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Possible deviations due to exotic-lepton mixings in the process $e^+e^- \rightarrow W^+W^-$

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The process $e^+e^- \rightarrow W^+W^-$ is reexamined with the inclusion of mixing between ordinary leptons and exotic leptons of the types (i) mirror, (ii) vector doublet, and (iii) vector singlet. The effects of mixings are discussed in terms of the expected relative deviations from the standard-model predictions for the total cross section, the forward-backward asymmetry, the differential angular distribution for unpolarized W's, and the inclusive angular distributions for longitudinally (L) and transversely (T) polarized W's with and without charge identification at energies of the CERN collider LEP II ($\sqrt{s} \approx 190$ GeV). These deviations are evaluated for three extreme cases of mixing: (I) mixing in the neutral-lepton sector only, (II) mixing in the charged-lepton sector only, and (III) equal mixings in both sectors. The expected relative deviations are presented as a function of the scattering angle θ as well as the exotic-neutrino mass m_N and the square of the mixing angle $\sin^2\psi$. For case (I), all three types of exotic leptons have identical mixing effects. For a run of 500-pb^{-1} data, with $m_N \geq 250$ GeV and $\psi = 10°$, the expected relative deviations are more than one standard deviation (SD) error in $d\sigma(\theta)$, $d\sigma_T(\theta)$ for all values of θ, in $d\sigma_{TT}(\theta)$ for $\cos\theta \geq 0.4$, and in $d\sigma_L(\theta)$ for $|\cos\theta| \leq 0.6$. For case (II), the deviations are almost the same for mirror and vector singlet type of exotic leptons. Further, the deviations in $d\sigma$, $d\sigma_T$, $d\sigma_L$, $d\sigma_{TT}$, and $d\sigma_{LT+TL}$ are not sensitive to m_N and θ. For a run of 500-pb^{-1} data, and for $m_N \geq 50$ GeV with $\sin^2\psi \approx 0.03$, the expected deviations are more than 2 SD error. For case (III), the mixing effects are negligible for vector doublet type of exotic leptons. For mirror and vector singlet leptons with $m_N \geq 50$ GeV and presently allowed upper limits $\sin^2\psi = 0.01-0.06$, the deviations in $d\sigma$, $d\sigma_T$, $d\sigma_L$, $d\sigma_{TT}$, and $d\sigma_{LT+TL}$ are between -7.9% and -18.3%, which would be discernible for a run of 500-pb^{-1} data.

I. INTRODUCTION

In recent years, the process $e^+e^- \rightarrow W^+W^-$ has attracted a good deal of attention[1-7] because of its experimental accessibility at the forthcoming collider LEP II at CERN. The production cross section reaches its maximum (≈ 20 pb) at $\sqrt{s} \approx 200$ GeV, and one expects to observe $\approx 10^4$ W-pair events per year, with a yearly integrated luminosity of ≈ 500 pb^{-1}. This has motivated the study of effects of non-standard-model physics in this process and had led to many evaluations of the possible deviations from standard-model (SM) predictions. For instance, the deviations caused by (i) anomalous moments κ or λ connected with WWZ or $WW\gamma$ vertices,[1-3] (ii) the extra Z boson predicted by superstring theory,[4] (iii) mixing of a new spin-1 state (strong electroweak sector) with ordinary gauge vector bosons,[5] (iv) inclusion of heavy fermions at the one-loop level,[6] and (v) residual e^+e^-VV' contact interactions[7] and new intermediate states[7] have been discussed in the recent literature. In this context, we report in this paper the possible deviations from SM results that would occur for this process due to the inclusion of mixing between ordinary and exotic leptons.

The exotic leptons, such as mirror leptons and vector doublet and vector singlet leptons with noncanonical $SU(2)\times U(1)$ transformation properties are predicted in many of the possible theories of new physics beyond the SM.[8-14] The mirror leptons appear in several theories of current interest, such as those involving family unification,[8] extended supersymmetry,[9] Kaluza-Klein theories,[10] and superstring-inspired models.[11] These leptons have opposite SU(2) assignments from those of the ordinary (known) leptons; i.e., they are left-handed singlets and right-handed doublets under SU(2). The vector doublet leptons, for which both the left- and right-handed fields are doublets, occur in E_6 grand unified theories.[12,13] On the other hand, vector singlet leptons, for which both the left- and right-handed fields are singlets under SU(2), are required in models incorporating a universal seesaw mechanism for the generation of masses of the leptons in the SM.[14] Theoretically, it is possible to construct extensions of the SM that may contain any one type of exotic lepton at a ≥ 100-GeV scale and at the same time be consistent with known low-energy phenomenology.[8,13,14] In models containing the known and new leptons, the mass eigenstates of the leptons are, in general, mixtures of the weak-interaction eigenstates by some mixing angle ψ (see Ref. 15). The phenomenological consequences of the mixings among the known and new leptons in low-energy processes (such as $\mu \rightarrow e\nu\nu; \tau \rightarrow \pi\nu$, $K\nu$, $e\nu\nu$, $\mu\nu\nu$; $\pi, K \rightarrow e\nu, \mu\nu$; and $\bar{\nu}l$ elastic scattering) have been studied previously,[15-18] and these provide useful constraints on the mixing angles. In the best cases, the bounds on the square of the mixing angle $\sin^2\psi$ are typically[15] $\leq 0.02-0.06$ (for the first generation), $\leq 0.002-0.005$ (for the second generation), and $\leq 0.10-0.22$ (for the third generation).

For the process $e^+e^- \rightarrow W^+W^-$, which in the SM occurs at the tree level through the dominant s-channel

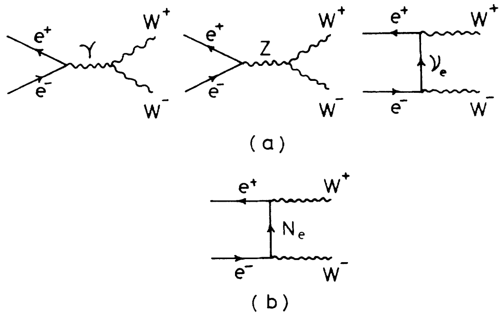

FIG. 1. (a) Lowest-order contributions to the process $e^+e^- \to W^+W^-$ in the standard model (SM) and (b) additional contribution due to exotic-neutrino mixing.

TABLE I. Possible types of new leptons (Ref. 15). Pairs of leptons enclosed in parentheses are SU(2) doublets, while other leptons (not enclosed) are SU(2) singlets. N_e^0 and E_e^0 refer to the first generation of any new leptons with charge 0 and -1, respectively. The superscript 0 is used to indicate that these are weak-interaction eigenstates.

(a) Mirror
$N_{eL}^0,\ E_{eL}^0,\ \begin{pmatrix} N_e^0 \\ E_e^0 \end{pmatrix}_R$
(b) Vector doublets
$\begin{pmatrix} N_e^0 \\ E_e^0 \end{pmatrix}_L,\ \begin{pmatrix} N_e^0 \\ E_e^0 \end{pmatrix}_R$
(c) Vector singlets
$N_{eL}^0,\ N_{eR}^0,\ E_{eL}^0,\ E_{eR}^0$

γ, Z-exchange and t-channel neutrino- (ν_e) exchange Feynman diagrams [Fig. 1(a)], the inclusion of mixing between ordinary and exotic leptons results in two effects: (i) The $We\nu_e$ and Zee couplings are modified and (ii) an additional Feynman diagram involving exotic-neutral-lepton (N_e) exchange [Fig. 1(b)] occurs (see Sec. II). We evaluate the effects of these modifications on various experimentally accessible parameters.

The plan of the paper is as follows. In Sec. II, for the sake of completeness and defining our notation, we discuss the exotic-lepton mixing model and obtain the mixing-modified $We\nu_e$, Zee couplings along with the mixing-induced WeN_e couplings. Then the helicity amplitudes for the process, with definite helicities for all the particles involved and with the inclusion of the effects of exotic-lepton mixings, are presented. In Sec. III, we evaluate, for unpolarized e^+e^- beams, the effects of exotic-lepton mixings on (i) total cross section, (ii) the differential cross section for unpolarized W's, (iii) forward-backward asymmetry, (iv) inclusive differential cross sections for the production of transversely and longitudinally polarized W's assuming the possibility of W charge identification, and (v) inclusive angular distributions for the detection of any one W (i.e., no charge identification) with longitudinal or transverse polarization, at LEP II ($\sqrt{s} \approx 190$ GeV) energies. The results are presented in terms of the expected relative percentage deviations from the SM predictions. The conclusions are summarized in Sec. IV.

II. EXOTIC-LEPTON MIXINGS AND HELICITY AMPLITUDES

A. Mixing model and modified couplings

The mixings between the presently known lepton fields and new lepton fields shown in Table I have been discussed recently, in a general framework, in Ref. 15. All left-handed SU(2)-doublet fields and right-handed SU(2)-singlet fields are defined to be *ordinary,* while all left-handed singlet fields and right-handed doublet fields are *exotic.* For the sake of simplicity and in order to reduce the number of free parameters, we assume that the first generation (and similarly the other generations) contains, in addition to the ordinary fields[19,20] $\binom{\nu_e^0}{e^0}_L$, e_R^0, and ν_{eR}^0, one new type of lepton field shown in Table I. Then, for the enriched first generation, the SU(2)×U(1) interaction Lagrangian involving electromagnetic, weak charged, and weak neutral currents of the charged leptons, relevant for ascertaining the $We\nu_e$, WeN_e, and Zee couplings, is given by

$$\mathcal{L}_{\text{int}} = eJ_{\text{EM}}^{\mu}A_{\mu} + \frac{g}{2\sqrt{2}}(J_W^{\mu}W_{\mu} + \text{H.c.}) + \frac{g}{2\cos\theta_W}J_Z^{\mu}Z_{\mu}\ , \tag{1}$$

where, in the weak-eigenstate basis,

$$J_{\text{EM}}^{\mu} = -(\bar{e}^{\,0}\gamma^{\mu}e^0 + \bar{E}^{\,0}\gamma^{\mu}E^0)\ , \tag{2a}$$

$$J_W^{\mu} = 2[\bar{\nu}_{eL}^{\,0}\gamma^{\mu}e_L^0 + \bar{N}_{eL}^{\,0}\gamma^{\mu}(-2t_{3L}^E)E_L^0 + \bar{N}_{eR}^{\,0}\gamma^{\mu}(-2t_{3R}^E)E_R^0]\ , \tag{2b}$$

$$J_Z^{\mu} = -\bar{e}_L^{\,0}\gamma^{\mu}e_L^0 + \bar{E}_L^{\,0}\gamma^{\mu}(2t_{3L}^E)E_L^0 + \bar{E}_R^{\,0}\gamma^{\mu}(2t_{3R}^E)E_R^0 - 2\sin^2\theta_W J_{\text{EM}}^{\mu}\ , \tag{2c}$$

with $f_{L(R)}^0 = \frac{1}{2}[1-(+)\gamma_5]f^0$ for $f^0 \equiv e^0$, ν_e^0, N_e^0, or E^0. The t_{3L}^E and t_{3R}^E are the T_3 eigenvalues of the new lepton fields E_L^0 and E_R^0, respectively. It should be noted that the use of $t_{3L(R)}$ in Eq. (2b) (the charged-current couplings) is only strictly correct when the relevant fermions are in singlet or doublet representations (as is the case here). However, the Clebsch-Gordan coefficient associated with the isospin-raising operator is not linearly related to $t_{3L(R)}$ for a representation of higher dimension, e.g., fermions in isotriplets.[21]

