Strange quarks in the deuteron

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We consider elastic electroweak lepton-deuteron scattering as a means to extract information on both vector and axial-vector isoscalar currents, and thus the strange- (and heavier-) quark content of the deuteron. Parity violation in electron-deuteron scattering is examined in some detail. Numerical estimates are made, using a simple model when required, to find the sensitivity to strange-quark components of the various form factors. We find that backward angles are in general best to isolate axial-vector and magnetic vector components. Figures of merit relevant to future experiments are given, using existing limits on s-quark contributions to nucleon form factors where possible.

I. INTRODUCTION

There has been a great deal of speculation recently on the strange-quark content of ordinary nucleons and nuclei.[1,2] This has been motivated primarily by several different classes of experiments, in particular deep-inelastic polarized muon scattering from the proton,[3] and measurements of the pion-nucleon Σ term.[4] Each of these measurements has yielded somewhat surprising results, which have been interpreted by many authors as a sign that the commonly held expectation[5] that s quarks contribute only very slightly to the low-energy structure of the nucleon may need to be revised.

The analysis and interpretation of these experimental results are subject to much debate.[1,2,6,7] Aside from improving the existing measurements, one must also consider independent tests, preferably in quite different regimes, which are also sensitive to strange- (or heavier-) quark currents. One such class of experiments is electroweak elastic-form-factor measurements.[1,8] Neutrino elastic scattering from the proton has already yielded possible indications of a nonzero strange axial-vector form factor.[9] These neutrino experiments are very difficult, due to the extremely small cross sections involved. There is, however, another means to measure the same weak form factors, namely, via polarized e^- scattering. The difference in cross sections for right- and left-handed longitudinally polarized electrons, scattering off an unpolarized target, arises from the interference between weak and electromagnetic amplitudes. Such measurements may yield highly accurate weak-form-factor measurements in the near future.

The sensitivity of elastic form factors to strange content arises because the underlying quark currents couple in different ways to the photon and Z^0. Electromagnetic interactions measure the matrix element of the current,

$$J_\mu^\gamma = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s + \cdots \tag{1.1}$$

(the ellipsis refers to still heavier quarks), while the weak neutral Z^0 boson couples to the current:

$$J_\mu = \bar{u}\gamma_\mu(1-\gamma_5)u - \bar{d}\gamma_\mu(1-\gamma_5)d - \bar{s}\gamma_\mu(1-\gamma_5)s + \cdots - 2\sin^2\theta_W J_\mu^\gamma \ . \tag{1.2}$$

The different coefficients, and different isospin structure, allow us in principle to separate the various $SU(3)_f$ components, and thus in particular the s-quark matrix elements.[1] Data from the proton alone will not suffice to isolate the strange-quark contributions to the nucleon structure, however; measurements on the neutron are necessary as well. They can be combined with the proton data under the assumption that SU(2)-isospin symmetry of the nucleon system is good.

In this paper elastic scattering of leptons from the deuteron is considered as a new means to extract information about s-quark effects. We focus primarily on the parity-violating asymmetry between left- and right-handed polarized electrons scattering off unpolarized deuterium. Parity violation in $\vec{e}$-d scattering has been analyzed previously,[10] but generally assuming negligible strange-quark contributions. In the standard model there is no axial-vector isoscalar current involving light (u and d) quarks. Since the deuteron is an isoscalar object, nonzero $\langle \bar{s}\gamma_\mu\gamma_5 s + \cdots \rangle$ contributions will thus show up directly in the axial-vector form factors. In addition, the isoscalar magnetic moment of the nucleon is relatively small, since $\kappa_p \approx -\kappa_n$, and so one might expect additional Lorentz-vector isoscalar current arising from s quarks to show up more clearly.

To make statements about the nucleon's properties from deuteron measurements, one needs of course a consistent model of the deuteron. In this paper only simple nonrelativistic deuteron wave functions are used. This yields reasonable predictions for electromagnetic form factors in the moderate q^2 range considered here,[11] and thus should be a reasonable first step. This range includes $0 \le -q^2 \lesssim 0.5-1.0$ GeV^2, below the first diffraction minima. Relativistic and meson-exchange corrections, among others, should not seriously modify any conclusions made about *general* sensitivity of deuteron cross sections to the s-quark content of the nucleon, although they may certainly alter some quantitative details.

The results of this paper are intended as a guide to future experiments. The existing knowledge and estimates of possible s-quark currents in the nucleon are used, whenever possible, to see where these will nontrivially modify deuteron measurements. Various possible kinematics are considered, and figures of merit in the asymmetry measurements are also computed, making some reasonable assumptions about experimental conditions. We find that anomalously large s-quark contents in the nucleon, not currently excluded by any electroweak data, would show up quite clearly in some kinematic regimes. There is some sensitivity to the choice of deuteron model, but because of the ratio involved in the definition of the asymmetry, this dependence is often quite weak.

In general, parity-violation measurements on the deuteron in the kinematic regime considered here will be difficult—the figure of merit is in many cases of interest fairly low, requiring long experiments. Systematic-error sources have *not* been investigated here. The anticipated errors discussed in Sec. III are purely statistical. Systematic-error considerations may provide strong preferences for particular beam energies, or e^- scattering angles, as well as the detailed detector specifications.

In the following section some details of the formalism are presented, including definitions of form factors, the weak response functions, and our model-dependent connection of deuteron to single-nucleon form factors. In Sec. III the resulting asymmetries and cross sections are calculated and discussed.

II. FORMALISM

In this section the steps required to estimate the contribution of s-quark currents to deuteron elastic observables are sketched. The kinematics are given by

$$D^\mu=(D_0,0,0,-\tfrac{1}{2}Q)_{\rm Breit},\quad (M,0,0,0)_{\rm lab}\ ;$$

$$D'^\mu=(D_0,0,0,\tfrac{1}{2}Q)_{\rm Breit},\quad \left[\left[M+\frac{Q^2}{2M}\right],0,0,Q\sqrt{1+Q^2/4M^2}\right]_{\rm lab}\ ;$$

$$q^\mu=(D'-D)^\mu=(0,0,0,Q)_{\rm Breit},\quad \left[\frac{Q^2}{2M},0,0,Q\sqrt{1+Q^2/4M^2}\right]_{\rm lab}\ ;$$

$$\xi^\mu(0)=\left[-\frac{Q}{2M},0,0,\frac{D_0}{M}\right]_{\rm Breit},\quad (0,0,0,1)_{\rm lab}\ ;$$

$$\xi^\mu(\pm 1)=\left[0,\mp\frac{1}{\sqrt{2}},-\frac{i}{\sqrt{2}},0\right]_{\rm Breit},\quad \left[0,\mp\frac{1}{\sqrt{2}},-\frac{i}{\sqrt{2}},0\right]_{\rm lab}\ .$$

Here D^μ is the deuteron momentum four-vector, $q^2=-Q^2$ is the four-momentum transfer, $D_0=\sqrt{M^2+Q^2/4}$ is the deuteron energy in the Breit frame, M is the deuteron rest mass, and $\xi^{(\prime)\mu}$ is the polarization vector for the initial (final) deuteron.

