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Singlet axial-vector current and the "proton-spin" question

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As a preliminary we consider the phenomenology of the European Muon Collaboration experimental result without assuming SU(3) invariance. The most reliable conclusion is that the singlet axial-vector current matrix element is rather small. We calculate this matrix element in a variety of chiral-soliton models in which the axial anomaly equation is satisfied. There are a number of subtleties which we explore. It turns out that reasonable chiral models including only pseudoscalar fields give a zero matrix element. Taking "short-distance" effects consistently into account via the inclusion of vector mesons or explicit quarks does give a small nonzero result. The situation is closely analogous to the prediction of a nonzero result for the nonelectromagnetic part of the neutron-proton mass difference in these chiral models.

I. INTRODUCTION AND SUMMARY

The recent European Muon Collaboration (EMC) experiment[1] has caused a sensation by measuring the proton form factor at zero-momentum transfer of a certain linear combination of the diagonal quark axial-vector currents $\bar{u}\gamma_\mu\gamma_5 u$, $\bar{d}\gamma_\mu\gamma_5 d$, and $\bar{s}\gamma_\mu\gamma_5 s$. The main reason is that combining their result with ordinary- and [using SU(3)] strange-baryon β-decay results leads to an approximately zero value for the singlet $J_\mu^5 = i\bar{u}\gamma_\mu\gamma_5 u + i\bar{d}\gamma_\mu\gamma_5 d + i\bar{s}\gamma_\mu\gamma_5 s$ matrix element. In turn, this implies that the quark-spin contribution $-\frac{1}{2}\int d^3x\, J_i^5$ to the proton angular momentum approximately vanishes. This then would rule out the naive nonrelativistic quark model in which the spin of the proton is *completely* given by combining quark spins. Another interesting apparent implication[2] of this analysis is that the strange-quark operator $\bar{s}\gamma_\mu\gamma_5 s$ has a nontrivial matrix element in the proton state.

Properly, there is already a sizable body of literature[3] on this problem. Most approaches are based on the parton or perturbative-QCD pictures. There it is quite natural that quark orbital contributions as well as gluonic contributions to the proton angular momentum should exist. However, it is not easy to calculate them from this viewpoint.

Another approach to this problem, on which we shall focus in the present paper, is based on the soliton or Skyrme description of the proton. This method emphasizes the role of spontaneously broken chiral symmetry and treats the proton as a collective excitation. Remarkably, the simplest version of the SU(3) Skyrme model predicts[2] the desired result: zero for the axial-vector singlet matrix element. Even though the simple Skyrme model provides only a rough description of most nucleon properties, it can be improved so that the present result suggests that it may be a desirable "zeroth-order" model.

The initial treatment[2] of the SU(3) Skyrme model implied that it gave a relatively large $\bar{s}\gamma_\mu\gamma_5 s$ matrix element in agreement with experiment plus SU(3) invariance. However, a more detailed treatment[4] showed that the predicted $\bar{s}\gamma_\mu\gamma_5 s$ matrix element is really small. Furthermore, it has been pointed out that it is possible for SU(3) to be badly broken for the present purpose without[5] badly disturbing the relatively good SU(3) predictions in the Cabibbo scheme.[6] With this background, we have started things off by presenting a model-independent phenomenology (see Sec. II) in which SU(3) invariance[7] is not assumed. The net result is that the EMC experiment still implies that the axial-vector singlet matrix element vanishes (with a somewhat large error), but that the $\bar{s}\gamma_\mu\gamma_5 s$ matrix element is not necessarily large.

Thus we will specialize our discussion to calculating the proton matrix element of the axial-vector singlet current J_μ^5 in various chiral models. J_μ^5 is of course the famous "U(1)-problem" current whose divergence possesses the Adler-Bell-Jackiw anomaly.[8] A discussion of the way in which the anomalous divergence equation of J_μ^5 can be realized as an operator relation in the chiral-Lagrangian framework was given some years ago. We will employ this mechanism, which involves a kind of

η' dominance of the gluon anomaly, in all our models. This provides a reasonable, *particular* solution of the U(1) problem. In the more complicated of our models, additional $U(3)_L \times U(3)_R$ *chiral-symmetric* terms may be added; these do not formally change the anomaly equation, but they may alter the relevant dynamics. This is reviewed in Sec. III, and the Goldberger-Treiman-type formula which follows from the anomaly equation is also noted and briefly discussed.

There is, of course, no known fundamental reason for the singlet matrix element to *exactly* vanish (although it is naturally small in chiral models). Hence its calculation is very interesting as a test of our understanding of chiral models of nucleon structure. [The present relatively large experimental uncertainty (see Fig. 1) is consistent with a reasonably small nonzero value for this quantity.] Now it turns out that the calculation involves a number of subtleties which have not always been recognized in the literature. We point out, in Sec. III, two conditions that are necessary in any model to get a nonzero axial-vector singlet matrix element at zero-momentum transfer ($q^2=0$). First, the current J^5_μ should not be a pure gradient. If it is, only the *induced* form factor can exist, which is irrelevant for the present problem. One proposal[2] for obtaining a nonzero result makes use of a derivative-type SU(3)-symmetry-breaking term, which has the desirable feature of splitting the decay constants F_π and F_k. But this scheme does not work because the axial-vector current[4] remains a pure gradient. Even if the axial-vector current is not a pure gradient, we have the additional condition that the η' meson (η meson in the two-flavor case) get excited in the nucleon subspace. This seems to be the difficulty with a proposed scheme[9] which in effect makes use of an additional chiral-invariant term which is normally discarded in the Skyrme model. In the chiral (i.e., zero quark mass) limit, which is appropriate for discussing the effects of chiral-symmetric terms, the η' does not get excited as shown in Sec. III. Furthermore, an η-type field does not get excited at the dominant two-flavor level.

It appears that there is a remarkable similarity between the calculation of the axial-vector matrix element and the calculation of the nonelectromagnetic part of the neutron-proton-mass difference in chiral-soliton models. A recent paper[10] demonstrated that the proper calculation of this quantity also required the excitation of an η-like meson. In reasonable chiral models involving only pseudoscalars, this could *not* be accomplished. It was necessary to take account of *short-distance* effects such as either the introduction of vector mesons or, in some way, explicit quarks in order to calculate the *n-p* mass difference satisfactorily.

We find that the same story repeats itself here. In Sec. IV we discuss the calculation of the axial-vector matrix element in a model in which vector mesons have been added in a somewhat "minimal" although experimentally realistic way. The same model gave[10] a good account of itself for the *n-p* mass difference. Here both conditions mentioned above are satisfied—the total current is not a pure gradient and an η-like meson gets excited. We find that the matrix element of J^5_μ at $q^2=0$ is 0.30 (for comparison the experimental value of the neutron β-decay matrix element $g_A=1.25$). The consistency of this value with experiment requires nontrivial SU(3)-symmetry breaking for the diagonal (i.e., non-Cabibbo) axial-vector current matrix elements and a corresponding reduced $\bar{s}\gamma_\mu\gamma_5 s$ matrix element. A similar value for the axial-vector singlet matrix element is estimated using the "chiral quark model" in Sec. V. This model is somewhat related to the "chiral bag model." For the present calculation it has the advantage that the chiral-anomaly equation can be simply enforced as an operator equation in the same way as discussed in Sec. III, and one may furthermore avoid the subtleties involved in bag boundary conditions.