Once we allow for the mixing among leptons, then, the mass eigenstates are not necessarily the same as the weak-interaction eigenstates. At present, the absence of any evidence of new charged as well as neutral leptons implies that they must be heavy (mass $\gtrsim 30$ GeV).[22] Further, in the lepton sector, there are stringent limits on the generation-changing neutral currents.[22] We, therefore, consider the mixing among the leptons for each generation separately. Defining[19]

$$\eta^0_{L,R} \equiv \begin{bmatrix} \nu_e^0 \\ N_e^0 \end{bmatrix}_{L,R}, \quad \epsilon^0_{L,R} \equiv \begin{bmatrix} e^0 \\ E^0 \end{bmatrix}_{L,R}, \tag{3}$$

the mass eigenstates (terms without superscript 0) are obtained from the unitary transformations

$$\epsilon_{L,R} = U(\phi^e_{L,R})\epsilon^0_{L,R}, \quad \eta_{L,R} = U(\phi^\nu_{L,R})\eta^0_{L,R}, \tag{4}$$

where the mixing matrices $U(\phi^e)$ and $U(\phi^\nu)$ diagonalize, respectively, charge -1 and charge 0 mass matrices. We choose, for simplicity,

$$U(\phi^i_a) = \begin{bmatrix} c^i_a & s^i_a \\ -s^i_a & c^i_a \end{bmatrix}, \tag{5}$$

with $c^i_a = \cos(\phi^i_a)$, $s^i_a = \sin(\phi^i_a)$, for $i = e$ or ν and $a = L$ or R. Then, in the mass-eigenstate basis, the currents in Eqs. (2) are given by

$$J^\mu_{\rm EM} = -\bar\epsilon\gamma^\mu\epsilon, \tag{6a}$$

$$J^\mu_W = \bar\eta\gamma^\mu\{(1-\gamma_5)[(K^\nu_L)^\dagger(K^e_L) - 2t^E_{3L}(Q^\nu_L)^\dagger(Q^e_L)] + (1+\gamma_5)[-2t^E_{3R}(Q^\nu_R)^\dagger(Q^e_R)]\}\epsilon, \tag{6b}$$

$$J^\mu_Z = \bar\epsilon\gamma^\mu[(2\sin^2\theta_W - \tfrac{1}{2} + \tfrac{1}{2}\gamma_5) + (1-\gamma_5)(t^E_{3L} + \tfrac{1}{2}) \times (Q^e_L)^\dagger(Q^e_L) + (1+\gamma_5)t^E_{3R}(Q^e_R)^\dagger(Q^e_R)]\epsilon, \tag{6c}$$

where the matrices K and Q are

$$K^i_a = (c^i_a, -s^i_a), \quad Q^i_a = (s^i_a, c^i_a) \tag{7}$$

for $i = e, \nu$ and $a = L, R$. The row matrices K and Q are introduced for the sake of convenience in Eqs. (6). $K^\dagger K$ and $Q^\dagger Q$ are 2×2 matrices whose off-diagonal terms induce the eN_eW, ν_eEW, eEZ, and ν_eN_eZ couplings. We note that (i) the electromagnetic current [Eq. (6a)] and (ii) the first term in square brackets in Eq. (6c), which describes the standard model weak neutral currents, remain diagonal in the mass-eigenstate basis and do not contain effects of the exotic-lepton mixing. In weak charged currents [Eq. (6b)] the exotic-lepton mixings not only modify the left-handed currents but also induce the right-handed currents admixture. Similarly, there occur mixing-induced left- and right-handed contributions in J^μ_Z [Eq. (6c)]. The off-diagonal terms in J^μ_W induce WeN_e couplings which allow an additional t-channel exotic-neutral-lepton-exchange contribution for the process $e^+e^- \to W^+W^-$ [Fig. 1(b)]. Parametrizing the mixing-modified Zee, $We\nu_e$, and mixing-induced WeN_e couplings as

$$\Gamma_{Zee} = \frac{ig}{2\cos\theta_W}\gamma^\mu(V^e_Z - A^e_Z\gamma_5), \tag{8a}$$

$$\Gamma_{Wen} = \frac{ig}{2\sqrt{2}}\gamma^\mu(V^n_W - A^n_W\gamma_5), \tag{8b}$$

where $n = \nu_e, N_e$, we find, using Eqs. (6) and (7),

$$V^e_Z = 2\sin^2\theta_W - \tfrac{1}{2} + (t^E_{3L} + \tfrac{1}{2})(s^e_L)^2 + t^E_{3R}(s^e_R)^2, \quad A^e_Z = -\tfrac{1}{2} + (t^E_{3L} + \tfrac{1}{2})(s^e_L)^2 - t^E_{3R}(s^e_R)^2, \tag{9a}$$

$$V^\nu_W = c^\nu_L c^e_L - 2t^E_{3L}s^\nu_L s^e_L - 2t^E_{3R}s^\nu_R s^e_R, \quad A^\nu_W = c^\nu_L c^e_L - 2t^E_{3L}s^\nu_L s^e_L + 2t^E_{3R}s^\nu_R s^e_R, \tag{9b}$$

$$V^N_W = -s^\nu_L c^e_L - 2t^E_{3L}c^\nu_L s^e_L - 2t^E_{3R}c^\nu_R s^e_R, \quad A^N_W = -s^\nu_L c^e_L - 2t^E_{3L}c^\nu_L s^e_L + 2t^E_{3R}c^\nu_R s^e_R. \tag{9c}$$

B. Helicity amplitudes

The helicity amplitudes for the process

$$e^-(k,\sigma) + e^+(\bar k,\bar\sigma) \to W^-(q,\lambda) + W^+(\bar q,\bar\lambda) \tag{10}$$

(where the symbols in parentheses represent the four-momentum and helicity of the respective particles), without the inclusion of mixing between ordinary and exotic leptons, have been discussed by Hagiwara *et al.*[1] Following the notation of Ref. 1 (neglecting the electron mass in comparison to the beam energy $\sqrt{s}/2$), we separate the contributions of the amplitude $\mathcal{M}_{\sigma\bar\sigma,\lambda\bar\lambda}(\theta)$ for the process [Eq. (10)] as[1]

$$\mathcal{M}_{\sigma\bar\sigma,\lambda\bar\lambda}(\theta) = \sqrt{2}e^2\tilde{\mathcal{M}}_{\sigma\bar\sigma,\lambda\bar\lambda}(\theta)d^{J_0}_{\Delta\sigma,\Delta\lambda}(\theta), \tag{11}$$

where

$$\tilde{\mathcal{M}} = \tilde{\mathcal{M}}^\gamma + \tilde{\mathcal{M}}^Z + \tilde{\mathcal{M}}^\nu + \tilde{\mathcal{M}}^N \tag{12}$$

is the sum of the respective contributions from γ, Z, ν, and N exchange diagrams (see Fig. 1). $d^{J_0}_{\Delta\sigma,\Delta\lambda}(\theta)$ are the d functions,[23] $\Delta\lambda = \lambda - \bar\lambda$, $\Delta\sigma = (\sigma - \bar\sigma)/2$ (lepton helicities are normalized to ± 1), $J_0 = \max(|\Delta\sigma|, |\Delta\lambda|)$, and θ denotes the c.m. scattering angle of W^- with respect to the e^- direction. With the inclusion of mixing-modified $We\nu_e$, Zee couplings, and mixing-induced WeN_e couplings, we find that the explicit form of the four contributions in Eq. (12) are given by[24]

$$\tilde{\mathcal{M}}^\gamma = -\beta\delta_{|\Delta\sigma|,1}\mathcal{A}^\gamma_{\lambda\bar\lambda}\delta_{J_0,1}, \tag{13a}$$

$$\tilde{\mathcal{M}}^Z = \beta G^{Zee}_{\Delta\sigma}\mathcal{A}^Z_{\lambda,\bar\lambda}\frac{s}{s - m_Z^2}\delta_{J_0,1}, \tag{13b}$$

$$\tilde{\mathcal{M}}^\nu = G^{We\nu e}_{\Delta\sigma}\left[-\frac{\sqrt{2}}{\sin^2\theta_W}\frac{\delta_{|\Delta\lambda|,2}}{1+\beta^2-2\beta\cos\theta} + \frac{\delta_{J_0,1}}{2\sin^2\theta_W\beta}\times\left[\mathcal{B}_{\lambda\bar\lambda} - \frac{\mathcal{C}_{\lambda\bar\lambda}}{1+\beta^2-2\beta\cos\theta}\right]\right], \tag{13c}$$

$$\tilde{\mathcal{M}}^N = \frac{1}{1+\beta^2+\xi_N^2\gamma^{-2}-2\beta\cos\theta} \times\left[G^{WNe}_{\Delta\sigma}\left[-\frac{\sqrt{2}}{\sin^2\theta_W}\delta_{|\Delta\lambda|,2} + \frac{\delta_{J_0,1}}{2\sin^2\theta_W\beta}\times[(1+\beta^2-2\beta\cos\theta)\mathcal{B}_{\lambda\bar\lambda} - \mathcal{C}_{\lambda\bar\lambda}]\right] + \frac{1}{\sqrt{2}\sin^2\theta_W}G_0^{'WNe}\xi_N\mathcal{D}_{\lambda\bar\lambda}\right], \tag{13d}$$

where $\zeta_N = m_N/m_W$, $\gamma=\sqrt{s}/2m_W$, $\beta=(1-4m_W^2/s)^{1/2}$,

$$G^{Zee}_{\Delta\sigma}=\{[V^e_Z-(\Delta\sigma)A^e_Z]/2\sin^2\theta_W\}\delta_{|\Delta\sigma|,1}\ , \tag{14a}$$

$$G^{Wne}_{\Delta\sigma}=\tfrac{1}{4}[(V^n_W)^2+(A^n_W)^2-2(\Delta\sigma)V^n_W A^n_W]\delta_{|\Delta\sigma|,1}\ , \tag{14b}$$

$$G'^{WNe}_{0}=\tfrac{1}{4}[(V^N_W)^2-(A^N_W)^2]\delta_{\Delta\sigma,0}\ , \tag{14c}$$

with $n=\nu$ or N, and V's and A's are given by Eqs. (9). The coefficients $\mathcal{A}^{\gamma}_{\lambda\bar\lambda}$, $\mathcal{A}^{Z}_{\lambda\bar\lambda}$, $\mathcal{B}_{\lambda\bar\lambda}$, and $\mathcal{C}_{\lambda\bar\lambda}$ are the same as those given in Ref. 1 and are reproduced in Table II. The explicit form of the additional coefficient $\mathcal{D}_{\lambda\bar\lambda}$ is given in Table III.[25] It may be noted that in the limit of no mixing between the ordinary and exotic leptons, i.e., $s^i_a=0$, $c^i_a=1$ for $i=e,\nu$ and $a=L,R$, the various couplings given by Eqs. (14) reduce to the SM values, namely,

$$(G^{Zee}_{\Delta\sigma})_{\rm SM}=\delta_{|\Delta\sigma|,1}-\frac{\delta_{\Delta\sigma,-1}}{2\sin^2\theta_W}\ , \tag{15a}$$

$$(G^{W\nu e}_{\Delta\sigma})_{\rm SM}=\delta_{|\Delta\sigma|,-1}\ , \tag{15b}$$

$$(G^{WNe}_{\Delta\sigma})_{\rm SM}=0=(G'^{WNe}_{0})_{\rm SM}\ , \tag{15c}$$

III. EFFECTS OF MIXINGS ON CROSS SECTIONS AND ANGULAR DISTRIBUTIONS

With unpolarized e^+e^- beams, we evaluate the effects of exotic-lepton mixings on the following parameters:

(i) The total cross section $\sigma(e^+e^-\to W^+W^-)$.

(ii) The differential cross section for unpolarized W's:[26]

$$\frac{d\sigma(\theta)}{d\cos\theta}=\sum_{\lambda,\bar\lambda=0,+,-}\frac{d\sigma(\lambda,\bar\lambda,\theta)}{d\cos\theta}\ , \tag{16}$$

where the nine differential cross sections are given by

$$\frac{d\sigma(\lambda,\bar\lambda,\theta)}{d\cos\theta}=\frac{\beta}{128\pi s}\sum_{\sigma,\bar\sigma}|\mathcal{M}_{\sigma\bar\sigma,\lambda\bar\lambda}(\theta)|^2\ , \tag{17}$$

with $\mathcal{M}_{\sigma\bar\sigma,\lambda\bar\lambda}(\theta)$ given by Eq. (11).

