)=T^{\alpha\beta\mu}(q_1,q_2,p,m)\ . \tag{2.4}$$

Incidentally, $F_3(u,t,z,h,m^2)=F_3(t,u,z,h,m^2)$ also holds.

Let us now concentrate on extreme quark masses. Using the large-m^2 expansions of the form factors calculated in Appendix B and Shouten's identity, we may write the heavy-quark contribution as

$$T^{\alpha\beta\mu}(q_1,q_2,p,m)=\frac{1}{6m^2}\left[\frac{s-z}{s-z+i\sqrt{z}\,\Gamma_Z}-1\right]\times(q_1+q_2)^\mu\epsilon^{\alpha\beta\rho\sigma}q_{1\rho}q_{2\sigma}+O\left[\frac{1}{m^4}\right]. \tag{2.5}$$

That is, the leading terms from the triangle and box diagrams are both proportional to $1/m^2$, but apart from finite-width effects, they cancel each other, leaving behind terms of $O(1/m^4)$. For this reason, the $ZHgg$ coupling is expected to be fairly insensitive to heavy quarks.

The contribution from a light quark circulating in the box is suppressed by a factor of m^2; one power stems from the Higgs-boson coupling and another one from the trace. Both features are absent in the case of the triangle-type graph, and taking the limit $m\to 0$ yields

$$T^{\alpha\beta\mu}(q_1,q_2,p,0)=-\frac{2}{s}\frac{s-z}{s-z+i\sqrt{z}\,\Gamma_Z}\times(q_1+q_2)^\mu\epsilon^{\alpha\beta\rho\sigma}q_{1\rho}q_{2\sigma}\ . \tag{2.6}$$

In the approximation that the top quark is ultraheavy and all other quarks are massless, this is already the full answer. It is amazing that the main effect arises from the only term in the whole set of form factors, which is bare of logarithms and Spence functions, namely, the first term in Eq. (B3).

III. DECAY WIDTH FOR $Z\to Hgg$

Let $\langle|\mathcal{T}|^2\rangle$ denote the absolute squared of the invariant decay amplitude $\mathcal{T}$ after averaging over the initial polarizations and summing over the final ones and over color. The differential decay rate is then given by

$$d\Gamma=\frac{1}{2}\frac{1}{2\sqrt{z}}\langle|\mathcal{T}|^2\rangle dx_{\rm PS}\ , \tag{3.1}$$

where $dx_{\rm PS}$ denotes the phase-space element, $2\sqrt{z}$ stands for the flux factor, and the supplementary factor of $\frac{1}{2}$ takes care of the fact that there are two identical particles in the final state. Depending on the experimental setup, it may be requested to consider the decay width as differential with respect to the Higgs-boson energy E and the angle between the two gluon jets, $c=\cos\theta$, in the center-of-mass system. An adequate phase-space parametrization reads

$$\begin{aligned} dx_{\rm PS} &= \frac{1}{64\pi^3}\frac{(z-t)^2(z-u)^2}{z^{3/2}s(u-t)}dE\,dc\,, \\ \sqrt{h} &\le E \le \frac{z+h}{2\sqrt{z}}\,, \\ -1 &\le c \le 1-\frac{2s}{(\sqrt{z}-E)^2}\,, \\ s &= z+h-2\sqrt{z}E\,, \\ t &= \sqrt{z}\left[E-\left[(\sqrt{z}-E)^2-\frac{2s}{1-c}\right]^{1/2}\right], \\ u &= z+h-s-t\,. \end{aligned} \tag{3.2}$$

Alternatively, to compute the total width, it is more convenient to integrate over the Mandelstam variables t and u using

$$dP=\frac{1}{128\pi^3 z}dt\,du\,,\quad h\le t\le z\,,\quad \frac{zh}{t}\le u\le z+h-t\,. \tag{3.3}$$

In the approximation of the top quark being infinitely heavy and the others being massless, the integration can be performed analytically and yields

$$\Gamma_0(Z\to Hgg)=\left[\frac{\alpha_s}{\pi}\right]^2\frac{G_F^2 z^{5/2}}{3072\pi^3}\times(1-8r+8r^3-r^4-12r^2\ln r)\,,\quad r=\frac{h}{z}\,, \tag{3.4}$$

where terms of the order Γ_Z^2/z have been neglected. Note that we may disregard the details of the fragmentation as long as we do not specify the jets, because the probability for the hadronization of the gluons totals 100%.