II. MODEL-INDEPENDENT PHENOMENOLOGY

The relevant quantities for discussing the results of the EMC "proton-spin" experiment[1] are the proton matrix elements of the flavor-conserving quark axial-vector currents. We define the "diagonal" pseudovector current of the ath quark q_a as

$$P^a_{a\mu}(x)=i\bar{q}_a\gamma_\mu\gamma_5 q_a \ , \tag{2.1}$$

where a color sum is implicit, and Pauli's γ matrix and metric conventions are being followed. The corresponding form factors are then

$$\frac{\sqrt{p_0 p_0'}}{M}\langle P(\mathbf{p}')|P^a_{a\mu}(0)|P(\mathbf{p})\rangle = i\bar{u}(\mathbf{p}')\left[\gamma_\mu\gamma_5 H_a(q^2)+\frac{iq_\mu}{2M}\gamma_5\tilde{H}_a(q^2)\right]u(\mathbf{p}) \ , \tag{2.2}$$

where M is the nucleon mass and the momentum transfer, $q_\mu=p_\mu-p'_\mu$. The EMC measurement implies, for the sum of the zero-momentum-transfer form factors weighted with the square of their electrical charges,

$$\tfrac{4}{9}H_1(0)+\tfrac{1}{9}H_2(0)+\tfrac{1}{9}H_3(0)=0.228\pm0.057 \ . \tag{2.3}$$

Here we have, considering the relatively large error, neglected some small QCD corrections.[2] Furthermore, the statistical and systematic uncertainties were combined in quadrature. Note that the EMC experiment does not provide any information on the induced "form factors" $\tilde{H}_a(0)$. It is natural to ask about the individual $H_a(0)$. To help disentangle these we may use the constraint from β-decay and isospin invariance:

$$H_1(0)-H_2(0)=g_A=1.25 \ , \tag{2.4}$$

wherein the uncertainty is negligible compared to that of (2.3). A third independent linear combination corresponds to the 8th component of an SU(3) octet:

$$H_1(0)+H_2(0)-2H_3(0)\equiv R \ . \tag{2.5}$$

Assuming SU(3) invariance, R may be obtained from the flavor-changing axial-vector currents which enter in the Cabibbo theory[6] of semileptonic hyperon decays. This yields $R=3F-D\approx0.68\pm0.08$, wherein F and D are conventional parameters.[11] If one accepts this estimate for (2.5), one finds[2] $H_1(0)=0.74\pm0.08$,

$H_2(0)=-0.51\pm0.08$, and $H_3(0)=-0.23\pm0.08$. This would imply a relatively large strange matrix element $\langle p|\bar{s}\gamma_\mu\gamma_5 s|p\rangle$ as well as an approximately vanishing singlet matrix element.

In the above analysis the use of the SU(3)-symmetric value of R in (2.5) has played a crucial role. How good is this assumption? In the literature it has been argued[3] that since SU(3) symmetry holds fairly well for relating the hyperon decay matrix elements to each other, it should also work well for calculating the flavor-conserving octet component in (2.5). One should recognize, though, that the flavor-conserving current involving strange quarks might have a response to SU(3)-symmetry breaking which is different from that of the strangeness-changing currents. In fact, this is precisely what happens in the SU(3) Skyrme model. This point was recently discussed[5] in detail in the framework of the SU(3) Skyrme model of pseudoscalars in which the collective Hamiltonian, including symmetry-breaking terms, was diagonalized exactly. It was found that while the flavor-changing Cabibbo matrix elements suffered corrections around 30%, the flavor-conserving current matrix element in (2.5) was drastically reduced to about one-quarter of its SU(3)-symmetric value. Regardless of how much one is willing to trust the precise prediction of the SU(3) Skyrme model, this effect is clearly a qualitative one (which could be understood, though not easily predicted, in the quark language as discussed in Ref. 5). Hence it seems desirable to analyze the phenomenology for an arbitrary value of R and see which conclusions still be can drawn.

We have solved (2.3), (2.4), and (2.5) simultaneously for the axial-vector singlet matrix element:

$$H(0)\equiv H_1(0)+H_2(0)+H_3(0)\ , \tag{2.6}$$

and the strange-quark matrix element $H_3(0)$ considering values of R in the reasonable range of 0–1. The results are displayed in Figs. 1 and 2, respectively. The solid line represents the average value and the dotted lines the two extremes allowing for the uncertainty in (2.3). It can be seen that the uncertainty in $H(0)$ at each R is 3 times that of $H_3(0)$. An immediate conclusion from Fig. 1 is that, within the relatively large error, the singlet current matrix element $H(0)$ is consistent with zero. It is encouraging that this main conclusion of the earlier analyses is independent of SU(3)-symmetry breaking. On the other hand, the conclusion that there is a relatively large $H_3(0)$ depends upon the choice of R. The possibility of $H_3(0)$ vanishing, which is the old "naive" expectation, is not ruled out, but can be achieved for $R<0.3$.

To summarize, it seems reasonable at the present stage to analyze[7] the data considering R as an arbitrary parameter. Various models make various predictions for R. The SU(3) Skyrme model treated according to the "bound-state" approach[12] will give R close to zero. The original approach[13] to the SU(3) Skyrme model, which included only first-order symmetry-breaking corrections to the energies, will give an SU(3)-symmetric value[14] for R. However, this approach was shown, on comparison with the exact diagonalization of the collective Hamiltonian, to be incorrect; the correct predictions for R in this model may be read off as a function of the symmetry-breaking parameter from Fig. 1 of Ref. 4 as $R=3g_A Z$, where Z is defined there. For the preferred range of the symmetry-breaking parameter, R is around 0.1–0.3.

Finally, we remark that additional information about the H_a may be obtained from experiments on elastic νp scattering. At present, the results[15] are not precise enough (and are also claimed[16] to be subject to some theoretical uncertainties of analysis) to force any firm conclusions. Specifically, one finds[17]

$$H_3(0)=-0.15\pm0.09\ .$$

It may be seen from Fig. 2 that no value of R shown can be excluded by this. Future progress in this experimental approach would clearly be important.

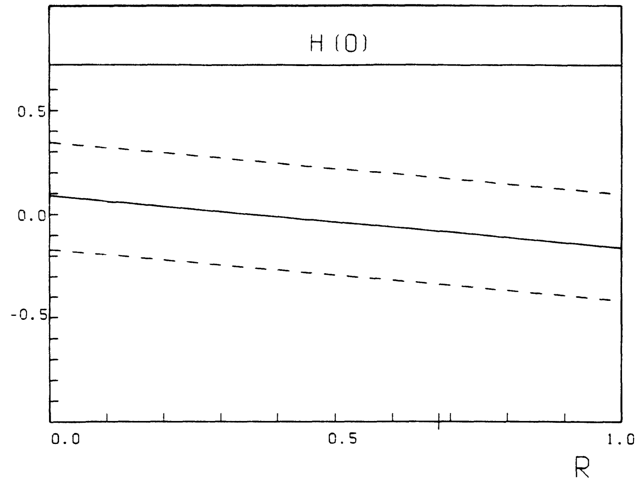

FIG. 1. Axial-vector singlet form factor $H(0)$ plotted against the "eighth component" of the axial-vector octet form factor R. The solid line represents the average value, while the dotted lines indicate the experimental uncertainty. For comparison SU(3) symmetry with the D/F ratio obtained in Ref. 11 is indicated by the mark at $R=0.68$.

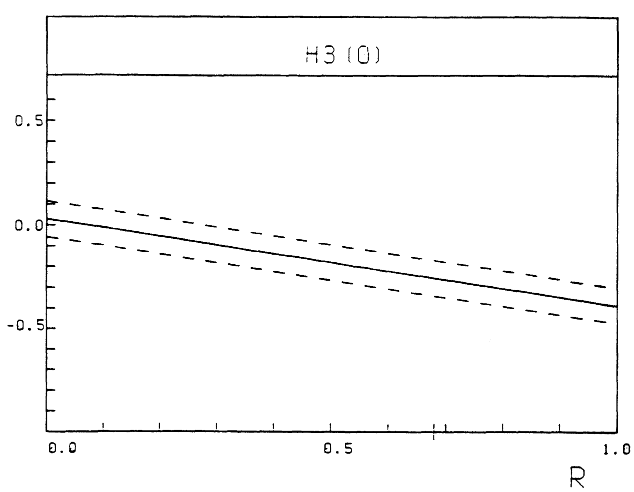

FIG. 2. Same as Fig. 1 for the strange axial-vector form factor $H_3(0)$.