(iii) The forward-backward asymmetry A_{FB}:

$$A_{\rm FB}=\frac{\int_{\theta=0}^{\pi/2}d\sigma(\theta)/d\cos\theta-\int_{\theta=\pi/2}^{\pi}d\sigma(\theta)/d\cos\theta}{\int_{\theta=0}^{\pi/2}d\sigma(\theta)/d\cos\theta+\int_{\theta=\pi/2}^{\pi}d\sigma(\theta)/d\cos\theta}\ . \tag{18}$$

(iv) The inclusive cross sections (with the charge identification of W^+,W^-, but the two transverse polarization of W's, i.e., $\lambda,\bar\lambda=+,-$, are not distinguished):[26]

TABLE II. Coefficients $\mathcal{A}^{\gamma,Z}_{\lambda\bar\lambda}$, $\mathcal{B}_{\lambda\bar\lambda}$, and $\mathcal{C}_{\lambda\bar\lambda}$ for the helicity amplitudes $\tilde{\mathcal{M}}^{\gamma,Z,\nu}_{\sigma\bar\sigma,\lambda\bar\lambda}(\theta)$.

$\Delta\lambda$	$(\lambda\bar\lambda)$	$\mathcal{A}^{\gamma,Z}_{\lambda\bar\lambda}$	$\mathcal{B}_{\lambda\lambda}$	$\mathcal{C}_{\lambda\bar\lambda}$
1	$(+0),(0-)$	2γ	2γ	$2(1+\beta)\gamma$
-1	$(0+),(-0)$	2γ	2γ	$2(1-\beta)\gamma$
0	$(++),(--)$	1	1	$1/\gamma^2$
0	(00)	$2\gamma^2+1$	$2\gamma^2$	$2/\gamma^2$

TABLE III. Coefficients $\mathcal{D}_{\lambda\bar\lambda}$ for the helicity amplitudes $\tilde{\mathcal{M}}^{N}_{\sigma\bar\sigma,\lambda\bar\lambda}(\theta)$.

$\Delta\lambda$	$(\lambda\bar\lambda)$	$\mathcal{D}_{\lambda\bar\lambda}$
0	$(++),(--)$	$(1+\sigma\lambda\cos\theta)/\gamma$
0	(00)	$-\gamma(1+\beta^2-2\beta\cos\theta)$
±1	$(+0),(0+)$	$(\beta+\sigma)$
±1	$(-0),(0-)$	$(\beta-\sigma)$

$$\frac{d\sigma_{TT}(\theta)}{d\cos\theta}=\sum_{\lambda,\bar\lambda=+,-}\frac{d\sigma(\lambda,\bar\lambda,\theta)}{d\cos\theta}\ , \tag{19a}$$

$$\frac{d\sigma_{LL}(\theta)}{d\cos\theta}=\frac{d\sigma(0,0,\theta)}{d\cos\theta}\ , \tag{19b}$$

$$\frac{d\sigma_{TL}(\theta)}{d\cos\theta}=\sum_{\lambda=+,-}\frac{d\sigma(\lambda,0,\theta)}{d\cos\theta}\ , \tag{19c}$$

$$\frac{d\sigma_{LT}(\theta)}{d\cos\theta}=\sum_{\bar\lambda=+,-}\frac{d\sigma(0,\bar\lambda,\theta)}{d\cos\theta}\ . \tag{19d}$$

(v) The inclusive angular distributions for transversely polarized W's (with no charge identification of W's together with nondistinguishability of the two transverse polarizations):[26]

$$\frac{d\sigma_T(\theta)}{d\cos\theta}=\left[\frac{d\sigma_{TT}(\theta)}{dz}+\frac{d\sigma_{TL}(\theta)}{dz}\right]_{z=\cos\theta}+\left[\frac{d\sigma_{TT}(\theta)}{dz}+\frac{d\sigma_{LT}(\theta)}{dz}\right]_{z=-\cos\theta}\ , \tag{20a}$$

$$\frac{d\sigma_L(\theta)}{d\cos\theta}=\left[\frac{d\sigma_{LL}(\theta)}{dz}+\frac{d\sigma_{LT}(\theta)}{dz}\right]_{z=\cos\theta}+\left[\frac{d\sigma_{LL}(\theta)}{dz}+\frac{d\sigma_{TL}(\theta)}{dz}\right]_{z=-\cos\theta}\ . \tag{20b}$$

For numerical evaluation, we set the left and right mixing angles equal [i.e., $\phi^e_L=\phi^e_R\equiv\phi_e$, $\phi^\nu_L=\phi^\nu_R\equiv\phi_\nu$] and consider three extreme cases of mixings:

$$\begin{aligned}&\text{case I},\ \ \phi_e=0,\ \ \phi_\nu=\psi\ ,\\ &\text{case II},\ \ \phi_e=\psi,\ \ \phi_\nu=0\ ,\\ &\text{case III},\ \ \phi_e=\psi,\ \ \phi_\nu=\psi\ ,\end{aligned} \tag{21}$$

where ψ is an arbitrary positive angle. Case I corresponds to the mixing in the neutrino sector only. Case II, perhaps less probable, corresponds to the mixing in the charged-lepton sector only. Case III corresponds to mixing in both the sectors. For the sake of reducing the number of free parameters, we have taken equal mixing angles for case III.[27] We use

$\alpha(m_W^2)=\frac{1}{128}$ (Ref. 28) ,

$m_Z=91.17$ GeV (Ref. 29) ,

$m_W=80.0$ GeV (Ref. 30) ,

$m_N = 50$–300 GeV (Refs. 31 and 32) .

$\sin^2\psi = 0.01$–0.06 (Refs. 15 and 33) .

For $\sin^2\theta_W$, we use the Marciano-Sirlin defining relation[34]

$$\sin^2\theta_W \equiv 1 - m_W^2/m_Z^2 , \tag{22}$$

where m_W and m_Z refer to the physical masses.[35]

The effects of exotic-lepton mixing on the total cross section (σ) for mixing case I is shown in Fig. 2. We evaluate the mixing effects in terms of the expected relative deviations from SM predictions. The relative percentage deviation $\Delta(\sigma)$ is defined through the relation

$$\Delta(\sigma) = [(\sigma - \sigma_{\rm SM})/\sigma_{\rm SM}] \times 100 . \tag{23}$$

We separate the deviation into two parts:

$$\Delta(\sigma) = \Delta_c(\sigma) + \Delta_t(\sigma) , \tag{24a}$$

where

$$\Delta_c(\sigma) = \frac{\sigma_{(\gamma+Z+\nu)(\gamma+Z+\nu)\ \text{with mixing}} - \sigma_{\rm SM}}{\sigma_{\rm SM}} \times 100 \tag{24b}$$

is the percentage deviation caused by the mixing-induced changes in the Zee and $We\nu$ couplings of Fig. 1(a), and

$$\Delta_t(\sigma) = \frac{\sigma_{NN} + \sigma_{(\gamma+Z+\nu)N}}{\sigma_{\rm SM}} \times 100 \tag{24c}$$

is the percentage deviation induced by the t-channel exotic-neutral-lepton (N_e) exchange [Fig. 1(b)] and its interference with the γ, Z, and ν exchange amplitudes. The variation of $\Delta(\sigma)$, $\Delta_c(\sigma)$, and $\Delta_t(\sigma)$ with $\sqrt{s}$, the mass of the exotic neutral lepton (m_N), and the square of the mixing angle ($\sin^2\psi$), for mixing cases I, II, and III, respectively, are shown in Figs. 3–5. For mixing case I (Fig. 3), all three types of exotic leptons, i.e., mirror, vector doublet, and vector singlet, have identical effects. For $\sqrt{s} = 165$–600 GeV, $m_N \le 250$ GeV, and $\sin^2\psi \le 0.06$, $\Delta_t(\sigma)$ is positive; i.e., σ increases due to an extra t-channel diagram. However, $\Delta_c(\sigma)$ is negative and is the dominant deviation, effectively decreasing the total cross section. Thus the *delay* in the cancellation expected due to the extra t-channel diagram is masked by the dominant $\Delta_c(\sigma)$. For mixing case II (Fig. 4), the deviations $\Delta_t(\sigma)$, $\Delta_c(\sigma)$, and $\Delta(\sigma)$ depend on the type of mixed exotic leptons. For this mixing case the negative $\Delta_c(\sigma)$ also is dominant over the positive $\Delta_t(\sigma)$ and the total cross section decreases due to mixing. For mixing case III (Fig. 5), for mirror and vector singlet types of exotic leptons, $\Delta_t(\sigma)$ is positive and small, while $\Delta_c(\sigma)$ is negative and dominant for the range of $\sqrt{s}$, m_N, and $\sin^2\psi$ considered here. However, for vector doublet types of exotic leptons, the sign of $\Delta_c(\sigma)$ and $\Delta_t(\sigma)$ reverses at high energies [see Fig. 5(a)]. At $\sqrt{s} = 190$ GeV, for heavy exotic neutral leptons ($m_N = 100$–300 GeV), the expected relative deviations $\Delta(\sigma)$, $\Delta_c(\sigma)$, and $\Delta_t(\sigma)$ are shown in Table IV for a mixing angle $\psi = 10°$. The magnitude of these deviations increases with the increase in the mixing angle.

The effects of exotic-lepton mixings on the differential cross section for mixing case I are shown in Fig. 6. For the differential cross section also we define the relative percentage deviation from SM expectations as

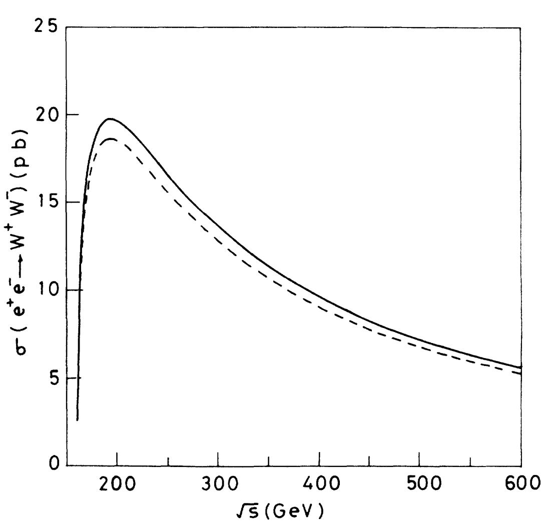

FIG. 2. Total cross section for $e^+e^- \to W^+W^-$ as a function of the c.m. energy. The solid curve is for SM, and the dashed curve is for mixing case I ($\psi = 10°$ and $m_N = 250$ GeV). The curves for the three types of exotic leptons coincide.