The matrix elements of weak currents between deuteron states[10,12,13] are written as

$$\langle D'|J_\mu^{(0)}|D\rangle\equiv\frac{-1}{\sqrt{4D_0D_0'}}\{G_1(q^2)(\xi'^*\cdot\xi)P_\mu+G_2(q^2)[\xi_\mu(\xi'^*\cdot q)-\xi_\mu'^*(\xi\cdot q)]-G_3(q^2)(\xi\cdot q)(\xi'^*\cdot q)P_\mu/2M^2\}\ ,\quad(2.1)$$

$$\langle D'|J_\mu^{(5)}|D\rangle\equiv\frac{-1}{\sqrt{4D_0D_0'}}\left[iG_4(q^2)\epsilon_{\mu\alpha\beta\gamma}\xi'^{*\alpha}\xi^\beta P^\gamma+\frac{iG_5(q^2)}{M^2}\epsilon_{\mu\alpha\beta\gamma}q^\alpha P^\beta[\xi^\gamma(\xi'^*\cdot q)-\xi'^{*\gamma}(\xi\cdot q)]\right.$$
$$\left.+\frac{iG_6(q^2)}{M^2}\epsilon_{\alpha\beta\gamma\delta}\xi'^{*\alpha}\xi^\beta P^\gamma q^\delta q^\mu\right]\ ,\quad(2.2)$$

where $P^\mu=D'^\mu+D^\mu$. These are the most general Lorentz structures one can write down for the vector (J) and axial-current $(J^{(5)})$ electroweak matrix elements, assuming Hermiticity and T invariance, and ignoring "second-class" currents.[10] Matrix elements of the electromagnetic current J_μ^γ have the same form as (2.1), and these form factors are denoted with superscript γ as G_i^γ. Form factors without superscript are understood to be weak and isoscalar, throughout this paper. It is often convenient to rewrite the three vector terms into different linear combinations defined by

$$G_C(q^2)\equiv G_1(q^1)-(q^2/6M^2)G_Q(q^2)\ ,$$
$$G_Q(q^2)\equiv G_1(q^2)-G_2(q^2)+(1-q^2/4M^2)G_3(q^2)\ ,\quad(2.3)$$
$$G_M(q^2)\equiv G_2(q^2)\ .$$

Note that *all* of the form factors in (2.2) are usually discarded, because the deuteron is an isoscalar system, and there are no axial-vector isoscalar currents in the standard model if one ignores strange (or heavier) quarks. In that case one would expect the exact relations[13]

$$G_{1,2,3}(q^2) = -2\sin^2\theta_W G^{\gamma}_{1,2,3}(q^2)\ ,\quad G_{4,5,6}(q^2) = 0\ . \tag{2.4}$$

We are interested here in the contributions of possibly nonzero G_4 and G_5 to observables, as well as deviations from this simple relation for $G_{1,2,3}$. (The G_6 term can be safely ignored, since its contribution to e^- and ν scattering is effectively zero due to the explicit factor of q^μ.)

Given the form-factor definitions in (2.1) and (2.2) one can derive in the usual manner[12,14] e^-d and νd elastic cross sections $\sigma_{e,e'}$ and $\sigma_{\nu,\nu'}$ and the parity-violating asymmetry[10,13] $A_d(q^2)$, defined as

$$A_d(q^2) \equiv \frac{\sigma(\mathbf{e}\uparrow,e') - \sigma(\mathbf{e}\downarrow,e')}{\sigma(\mathbf{e}\uparrow,e') + \sigma(\mathbf{e}\downarrow,e')}\ . \tag{2.5}$$

Here $\mathbf{e}\uparrow$ denotes a positive-helicity (right-hand polarized) electron, $\mathbf{e}\downarrow$ a negative-helicity electron, and $\sigma(\mathbf{e}\uparrow,e')$ is the inclusive elastic differential cross section on unpolarized deuterons.

These quantities are given in the laboratory frame by

$$\frac{d\sigma_{e,e'}}{d\Omega} \equiv \frac{4\alpha^2 e_2^2}{q^4} r\left[2\sin^2\frac{\theta}{2}W_1^{\gamma} + \cos^2\frac{\theta}{2}W_2^{\gamma}\right],\quad \frac{d\sigma_{\nu,\nu'}}{d\Omega} \equiv \frac{G_F^2 e_2^2}{2\pi^2} r\left[2\sin^2\frac{\theta}{2}W_1^{(0)} + \cos^2\frac{\theta}{2}W_2^{(0)} \mp \frac{2}{M}(e_1+e_2)\sin^2\frac{\theta}{2}W_8^{(0)}\right],$$

$$A_d \equiv \frac{-G_F Q^2}{4\pi\alpha\sqrt{2}}\left[\frac{2\sin^2\frac{\theta}{2}W_1^{\text{int}} + \cos^2\frac{\theta}{2}W_2^{\text{int}} - W_8^{\text{int}}(1-4x)\frac{2}{M}\sin^2\frac{\theta}{2}(e_1+e_2)}{2\sin^2\frac{\theta}{2}W_1^{\gamma} + \cos^2\frac{\theta}{2}W_2^{\gamma}}\right], \tag{2.6}$$

where θ is the lepton scattering angle in the laboratory frame, e_1 (e_2) the initial (final) lepton energies, the lower sign in the middle expression is for incident antineutrinos, $x \equiv \sin^2\theta_W$, and the nuclear recoil factor $r = [1 + (1-\cos\theta)e_1/M]^{-1}$.