For completeness, we also list the Born results for the decay mode $Z\to Hq\bar{q}$. In view of $m_t\ge 77$ GeV $>M_Z/2$,[5] only the five light-quark flavors can be present in the final state. In the massless-quark approximation, the Higgs boson can be emitted only from the Z boson and

$$\langle|\mathcal{T}|^2\rangle=4G_F^2\sum_{q\neq t}(V_q^2+A_q^2)z^2\frac{2zs+N}{(s-z)^2+z\Gamma_Z^2}\,, \tag{3.5}$$

where $V_q=2I_q-4\sin^2\theta_W Q_q$, $A_q=2I_q$, and θ_W denotes the weak mixing angle. Up to terms of the order Γ_Z^2/z, we find, for the integrated width $(r=h/z)$,

$$\Gamma_0(Z\to Hq\bar{q})=\frac{G_F^2 z^{5/2}}{64\pi^3}\sum_{q\neq t}(V_q^2+A_q^2)\left[-\frac{47}{12}+5r-\frac{5}{4}r^2+\frac{r^3}{6}-\left[1-\frac{3}{2}r+\frac{r^2}{4}\right]\ln r + \left[5-2r+\frac{r^2}{4}\right]\left[\frac{r}{4-r}\right]^{1/2}\left[\pi-6\arcsin\frac{\sqrt{r}}{2}\right]\right]. \tag{3.6}$$

In the numerical analysis we set $M_Z=91.15$ GeV, $\Gamma_Z=2.55$ GeV,[6] $\sin^2\theta_W=0.23$,[7] $m_s=0.5$ GeV, $m_c=1.6$ GeV, $m_b=5$ GeV, $m_t=100$ GeV, and assume mass degeneracy in the ud doublet. For $\alpha_s(\mu^2)$ we employ the representation in the modified minimal-subtraction ($\overline{\rm MS}$) scheme as of Eq. (6) in Ref. 8 with $\Lambda_{\overline{\rm MS}}^{(5)}=240$ MeV,[9] and we choose $\mu=M_Z$, which yields $\alpha_s(M_Z^2)/\pi=0.038$.

Figure 2 compares the contributions from the individual channels $Z\to Hq\bar{q}$ (dashed line) and $Z\to Hgg$ (solid line) to the differential decay rate $d^2\Gamma(Z\to Hjj)/(dE_H d\cos\theta_{jj})$, for $M_H=60$ GeV at the fixed value $E_H=63$ GeV as a function of the angle defined by the two jets θ_{jj} in the center-of-mass system. The angular distributions being essentially parallel, the gluonic process generates a homogeneous background which is down by more than 4 orders of magnitude. The pure box contribution (dot-dashed line) amounts to less than 0.1% of the complete prediction for $Z\to Hgg$. This is qualitatively well understood, as the box amplitude is suppressed for both light and heavy quarks, whereas the triangle amplitude converges towards a finite value for $m\to 0$. On the other hand, this explains why there is only a minor support from a light-quark doublet even if the mass splitting is substantial, as applies typically to the cs doublet. Be-

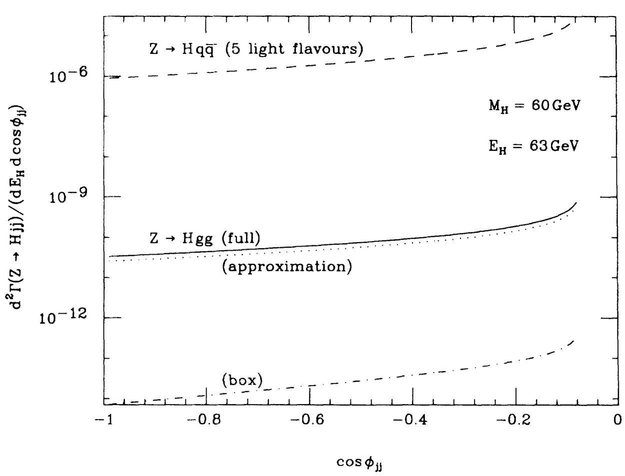

FIG. 2. Angular distribution of the partial decay rates $d^2\Gamma(Z\to Hjj)/(dE_H d\cos\theta_{jj})$, from $Z\to Hq\bar{q}$ (dashed line) and $Z\to Hgg$ (solid line) for $M_H=60$ GeV at $E_H=63$ GeV. For comparison, also the contribution from the box diagrams alone (dot-dashed line) and the approximation $m_q=0$ $(q=u,d,c,s,b)$, $m_t\to\infty$, $\Gamma_Z=0$ (dotted line) are shown.