III. AXIAL-VECTOR U(1) CURRENT

We have just seen that the most solid conclusion which may be presently drawn from experiment is that the axial-vector U(1) current $J_\mu^5 \equiv P_{1\mu}^1 + P_{2\mu}^2 + P_{3\mu}^3$ in the notation of (2.1) has a small matrix element $H(0)$ between proton states. It is well known that the conservation of J_μ^5 is broken not only by the quark mass terms, but also by a gluonic anomaly term which does not appear at the classical level. This may be expressed by the equation

$$\partial_\mu J_\mu^5 = 2i \sum_a m_a \bar{q}_a \gamma_5 q_a + G \ , \tag{3.1}$$

where the m_a are the quark masses, and the anomaly G is related to the QCD field strength tensor $F_{\mu\nu}$ and coupling constant g by

$$G = -\frac{ig^2 N_F}{16\pi^2} \epsilon_{\mu\nu\alpha\beta} \mathrm{Tr}(F_{\mu\nu} F_{\alpha\beta}) \ .$$

$N_F = 3$ is the number of flavors relevant for low-energy physics. Introducing form factors for the operators on the right-hand side (RHS) of (3.1),

$$\begin{aligned} &\frac{\sqrt{p_0 p_0'}}{M} \langle P(\mathbf{p}') | 2m_a \bar{q}_a \gamma_5 q_a | P(\mathbf{p}) \rangle = D_a(q^2) \bar{u}(\mathbf{p}') \gamma_5 u(\mathbf{p}) , \\ &\frac{\sqrt{p_0 p_0'}}{M} \langle P(\mathbf{p}') | G | P(\mathbf{p}) \rangle = iE(q^2) \bar{u}(\mathbf{p}') \gamma_5 u(\mathbf{p}) \ , \end{aligned} \tag{3.2}$$

and using (3.1) with (2.2) and (2.6) yields the relation

$$H(0) = \frac{1}{2M} \left[\sum_{a=1}^{3} D_a(0) + E(0) \right] . \tag{3.3}$$

Here we have assumed that $m(\eta') \neq 0$ so that the induced form factor has no pole at $q^2 = 0$. While $m(\eta') \to 0$ in the large-N_c limit, it is far from true experimentally. Equation (3.3) has been discussed from different points of view in the recent literature.[18] The interesting question would seem to be how to calculate $H(0)$ as a test of our knowledge of the structure of the proton.

In this paper we shall discuss $H(0)$ in several models in which the proton is treated as a soliton. Roughly speaking, this treatment emphasizes the importance of spontaneously broken chiral symmetry by attributing a good deal of the proton's structure to the "pion cloud" which surrounds it.

There are two simple features of such models which act to suppress their contributions to $H(0)$. First, we see[4] from (2.2) and (2.6) that any term in J_μ^5 which is a pure four-gradient [i.e., $\partial_\mu F(x)$] will be proportional to q_μ in momentum space and will thus contribute to the *induced* form factor $\tilde{H}(q^2) \equiv \tilde{H}_1 + \tilde{H}_2 + \tilde{H}_3$ rather than to $H(q^2)$. Second, the operators which can contribute on the RHS of (3.3) must represent isoscalars with η and η' quantum numbers. These matrix elements are typically suppressed in the Skyrme model. The net result is that one is forced to go beyond the basic Skyrme model, e.g., by including (higher-mass) vector mesons or explicit quarks to find nonzero $H(0)$. Thus $H(0)$ may be considered as a probe of "short-range" effects. This is analogous to the situation concerning the neutron-proton-mass difference, which also requires[10] an extension of the Skyrme model.

We will require for our chiral-Lagrangian models that the axial anomaly equation (3.1) be reproduced. This can be accomplished by using an effective gluonic composite field G together with the chiral nonet field U. The basic Lagrangian[19,20] needed to mock up (3.1) is simply

$$\begin{aligned} \mathcal{L} = &-\frac{F_\pi^2}{8} \mathrm{Tr}(\partial_\mu U \, \partial_\mu U^\dagger) + \frac{1}{6F_\pi^2 m_{\eta'}^2} G^2 \\ &+ \frac{i}{12} G(\ln \det U - \ln \det U^\dagger - 2i\theta) + \mathcal{L}_{\mathrm{SB}} \ . \end{aligned} \tag{3.4}$$

Here F_π is a bare pion decay constant (≈ 132 MeV) and $m_{\eta'}$ is a bare η' mass. θ is the QCD vacuum angle. The unmixed η' field may be extracted from U by writing

$$U = e^{i\chi} \tilde{U}, \quad \det \tilde{U} = 1, \quad \eta' = \frac{\sqrt{3} F_\pi}{2} \chi \ . \tag{3.5}$$

Neglecting the chiral-SU(3)-symmetry-breaking piece $\mathcal{L}_{\mathrm{SB}}$, (3.4) yields an axial-vector U(1) current

$$J_\mu^5 = -2 \frac{\partial \mathcal{L}}{\partial(\partial_\mu \chi)} = \frac{3F_\pi^2}{2} \partial_\mu \chi \ . \tag{3.6}$$

Using the equation of motion, we see $\partial_\mu J_\mu^5 = G$ as desired. Clearly the "ln" term in (3.4) is the one which reproduces the anomaly and, apart from $\mathcal{L}_{\mathrm{SB}}$, is the only one which is not $\mathrm{U}(1)_A$ invariant. We would like to stress, for what comes later, that any additional terms involving possibly new fields may be added to (3.4) without spoiling the anomaly equation so long as those terms are chiral U(3)×U(3) invariant. Furthermore, note from (3.6) that as the model now stands, J_μ^5 is a pure gradient, and so it will not contribute to $H(q^2)$.

By design, there is no kinetic term for the composite field G. This ensures that it may be eliminated by its equation of motion as $G = \sqrt{3} F_\pi m_{\eta'}^2 [\eta' - (\sqrt{3}/2) F_\pi \theta]$ to yield

$$\begin{aligned} \mathcal{L} = &-\frac{1}{2}(\partial_\mu \eta')^2 - \frac{F_\pi^2}{8} \mathrm{Tr}(\partial_\mu \tilde{U} \partial_\mu \tilde{U}^\dagger) \\ &- \frac{1}{2} m_{\eta'}^2 \left[\eta' - \frac{\sqrt{3}}{2} F_\pi \theta \right]^2 + \mathcal{L}_{\mathrm{SB}} \ . \end{aligned} \tag{3.7}$$

The G elimination mechanism which supplies the η' with a mass may be thought of as the η' field effectively dominating the G operator. It is amusing to contrast this with the analogous situation in the 0^+ channel where the "trace anomaly" is relevant. There the analogous mechanism *lowers*[21] the mass of the singlet.

This model has a number of useful features in addition to describing η' mass generation. For example it satisfies the large-N_c counting rules,[22] it gives (with the minimal $\mathcal{L}_{\mathrm{SB}}$) a reasonable picture of $\eta\eta'$ mixing, it simply[19,20,23] explains the θ dependences of physical amplitudes, and (if U is replaced by a linear σ-model field M) it can display a trigger mechanism[24] for spontaneous breakdown of chiral symmetry. However, an objection against its use has been raised[3] since it predicts a too small rate for the decay $\eta' \to \eta 2\pi$. This objection does not take account of the complication that calculation of this process should include poles near the physical region expected from general dispersion theory arguments. These poles are due to the exchange of δ (scalar and isovector) and σ (scalar and isoscalar) particles. It has been shown[25] that for a Lagrangian such as (3.7), but where U has been replaced by the *linear* sigma model field M so that scalars are included, the too small "current algebra" result for $\eta' \to \eta 2\pi$ can be understood as a delicate cancellation involving the σ- and δ-exchange diagrams in the limit where these masses are set equal to each other and sent to infinity. For physical values of the σ and δ masses, this great cancellation does not hold. It would be interesting to calculate soliton properties in a more complicated model including the σ and δ particles but that is beyond the scope of the present work.