$$\sum_{i,f} \frac{D_0 D'_0}{M^2}[\langle D'|J^{\gamma}_{\mu}|D\rangle\langle D|J^{(0)}_{\nu}|D'\rangle + \langle D'|J^{(0)}_{\mu}|D\rangle\langle D|J^{\gamma}_{\nu}|D'\rangle]\delta_{\mathbf{D}',\mathbf{D}+\mathbf{q}} \equiv W_1^{\text{int}}\left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right] + \frac{W_2^{\text{int}}}{M^2}\left[D_\mu - \frac{D\cdot q}{q^2}q_\mu\right]\left[D_\nu - \frac{D\cdot q}{q^2}q_\nu\right],$$

$$\sum_{i,f} \frac{D_0 D'_0}{M^2}[\langle D'|J^{\gamma}_{\mu}|D\rangle\langle D|J^{(0)}_{\nu 5}|D'\rangle + \langle D'|J^{(0)}_{\mu 5}|D\rangle\langle D|J^{\gamma}_{\nu}|D'\rangle]\delta_{\mathbf{D}',\mathbf{D}+\mathbf{q}} \equiv -\frac{W_8^{\text{int}}}{M^2} i\epsilon_{\mu\nu\alpha\beta}D^\alpha q^\beta\ , \tag{2.7}$$

and similarly for the purely weak or purely electromagnetic cases.[15]

Working out the required spin sums in (2.7), using the form factors from (2.1), (2.2), and (2.3), gives,[10,12,13] e.g.,

$$W_1^{\gamma} = \frac{Q^2}{6M^2}(1+Q^2/4M^2)G_M^{\gamma 2},\quad W_1^{(0)} = \frac{Q^2}{6M^2}(1+Q^2/4M^2)G_M^2 + \tfrac{2}{3}(1+Q^2/4M^2)^2\left[G_4 + \frac{Q^2}{M^2}G_5\right]^2,$$

$$W_1^{\text{int}} = \frac{Q^2}{3M^2}(1+Q^2/4M^2)(G_M^{\gamma}G_M),\quad W_2^{(0)} = \left[G_C^2 + \frac{Q^2}{6M^2}G_M^2 + \frac{Q^4}{18M^4}G_Q^2 + \frac{2}{3}(1+Q^2/4M^2)\left[G_4 + \frac{Q^2}{M^2}G_5\right]^2\right],$$

$$W_2^{\text{int}} = 2\left[G_C^{\gamma}G_C + \frac{Q^2}{6M^2}G_M^{\gamma}G_M + \frac{Q^4}{18M^4}G_Q^{\gamma}G_Q\right],\quad W_8^{\text{int}} = \tfrac{2}{3}(1+Q^2/4M^2)\left[G_M^{\gamma}\left[G_4 + \frac{Q^2}{M^2}G_5\right]\right], \tag{2.8}$$

and W_2^{γ} and $W_8^{(0)}$ follow similarly.

In order to make numerical estimates for the cross sections, these deuteron form factors must be connected to the better known nucleon form factors. This can be done by using a simple nonrelativistic approximation for the currents and wave functions. First, the form-factor definitions (2.1) and (2.2) are cast into a nonrelativistic appearance[16] by rewriting them in the Breit frame in terms of the simpler rest-frame deuteron polarization vectors $\eta^\mu \equiv \xi^\mu_{\text{lab}}$. In this way,

$$\langle D'|J^0|D\rangle_{\rm Breit} \equiv \left[G_C(\boldsymbol{\eta}'^*\cdot\boldsymbol{\eta}), \quad + \frac{G_Q}{2M^2}\left[(\boldsymbol{\eta}'^*\cdot\mathbf{q})(\boldsymbol{\eta}\cdot\mathbf{q}) - \frac{Q^2}{3}(\boldsymbol{\eta}'^*\cdot\boldsymbol{\eta}) \right] \right] ,$$

$$\langle D'|J^k|D\rangle_{\rm Breit} \equiv \left[\frac{G_M}{2M} \right] [\eta^k(\boldsymbol{\eta}'^*\cdot\mathbf{q}) - \eta'^{*k}(\boldsymbol{\eta}\cdot\mathbf{q})], \quad \langle D'|J^{(5)0}|D\rangle_{\rm Breit} \equiv 0 , \tag{2.9}$$

$$\langle D'|J^{(5)k}|D\rangle_{\rm Breit} \equiv \Bigg\{ iG_4 \left[(\boldsymbol{\eta}'^*\times\boldsymbol{\eta})^k - \left[\frac{1/4M^2}{1+D_0/M} \right] [(\boldsymbol{\eta}'^*\cdot\mathbf{q})(\boldsymbol{\eta}\times\mathbf{q})^k - (\boldsymbol{\eta}\cdot\mathbf{q})(\boldsymbol{\eta}'^*\times\mathbf{q})^k] \right]$$
$$- i\frac{G_5}{M^2}\frac{D_0}{M}[(\boldsymbol{\eta}'^*\cdot\mathbf{q})(\boldsymbol{\eta}\times\mathbf{q})^k - (\boldsymbol{\eta}\cdot\mathbf{q})(\boldsymbol{\eta}'^*\times\mathbf{q})^k] \Bigg\} .$$

The left-hand sides above will be evaluated using approximate nonrelativistic wave functions for $|D\rangle$ and the usual nonrelativistic reduced single-nucleon current operators[12,17] (in the Breit frame)

$$J^0 = F_E(q^2), \quad J^{(5),0} = \frac{F_A(q^2)}{2m}\boldsymbol{\sigma}\cdot(\mathbf{p}+\mathbf{p}') ,$$
$$\mathbf{J} = \left[F_M(q^2)\frac{i}{2m}\boldsymbol{\sigma}\times\mathbf{q} + \frac{F_E(q^2)}{2m}(\mathbf{p}+\mathbf{p}') \right] , \tag{2.10}$$
$$\mathbf{J}^{(5)} = F_A(q^2)\boldsymbol{\sigma} ,$$

where m is the nucleon mass, and F_M, F_E, and F_A are the free, single-nucleon form factors. (We use F in place of the usual G to avoid confusion with deuteron elastic form factors.) Neutron and proton currents are simply added; this is just the impulse approximation.