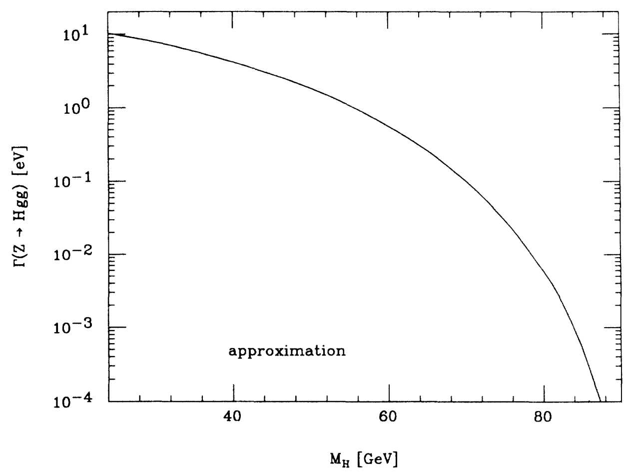

FIG. 3. Exact result (solid line) and approximation of idealized quark masses and narrow Z-boson width (dotted line) for the integrated decay rate $\Gamma(Z\to Hgg)$ as a function of M_H.

cause of the cancellation of the leading terms for $m\to\infty$ from the triangle and box graphs, the full result depends only insignificantly on the top-quark mass. Consequently, it is fair to say that the whole process is essentially induced by the nonsaturated bottom quark alone. The correctness of this picture is nicely confirmed by the fact that the extreme quark mass approximation (dotted line) deviates by less than 25% from the exact answer.

Figure 3 displays the Higgs-boson-mass dependence of the total rate $\Gamma(Z\to Hgg)$ (solid line), together with the idealized quark mass approximation $\Gamma_0(Z\to Hgg)$ (dotted line). For increasing M_H the decay is more and more phase-space suppressed and does not happen at all for $M_H=M_Z$. We are dealing with a truly rare Z-boson decay: At $M_H=24$ GeV, $\Gamma(Z\to Hgg)=10$ eV is of the same order of magnitude as $\Gamma(Z\to W^+e^-\bar{\nu}_e)=24$ eV,[10] and at $M_H\approx 60$ GeV it is comparable to $\Gamma(Z\to\gamma\gamma\gamma)=0.7$ eV.[11]

IV. CONCLUSIONS

We have calculated the $ZHgg$ vertex to lowest order assuming that the gauge bosons are real. As an application, we have studied the decay $Z\to Hgg$. We find that it is essentially insensitive to the top quark, but dominated by the unbalanced bottom quark (beauty predominance). It can be considered as a second-order QCD correction to the Bjorken process $Z\to Hq\bar{q}$. We obtain $\Gamma(Z\to Hgg)/\Gamma(Z\to\text{all})<5\times 10^{-9}$ for $M_H\geq 24$ GeV, which, in view of an expected yield of 10^7 Z events per 1 yr of running at LEP, renders it unlikely for this channel to be of practical relevance in the near future.

Note added in proof. An erratum[14] to Ref. 3 has been published, in which the authors confirm that due to a numerical error the box-diagram contribution to the decay amplitude of $Z\to Hgg$ was overestimated in their original work.

ACKNOWLEDGMENTS

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APPENDIX A: SCALAR ONE-LOOP INTEGRALS

In this paper we essentially adopt the conventions of Ref. 4, except that we use the Minkowskian metric with $g^{\mu\nu}=\text{diag}(1,-1,-1,-1)$. We define the scalar one-loop integrals of interest as

$$C_0(s_1,s_2,s_5,m^2)=-\int\frac{d^4q}{i\pi^2}\{(q^2-m^2+i\epsilon)[(q+p_1)^2-m^2+i\epsilon][(q+p_5)^2-m^2+i\epsilon]\}^{-1},$$
$$D_0(s_1,s_2,s_3,s_4,s_5,s_6,m^2)=\int\frac{d^4q}{i\pi^2}\{(q^2-m^2+i\epsilon)[(q+p_1)^2-m^2+i\epsilon][(q+p_5)^2-m^2+i\epsilon][(q+p_4)^2-m^2+i\epsilon]\}^{-1},\tag{A1}$$