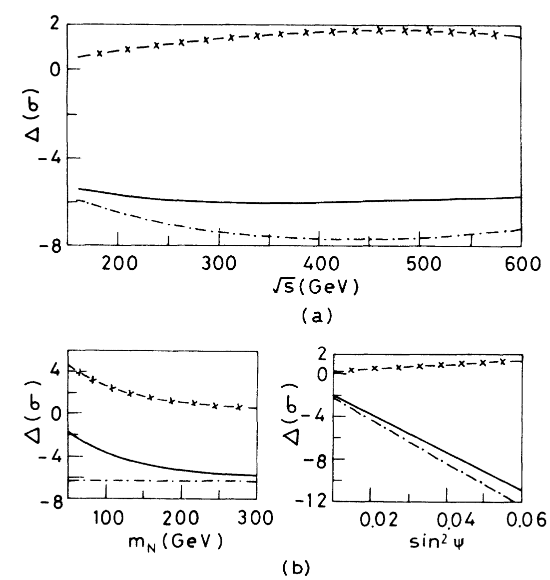

FIG. 3. Variation of the expected relative percentage deviations in $\Delta(\sigma)$ (solid curve), $\Delta_c(\sigma)$ (dot-dashed curve), and $\Delta_t(\sigma)$ (cross-dashed curve) for mixing case I with (a) $\sqrt{s}$ and (b) m_N (first column) and $\sin^2\psi$ (second column). For (a) $m_N = 250$ GeV and $\psi = 10°$ and for (b) $\sqrt{s} = 190$ GeV and $\psi = 10°$ for curves in the first column and $m_N = 250$ GeV for the curve in the second column. The curves for mirror, vector doublet, and vector singlet mixings coincide in this case.

$$\Delta(d\sigma)=\left\{\left[\frac{d\sigma}{d\cos\theta}-\left[\frac{d\sigma}{d\cos\theta}\right]_{\rm SM}\right]\Big/\left[\frac{d\sigma}{d\cos\theta}\right]_{\rm SM}\right\}\times 100\;. \tag{25}$$

We separate again the deviation into two parts:

$$\Delta(d\sigma)=\Delta_c(d\sigma)+\Delta_t(d\sigma)\;, \tag{26a}$$

where

$$\Delta_c(d\sigma)=\frac{(d\sigma_{(\gamma+Z+\nu)(\gamma+Z+\nu)\ \text{with mixing}}/d\cos\theta)-(d\sigma/d\cos\theta)_{\rm SM}}{(d\sigma/d\cos\theta)_{\rm SM}}\times 100 \tag{26b}$$

is the percentage deviation caused by the mixing-induced changes in the Zee and $We\nu_e$ couplings of Figs. 1(a), and

$$\Delta_t(d\sigma)=\frac{(d\sigma_{NN+(\gamma+Z+\nu)N}/d\cos\theta)-(d\sigma/d\cos\theta)_{\rm SM}}{(d\sigma/d\cos\theta)_{\rm SM}}\times 100 \tag{26c}$$

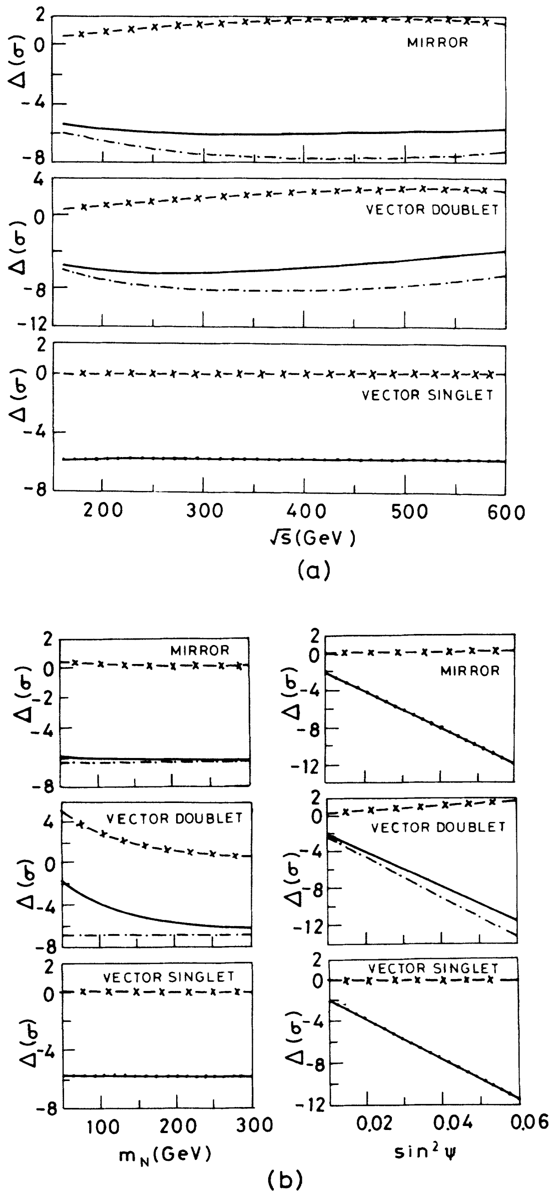

FIG. 4. Same as Fig. 3, but for mixing case II.

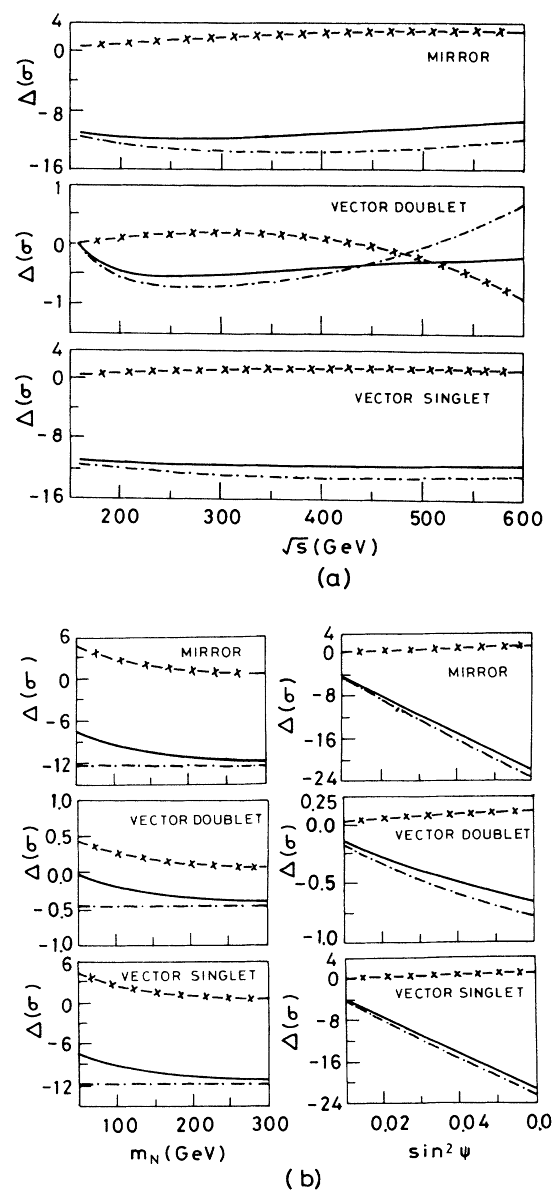

FIG. 5. Same as Fig. 3, but for mixing case III.

TABLE IV. Expected relative percentage deviations $\Delta(\sigma)$, $\Delta_c(\sigma)$, and $\Delta_t(\sigma)$ in total cross section at $\sqrt{s}=190$ GeV and $\psi=10°$ with exotic-neutrino mass $m_N=100$–300 GeV.

Case	Mirror	Vector doublet	Vector singlet
		$\Delta(\sigma)$	
I	-3.6 to -5.8	-3.6 to -5.8	-3.6 to -5.8
II	-6.0 to -6.2	-3.7 to -6.2	-5.8
III	-9.3 to -11.6	-0.2 to -0.4	-9.2 to -11.3
		$\Delta_c(\sigma)$	
I	-6.3	-6.3	-6.3
II	-6.3	-6.8	-5.8
III	-12.2	-0.5	-11.8
		$\Delta_t(\sigma)$	
I	2.7 to 0.5	2.7 to 0.5	2.7 to 0.5
II	0.3 to 0.1	3.1 to 0.6	0.0
III	2.9 to 0.6	0.3 to 0.1	2.6 to 0.5

is the percentage deviation induced by the t-channel exotic-neutral-lepton exchange [Fig. 1(b)]. The variations of $\Delta(d\sigma)$, $\Delta_c(d\sigma)$, and $\Delta_t(d\sigma)$ at $\sqrt{s}=190$ GeV with (a) $\cos\theta$, (b) m_N, and (c) $\sin^2\psi$ for the three mixing cases I, II, and III are shown, respectively, in Figs. 7–9. The error bars in Figs. 7–9 represent the one-standard-deviation error expected in the SM for a run of 500-pb^{-1} data.[36] For mixing case I, all three types of exotic leptons have identical mixing effects. $\Delta_t(d\sigma)$ is positive and depends weakly on the mixing angle $\sin^2\psi$ [see Fig. 7(b), second column]. $\Delta_c(d\sigma)$ (which is not sensitive to the value of m_N [see Figs. 7(b), first column]) is negative and is the dominant contribution for all θ values. For mixing cases II and III, $\Delta_t(d\sigma)$ and $\Delta_c(d\sigma)$ depend on the type of exotic lepton (Figs. 8 and 9). Here also we find that $\Delta_c(d\sigma)$ is negative and dominant over $\Delta_t(d\sigma)$, which is positive, except for mixing case III of vector doublet types of exotic leptons where $\Delta_t(d\sigma)$ is also negative in backward directions. Further, we note that for all three cases of mixings the magnitudes of $\Delta_t(d\sigma)$ and $\Delta_c(d\sigma)$ are greater in the backward direction than in the forward direction. At $\sqrt{s}=190$ GeV, for a heavy exotic neutrino ($m_N=100$–300 GeV), the presently allowed limits on exotic-lepton mixings ($\psi=10°$) allow large deviations ranging from $\Delta(d\sigma)=-3.4\%$ to -10.3% except for case III of vector doublet types where the deviations are negligible (see Table V). The magnitudes of these deviations further increase with the increase in the mixing angle ψ.

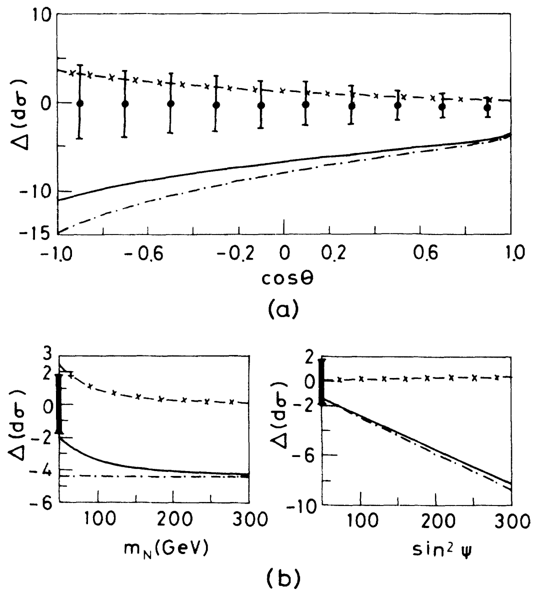

FIG. 7. Variation of the expected relative percentage deviations in $\Delta_c(d\sigma)$ (dot-dashed curve), $\Delta_t(d\sigma)$ (cross-dashed curve), and $\Delta(d\sigma)$ (solid curve) for mixing case I with (a) $\cos\theta$ and (b) m_N (first column) and with $\sin^2\psi$ (second column). For (a) $m_N=250$ GeV and $\psi=10°$, and the error bars represent one standard deviation error for a run of 500-pb^{-1} data. For (b) $\cos\theta=0.9$ and $\psi=10°$ for the curves in the first column and $m_N=250$ GeV for the curves in the second column, and error bars represent the 1σ error, expected in the tenth bin $(0.8\le\cos\theta\le 1.0)$.

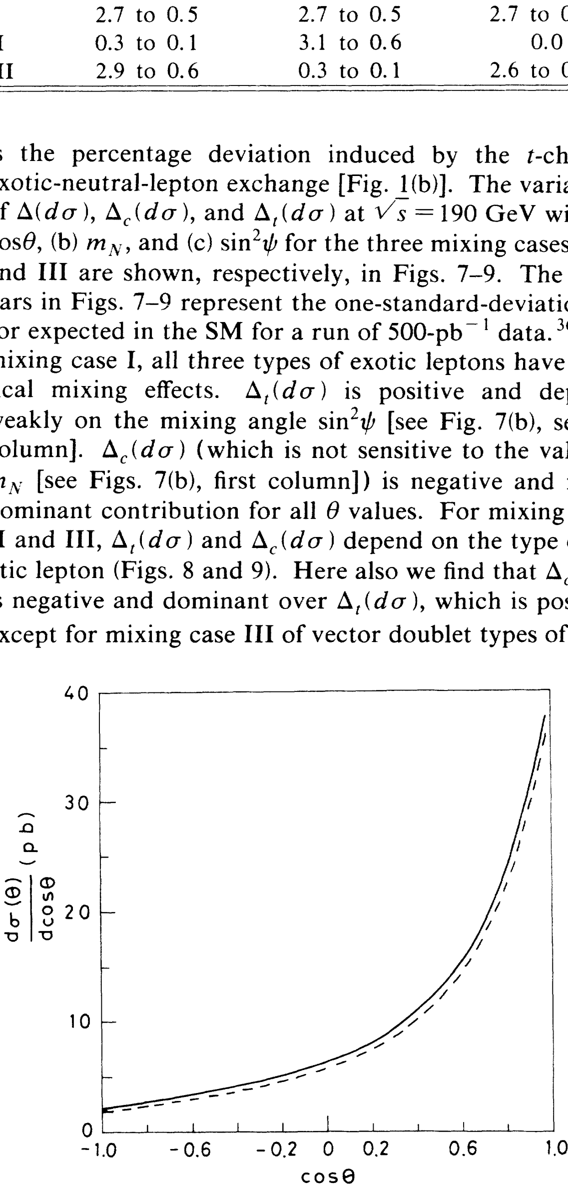

FIG. 6. Differential cross section $d\sigma/d\cos\theta$ as a function of $\cos\theta$. The solid curve is for SM, and the dashed curve is for mixing case I ($\psi=10°$ and $m_N=250$ GeV) for which the curves of the three types of exotic leptons coincide.