The deuteron wave functions in (2.9) are written as[12]

$$\Psi_d = \chi^{(n)\dagger}\psi_d\chi^{(p)} , \tag{2.11}$$

with $\chi^{(p),(n)}$ nucleon spinors. ψ_d is a 2×2 matrix given by

$$\psi_d(\mathbf{r}) = \frac{1}{\sqrt{4\pi}}\left[\frac{u(r)}{r}\boldsymbol{\sigma}\cdot\boldsymbol{\eta}_m - \frac{1}{\sqrt{2}}\frac{w(r)}{r}[\,3(\boldsymbol{\sigma}\cdot\hat{\mathbf{r}})(\hat{\mathbf{r}}\cdot\boldsymbol{\eta}_m) - \boldsymbol{\sigma}\cdot\boldsymbol{\eta}_m] \right] \frac{i\sigma^y}{\sqrt{2}} , \tag{2.12}$$

where $\boldsymbol{\sigma}$ are the usual Pauli spin matrices and $u(r)$ and $w(r)$ are the S- and D-state deuteron radial wave functions (see Appendix A).

Sandwiching the currents (2.10) between wave functions (2.11) and (2.12), and comparing with the general forms in (2.9) yields, after some algebra,[11,12,18]

$$G_C(q^2) = F_E(q^2)\int_0^\infty (u^2+w^2)j_0\left[\frac{qr}{2}\right]dr ,$$
$$G_Q(q^2) = F_E(q^2)\frac{6\sqrt{2}M^2}{Q^2}\int_0^\infty \left[uw - \frac{w^2}{\sqrt{8}} \right] j_2\left[\frac{qr}{2}\right]dr ,$$
$$G_M(q^2) = F_E(q^2)D_M^E(q^2) + F_M(q^2)D_M^M(q^2) ,$$
$$D_M^E(q^2) = \tfrac{3}{2}\int_0^\infty w^2\left[j_0\left[\frac{qr}{2}\right] + j_2\left[\frac{qr}{2}\right] \right]dr , \tag{2.13}$$
$$D_M^M(q^2) = 2\int_0^\infty (u^2 - \tfrac{1}{2}w^2)j_0\left[\frac{qr}{2}\right] + \sqrt{2}\int_0^\infty \left[uw + \frac{w^2}{\sqrt{2}} \right] j_2\left[\frac{qr}{2}\right]dr ,$$
$$G_4 + \frac{Q^2}{M^2}G_5 = F_A(q^2)\frac{M}{D_0}\int_0^\infty (u^2 - \tfrac{1}{2}w^2)j_0\left[\frac{qr}{2}\right] + \tfrac{1}{2}(w^2 + \sqrt{2}\,uw)j_2\left[\frac{qr}{2}\right]dr ,$$

with $q \equiv +\sqrt{Q^2}$.

We use deuteron radial wave functions derived from several popular nucleon-nucleon potentials.[19,20] The specific choice of wave function is not critical here, as we wish to see only the general effects of nonzero F_A, and extra contributions to F_M and F_E from strange quarks, in the asymmetry. The form factors $F_{E,M,A} \equiv F_{E,M,A}^p + F_{E,M,A}^n$ are simply the nucleon *isoscalar* electric, magnetic, and axial form factors. These are normalized to $F_E^\chi(0)=1$, $F_M^\chi(0)=\mu_p+\mu_n=2.793-1.913=0.88$, and $F_A(0)=0$, if one ignores strange quarks.

The presence of strange quarks in the nucleon modifies the weak isoscalar form factors, of course. Recent neutrino scattering experiments[9] have shown that $F_A(0)\approx -0.15\pm0.08$ is allowed. This result is also consistent with a naive interpretation of the deep-inelastic sum-rule measurements,[1] if one ignores the gluon contribution to the evolution of the spin distributions.[21] A nonzero axial-vector isoscalar form factor was anticipated some time ago by, e.g., Collins, Wilczek, and Zee[22] because of the axial anomaly, but such a large value for a ground-state nucleon is still widely considered something of a surprise.

The vector form factors may also receive contributions from strange quarks.[1] This does not, however, have the same strong theoretical motivation as in the axial-vector case, since the vector currents are conserved, and anomaly-free. We define strange form factors for the nucleon in exact analogy with the usual electromagnetic form factors, so, e.g.,

$$\langle p'|\bar{s}\gamma_\mu s|p\rangle \equiv \bar{u}(p')[\gamma_\mu F_1^{\rm st}(q^2)+i\sigma_{\mu\nu}q^\nu F_2^{\rm st}(q^2)]u(p) , \tag{2.14}$$

where u is just a Dirac spin-$\frac{1}{2}$ spinor. With the above definitions, and using the underlying quark currents from Eqs. (1.1) and (1.2), the weak nucleon form factors are immediately given by

$$F_i^{\rm proton}=F_i^{\gamma,\rm proton}(\tfrac{1}{2}-2\sin^2\theta_W)-\tfrac{1}{2}F_i^{\gamma,\rm neutron}-\tfrac{1}{2}F_i^{\rm st} , \tag{2.15}$$

where $i=1,2$ (or E,M). This formula is the basis for extracting s-quark information from free-nucleon data. On the right-hand side, everything is in principle well-known experimentally except $F_i^{\rm st}$. For our purposes, forming the isoscalar combination immediately gives the weak form factor

$$F_i=F_i^\gamma(-2\sin^2\theta_W)-F_i^{\rm st} . \tag{2.16}$$

The presence of s quarks thus violates the simple relation (2.4), and a measurement of such weak vector form factors can be directly connected, via the definition (2.14), to the strange-quark distribution in the nucleon.

Some recent estimates for the strange contribution to the magnetic form factor include $F_M^{\rm st}(0)=0.45$ [SU(2) Skyrme model], ~ 0.2 (chiral quark model), $\sim(1)$ (baryon chiral Lagrangian), and -0.24 to -0.43 (dispersion[23] fits). There is currently little detailed experimental evidence directly measuring such strange isoscalar Lorentz-vector current matrix elements, although a reanalysis of the Ahrens proton data[9] may allow some limits to be placed. Our crude analysis shows that values of $|F_M^{\rm st}(0)|\lesssim 0.4$ are still consistent with their published data, assuming a dipole q^2 dependence, and assuming that the strange electric form factor behaves roughly like the neutron electric form factor, namely, $F_E^{\rm st}(q^2)\approx q^2F_M^{\rm st}/4m^2$ (this assumption is discussed below). A full reanalysis still needs to be done in a more sophisticated way, including simultaneous fits of the data to $F_M^{\rm st}$, $F_E^{\rm st}$, and $F_A^{\rm st}$. For the purposes of comparison, in this paper we will use a maximum value of $F_M^{\rm st}(0)=0.4$, to see the effects of large vector s-quark contributions.