where $p_4=p_1+p_2+p_3$, $p_5=p_1+p_2$, $p_6=p_2+p_3$, and $s_i=p_i^2$ $(i=1,\ldots,6)$. For $m^2\gg\max_i|s_i|$ we may expand:

$$C_0(s_1,s_2,s_5,m^2)=\frac{1}{2m^2}\left[1+\frac{1}{12m^2}\sum_{i=1,2,5}s_i+O\left[\frac{s_is_j}{m^4}\right]\right],$$
$$D_0(s_1,s_2,s_3,s_4,s_5,s_6,m^2)=\frac{1}{6m^4}\left[1+\frac{1}{10m^2}\sum_{i=1}^{6}s_i+O\left[\frac{s_is_j}{m^4}\right]\right].\tag{A2}$$

For positive $\lambda=\lambda(s_1,s_2,s_5)=s_1^2+s_2^2+s_5^2-2(s_1s_2+s_1s_5+s_2s_5)$, the exact result for the three-point function reads[12]

$$C_0(s_1,s_2,s_5,m^2)=\frac{1}{\sqrt{\lambda}}\sum_{i=1,2,5}\left[\text{Li}_2\left[\frac{1+b_i}{a_i+b_i}\right]-\text{Li}_2\left[\frac{1-b_i}{a_i-b_i}\right]+\text{Li}_2\left[\frac{-1-b_i}{a_i-b_i}\right]-\text{Li}_2\left[\frac{-1+b_i}{a_i+b_i}\right]\right],\tag{A3}$$

where Li_2 denotes the dilogarithm[13] and

$$a_i=\left[1-4\frac{m^2-i\epsilon}{s_i}\right]^{1/2},\quad b_i=\frac{1}{\sqrt{\lambda}}\left[s_i-\sum_{j\neq i}s_j\right]\quad(i,j=1,2,5).$$

From $m^2=0$ this simplifies to

$$C_0(s_1,s_2,s_5,0)=\frac{1}{\sqrt{\lambda}}\sum_{i=1,2,5}\left[\mathrm{Li}_2\left[-\frac{1+b_i}{1-b_i}+i\epsilon\frac{1+b_i}{s_i}\right]-(b_i\leftrightarrow -b_i)\right] . \quad \text{(A4)}$$

For $s_1=s_2=0$ we obtain

$$C_0(0,0,s_5,m^2)=-\frac{1}{2s_5}\ln^2\frac{a_5+1}{a_5-1} , \quad \text{(A5)}$$

which is logarithmically divergent for $m^2\to 0$:

$$C_0(0,0,s_5,m^2)=-\frac{1}{2s_5}\ln^2\frac{-s_5}{m^2-i\epsilon},\quad m^2\ll |s_5| . \quad \text{(A6)}$$

We do not attempt to derive a general expression for $D_0(s_1,s_2,s_3,s_4,s_5,s_6,m^2)$, but take advantage of the masslessness of the gluons. Thereby we do not encounter any infrared complications. It is convenient to introduce the following basic integrals for real a,b,c:

$$\begin{aligned} I(a,b,c)&=\int_0^1\frac{dx}{x-c}\ln(ax+b)=\ln(ac+b)\ln\left[1-\frac{1}{c}\right]-\mathrm{Li}_2\left[1-\frac{a+b}{ac+b}\right]+\mathrm{Li}_2\left[1-\frac{b}{ac+b}\right] ,\\ J(a,b,c)&=\int_0^1\frac{dx}{x-c}\ln[ax(1-x)-b+i\epsilon]\\ &=\ln[ac(1-c)-b+i\epsilon]\ln\left[1-\frac{1}{c}\right]-\mathrm{Li}_2\left[\frac{c-1}{c-a_+}\right]+\mathrm{Li}_2\left[\frac{c}{c-a_+}\right]-\mathrm{Li}_2\left[\frac{c-1}{c-a_-}\right]+\mathrm{Li}_2\left[\frac{c}{c-a_-}\right] , \end{aligned} \quad \text{(A7)}$$

where

$$a_\pm=\frac{1}{2}\left[1\pm\left[1-4\frac{b-i\epsilon}{a}\right]^{1/2}\right] .$$