The effects of exotic-lepton mixings on the forward-backward asymmetry (A_{FB}) for mixing case I is shown in Fig. 10. The expected relative percentage deviation from SM expectation in A_{FB} is defined through the relation

$$\Delta(A_{FB})=\{[A_{FB}-(A_{FB})_{SM}]/(A_{FB})_{SM}\}\times 100\ . \tag{27}$$

As pointed out earlier, the percentage deviation in the differential cross section due to the additional t-channel diagram $\Delta_t(d\sigma)$ is positive and greater in backward directions than in forward directions [Fig. 7(a)]. Thus, in principle, it should tend to soften the peaking in the forward direction and reduce A_{FB}. However, for the range of $\sqrt{s}$, m_N, and $\sin^2\psi$ considered in this paper, the relative deviation $\Delta_c(d\sigma)$ induced by mixing-modified couplings of Fig. 1(a) is negative and the dominant one at all angles. These negative deviations in the differential cross

TABLE V. Expected relative percentage deviations in differential cross section [$\Delta(d\sigma)$] at $\cos\theta=0.9$ for $\sqrt{s}=190$ GeV and $\psi=10°$ with exotic-neutrino mass $m_N=100$–300 GeV.

Case	Mirror	Vector doublet	Vector singlet
I	-3.4 to -4.2	-3.4 to -4.2	-3.4 to -4.2
II	-6.3	-3.6 to -4.4	-6.1
III	-9.5 to -10.3	-0.2	-9.3 to -10.1

section are also more in the backward direction in comparison to those in the forward direction, thereby masking the effects of $\Delta_t(d\sigma)$ and effectively increasing the asymmetry. The variation of $\Delta(A_{FB})$ with $\sqrt{s}$, m_N, and $\sin^2\psi$ is shown in Figs. 11(a) and 11(b). We note that the percentage deviation in the asymmetry $\Delta(A_{FB})$ at a given $\sqrt{s}$ depends on m_N and $\sin^2\psi$. Typically, for $\sqrt{s}=190$ GeV, $\psi=10°$, for mixing case I, $\Delta(A_{FB})$ is negative (A_{FB} decreases) for light m_N and positive (A_{FB} increases) for heavy m_N. The range of $\Delta(A_{FB})$ for (a) $m_N=10$–100 GeV and (b) $m_N=100$–300 GeV, at $\sqrt{s}=190$ GeV, and $\psi=10°$, for the three cases of mixing is shown in Table VI. The magnitude of these deviations increases with the increase in the mixing angle.

The effects of exotic-lepton mixings on the inclusive angular distributions [Eqs. (19) and (20)] for mixing case I at $\sqrt{s}=190$ GeV are shown in Fig. 12. The relative percentage deviations in the inclusive angular distributions are also defined like $\Delta(d\sigma)$ in Eq. (25). The variation of expected relative percentage deviations in the inclusive angular distributions, i.e., $\Delta(d\sigma_{TT})$, $\Delta(d\sigma_{LT+TL})$, $\Delta(d\sigma_{LL})$, $\Delta(d\sigma_L)$, and $\Delta(d\sigma_T)$, with $\sqrt{s}$, m_N, and $\sin^2\psi$, are shown in Figs. 13–17, respectively. At $\sqrt{s}=190$ GeV, for $m_N=100$–300 GeV and $\psi=10°$, the range of ex-

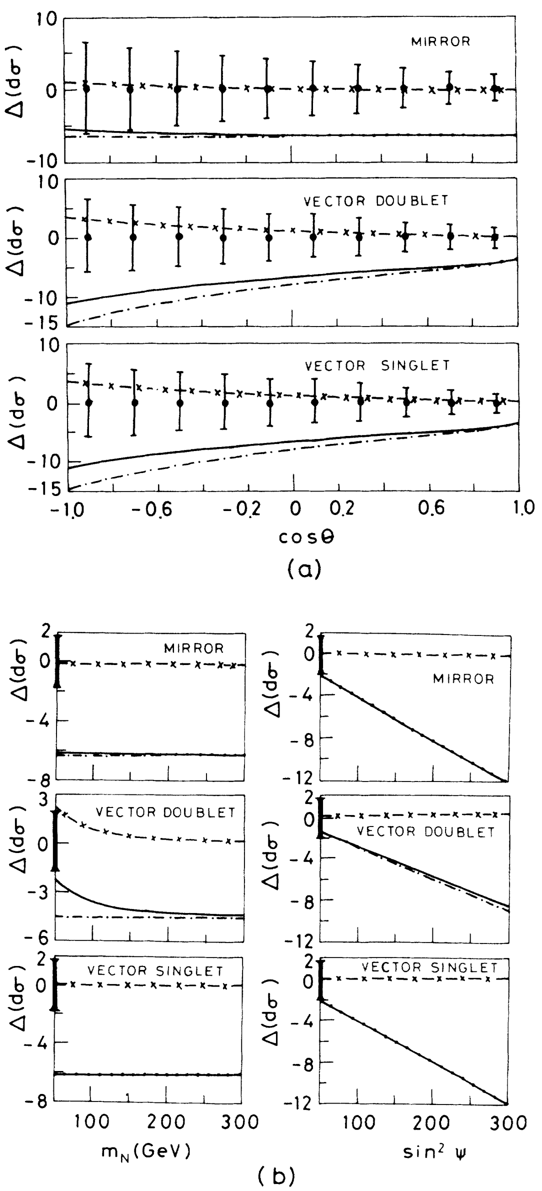

FIG. 8. Same as Fig. 7, but for mixing case II.

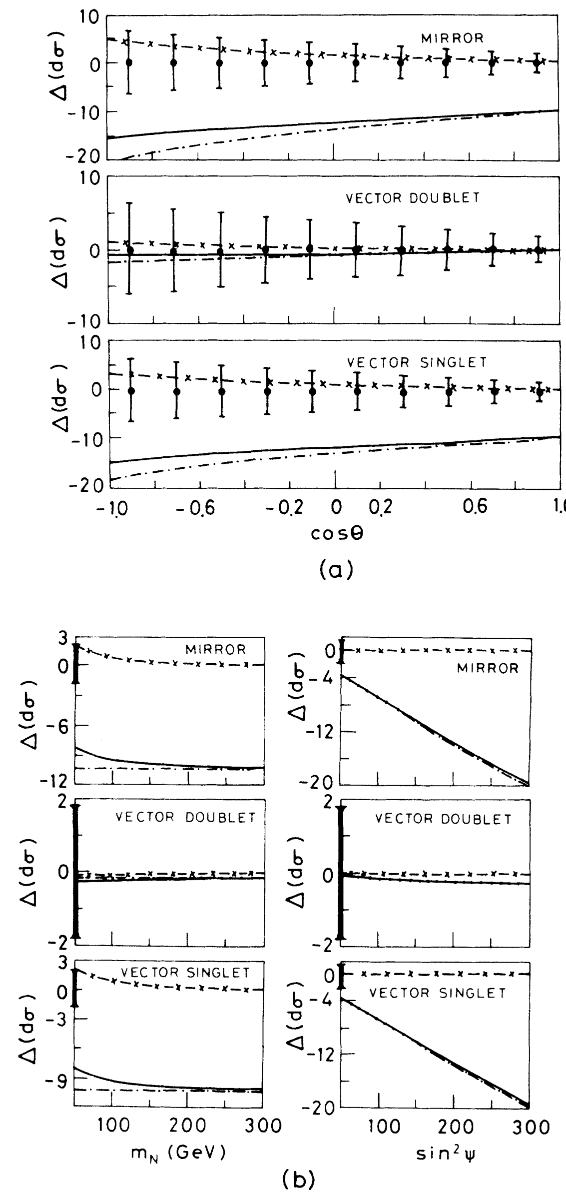

FIG. 9. Same as Fig. 7, but for mixing case III.

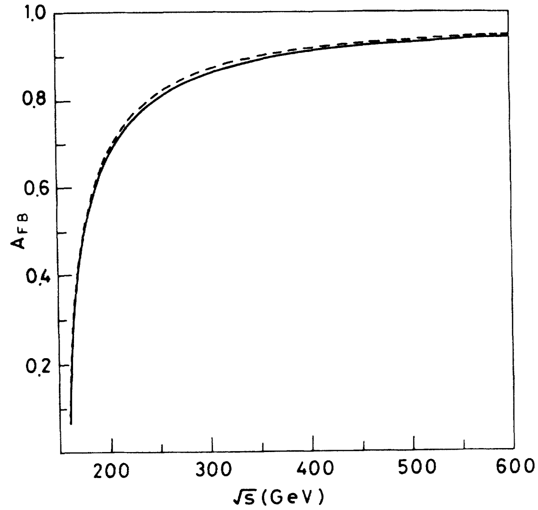

FIG. 10. Forward-backward asymmetry for $e^+e^- \to W^+W^-$ as a function of $\sqrt{s}$. The description of the curves is the same as that in Fig. 2.

pected relative deviations for the mirror, vector doublet, and vector singlet exotic-lepton mixings for the three cases I, II, and III is given in Table VII. The deviations range from -4.4% to -11.2% for $d\sigma_{TT}$ at $\cos\theta=0.9$, from -4.0% to -11.0% for $d\sigma_T$ at $\cos\theta=0.9$, from -5.0% to -16.0% for $d\sigma_L$ at $\cos\theta=0$, from -2.7% to -18.3% for $d\sigma_{LT+TL}$ at $\cos\theta=-0.7$, and from -4.6% to -16.3% for $d\sigma_{LL}$ at $\cos\theta=-0.5$ except for case III of vector doublet type where these deviations are small.

For the specific mixing cases I–III, we notice the following.

Case I: *Mixing among neutral leptons only.* In this case, the couplings given by Eq. (9) reduce to

$$V_Z^e=2\sin^2\theta_W-\tfrac{1}{2},\quad A_Z^e=-\tfrac{1}{2}\,,$$

$$V_W^\nu=\cos\psi,\quad A_W^\nu=\cos\psi\,,$$

$$V_W^N=-\sin\psi,\quad A_W^N=-\sin\psi\,.$$

As such, all three types of exotic leptons have identical mixing effects. This is reflected in Figs. 2, 3, 6, 7, and 10–17. For a run of 500-pb^{-1} data at $\sqrt{s}=190$ GeV and for $m_N=250$ GeV and $\psi=10°$, the expected relative deviations in $d\sigma(\theta)$ and $d\sigma_T(\theta)$ are in error by more than one standard deviation (1σ) for all values of θ. In $d\sigma_{TT}(\theta)$ these deviations are in error by more than 1σ in the forward direction ($\cos\theta\geq 0.4$), and in $d\sigma_L(\theta)$ the deviations are more than 1σ in the range $-0.6\leq\cos\theta\leq 0.6$. However, in $d\sigma_{LL}(\theta)$ and $d\sigma_{LT+TL}(\theta)$ the expected deviations are within 1σ of error and would be difficult to discern with 500-pb^{-1} data. Furthermore, for a light exotic neutrino ($m_N\leq 50$ GeV), for a run of 500-pb^{-1} data,

TABLE VI. Expected relative percentage deviation in the forward-backward asymmetry [$\Delta(A_{FB})$] at $\sqrt{s}=190$ GeV and $\psi=10°$, for (a) $m_N=10$–100 GeV and (b) $m_N=100$–300 GeV.