III. RESULTS AND DISCUSSION

In order to look for sensitivities of the asymmetry to the various possible strange-quark contributions, we have systematically scanned through reasonable kinematics for elastic e^-d and νd scattering. This is guided in part by the likely experimental facilities available for such experiments. For example, in the case of electrons, energies in the 100–1000-MeV range will be accessible at NIKHEF, Bates, and Mainz, while energies from 1 to 5 GeV may be available in the near future at CEBAF. Small-angle measurements will probably be constrained to smaller solid angle, affecting our figure of merit estimates. In general, we have considered only low-q^2 cases. First, this keeps the dependence on our simple nonrelativistic deuteron models reasonably small. More importantly, as one approaches the first deuteron diffraction minima, and beyond, the cross sections (and thus the figure of merit) drop precipitously. We have found good sensitivities to s quarks in both axial and magnetic form factors even at very small $q^2\approx -0.01$ GeV.^{2} The strange contributions to the electric form factor become harder to see as $q^2\to 0$, since $F_E^{\rm st}(q^2=0)\equiv 0$ is *constrained* by an exact counting rule.

A. Calculations and assumptions

To examine the effects of possible s quarks, we have calculated the asymmetry in several ways. First, we ignore all strange contributions and use the standard model relations between isoscalar form factors [Eq. (2.4)], along with the electromagnetic structure functions derived in Eqs. (2.8) and (2.13). These can be compared with existing electron-scattering data, and fit quite well. This gives the "base" calculation, and in fact reduces to an extremely simple form[10,13,15] at fixed q^2, a constant function of θ. We then recalculate allowing only an extra axial-vector isoscalar contribution by giving $F_A(0)$ a nonzero value. The third calculation assumes that s quarks appear *only* via the magnetic vector form factor, i.e., $F_M^{\rm st}(0)\neq 0$. The fourth case has strange quarks appearing only in the electric vector form factor $F_E^{\rm st}(q^2)$, with q^2 dependence described below.

In all these calculations we let the strange form factors have a phenomenological dipole q^2 dependence exactly the same as their isovector partners. Thus, e.g.,

$$F_M^{\rm st}(q^2)=F_M^{\rm st}(0)g_d(q^2) \equiv \mu^{\rm st}\frac{1}{(1-q^2/0.71\ {\rm GeV}^2)^2} . \tag{3.1}$$

This q^2 dependence is not theoretically required, but serves as a rough guide, without adding too many extra parameters into the calculations. $\mu^{\rm st}$ is the fundamental parameter of interest, measuring the contribution of s quarks to the nucleon's static magnetic moment. We choose the value of $+0.4$ for this quantity in our calculations, simply to see its effects. The resulting deviation of the asymmetry from its base value is linear in this choice. As mentioned earlier, such a value is still not quite pushing the limits allowed by νp experiments.[9] Several theoretical models[1,23] predict values around -0.4, the results of which can be easily seen from our curves simply by reversing the sign of the deviations we calculate.

To see the effects of a strange contribution to F_E is more difficult, since it is constrained to 0 at $q^2=0$. We choose the form

$$F_E^{st}(q^2)=\frac{q^2}{4m^2}\mu^{st}g_d(q^2) . \tag{3.2}$$

This is similar to a common parametrization of the neutron electric form factor. It corresponds to a strange Dirac charge form factor identically zero. There is no pressing theoretical reason to assume exactly such a form, but it is reasonable, and allows us to see where the asymmetry may be expected to be sensitive to electric contributions. In fact, recent predictions[23] based on dispersion theory give large values for the strange electric radius, corresponding to using a form factor which rises even more quickly with q^2 than Eq. (3.2), and with opposite sign. Again, the deviations we calculate assuming only electric s-quark contributions are linear in the overall size of (3.2), and can thus be easily scaled with the strength and sign of the electric form factor.

To quantitatively estimate the statistical error expected, some fairly detailed assumptions about the experiment are required. For comparison purposes we use values similar to those recently discussed in relation to proton-asymmetry measurements. We have assumed 100 μA currents of 100% polarized e^-'s, 100 hours of beam time, 1.3×10^{24} nucleons/cm^2 [this is roughly 30 cm of liquid deuterium (LD_2)], and a 1-sr (100-msr) solid angle for backward- (forward-) angle experiments. Error estimates can be easily scaled when any of the assumed quantities here are different.

There are, of course, other quantities to which the asymmetry may be sensitive. The most obvious two are $\sin^2\theta_W$ and F_E^n. We have also allowed these to vary, to check that uncertainties in them will not dominate the deviations of the asymmetry from the base predictions. At low q^2, the large existing experimental uncertainties in F_E^n are quite unimportant, since it is the isoscalar quantity F_E which dominates the asymmetry. The dependence of the weak mixing angle is essentially direct, $A\propto\sin^2\theta_W$. For backward-angle kinematics, the experimental uncertainities in A turn out to be far greater than the $\approx1\%$ that $\sin^2\theta_W$ could contribute. At forward angles, however, this uncertainty can be comparable to the deviations caused by the assumed s-quark contributions. Other possible modifications of the asymmetry, due, e.g., to higher-order diagrams, isospin breaking, or odd-parity admixtures in the deuteron wave functions, have not been considered here.

B. Numerical results: angular dependence

Figure 1 shows the predicted asymmetry as a function of scattered electron angle, for fixed $q^2=-0.4$ GeV^2. This plot is on a linear scale, and the base asymmetry is roughly an order of magnitude larger than was recently measured on ^{12}C at Bates. At this q^2, 10° scattering corresponds to a beam energy of 3.7 GeV, 90° to 500 MeV, and backscattering to 375 MeV.