The symmetry-breaking piece $\mathcal{L}_{SB}$ is explicitly given by (assuming isospin invariance)

$$\mathcal{L}_{SB} = \mathrm{Tr}[(\beta' T + \beta'' S)(\partial_\mu U\, \partial_\mu U^\dagger U + U^\dagger \partial_\mu U\, \partial_\mu U^\dagger) + (\delta' T + \delta'' S)(U + U^\dagger - 2)] \,, \tag{3.8}$$

where $T = \mathrm{diag}(1,1,0)$ and $S = \mathrm{diag}(0,0,1)$. The nonderivative part is the usual one, while the derivative part splits the decay constants so $F_k \neq F_\pi$. The constants are found[26] to be $\beta' \approx -26.4$ MeV^2, $\beta'' \approx -985$ MeV^2, $\delta' \approx 4.15 \times 10^{-5}$ GeV^4, and $\delta'' \approx 1.55 \times 10^{-3}$ GeV^4. Clearly, the nonderivative part does not contribute to J_μ^5. The derivative piece does, but is easily seen[4] to give a contribution which is a pure gradient. Hence there is no contribution to $H(q^2)$ from (3.8). The way in which this turns out to be consistent with the RHS of the anomaly equation (3.3) is that the piece $\sum_a D_a(0)$ coming from the symmetry-breaking terms cancels the piece $E(0)$ corresponding to the excitation of the G field or equivalently the η' field. Verification of this point requires the exact solution of the equations of motion. Note also the existence of an excitation of η' due to the presence of SU(3) breaking. This does not exist at the dominant (for the proton) two-flavor level. Actually, experience with the related *n-p* mass-difference problem suggests that this type of η' excitation is not quantitatively important. Two additions to the Lagrangian (3.7) are required to understand the baryon as a soliton. Can these contribute to $H(0)$? First, the Wess-Zumino term[27] must be added to the action. This term is easily seen[28] to give no contribution to the U(1) axial-vector current. Second, the Skyrme term

$$\mathcal{L}_{Sk} = \frac{1}{32e^2} \mathrm{Tr}([\alpha_\mu, \alpha_\nu]^2) \,, \qquad \alpha_\mu \equiv \partial_\mu U\, U^\dagger \tag{3.9}$$

must be included to stabilize the soliton. However, because of the commutator in (3.9), we see from the substitution $U = e^{i\chi}\tilde{U}$, (3.5) together with (3.6), that the Skyrme term also does not contribute to the U(1) axial-vector current.

To partially sum up, in the Skyrme model of pseudoscalars, even including a derivative-type symmetry breaker, one has $H(0) = 0$. Certainly, many additional chiral-invariant terms can be added. These are somewhat *ad hoc,* but so is the Skyrme term. Consider, for example, the addition

$$\mathcal{L}_{\text{extra}} = f\, \mathrm{Tr}(\alpha_\mu \alpha_\mu \alpha_\nu \alpha_\nu) \,, \tag{3.10}$$

where f is an *arbitrary* constant. Because (3.10) is chiral U(3)×U(3) invariant, it does not contribute to the U(1) anomaly. Setting again $U = e^{i\chi}\tilde{U}$, we see that (3.10) contains, in addition to a term independent of χ, linear, quadratic, and quartic terms in $\partial_\mu \chi$. Strictly speaking, it is not necessary to always keep the product $e^{i\chi}$ and $\tilde{U}$ together (this, however, violates the quark-line rule if U is believed descended from a linear-σ-model field M), and so any term in the expansion of (3.10) is chiral invariant by itself. The term linear in $\partial_\mu \chi$ gives a contribution to the current

$$\Delta J_\mu^5 = -8if\, \mathrm{Tr}(\tilde{\alpha}_\nu \tilde{\alpha}_\nu \tilde{\alpha}_\mu) \,, \qquad \tilde{\alpha}_\mu = \partial_\mu \tilde{U}\, \tilde{U}^\dagger \,, \tag{3.11}$$

which is to be *added* to (3.6). Recently, it has been claimed[9] that a term such as (3.11) should represent the *entire* U(1) axial-vector current with the coefficient determined from an anomalylike argument. Clearly, in the class of models under consideration, ΔJ_μ^5 is just part of the axial-vector U(1) current. Furthermore, it is claimed[9] that (3.11) gives a nonzero contribution to $H(0)$ in the Skyrme model. It is true that (3.11) is not a gradient, and so it looks encouraging. But a closer analysis reveals some difficulties. In the Skyrme model the nucleon is treated[23] as a collective excitation of the coordinates $A(t)$ introduced[29] by setting $U = A(t) U_0(\mathbf{r}) A^\dagger(t)$, where $U_0(\mathbf{r})$ is the Skyrme soliton solution. Specifically, the collective Lagrangian is expressed in terms of the eight angular velocity operators $\Omega_a = -i\, \mathrm{Tr}(\lambda_a A^\dagger \dot{A})$, where the λ_a are the Gell-Mann SU(3) matrices. Terms higher than quadratic in the Ω_a's are neglected (these are higher order in $1/N_c$). One can compute the vector charges from this Lagrangian by varying with respect to Ω_a. These will therefore come out to be at most linear in Ω_a. Keeping terms in the vector currents which are quadratic in the Ω_a is inconsistent. It is also therefore highly suspicious for terms in the axial-vector current. However, this is precisely the situation for (3.11); introducing $A(t)$ into this expression leads to a U(1) axial-vector current which is quadratic in $\mathbf{\Omega}$. An interesting attempt to overcome this problem was made in Ref. 9. However, it seems to us there is a further subtlety. To see this let us consider the first term in Eq. (15) of Ref. 9 which corresponds to the two-flavor reduction of ΔJ_μ^5. Introducing $P_\mu = U^{-1/2} \alpha_\mu U^{1/2}$ then gives a piece proportional to

$$\mathrm{Tr}(P_\nu P_\nu P_\mu) = P_\nu^a P_\nu^b P_\mu^c\, \mathrm{Tr}(\tau^a \tau^b \tau^c) = 2i\epsilon^{abc} P_\nu^a P_\nu^b P_\mu^c \,. \tag{3.12}$$

Evidently, this vanishes at the classical level in which the fields P_ν^a commute with each other. When we quantize (3.12) the order of the operators may be significant, and so (3.12) does not obviously vanish. In particular, the commutator $[\Omega_a, \Omega_b]$ might appear and this would give a piece linear in $\mathbf{\Omega}$ yielding the first term in Eq. (15) of Ref.

9. It seems to us, however, that (3.21) should be quantized in such a way that it vanishes. This is because the P_ν^a are negative under G parity, and so (3.12) is also negative under G parity. On the other hand, J_μ^5 has positive G parity. The quantization should preserve the G parity, and so (3.12) should also vanish in the quantum theory. The natural conclusion is that in order to obtain the quantum expression for ΔJ_μ^5 from the classical one, we should symmetrize the terms quadratic in Ω. At the classical level we find (in agreement with Ref. 9)

$$\Delta J_i^5 = \frac{-64\pi^2 f}{3}\langle r^2\rangle_0[i\epsilon_{ilm}\Omega_l\Omega_m+(if_{i\alpha\beta}+d_{i\alpha\beta})\Omega_\alpha\Omega_\beta] ,$$

where $\langle r^2\rangle_0$ is the isoscalar squared radius of the nucleon and the Latin indices run from 1 to 3, while the Greek indices run from 4 to 7. The quantization rule is to replace $\Omega_i \to -R_i/\alpha^2$, $\Omega_\alpha \to -R_\alpha/\beta^2$, α^2 and β^2 denoting moments of inertia and the R's "right" SU(3) generators. Now it is easy to see that Eq. (15) of Ref. 9 follows if we *antisymmetrize* the above terms in Ω. However, that was just noted above to be an inconsistent prescription. The correct formula would result from symmetrization:

$$\Delta J_i^5 = \frac{-32\pi^2 f}{3(\beta^2)^2}\langle r^2\rangle_0 d_{i\alpha\beta}[R_\alpha,R_\beta]_+ .$$