Case	Mirror	Vector doublet	Vector singlet
		(a)	
I	-0.2 to 0.3	-0.2 to 0.3	-0.2 to 0.3
II	-1.1 to -0.7	-0.8 to 0.0	-0.4
III	-1.3 to -0.5	-0.7 to 0.3	-0.6 to -0.1
		(b)	
I	0.3 to 2.1	0.3 to 2.1	0.3 to 2.1
II	-0.7 to -0.1	0.0 to 2.3	-0.4
III	-0.5 to 1.9	0.3 to -0.3	-0.1 to 1.6

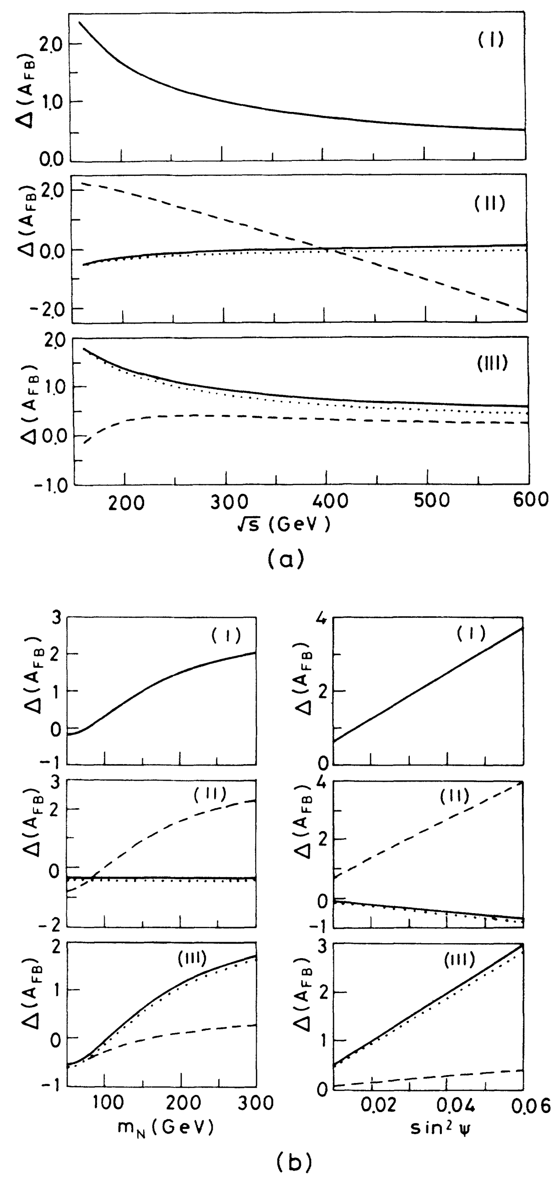

FIG. 11. Variation of the expected relative percentage deviation in the forward-backward asymmetry $\Delta(A_{FB})$ due to exotic-lepton mixings, with (a) $\sqrt{s}$ and (b) m_N and $\sin^2\psi$. The solid, dashed, and dotted curves are, respectively, for mirror, vector doublet, and vector singlet type of exotic-lepton mixings. For (a) $m_N=250$ GeV and $\psi=10°$, and for (b) $\sqrt{s}=190$ GeV, and $\psi=10°$ for the curves in the first column and $m_N=250$ GeV for the curves in the second column.

the expected deviations are within 1σ of error in all angular distributions.

Case II: *Mixing among charged leptons only.* In this case, we note that the expected relative percentage deviations in the various parameters are almost the same for mirror and vector singlet types of exotic leptons. In the total cross sections, the deviations have a weak dependence on m_N and $\sqrt{s}$. In the differential angular distributions $d\sigma(\theta)$, $d\sigma_L(\theta)$, $d\sigma_T(\theta)$, $d\sigma_{TT}(\theta)$, and $d\sigma_{LT+TL}(\theta)$ the deviations for mirror and vector singlet types of exotic leptons are not sensitive to m_N and $\cos\theta$ (see Figs. 8, 13, 14, 16, and 17). For a heavy exotic neutrino ($m_N=250$ GeV) and $\psi=10°$, the deviations in $d\sigma_L(\theta)$ and $d\sigma_{LT+TL}(\theta)$ are within 1σ of error for a run of 500-pb^{-1} data at $\sqrt{s}=190$ GeV, but are in error by more than 1σ in $d\sigma(\theta)$, $d\sigma_T(\theta)$, and $d\sigma_{TT}(\theta\geq 0)$. We note that the deviations in (i) $d\sigma(\theta)$ at $\cos\theta=0.9$ [Fig. 8(b)] and (ii) $d\sigma_T(\theta)$ at $\cos\theta=0.9$ [Fig. 17(b)], for $m_N \geq 50$ GeV and $\sin^2\psi \approx 0.03$, are in error by greater than about 2σ for a run of 500-pb^{-1} data at $\sqrt{s}=190$

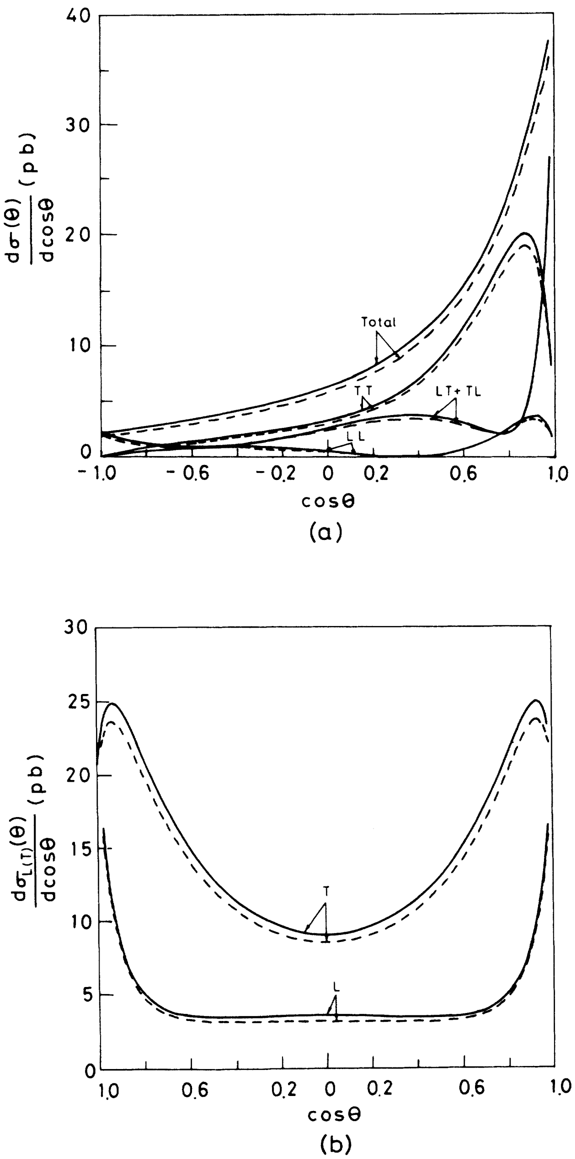

FIG. 12. Differential cross section ($d\sigma/d\cos\theta$) and the inclusive angular distributions ($d\sigma_{LL}/d\cos\theta$), ($d\sigma_{TT}/d\cos\theta$), ($d\sigma_{LT+TL}/d\cos\theta$), ($d\sigma_L/d\cos\theta$), and ($d\sigma_T/d\cos\theta$) for production of polarized W's ($T\equiv$transverse, $L\equiv$longitudinal) at $\sqrt{s}=190$ GeV. The solid curves are for the SM, and the dashed curves are for mixing case I with $\psi=10°$ and $m_N=250$ GeV (curves for the three types of exotic leptons coincide).

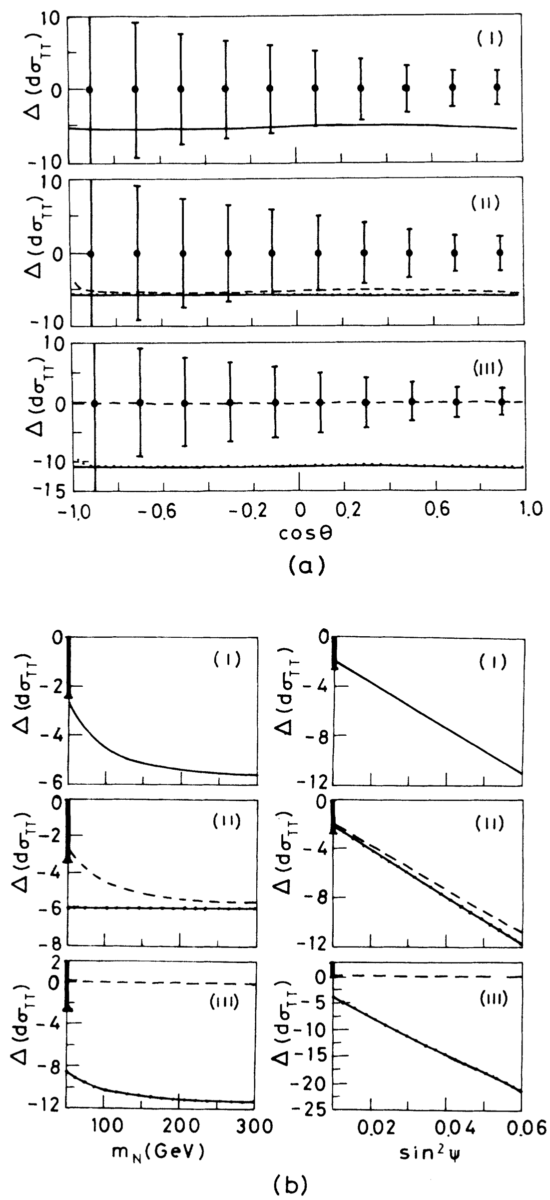

FIG. 13. Variation of expected relative percentage deviation $\Delta(d\sigma_{TT})$ at $\sqrt{s}=190$ GeV with (a) $\cos\theta$ and (b) m_N and $\sin^2\psi$. The solid, dashed, and dotted curves are, respectively, for mirror, vector doublet, and vector singlet lepton mixings. For case I, the three types coincide. For (a) $m_N=250$ GeV and $\psi=10°$, and the error bars represent one-standard-deviation error for a run of 500-pb^{-1} data. For (b) $\cos\theta=0.9$ and $\psi=10°$ for the curves in the first column and $m_N=250$ GeV for the curves in the second column, and error bars represent the 1σ error, expected in the tenth bin ($0.8\leq\cos\theta\leq 1.0$).

GeV. As such it would be of interest to look for exotic-lepton-mixing effects in these decay parameters.