Here it is understood that c is assigned a small imaginary part if $0<c<1$. With this notation we obtain

$$D_0(0,z,0,h,t,u,m^2)=\frac{2}{Nr}[J(z,m^2,r_+)+J(h,m^2,r_+)-J(t,m^2,r_+)-J(u,m^2,r_+)] , \quad \text{(A8)}$$

where

$$r=\left[1+4\frac{s}{N}(m^2-i\epsilon)\right]^{1/2} ,\quad r_+=\frac{1+r}{2} .$$

The following symmetry relations hold:

$$D_0(0,h,0,z,t,u,m^2)=D_0(0,z,0,h,u,t,m^2)=D_0(0,z,0,h,t,u,m,^2) . \quad \text{(A9)}$$

In the limit $m^2\to 0$, logarithmic divergences occur:

$$D_0(0,z,0,h,t,u,m^2)=\frac{2}{N}\left[J_0\left[z,m^2,\frac{N}{s}\right]+J_0\left[h,m^2,\frac{N}{s}\right]-J_0\left[t,m^2,\frac{N}{s}\right]-J_0\left[u,m^2,\frac{N}{s}\right]\right] , \quad \text{(A10)}$$

where

$$J_0(a,b,c)=\ln\frac{b-i\epsilon}{c}\ln\left[-\left[1+\frac{a}{c}\right]b+i\epsilon\right]-\frac{1}{2}\ln^2\left[-\left[\frac{1}{a}+\frac{1}{c}\right]b+\frac{i\epsilon}{a}\right]-\mathrm{Li}_2\left[\frac{1}{1+c/a}+i\epsilon\frac{c}{a}\right] . \quad \text{(A11)}$$

Moreover, we find

$$\begin{aligned} D_0(0,0,z,h,s,u,m^2)=\frac{1}{sux}\Bigg[&I(s,0,x_+)-I\left[t-h,s,\frac{1-x_-}{1-\alpha}\right]-I\left[z-u,0,\frac{x_+}{\alpha}\right]+I\left[z-u,0,\frac{x_+}{\beta}\right]-I\left[t-h,s,\frac{x_+}{\beta}\right]\\ &-J(s,m^2,x_+)+J\left[h,m^2,\frac{1-x_-}{1-\alpha}\right]+J\left[z,m^2,\frac{x_+}{\alpha}\right]\\ &-J\left[u,m^2,\frac{x_+}{\beta}\right]+J\left[h,m^2,\frac{x_+}{\beta}\right]-(x_+\leftrightarrow x_-)\Bigg] , \end{aligned} \quad \text{(A12)}$$

where $\lambda=\lambda(s,z,h)$:

$$x=\left[1+4\frac{N}{su^2}(m^2-i\epsilon)\right]^{1/2},\quad x_\pm=\frac{-u}{t-u+\sqrt{\lambda}}(1\pm x),\quad \alpha=\frac{s+z-h+\sqrt{\lambda}}{2s},\quad \beta=\frac{u-t+\sqrt{\lambda}}{2s}\ .$$

This involves 54 Spence functions of distinct arguments [not counting $\mathrm{Li}_2(1)=\pi^2/6$] and exhibits the symmetry property

$$D_0(0,0,h,z,s,u,m^2)=D_0(0,0,z,h,s,u,m^2)\ , \tag{A13}$$

which is, however, somewhat hidden in our representation. The small-m^2 expansion of $D_0(0,0,z,h,s,u,m^2)$ is slightly messy, and we do not list it here, but only mention that it is also logarithmically divergent.

APPENDIX B: FORM FACTORS

We write the form factors as a superposition of triangle and box contributions:

$$F_i(t,u,z,h,m^2)=m^2\left[\frac{s-z}{s-z+i\sqrt{z}\,\Gamma_Z}T_i(s,m^2)+B_i(t,u,z,h,m^2)\right]\quad (i=1,\ldots,4)\ , \tag{B1}$$

where we allow for a finite width Γ_Z in the Z-boson propagator. For convenience, we introduce the following short-hand notation for the scalar one-loop integrals presented in Appendix A:

$$\begin{aligned}&C_0(v)=C_0(0,0,v,m^2)\quad (v=s,t,u,z,h)\ ,\\ &C_1(s)=C_0(s,z,h,m^2)\ ,\\ &D_0(t,u)=D_0(0,z,0,h,t,u,m^2)\ ,\\ &D_1(s,v)=D_0(0,0,z,h,s,v,m^2)\quad (v=t,u)\ .\end{aligned} \tag{B2}$$