However, as we remarked, the usual collective quantization of the Skyrme Lagrangian does not permit us to reliably calculate the matrix elements of current operators higher than linear order in the generators R_a. These terms should therefore be neglected, and we conclude that the current (3.11) does not contribute to $H(0)$ in the Skyrme model of pseudoscalars. The above argument can be roughly summarized in particle-physics language (noting that $P_\mu \propto \partial_\mu\varphi + \cdots$, where φ is the 3×3 matrix of pseudoscalars) by saying that a $G=+$ operator cannot be constructed as a product of three pions. The choice $\eta 2\pi$ does not violate G parity, but the η field does not get excited in this model in the chiral (zero-quark-mass) limit. That limit is a reasonable one for understanding the effects of a chiral-symmetric addition to the Lagrangian and was the one considered in Ref. 9. At the dominant two-flavor level (roughly, the two-flavor level should be adequate for computing properties of the nucleon), the η field will not be excited either. The choice $K\bar{K}\pi$ is another allowed possibility, but it results in a contribution quadratic in $\mathbf{\Omega}$, which is just the remaining term discussed above. Note that an alternative argument, besides G parity, against the antisymmetrization procedure is that it contains an undecidable sign ambiguity depending on the factor order one starts with at the classical level.

Another possibility for extending the Skyrme model which has been discussed[30] in the literature is to allow, for example, an eight-derivative chiral-invariant term which imitates the structure of the well-known vector-vector-pseudoscalar vertex: $\epsilon_{\mu\nu\sigma\tau}\partial_\mu\eta' Z_\nu \partial_\sigma Z_\tau$ with $Z_\tau = \epsilon_{\tau\mu\nu\rho}\mathrm{Tr}(\alpha_\mu\alpha_\nu\alpha_\rho)$. This type of term will suffer additional suppression from the quark line rule because it is not a single trace in flavor space. It seems better to us to directly use the vector mesons, as we shall do in Sec. IV, since the strengths of their couplings can be deduced from our experimental knowledge of meson decays.

Finally, let us briefly discuss the axial divergence equation (3.1) and its proton matrix elements (3.3) again. For the kind of treatment of the η' meson [U(1) problem] being considered here, it is usually[31] a good approximation to go to the chiral limit where the quark masses vanish. In this limit (3.3) amounts to a kind of Goldberger-Treiman relation. Noting the proportionality between G and η' discussed in connection with (3.7), we may write (3.3) in the chiral limit as

$$H(0) = \frac{\sqrt{3}F_\pi}{2M}\tilde{g}_{pp\eta'} , \tag{3.13}$$

where $\tilde{g}_{pp\eta'}$ is the off-shell η' nucleon Yukawa coupling constant. Equation (3.13) holds for the chiral limit of all the models considered in this paper. An equivalent statement to the vanishing of $H(0)$ in the chiral limit is evidently the vanishing of the Yukawa coupling constant $\tilde{g}_{pp\eta'}$. This viewpoint illustrates the relevance of exciting the η' (or η in a two-flavor treatment) meson in order to achieve nonvanishing $H(0)$. One might think that it is more convenient to calculate the RHS of (3.13) than the LHS directly. However, that turns out not necessarily true in practice. In fact, focusing on the RHS of (3.13) might occasionally lead to confusion. For example, if one considers a truncated model in which the η' field has not been included but in which, if it were to be included, it could get excited, then the LHS would properly give a nonzero value, while the RHS would give zero.

IV. VECTOR MESONS

We will now consider the computation of $H(0)$ in a model where vector mesons are included. Apart from the SU(3)-symmetry-breaking terms, the other terms involving vector mesons will be chiral U(3)×U(3) invariant, and so the mechanism discussed in Sec. III for satisfying the axial U(1) anomaly equation will still hold. Here, as previously shown,[32] the η-like field can get excited, and noting Eq. (3.3), we expect [since $E(q^2)$ is proportional to the η' matrix element] a nonzero prediction for $H(0)$. This is comparable to the situation[10] regarding the neutron-proton-mass difference. Some readers may feel that it is better to keep only pseudoscalars in the low-energy Lagrangian and to allow many higher-order derivative terms, if necessary. It therefore seems relevant to point out that proponents of this approach have recently concluded[33] that the vector mesons generate the main structure of such Lagrangians.

Of course, chiral Lagrangians involving vector mesons have been intensively studied for at least 20 years.[34] We shall adopt a particular version[35] in which the vector-meson nonet field ρ_μ is related to auxiliary "gauge fields" A_μ^L and A_μ^R by

$$\begin{aligned} A_\mu^L &= \xi\rho_\mu\xi^\dagger + \frac{i}{g}\xi\partial_\mu\xi^\dagger , \\ A_\mu^R &= \xi^\dagger\rho_\mu\xi + \frac{i}{g}\xi^\dagger\partial_\mu\xi , \end{aligned} \tag{4.1}$$

where $\xi = U^{1/2}$ and g is a coupling constant. The advantage of this formulation is that A_μ^L and A_μ^R transform linearly under global $U(3)_L$ and global $U(3)_R$ so that chiral invariants may be easily constructed. The total action then contains a number of pieces. First, there is a "gauge-invariant" kinetic term for the vectors

$$\mathcal{L}_1=-\frac{1}{4}\mathrm{Tr}[F_{\mu\nu}(\rho)F_{\mu\nu}(\rho)] ,$$
$$F_{\mu\nu}(\rho)=\partial_\mu\rho_\nu-\partial_\nu\rho_\mu-ig[\rho_\mu,\rho_\nu] . \tag{4.2}$$

Next, there are two "mass-type" terms for the vectors which give the vector-pseudoscalar interactions:

$$\mathcal{L}_2=-m_0^2\,\mathrm{Tr}(A_\mu^L A_\mu^L+A_\mu^R A_\mu^R)+B\,\mathrm{Tr}(A_\mu^L U A_\mu^R U^\dagger) , \tag{4.3}$$

where m_0^2 and B are constants. The terms in the action proportional to $\epsilon_{\mu\nu\alpha\beta}$ are conveniently written treating A_L and α as one-forms:

$$\Gamma_3=\Gamma_{\mathrm{WZ}}(U)+\int \mathrm{Tr}\left[ic_1(A_L\alpha^3)+c_3(dA_L\alpha A_L-A_L\alpha dA_L+A_L\alpha A_L\alpha)+c_3\left[-2iA_L^3\alpha+\frac{1}{g}A_L\alpha A_L\alpha\right]\right] , \tag{4.4}$$

where c_1, c_2, and c_3 are constants, two of which may be determined from vector-meson decays and the third may be treated as a parameter which, however, has only a relatively minor effect on the soliton properties. The c_1, c_2, and c_3 terms stabilize[34] the classical soliton against collapse without the need for the *ad hoc* Skyrme term. All three terms above are U(3)×U(3) invariant. To them we should add the second and third terms of (3.4) in order to satisfy the anomaly equation and an SU(3)-symmetry-breaking part $\mathcal{L}_{\mathrm{SB}}$ given in Eq. (2.4) of Ref. 10 and discussed there in detail.

The new features of this pseudoscalar-vector chiral Lagrangian compared to the older ones include the terms proportional to $\epsilon_{\mu\nu\alpha\beta}$ and the explicit demonstration of how the axial-vector mesons get eliminated by a nonlinear constraint (analogous to the elimination of the scalar field σ in going from the linear to the nonlinear σ model). The parameters of the model were determined in Refs. 35 and 36, baryon properties were discussed in Refs. 36 and 32, and meson-baryon scattering in Ref. 37. The baryon properties and scattering amplitudes are both improved compared to the Skyrme model of pseudoscalars only.