Case III: *Equal mixing in the charged- and neutral-lepton sectors.* In this case, the mixing effects of vector doublet types of exotic leptons are negligible, and the deviations are less than 1.8% even for a heavy exotic neutrino ($m_N \approx 250$ GeV) and large mixing ($\sin^2\psi \approx 0.06$). Therefore, the effects of vector doublet types of mixed heavy exotic neutrinos are unlikely to be discernible. However, for the mixing of mirror or vector singlet types of exotic leptons, the deviations are quite large (i.e., in the range from -7.9% to -18.3%), as shown in Tables IV–VII. For heavy exotic neutrinos ($m_N \geq 100$ GeV) and $\sin^2\psi \geq 0.02$, the deviations in $d\sigma$, $d\sigma_T$, $d\sigma_L$, $d\sigma_{TT}$, and $d\sigma_{LT+TL}$ are in error by greater than 1σ SM for a run of 500-pb^{-1} data. To be specific, for $m_N \geq 100$ GeV, $\psi = 10°$, and $\sqrt{s} = 190$ GeV, the deviations are

(i) $|\Delta(\sigma)| \geq 9.2\%$,

(ii) $|\Delta[d\sigma(\cos\theta=0.9)]| \geq 9.3\%$,

(iii) $|\Delta[d\sigma_T(\cos\theta=0.9)]| \geq 9.7\%$,

(iv) $|\Delta[d\sigma_{TT}(\cos\theta=0.9)]| \geq 10.1\%$,

(v) $|\Delta[d\sigma_L(\cos\theta=0)]| \geq 10.1\%$,

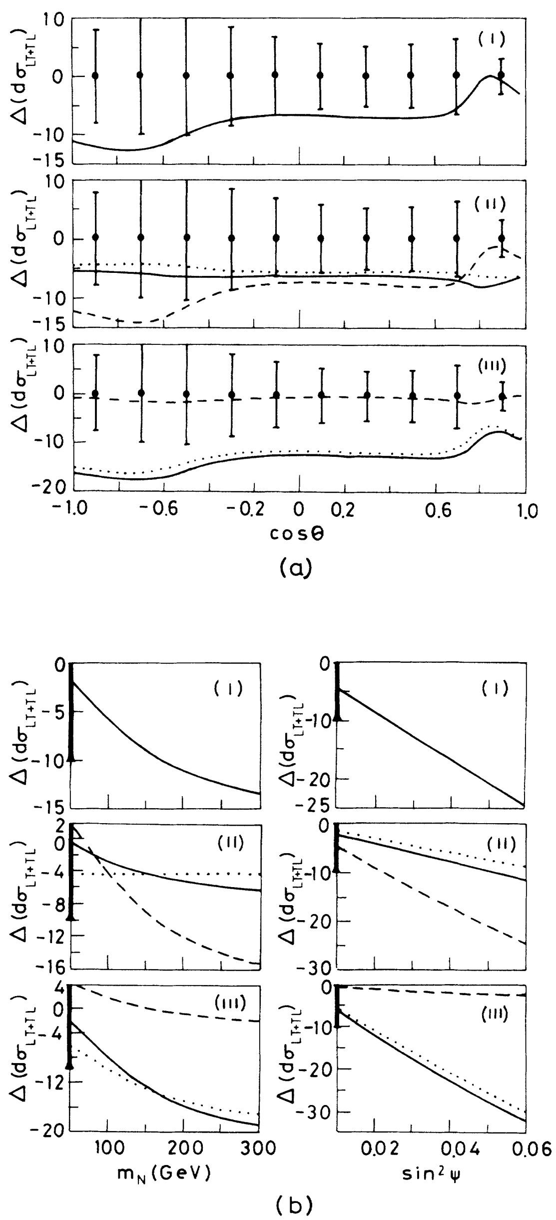

FIG. 14. Variation of $\Delta(d\sigma_{LT+TL})$ at $\sqrt{s} = 190$ GeV with (a) $\cos\theta$ and (b) m_N and $\sin^2\psi$. For (b) $\cos\theta = -0.7$ and the error bars represent one-standard-deviation error in the second bin ($-0.8 \leq \cos\theta \leq -0.6$) for a data of 500 pb^{-1}. The other description of the curves is same as that in Fig. 13.

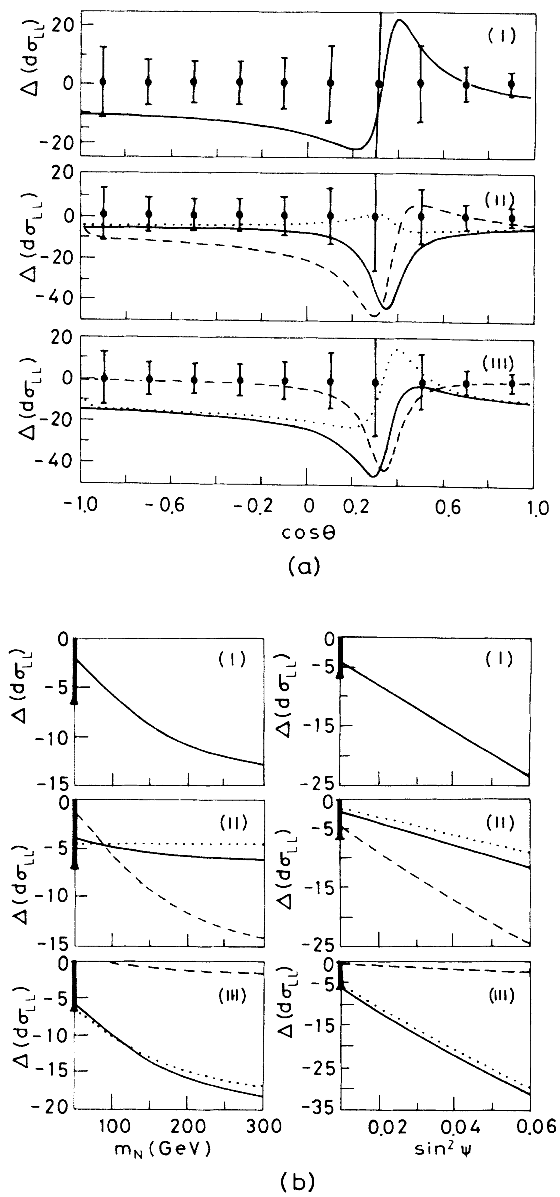

FIG. 15. Variation of $\Delta(d\sigma_{LL})$ at $\sqrt{s} = 190$ GeV with (a) $\cos\theta$ and (b) m_N and $\sin^2\psi$. The error bars represent one-standard-deviation error for a run of 1000-pb^{-1} data. For (b) $\cos\theta = -0.5$ and the error bars represent one-standard-deviation error in the third bin ($-0.6 \leq \cos\theta \leq -0.4$). The other description of the curves is the same as that in Fig. 13.

TABLE VII. Expected relative percentage deviations in the inclusive distributions [$\Delta(d\sigma_{LL})$, $\Delta(d\sigma_{TT})$, $\Delta(d\sigma_{LT+TL})$, $\Delta(d\sigma_L)$, and $\Delta(d\sigma_T)$] at $\sqrt{s}=190$ GeV and $\psi=10°$ with exotic neutrino mass $m_N=100$–300 GeV.

	$\cos\theta$	Case I All three types	Case II Mirror	Case II Vector doublet	Case II Vector singlet	Case III Mirror	Case III Vector doublet	Case III Vector singlet
$\Delta[d\sigma_{TT}(\theta)]$	0.9	-4.5 to -5.6	-6.0	-4.4 to -5.6	-6.0	-10.1 to -11.2	0.0	-10.1 to -11.2
$\Delta[d\sigma_{LL}(\theta)]$	-0.5	-5.7 to -12.4	-4.8 to -6.0	-5.5 to -13.6	-4.6	-9.8 to -17.5	-0.2 to -1.3	-10.0 to -16.3
$\Delta[d\sigma_{LT+TL}(\theta)]$	-0.7	-5.7 to -13.0	-2.7 to -6.2	-3.9 to -14.8	-4.3	-7.9 to -18.3	1.8 to -1.7	-9.7 to -16.6
$\Delta[d\sigma_T(\theta)]$	0.9	-4.0 to -5.2	-6.1	-4.0 to -5.3	-6.0	-9.7 to -11.0	-0.1	-9.7 to -10.8
$\Delta[d\sigma_L(\theta)]$	0.0	-5.3 to -10.0	-5.8 to -6.8	-5.8 to -11.6	-5.0	-10.5 to -16.0	-0.7 to -1.7	-10.1 to -14.4

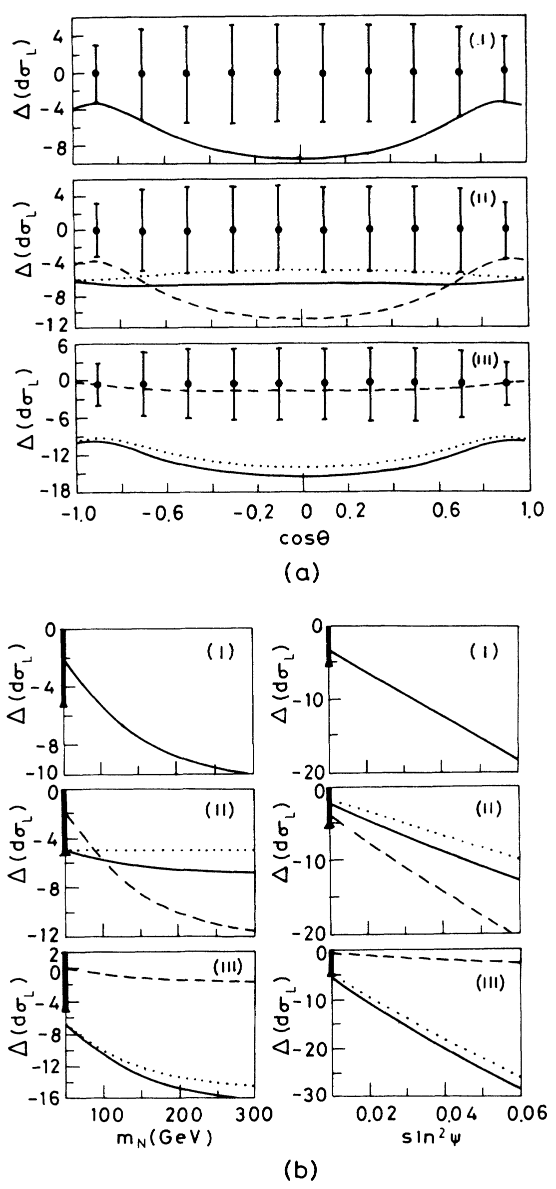

FIG. 16. Variation of $\Delta(d\sigma_L)$ at $\sqrt{s}=190$ GeV with (a) $\cos\theta$ and (b) m_N and $\sin^2\psi$. For (b) $\cos\theta=0$ and the error bars are the expected one-standard-deviation error for a run of 500-pb^{-1} data. The other description of the curves is same as that in Fig. 13.

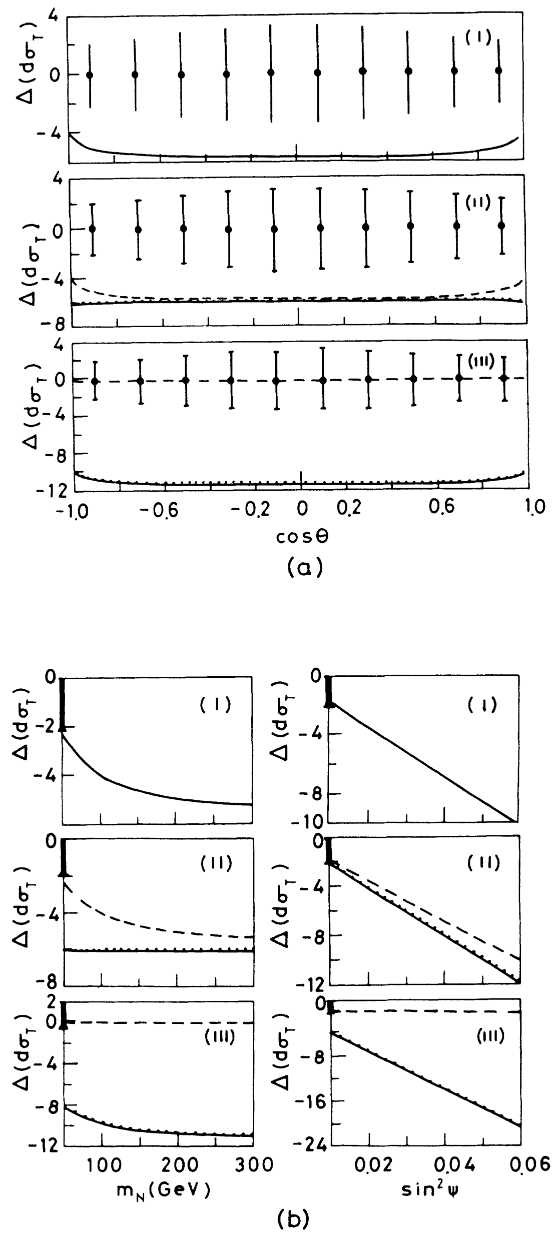

FIG. 17. Same as Fig. 13, but for $\Delta(d\sigma_T)$.

for mirror and vector singlet types of mixed exotic leptons. This allows the possibility of discerning the mixing effects of mirror and vector singlet types of mixed exotic leptons in these decay parameters.