At backward angles, the dependence on the electric strange contributions is fairly small, primarily because the momentum transfer is quite low, and kinematic

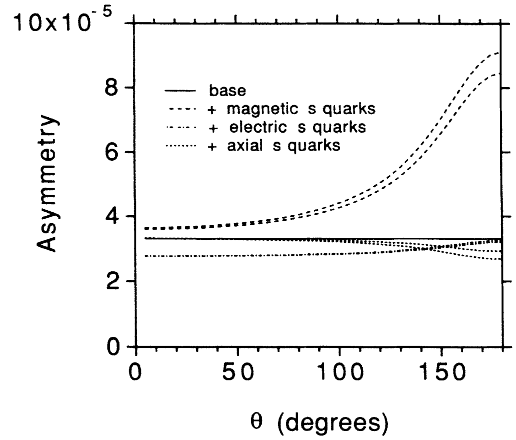

FIG. 1. Deuteron asymmetry vs scattering angle, at fixed $q^2=-0.4$ GeV^2. The solid curve is the base calculation with no strange contributions. The two long-dashed curves include a magnetic s-quark magnetic-moment contribution of $+0.4$. They differ in assumed nucleon-nucleon interaction potential (see text). The dotted-dashed curves include only an extra electric s-quark form factor. The short-dashed curves include only an extra axial-vector isoscalar form factor with a value of -0.15 at $q^2=0$.

coefficients inhibit it as well. The axial-vector strange-quark contribution, however, *only* comes in at backward angles. This can be seen directly from Eq. (2.6). The assumed value of $F_A(0)=-0.15$ has caused a 15% drop in the asymmetry at 180°. However, the estimated statistical error after 100 hours of beam time here is roughly 75% (see also Fig. 5), so this deviation will in practice be quite difficult to measure. The calculation with a strange magnetic moment of $\mu^{st}=+0.4$ also has a dramatic effect at backward angles—in this case increasing the asymmetry by about a factor of 3. This deviation is linear in the assumed value of μ^{st}, so, e.g., a choice of $\mu^{st}=-0.4$ actually brings the asymmetry down to negative values here. Thus, despite the low cross section and bad figure of merit, such backward-angle measurements appear quite sensitive to the strange-quark content, especially in the vector magnetic form factor, of the nucleons.

At forward angles, the situation is quite different. The deviation of the asymmetry at 10° due only to the magnetic s-quark contribution is about $+10\%$, and from the electric part about -16%. Although these are less dramatic, they may also still be marginally measurable, if systematic errors can be kept tightly controlled. Our estimated statistical error, as discussed above, is 6% after 100 hours at these kinematics. Of course, the measured asymmetry will include contributions from *both* electric and magnetic form factors, and these partially cancel with our assumed forms for $F_{E,M}^{st}$. Such a cancellation depends on the relative signs of the form factors, however, and thus low-angle measurements may be very helpful in separating electric and magnetic s-quark contributions.

The model sensitivity is seen schematically from our choice of two different potentials, Reid soft-core and Bonn,[19,20] giving rise to a pair of curves for each calculation. In general, the Reid soft-core potential causes slightly smaller deviations in the asymmetry than the Bonn potential does. The base result is, however, completely deuteron-model independent, since it involves a ratio. This is a direct consequence of the standard model, assuming no anomalous isoscalar contributions, or isospin mixing [see Eq. (2.4)]. We have also examined the sensitivity to $\sin^2\theta_W$ by increasing it 1% in the base calculation. This shows the scale of corrections in this figure—at both forward and backward angles, a 1% deviation is negligible.

Figure 2 shows the same calculations at a lower $q^2=-0.02$ GeV^2. The base asymmetry is linear in q^2, and is correspondingly much smaller, roughly the same as the recent ^{12}C experiment. 10° corresponds here to an incident energy of 810 MeV, while 180° is 73 MeV. This figure shows the same general behavior as in Fig. 1, although the model dependence is becoming less important. Since the models are all fit to static ($q^2=0$) deuteron properties, this clearly makes sense. The electric s-quark contribution is almost zero at backward angles. The axial deviation is more significant at 180°, but is not as strong as the magnetic vector modifications. The statistical error on the base calculation is around 70% here, while the deviation due to the F_A term is a remarkable 60% or so. The magnetic piece with $\mu^{\rm st}=+0.40$ causes an increase of almost a factor 3. We again conclude that backward-angle A_d measurements are sensitive to both axial-vector and vector s-quark contributions, while the figure of merit is unfortunately correspondingly small.

At forward angles, the predicted statistical error for the base calculation is about 1.4%, while the deviations due to the s-quark contributions to the vector form factors are a bit smaller than this—about 0.5% for the magnetic piece, and about 1% for the electric. The axial-vector form-factor contribution is completely negligible here. A forward-angle extraction of these quantities would thus require better statistics than we have assumed, and very tight control of the systematics. These deviations are quite comparable to the error bar coming from a 1% uncertainty in $\sin^2\theta_W$.

C. Numerical results: q^2 dependence and statistical errors

The detailed q^2 dependence can be seen more clearly in Figs. 3 and 4. In the forward-angle (10°) case shown in Fig. 3, the range of $0.01 \le -q^2 \le 1.0$ GeV^2 corresponds to beam energies of 575 MeV to almost 6 GeV. The statistical error estimates (cf. Fig. 5) range from around 1% at low q^2 to over 10% at the highest energies. The assumed axial-vector form factor is totally negligible over this entire q^2 range. The assumed large s-quark contributions in the vector form factors, however, show up noticeably for $-q^2 \gtrsim 0.05$ GeV^2. At the larger q^2, the strange electric piece actually dominates the correction, reducing the asymmetry prediction by 40%. This correction appears fairly deuteron-model independent. Using a larger strange electric form factor would of course increase this deviation even further. [The predicted values of $\langle r^2\rangle^{\rm st}=4.1$ GeV^{-2} and $\mu^{\rm st}=-0.43$ from dispersion results[23] correspond to an increase of $F_E^{\rm st}(q^2)$ over that given in (3.2) by a factor of more than -5.]

For backward angles (Fig. 4), the corresponding beam energies run from 51 MeV ($q^2=-0.01$) to 650 MeV ($q^2=-1.0$). Here, while the figure of merit is much worse, the effects of the strange quarks are also much stronger. The magnetic part is most important, dominating both axial-vector and electric contributions. The model dependence is also fairly large here, but considering the likely level of experimental error, should not eliminate the possibility of observation of these extra effects, assuming the s-quark content is indeed fairly large.