The above Lagrangian is a minimal one in the sense that [except for some SU(3)-breaking terms in $\mathcal{L}_{\mathrm{SB}}$] all the relevant terms with the minimal number of derivatives have been included. It is well known that the same chiral Lagrangian may be presented in many different ways. In particular the "hidden symmetry" Lagrangian of Ref. 38 is *identical*[36] to (4.2)+(4.3)+(4.4) when redundant fields are eliminated. In order to verify this statement one should note that *CP* invariance must be imposed on the $\epsilon_{\mu\nu\alpha\beta}$ terms in Ref. 38. Also the SU(3)-breaking pieces $\mathcal{L}_{\mathrm{SB}}$ differ somewhat; in the hidden-symmetry model, the $(3,3^*)+(3^*,3)$ transformation property is not respected.

The U(1) axial-vector current may be derived from the Lagrangian, for example, by "gauging" it with an external U(1) axial-vector field and picking up the term linear in that field. In this way we find in *addition* to (3.6) the following terms in J_μ^5:

$$\tilde{J}_\mu^5=\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}\left[\frac{i}{2\sqrt{2}g}\left[\frac{\gamma_1}{3}+\frac{\gamma_2}{2}\right]p_\nu p_\alpha(2g\rho-iv)_\beta\right.$$
$$-\frac{\gamma_2}{\sqrt{2}}F_{\nu\alpha}(\rho)(2g\rho-iv)_\beta$$
$$-\frac{i}{2\sqrt{2}g}\left[\gamma_3+\frac{\gamma_2}{2}\right](2g\rho-iv)_\nu$$
$$\left.\times(2g\rho-iv)_\alpha(2g\rho-iv)_\beta\right] , \tag{4.5}$$

where $p_\mu=\xi^+\partial_\mu\xi+\partial_\mu\xi\xi^\dagger$, $v_\mu=\xi^\dagger\partial_\mu\xi-\partial_\mu\xi\xi^\dagger$, and

$$\gamma_1=3\sqrt{2}i\left[2c_1-\frac{2c_2}{g}-\frac{c_3}{g^2}\right] ,$$
$$\gamma_2=\frac{2\sqrt{2}i}{g}c_2 , \tag{4.6}$$
$$\gamma_3=\frac{\sqrt{2}i}{g^2}c_3 .$$

It is obvious (e.g., ρ^3 terms are present) that this new contribution is not a pure gradient. Even though we have a guarantee from the arguments of Sec. III that the axial anomaly equation holds, it is amusing to check this explicitly. Using the η' equation of motion $(-\Box+m_{\eta'}^2)\eta'=\partial_\mu\tilde{J}_\mu^5/\sqrt{3}F_\pi$ after taking the divergence of the axial-vector current $J_\mu^5=\tilde{J}_\mu^5+\sqrt{3}F_\pi\partial_\mu\eta'$, we simply get $\partial_\mu J_\mu^5=\sqrt{3}F_\pi m_{\eta'}^2\eta'$ as expected. Also notice that the condition for η' to get excited is that $\partial_\mu\tilde{J}_\mu^5$ have a piece *not* containing η'.

In order to evaluate the proton matrix element of (4.6), we introduce the classical soliton and quantize its collective coordinates. This is discussed in great detail in Ref. 32. At the classical level the fields ξ, ρ_i^a, and ω_0 are nontrivial:

$$\xi(\mathbf{r})=\begin{bmatrix} & & 0\\ \exp[i\tau\cdot\hat{\mathbf{r}}F(r)/2] & & 0\\ 0 & 0 & 1\end{bmatrix},$$
$$\rho_i^a=\epsilon_{ika}G(r)/gr\ ,\tag{4.7}$$
$$\omega_0=-\omega(r)\ .$$

After introducing collective coordinates $A(t)$ by $\xi(\mathbf{r},t)=A(t)\xi(\mathbf{r})A^\dagger(t)$, etc., we take account of dynamical readjustment of the parameters of the collective Lagrangian. Then also the fields ω_i, ρ_0^a, η, η', k, and k_μ^* develop nonzero components. Since the collective Lagrangian is universally expanded only to quadratic terms in the angular velocity $\Omega_a=-i\,\mathrm{Tr}(\lambda_a A^\dagger \dot{A})$, we will only keep terms in J_μ^5 up to and including order $\mathbf{\Omega}$. The k and k_μ^* fields make no contribution to J_μ^5 in this order. The additional fields are given by

$$\tau\cdot\rho_0(\mathbf{r},t)=\frac{4}{g}A(t)\tau\cdot[\mathbf{\Omega}\zeta_1(r)+\hat{\mathbf{r}}(\mathbf{\Omega}\cdot\hat{\mathbf{r}})\zeta_2(r)]A^\dagger(t)\ ,$$
$$\omega(\mathbf{r},t)=\frac{2\varphi(r)}{r}\hat{\mathbf{r}}\times\mathbf{\Omega}\ ,\tag{4.8}$$
$$\text{“}\eta(\mathbf{r})\text{”}=2(\mathbf{\Omega}\cdot\hat{\mathbf{r}})\eta(r)\ .$$

In (4.8) the matrices $A(t)$ are restricted to SU(2) since "rotations" into the strange directions do not contribute to order $\mathbf{\Omega}$. Furthermore, "η" represents the pure nonstrange isosinglet field.

Substituting (4.7) and (4.8) into (4.5) gives, for the singlet axial-vector current expressed in collective variables,

$$\tilde{J}_i^5=R_1(r)(\Omega_i-\hat{\mathbf{r}}_i\hat{\mathbf{r}}\cdot\mathbf{\Omega})+R_2(r)\hat{r}_i\hat{\mathbf{r}}\cdot\mathbf{\Omega}\ ,\tag{4.9}$$

where the radial functions are given by

$$R_1(r)=\frac{1}{\sqrt{2}gr}\left[\left[\frac{2}{3}\gamma_1+\gamma_2\right](\zeta_1+\zeta_2)(G+\zeta_1)F'\sin F-\gamma_2[2g^2(\omega'\varphi-\omega\varphi')+\zeta_1G'-G\zeta_1'-(G'+\zeta_1')(1-\cos F)]\right]\ ,\tag{4.10a}$$

$$R_2=\frac{\sqrt{2}}{gr^2}\left[\left[\frac{1}{3}\gamma_1+\frac{1}{2}\gamma_2\right](\zeta_1+\zeta_2)\sin^2F+3\gamma_3(\zeta_1+\zeta_2)(1+G-\cos F)^2\right.$$
$$\left.+\gamma_2\left[2g^2\omega\varphi+G(G-\zeta_1-2\zeta_2)+(G-\zeta_2)(1-\cos F)+2G(\zeta_1+\zeta_2)(1-\cos F)+\frac{3}{2}(\zeta_1+\zeta_2)(1-\cos F)^2\right]\right]\ .\tag{4.10b}$$

Here a prime denotes differentiation with respect to r. Finally, quantizing the theory using the angular momentum operator $\mathbf{J}=\mathbf{\Omega}\alpha^2$ (α^2 being the moment of inertia for spatial rotations, denoted as θ in Ref. 32), we obtain for $H(0)$ the formula

$$H(0)=\frac{\sqrt{2}\pi}{3g\alpha^2}\int_0^\infty dr\,r^2(2R_1+R_2)\ .\tag{4.11}$$

The final numerical result turns out to be

$$H(0)=0.30\ .\tag{4.12}$$

This result changes only by about 10% on the variation of the parameters γ_1, γ_2, and γ_3 [see (4.6) and (4.4)] within their allowed ranges. Referring to Fig. 1 shows that the parameter R introduced in (2.5) must be less than about 0.25 in order that our determination of $H(0)$ be consistent with experiment.