As a final remark, we may point out that the radiative corrections[37–39] introduce deviations from tree-level SM predictions. The biggest contribution would be from initial-state radiation which is a pure QED effect and depend on the actual experimental condition.[40] At LEP II energies ($\sqrt{s} \approx 190$ GeV), the one-loop, bremsstrahlung, and renormalization term corrections are typically $\approx 2\%$ to 5% (for Higgs-boson mass $m_H = 10$–1000 GeV).[37] The logarithmic and heavy-quark one-loop corrections produce no drastic change in the total cross section even for $m_t = 300$ GeV.[39] For $\sqrt{s} = 180$–300 GeV and with $m_t = 150$ GeV, these corrections are $\leq 2.7\%$. Therefore, for light exotic neutrinos ($m_N \leq 100$ GeV), the radiative corrections would mask the exotic-lepton mixing effects.

IV. CONCLUSIONS

In this paper we have considered the process $e^+e^- \rightarrow W^+W^-$ with the inclusion of mixing between ordinary and exotic leptons of (i) mirror, (ii) vector doublet, and (iii) vector singlet types. The effects of mixings on (i) the total cross section (σ), (ii) differential cross section $[d\sigma(\theta)/d\cos\theta]$, (iii) forward-backward asymmetry (A_{FB}), and (iv) inclusive cross sections $d\sigma_{LL}(\theta)/d\cos\theta$, $d\sigma_{TT}(\theta)/d\cos\theta$, $d\sigma_{LT+TL}(\theta)/d\cos\theta$, $d\sigma_T(\theta)/d\cos\theta$, and $d\sigma_L(\theta)/d\cos\theta$ for the production of longitudinally (L) and transversely (T) polarized W's have been evaluated for the presently allowed upper limits on mixing angles ($\sin^2\psi \approx 0.02$–0.06) and heavy exotic neutrinos ($m_N = 50$–300 GeV). We have considered three extreme cases of mixings [Eqs. (21)]. Our conclusions at $\sqrt{s} = 190$ GeV and for an expected run of 500-pb^{-1} data are as follows.

For case I of mixing, i.e., mixing in the neutral-lepton sector only, we find that all three types of exotic leptons have identical mixing effects. For a heavy exotic neutrino $m_N \geq 100$ GeV, the expected deviations in the differential cross section $d\sigma(\theta)$ and $d\sigma_T(\theta)$ are more than 1σ for the currently allowed upper limits on mixing angles at all angles θ. In the case of $d\sigma_{TT}(\theta)$, the expected deviations are more than 1σ only in the forward directions ($\cos\theta \geq 0.4$). However, in $d\sigma_{LL}(\theta)$ and $d\sigma_{LT+TL}(\theta)$, the expected deviations are within 1σ.

For case II of mixing, i.e., mixing in the charged-lepton sector only, for heavy exotic neutrinos ($m_N \geq 150$ GeV), the effects of mixing of mirror, vector doublet, or vector singlet are almost the same. For $m_N \geq 100$ GeV and with the presently available upper limits on the mixing angle, the deviations in $d\sigma(\theta)$, $d\sigma_L(\theta)$, $d\sigma_T(\theta)$, $d\sigma_{TT}(\theta)$, and $d\sigma_{LT+TL}(\theta)$ are almost independent of $\cos\theta$ and are more than 1σ. As such, it would be of interest to look for the exotic-lepton mixing effects in these angular distributions.

For case III of mixing, i.e., equal mixing in the charged- and neutral-lepton sectors, the effects of vector doublet type of mixed exotic leptons are very small. The expected deviations are less than 1σ in all the parameters studied. As such, it would be difficult to discern this type of mixing effect. For the mirror and vector singlet type of mixed exotic leptons, the expected deviations in $d\sigma$ ($\cos\theta = 0.9$), $d\sigma_{TT}$ ($\cos\theta = 0.9$), $d\sigma_T$ ($\cos\theta = 0.9$), and $d\sigma_L$ ($\cos\theta = 0$) have an error even larger than 2σ. As such, it would be of interest to look for the mixing effects in the measurements of these differential parameters. However, since the effects of mirror and vector singlet mixings are of the same order (see Figs. 9–17), it would not be possible to distinguish between the type of mixed exotic leptons.[41,42]

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[1]K. Hagiwara, R. D. Peccei, D. Zeppenfeld, and K. Hikasa, Nucl. Phys. **B282**, 253 (1987). The references for earlier important contributions to the subject are given in this paper.

[2]*Physics at LEP*, LEP Jamboree, Geneva, Switzerland, 1985, edited by J. Ellis and R. D. Peccei (CERN Yellow Report No. 86-02, Geneva, 1986), Vol. 2, p. 72.

[3]D. Zeppenfeld, Phys. Lett. B **183**, 380 (1987); H. Neufeld, J. D. Stroughair, and D. Schildknecht, *ibid.* **198**, 563 (1987); C. L. Bilchak and J. D. Stroughair, Phys. Rev. D **41**, 2233 (1990).

[4]R. Najima and S. Wakaizumi, Phys. Lett. B **184**, 410 (1987); R. Najima, Prog. Theor. Phys. **77**, 926 (1987); S. Nandi, Phys. Lett. B **188**, 159 (1987).

[5]P. Chiappeta and F. Fergulio, Phys. Lett. B **213**, 95 (1988).

[6]C. Ahn and M. E. Peskin, Nucl. Phys. **B309**, 221 (1988); C. Ahn, Ph.D. thesis, SLAC Report No. 349, 1989.

[7]P. Mery, M. Perrottet, and F. M. Renard, Z. Phys. C **36**, 249 (1987).

[8]J. Bagger and S. Dimopoulos, Nucl. Phys. **B244**, 247 (1984); J. Bagger *et al.*, Nucl. Phys. **B258**, 565 (1985).

[9]F. de Augula *et al.*, Nucl. Phys. **B250**, 225 (1985).

[10]E. Witten, Nucl. Phys. **B186**, 412 (1981).

[11]A. L. Kagan and C. H. Albright, Phys. Rev. D **38**, 917 (1988).

[12]F. Gursey and P. Sikivie, Phys. Rev. D **16**, 816 (1977).

[13]T. G. Rizzo, Phys. Rev. D **34**, 1438 (1986); V. Barger *et al.*, *ibid.* **33**, 1912 (1986).

[14]S. Rajpoot, Phys. Rev. D **39**, 351 (1989); **36**, 1479 (1987); Phys. Rev. Lett. **60**, 2003 (1988); A. Davidson and K. C. Wali, *ibid.* **60**, 1813 (1988).

[15]P. Langacker and D. London, Phys. Rev. D **38**, 886 (1988). A general discussion of mixing between ordinary and exotic fermions is given in this paper.

[16]For mirror-lepton mixings, see, e.g., S. Singh and N. K. Sharma, Phys. Rev. D **36**, 160 (1987); J. Maalampi and K. Mursula, Phys. Lett. **157B**, 199 (1985); K. Enqvist, K. Mursula, and M. Roos, Nucl. Phys. **B226**, 121 (1983); S. Nandi, A. Stern, and E. C. G. Sudarshan, Phys. Rev. D **26**, 2522 (1982).

[17]For E_6 lepton mixings, see, e.g., S. Singh and N. K. Sharma, Phys. Rev. D **36**, 3387 (1987); T. G. Rizzo, *ibid.* **33**, 3329 (1986); **34**, 218 (1986); **34**, 1438 (1986); **34**, 2076 (1986); **34**, 2163 (1986); R. W. Robinett *ibid.* **33**, 1908 (1986); V. Barger *et al.*, *ibid.* **33**, 1912 (1986).

[18]P. Langacker and D. London, Phys. Rev. D **38**, 907 (1988); **39**, 266 (1989).

[19]The superscript 0 is used to indicate that these are the weak-interaction eigenstates.

[20]The right-handed neutrino field ν_{eR}^0 is included to allow for the possibility of nonzero Dirac neutrino mass.

[21]R. Foot, H. Lew, and G. C. Joshi, Phys. Lett. B **212**, 67 (1988), and references therein.

[22]Particle Data Group, G. P. Yost *et al.*, Phys. Lett. B **204**, 1 (1988).

[23]M. E. Rose, *Elementary Theory of Angular Momentum* (Wiley, New York, 1957). The d functions are also given in Ref. 22, p. 86.

[24]We omit a sign factor ϵ which is not required in the Jacob-Wick phase convention; see footnote on p. 263 of Ref. 1.

[25]In the extreme high-energy limit $\gamma \equiv \sqrt{s}/2m_W \rightarrow \infty$ and $\beta \rightarrow \infty$ for the production of longitudinal W's, the $\mathcal{D}_{00}$ term would give contributions proportional to $(m_N/m_W)\gamma$. This soft divergent term is, however, canceled when contribution from s-channel Higgs-boson exchange are included; see, e.g., V. D. Barger and R. J. N. Phillips, *Collider Physics* (Frontiers in Physics, Vol. 71) (Addison-Wesley, New York, 1987), p. 429. For this, in fact, one must also consider the contributions from Fig. 1 obtained by retaining terms proportional to the electron mass. We do not consider this case here because ignorance of terms proportional to m_e does not affect in a significant manner our result in Sec. IV.

[26]*Physics at LEP* (Ref. 2), pp. 70–72.

[27]For the type of mixing considered here, the lepton family (generation) number and the total lepton number is conserved for all the three cases.

[28]See footnote on p. 51 of Ref. 22.

[29]SLC Mark II Collaboration, G. S. Abrams *et al.*, Phys. Rev. Lett. **63**, 724 (1989).

[30]CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **63**, 720 (1989).

[31]The masses of mirror leptons are predicted to be ≤ 250 GeV in family unification models; see, e.g., G. Senjanvoic, F. Wilczek, and A. Zee, Phys. Lett. **141B**, 389 (1984); J. Bagger and S. Dimopoulos, Nucl. Phys. **B244**, 247 (1984); J. Bagger *et al.*, *ibid.* **B258**, 565 (1985).

[32]The singlet fermion masses are predicted to be in the range 100 GeV to 1 TeV; see, e.g., S. Rajpoot, Phys. Rev. D **36**, 1479 (1987).

[33]For numerical evaluation we take all the lepton mixing angles to be real.

[34]W. J. Marciano, Phys. Rev. D **20**, 274 (1979); A. Sirlin, *ibid.* **22**, 971 (1980); W. J. Marciano and A. Sirlin, *ibid.* **22**, 2695 (1980); **29**, 945 (1984).

[35]With this scheme, any other determination beyond the ratio of the physical boson masses requires correction for higher-order contributions.

[36]The error bars are obtained by dividing the range of $\cos\theta$ in ten equal bins and integrating the ordinate in respective bins.

[37]M. Lemoine and M. Veltman, Nucl. Phys. **B164**, 445 (1980).

[38]R. Philippe, Phys. Rev. D **26**, 1588 (1982).

[39]B. Grzadkowski and Z. Hioki, Phys. Lett. B **197**, 213 (1987).

[40]It is safe to ask the experimentalist to correct for this effect and then do the comparison with the effects of exotic-lepton mixings.

[41]In an earlier analysis, we have found that the meaurements of the differential angular distribution $d\sigma_L(\theta)$, $d\sigma_T(\theta)$, and the differential polarization asymmetry $\alpha_{\text{pol}}(\theta)$ in the process $e_{L(R)}^- e^+ \rightarrow Z^0 Z^0$ could be used to distinguish between the three types of mixed exotic leptons; see S. Singh, A. K. Nagawat, and N. K. Sharma, Phys. Rev. D **41**, 1438 (1990).

[42]We may point out that deviations from SM results can also occur in other theoretical frameworks. For instance, an anomalous moment κ or λ connected with WWZ or $WW\gamma$ vertices affects the differential angular distributions (see Ref. 2).