There is only one nonvanishing triangle term, which can be read off from the general result for the Zgg vertex derived in Ref. 12:

$$T_1(s,m^2)=\frac{2}{s}\left[-\frac{1}{m^2}+2C_0(s)\right]\ . \tag{B3}$$

For the box terms we obtain

$$\begin{aligned}B_1(t,u,z,h,m^2)=\frac{2}{s}\Bigg\{&\frac{h-s+t}{N}sC_0(s)-\left[\frac{2}{s}+\frac{h-s+t}{N}\right][zC_0(z)+hC_0(h)]\\ &+2\left[\frac{1}{s}+\frac{t}{N}\right]tC_0(t)+2\left[\frac{1}{s}+\frac{h-s}{N}\right]uC_0(u)-\left[2+\frac{(t-u)(z-u)}{N}\right]C_1(s)\\ &+\left[4m^2-s\left[1-u\frac{h-s}{N}\right]\right]D_1(s,u)+\left[4m^2+\frac{st^2}{N}\right]D_1(s,t)+\left[4m^2+\frac{N}{s}\right]D_0(t,u)\Bigg\}\ ,\\ B_2(t,u,z,h,m^2)=\frac{1}{N}\Bigg\{&-2(t+u)\frac{z-u}{N}[sC_0(s)-zC_0(z)-hC_0(h)]-2\left[1+2t\frac{z-u}{N}\right]tC_0(t)+2\left[1-2u\frac{z-u}{N}\right]uC_0(u)\\ &+2\left[t-u+(z-u)\left[2+\frac{(t-u)^2}{N}\right]\right]C_1(s)-\left[4(z-u)m^2-su\left[1-2u\frac{z-u}{N}\right]\right]D_1(s,u)\\ &-\left[4(z-u)m^2+st\left[1+2t\frac{z-u}{N}\right]\right]D_1(s,t)\Bigg\}-\left[1+4\frac{z-u}{N}m^2\right]D_0(t,u)\ ,\\ B_3(t,u,z,h,m^2)=\frac{1}{N}\Bigg\{&-2\left[1+s\frac{t+u}{N}\right][sC_0(s)-zC_0(z)-hC_0(h)]\\ &-2\left[1+2\frac{st}{N}\right]tC_0(t)-2\left[1+2\frac{su}{N}\right]uC_0(u)+2s\left[2+\frac{(t-u)^2}{N}\right]C_1(s)\\ &-s\left[4m^2+u\left[1+2\frac{su}{N}\right]\right]D_1(s,u)-s\left[4m^2+t\left[1+2\frac{st}{N}\right]\right]D_1(s,t)\Bigg\}-\left[1+4\frac{s}{N}m^2\right]D_0(t,u)\ ,\end{aligned} \tag{B4}$$

$$B_4(t,u,z,h,m^2)=\frac{2}{N}[tC_0(t)-uC_0(u)+(u-t)C_1(s)]-\left(1+\frac{su}{N}\right)D_1(s,u)+\left(1+\frac{st}{N}\right)D_1(s,t)\ .$$

Note that B_3 is symmetric and B_4 is antisymmetric with respect to t and u. It is remarkable that the final result does not contain any two-point functions B_0.

The leading terms for $m^2\to\infty$ read

$$\begin{aligned}
m^2T_1(s,m^2)&=\frac{1}{6m^2}+O\left(\frac{1}{m^4}\right),\\
m^2B_1(t,u,z,h,m^2)&=\frac{u-h}{6sm^2}+O\left(\frac{1}{m^4}\right),\\
m^2B_2(t,u,z,h,m^2)&=-\frac{1}{6m^2}+O\left(\frac{1}{m^4}\right),\\
m^2B_3(t,u,z,h,m^2)&=O\left(\frac{1}{m^4}\right),\\
m^2B_4(t,u,z,h,m^2)&=O\left(\frac{1}{m^4}\right).
\end{aligned}\tag{B5}$$

For $m^2=0$ only the triangle contribution survives:

$$m^2T_1(s,m^2)\Big|_{m^2=0}=-\frac{2}{s}\ .\tag{B6}$$

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