Figure 5 shows our estimated statistical errors:

$$\left[\frac{\Delta A_d}{A_d}\right]_{\rm stat} \approx \frac{1}{\sqrt{A_d^2 N}} \equiv \frac{1}{\sqrt{F_M}}\ , \tag{3.3}$$

where N is the total (electromagnetic) counting rate, and F_M is the "'figure of merit" quantity often referred to by other authors. N scales linearly with the counting time, target size, solid angle, and beam current. A_d scales linearly with the beam polarization. The kinematics from the previous two figures have been combined in Fig. 5.

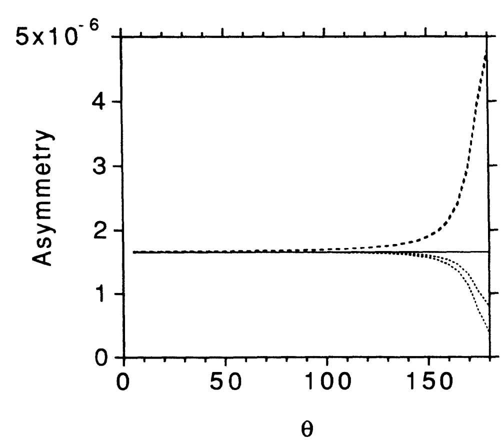

FIG. 2. Same as in Fig. 1, with $q^2=-0.02$ GeV^2.

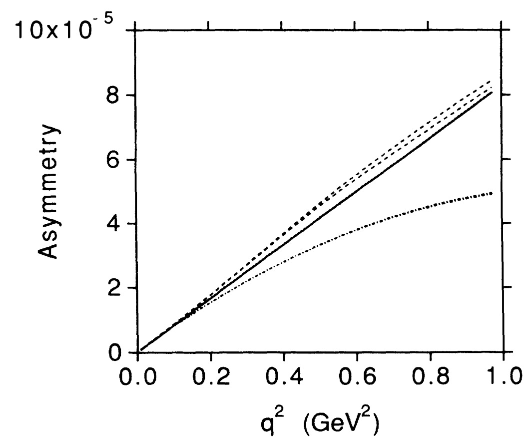

FIG. 3. Deuteron asymmetry vs q^2, at fixed scattering angle of 10°. The curve descriptions match those in Fig. 1.

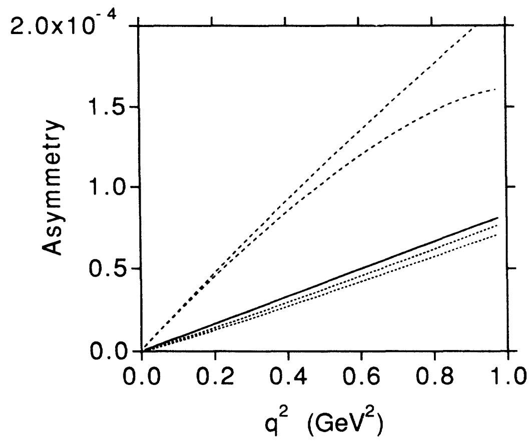

FIG. 4. Same as Fig. 3, with $\theta = 180°$. The curves with electric s-quark contributions are not shown, as they lie fairly close to the base.

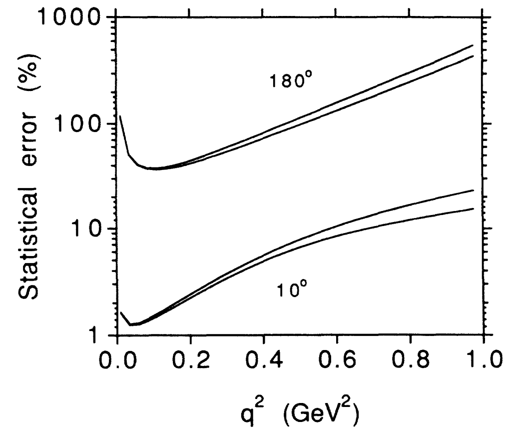

FIG. 5. Predicted statistical error vs q^2, at scattering angles of 10° (lower curves) and 180° (upper curves). Detailed assumptions are discussed in the text. At each angle we have used both Reid soft-core (Ref. 19) and Bonn (Ref. 20) potentials.

At backward angles (upper set of curves), the errors are larger, coming primarily from the small electromagnetic cross sections. We have calculated the same four cases as in the previous plots, but show only the base calculation, which should serve as a rough guide for experimental purposes. The figure of merit peaks at very low q^2, between 0.02 and 0.1 GeV^2, depending on scattering angle. Below this, the asymmetry is going to 0, forcing the statistical error to blow up. Above this, while the asymmetry itself is rising, the deuteron body form factor kills the counting rate, so $N \to 0$.

We have also examined the *relative* size of the predicted deviations in A_d due to s quarks, compared to the statistical error in the base calculation. This tells the "signal-to-noise" ratio, and should clearly be as large as possible. This ratio is plotted in Fig. 6, for several different cases. It is formally defined as

$$\text{signal/noise} \equiv \left[\frac{A_d(\text{including } s \text{ quarks}) - A_d(\text{base})}{A_d(\text{base})} \right] \Bigg/ \left[\frac{\Delta A_d(\text{base})}{A_d(\text{base})} \right]_{\text{stat}} \tag{3.4}$$

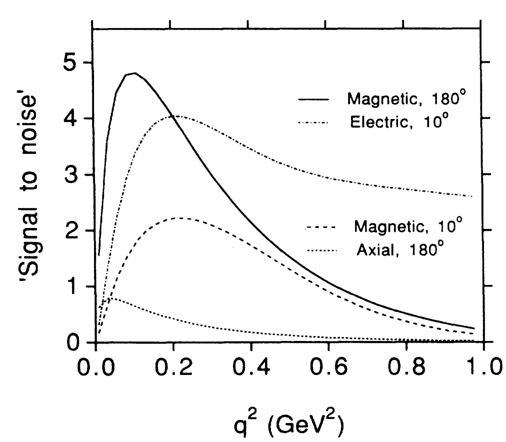

FIG. 6. Ratio of the deviation in A_d caused by s-quark contributions, to the statistical error, as discussed in the text. The solid curve assumes only a magnetic s-quark contribution at 180°, the long-dashed curve is 10°. The dotted-dashed curve has only electric contributions at 10°, the short-dashed curve assumes just an axial-vector contribution at 180°. (We show only Reid soft-core, for simplicity.)

and shows a characteristic peak roughly following the figure of merit. In the forward-angle case, the deviations due to strange vector form factors exceed the predicted statistical errors over the range $0.05 \lesssim -q^2 \lesssim 0.5$ GeV^2. The electric term is always the most important, and peaks for $q^2 \approx -0.2$ GeV^2 at roughly four times the statistical error. The magnetic s-quark deviation at this q^2 is roughly twice the estimated statistical error. The absolute scale here is not fully fixed—it depends on our detailed assumptions about experimental conditions. Lowering the statistical error (e.g., by running for longer times) increases the "signal-to-noise" ratio proportionally.