To discuss the parameter dependence more explicitly, we first note that g, m_0^2, and B introduced in (4.1) and (4.3) are quite well fixed from the meson sector. γ_1 has the interpretation of a $V\phi^3$ coupling constant (it is convenient to use instead $\tilde{h}=-2\sqrt{2}\gamma_1/3$), while γ_2 has the interpretation of a $VV\phi$ coupling constant (it is convenient to use $\tilde{g}_{VV\phi}=\sqrt{2}g\gamma_2$). These are reasonably but not precisely determined[36] from the meson sector, yielding central values $\tilde{g}_{VV\phi}=1.9$ and $\tilde{h}=0.4$. On the other hand, γ_3 (we instead use $\kappa=\gamma_3/\gamma_2$) is not at all determined from the meson sector. From calculations in the baryon sector of the SU(2) model we require[32,36] κ to be very roughly around 1 in order to reproduce the electromagnetic and axial low-energy form factors. The result (4.12) was obtained with $\tilde{g}_{VV\phi}=1.9$, $\tilde{h}=0.4$, and $\kappa=1$. In Table I we show the results for some other parameter choices. We also show the predictions for g_A, the axial-vector coupling constant in neutron β decay. Actually, the value shown for g_A corresponds to an SU(2) rather than an SU(3) treatment of the vector-meson chiral Lagrangian. This is because a full SU(3) treatment, including the effects of "cranking" the k and k^* fields, has not yet been carried out. Based on the experience[26] with the pseudoscalar Lagrangian, we would expect the SU(3) values for g_A to be reduced by 10–15 % from the SU(2)

TABLE I. Singlet axial form factor $H(0)$ as well as the neutron β decay constant g_A (computed at the two-flavor level) for some alternative allowed parameter sets in the vector-meson chiral Lagrangian.

$(\tilde{h},\tilde{g}_{VV\phi},\kappa)$	g_A(SU(2))	$H(0)$
(0.4,1.9,0)	0.76	0.34
(0.4,1.9,0.5)	0.85	0.33
(0.4,1.9,1.0)	0.91	0.30
(0.7,2.2,0)	0.53	0.29
(0.5,2.0,0)	0.67	0.34
(0.2,1.7,0)	0.99	0.32
(0,1.5,0)	1.24	0.28

values. In contrast to $H(0)$, g_A has a relatively severe dependence on the parameters of the Lagrangian. We furthermore note that a full SU(3) treatment of the vector chiral Lagrangian would not change the prediction for $H(0)$. Such a treatment is, however, required to compute $H_3(0)$.

V. QUARKS

Another popular approach to the short-distance description of the nucleon is the inclusion of explicit quarks. The traditional way to include the quarks is to couple the chiral pseudoscalar fields to a "bag". A number of calculations[39] of $H(0)$ have been performed in models of this sort, with results around the same as our "vector-meson" result (4.12). In this approach the implementation of the axial anomaly equation (3.1) is rather delicate because of the need to match the axial-vector current across the bag boundary. One way to circumvent this problem is to include the quarks via a generalized type of Gell-Mann–Levy σ model (=chiral quark model).[40] Two calculations[41,42] of $H(0)$ using variants of this method have appeared. Again, the results are in general agreement with (4.12); Ref. 41 finds that $H(0)$ should lie in the range 0.2–0.4, while Ref. 42 finds the range 0.15–0.60.

Here we would like to also briefly discuss $H(0)$ in the chiral quark model since this model seems to provide the beginnings of a "bridge" to the fundamental QCD description. We will make some relatively minor new points and content ourselves with a quick *estimate* of $H(0)$. Mainly, we will emphasize the aspects of this model which are related to the concerns raised in our previous discussion.

The U(3)×U(3)-invariant part of the Lagrangian will be taken to be

$$\begin{aligned}\mathcal{L}=&-\frac{F_\pi^2}{8}\mathrm{Tr}(\partial_\mu U\,\partial_\mu U^\dagger)-\bar{q}\gamma_\mu\partial_\mu q\\&-\frac{gF_\pi}{\sqrt{2}}\bar{q}(Uq_R+U^\dagger q_L)\\&-\frac{\tilde{g}}{2}(\bar{q}_L\gamma_\mu\partial_\mu U\,U^\dagger q_L-\bar{q}_R\gamma_\mu U^\dagger\partial_\mu U\,q_R)\,,\end{aligned}\tag{5.1}$$

where q is the column vector of quark spinors. g is the conventional "Yukawa" coupling constant in this approach, while $\tilde{g}$ measures the strength of another allowed Yukawa coupling. In contrast with the Skyrme model, which gives a somewhat too small value for g_A, the chiral quark model gives a too large value. It was recently shown[43] that $\tilde{g}$ could be used to adjust g_A to its experimental value, but such a term was not included in the previous calculations of $H(0)$. In order to mock up the U(1) anomaly, we shall also include the second and third terms of (3.4). Actually, this is not a unique choice in the present case since a term

$$\frac{iG}{12}\left[\lambda_1\ln\frac{\det U}{\det U^\dagger}+\lambda_2\ln\frac{\det\bar{q}_R q_L}{\det\bar{q}_L q_R}\right],\tag{5.2}$$

with $\lambda_1+\lambda_2=1$, could also be used. For the SU(3)-symmetry-breaking terms, we may consider the sum of (3.8) and $-\sum_a m_a\bar{q}_a q_a$. A certain amount of double counting seems inevitable in this model, but this is mitigated because the quarks make their main contributions at short distances and the pseudoscalars at larger distances. For example, it turns out that the pion-mass term has a larger effect on the nucleon properties than does the analogous quark-mass coefficient (m_1+m_2).

The U(1) axial-vector current derived from (5.1) and (3.8) contains both quark and meson pieces:

$$J_\mu^5=i(1+\tilde{g})\sum_a\bar{q}_a\gamma_\mu\gamma_5 q_a+\tfrac{3}{2}F_\pi^2\partial_\mu\chi+\cdots\,,\tag{5.3}$$

where some meson symmetry-breaking terms[4] proportional to β' and β'' were not written. We see that the current (5.3) passes the first test mentioned in Sec. III for contributing to $H(0)$. That is, while the meson terms in J_μ^5 are pure gradients, the quark term clearly is not. But we still must check that a (possibly composite) field with η or η' quantum numbers gets excited so that J_μ^5 can have a nonvanishing expectation value in the proton state. This does not happen at the simplest level[29] of collective quantization. To explain this feature we will next assume that the problem can be treated at the two-flavor rather than the three-flavor level. This implies that we are considering the proton matrix element of $\bar{s}\gamma_\mu\gamma_5 s$ to be negligible. As previously[4] discussed in detail, this holds fairly well even with the "collective" approach to strangeness in the three-flavor Skyrme model. The simplest approach to collective quantization involves expanding around the classical "hedgehog" solution U_0 and the associated solution of the Dirac equation for the quark ansatz:

$$q_S=\frac{1}{\sqrt{2}}\begin{pmatrix}\dfrac{iu(r)}{r}\\ \boldsymbol{\sigma}\cdot\hat{\mathbf{x}}\dfrac{v(r)}{r}\end{pmatrix}(\alpha_1\beta_2-\alpha_2\beta_1)\,,\tag{5.4}$$

α_i and β_i being two-component spin and isospin wave functions, respectively. The ansatz (5.4) is characterized by a "grand spin" $\mathbf{K}\equiv\mathbf{J}+\mathbf{I}$ quantum number of zero. However, the relevant operator for evaluating the first term of (5.3), $A^\dagger(t)\gamma_4\gamma_i\gamma_5 A(t)=\gamma_4\gamma_i\gamma_5$ [where $A(t)$ is the collective-coordinate isospin matrix], clearly transforms as $K=J=1$. This means we will get zero for the J_i^5 matrix element since $\langle K=0|K=1|K=0\rangle=0$.

In order to obtain nonzero $H(0)$, it is necessary to employ a more sophisticated collective quantization procedure ("cranking") in which the nucleon's moment of inertia is readjusted due to excitation of new field components by rotation. Then the quark wave function can pick up a $K=1$ piece which yields nonvanishing $H(0)$. The complete "cranking" has been worked out for the model with $\tilde{g}=0$ and neglect of the β' and β'' derivative-type symmetry-breaking terms in Ref. 44.

One finds when substituting $q(\mathbf{x},t)=A(t)q_0$ into (5.1) the equation of motion for q_0:

$$(h_s+h')q_0=\epsilon_0 q_0\,,\tag{5.5a}$$

where the "static Hamiltonian" h_s is

$$h_s = -i\boldsymbol{\alpha}\cdot\nabla + \frac{g\beta F_\pi}{\sqrt{2}}(\cos F - i\gamma_5 \hat{\mathbf{x}}\cdot\boldsymbol{\tau}\sin F) + \tilde{g}\boldsymbol{\sigma}\cdot\left[-\frac{F'}{2}\hat{\mathbf{x}}(\hat{\mathbf{x}}\cdot\boldsymbol{\tau}) - \frac{\sin 2F}{4r}[\boldsymbol{\tau}-\hat{\mathbf{x}}(\hat{\mathbf{x}}\cdot\boldsymbol{\tau})] - \frac{\gamma_5}{2}\frac{\sin^2 F}{r}\hat{\mathbf{x}}\times\boldsymbol{\tau}\right], \tag{5.5b}$$

and the perturbation piece h' is given by

$$h' = \frac{1}{2}\boldsymbol{\Omega}\cdot\left[\boldsymbol{\tau} + \tilde{g}\sin^2 F[\boldsymbol{\tau}-\hat{\mathbf{x}}(\hat{\mathbf{x}}\cdot\boldsymbol{\tau})] - \frac{\tilde{g}}{2}\sin 2F\gamma_5\hat{\mathbf{x}}\times\boldsymbol{\tau}\right]. \tag{5.5c}$$

$\boldsymbol{\Omega}$ is the angular velocity operator, as before while $\boldsymbol{\alpha}$ and β are here the usual matrices introduced by Dirac, $\gamma_5 = -\begin{pmatrix}0&1\\1&0\end{pmatrix}$, and $F(r)$ is the chiral profile in (4.7). Working in the adiabatic limit of small $\boldsymbol{\Omega}$, it is consistent to consider a first-order change δq to the static solution q_s; i.e.,

$$q_0 = q_s + \delta q \ . \tag{5.6}$$

Upon substitution of (5.6) into (5.5a) and keeping terms to first order in $\boldsymbol{\Omega}$, one obtains the "cranking equation" for δq:

$$(h_s - \epsilon_s)\delta q = -h' q_s \ , \tag{5.7}$$

from which one can deduce that δq must be a $K=1$ spinor. Finally, the Lagrangian in (5.1) can be written to second order in $\boldsymbol{\Omega}$ as

$$L = -M_0 - N_q \epsilon_s + \tfrac{1}{2}(\theta+\theta_q)\Omega^2 \ , \tag{5.8}$$

displaying the quark contribution θ_q to the moment of inertia in addition to the contribution dependent on the meson fields θ.

The matrix element of the axial-vector singlet current in (5.3) is easily seen to be related to the quark-spin operator. For our discussion we define spin and orbital parts of the quark moment of inertia as follows $(\theta_q = \theta_s + \theta_L)$:

$$\begin{aligned} \theta_s \mathbf{J} &= (\theta+\theta_q)\int q^\dagger \frac{\boldsymbol{\sigma}}{2} q \, d^3 r \ , \\ \theta_L \mathbf{J} &= (\theta+\theta_q)\int q^\dagger(-i\mathbf{r}\times\nabla) q \, d^3 r \ . \end{aligned} \tag{5.9}$$

The expression for $H(0)$ in the chiral quark model is then given by

$$H(0) = (1+\tilde{g})\left[\frac{\theta_q}{\theta+\theta_q}\right]\left[\frac{\theta_s}{\theta_q}\right], \tag{5.10}$$

and has the following interpretation. The first factor is the axial-vector coupling of the "constituent" quarks. The second factor is the fraction of the proton's spin carried by the quarks. The third is a relativistic suppression factor which is the fraction of the quarks' angular momentum due to their intrinsic spin as opposed to their orbital motion. Using a simplified treatment[45] in which the change in the quark field is taken as

$$\delta q = -\frac{i}{2}\xi(\boldsymbol{\tau}\cdot\boldsymbol{\Omega})q_S \ , \tag{5.11}$$

where ξ is a parameter to be determined self-consistently, it can be shown that the relativistic correction is given by

$$\frac{\theta_s}{\theta_q} \simeq \int \left[u^2 - \frac{1}{3}v^2\right] dr \ . \tag{5.12}$$

This is analogous to the relativistic "depolarizing" factor[3] in bag models. This suppression factor appears to have been left out in the treatment of the cloudy bag model in the first article of Ref. 39.

For the case $\tilde{g}=0$ the prediction for $H(0)$ can be obtained from the numerical solution of Ref. 44 where it is found $(g \simeq 5.4)$

$$\frac{\theta_q}{\theta+\theta_q} = 0.38, \quad \frac{\theta_s}{\theta_q} \simeq 0.68 \ , \tag{5.13}$$

and so

$$H(0) = 0.26 \ . \tag{5.14}$$

In this model the value of g_A for the nucleon comes out about 15% too high. As pointed out earlier, this can be corrected by adjusting the quarks' axial-vector coupling with a negative value for $\tilde{g}$. At first, one might conclude from (5.10) that $H(0)$ would then also be decreased by a similar factor $1+\tilde{g}$. However, including a nonzero $\tilde{g}$ dramatically increases the role of the quarks in the nucleon. For example, taking $\tilde{g} = -0.2$, the preferred solution is around $g \simeq 3.8$ and predicts

$$\frac{\theta_q}{\theta+\theta_q} \simeq 0.6, \quad \frac{\theta_s}{\theta_q} \simeq 0.69 \ , \tag{5.15}$$

and from (5.10) we find

$$H(0) \simeq 0.33 \ . \tag{5.16}$$

Our estimates in (5.15) were computed using an approximation to the exact cranking based on that of Ref. 45, but with (5.11) generalized to

$$\delta q = -\frac{i}{2}\xi(\boldsymbol{\tau}\cdot\boldsymbol{\Omega})q_s - \frac{i}{2}\xi'(\boldsymbol{\tau}\cdot\mathbf{x})(\boldsymbol{\Omega}\cdot\mathbf{x})q_s \ . \tag{5.17}$$

This approximation was found to be in excellent agreement with the exact cranking results of Ref. 44 for the case $\tilde{g}=0$. The predictions for $H(0)$ in the chiral quark model are seen to be consistent with our vector-meson result (4.12).

In the chiral quark model we have been discussing, the η excitation does not play a direct role since it contributes to J^5_μ only through a pure gradient. It can, however, affect the prediction for the quark moment of inertia and thus indirectly contribute to $H(0)$. A rough approximation outlined in Ref. 10 indicates that inclusion of the η boosts the quark moment of inertia slightly and hence should slightly boost the prediction for $H(0)$. Calculations[39] for $H(0)$ in the cloudy bag model get a direct contribution from the four-divergence of the η field. This is possible in the cloudy bag model because of the contribution of the integral around the surface of the bag. The bag is, however, an artificial construction intended to mock up confinement, and so any effects of the bag surface are somewhat difficult to interpret. It should be noted also that (5.10) is incomplete if one includes scalar-meson fields as proposed in the SU(2) model of Ref. 41 where it was found that scalars could contribute as much as one-third of the total for $H(0)$.

Note added. The remark in Sec. II that SU(3) breaking for R in (2.5) might be large without adversely affecting the successful Cabibbo scheme has been strengthened in N. W. Park, J. Schechter, and H. Weigel, Syracuse Report No. SU-4228-441, 1990 (unpublished). Furthermore, a small extension of the present treatment which yields an interesting connection with the operator-product-expansion approach of the last three papers in Ref. 8 has been given in J. Schechter, V. Soni, A. Subbaraman, and H. Weigel, Syracuse Report No. SU-4228-452, 1990 (unpublished).

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