For backward angles, it is the deviations from strange magnetic and axial-vector terms which can exceed the predicted errors, again over roughly the same q^2 range. The magnetic deviation at $q^2 = -0.1$ GeV^2 is almost five times larger than the predicted statistical error, while the deviation from an axial-vector form factor is slightly less than the predicted error there. Systematic error considerations will clearly be important for a precise determination of which q^2 to study, but the low-q^2 values considered here will be strongly statistically favored.

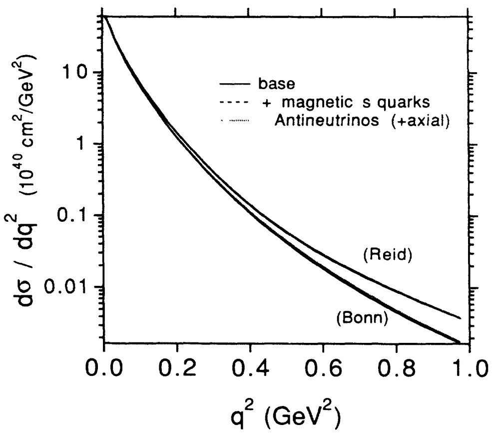

FIG. 7. Neutrino and antineutrino elastic cross sections vs q^2. Incident beam energy is fixed at 1.0 GeV. The upper set of curves correspond to Reid soft-core potential calculations, the lower curves use the Bonn potential. The s-quark contributions are clearly negligible.

D. Numerical results: νd cross section

Since the only existing direct low-energy *measurement* of an axial-vector isoscalar nucleon form factor involves elastic νp scattering, we present a final curve (Fig. 7) which shows some predicted elastic νd cross sections. In the recent νp scattering experiments,[9] a neutrino beam with energy ranging from roughly 0.2 to 5.0 GeV was used. In the figure shown we assume 1 GeV for simplicity, rather than integrating over a ν spectrum. It is immediately clear that the effects of strange quarks are much more difficult to see than in the free nucleon case. First, the deuteron cross sections are considerably smaller, especially at the larger q^2 values, than the corresponding proton cross sections. More importantly, the spread into two sets of curves, due just from the different deuteron model assumptions, is larger than the spreading coming from the assumed strange-quark contributions. (If one chooses backward-angle, fixed-q^2 kinematics, the effects of the s quarks become much more distinct, standing out well beyond the model uncertainties. However, unlike the parity-violation case, monochromatic neutrino beams with backward-angle detectors are experimentally more unreasonable.) The conclusion here is that elastic neutrino scattering from deuterium is not likely to yield useful information on the s-quark content of the nucleons.

E. Summary and conclusions

Parity-violating asymmetries in polarized electron scattering have been considered over the years as a means to test the standard model, quantitatively measure nucleon and nuclear weak form factors, and even to extract electromagnetic structure of nucleons.[13–24] In this paper we have examined the effects of possible large strange content in nucleons on elastic electroweak lepton-deuteron scattering. There is currently very little experimental evidence at all on vector s-quark matrix elements, while some evidence exists hinting at possible large axial-vector s-quark matrix elements. The deuteron, being an isoscalar target, provides a strong candidate for cleanly observing any possible extra isoscalar s-quark form factors. Because the deuteron has spin 1, it also has a magnetic form factor, and thus low-q^2 experiments can allow significant observation of both vector as well as axial-vector pieces.

We have shown that, assuming fairly large strange-quark components in the nucleons which are not *excluded* by current experimental limitations, the parity-violating asymmetry in elastic e^-d scattering is significantly modified from its value when strangeness is ignored. Such modifications, while slightly deuteron-model dependent, could provide a strong means to measure and separate the various pieces of the strange currents in nucleons. This would be further aided by combining data from various angles, as well as with complementary direct data from free nucleons.

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APPENDIX A

The deuteron wave function ψ in (2.11) and (2.12) can in fact be obtained from the more commonly used deuteron wave function[25,26]

$$\phi_d(\mathbf{r})=\frac{1}{\sqrt{4\pi}}\left[\frac{u(r)}{r}+\frac{1}{\sqrt{8}}\frac{w(r)}{r}S_{12}\right]\chi_m^{(1)}\,, \tag{A1}$$

where $S_{12}=3(\boldsymbol{\sigma}_p\cdot\hat{\mathbf{r}})(\boldsymbol{\sigma}_n\cdot\hat{\mathbf{r}})-\boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_n$, and $\chi_m^{(1)}$ is a spin-1 (triplet) spinor with $m=0,\pm 1$. The connection is shown in Refs. 12 and 27: the step required to go from this usual expression in terms of a deuteron spinor $\chi_m^{(1)}$, to the matrix form (2.11) with deuteron rest-frame polarization vectors $\boldsymbol{\eta}_m$, is to note the identity[27]

$$A_pB_n\chi_m^{(1)}=\left[B\frac{\boldsymbol{\sigma}\cdot\boldsymbol{\eta}_m}{\sqrt{2}}\tilde{A}i\sigma_y\right]_{sr}\,, \tag{A2}$$

with $\tilde{A}=\sigma^yA^T\sigma^y$, and the indices s and r of the matrix correspond to the indices of the decomposition of the spin-1 spinor into spin-$\frac{1}{2}$ Pauli spinors of the nucleons, e.g.,

$$\chi_0^{(1)}=\frac{1}{\sqrt{2}}(\chi_r^\uparrow\chi_s^\downarrow+\chi_r^\downarrow\chi_s^\uparrow)\,. \tag{A3}$$

Inserting the identity (A2) into the wave function (A1) then yields directly the matrix forms (2.11) and (2.12) used in this paper.

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