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Proposed room-temperature detector for gravitational radiation from galactic sources

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A room-temperature resonant-bar gravitational-radiation detector instrumented with a tunneling transducer may be sensitive enough to register supernovae events from the Galaxy. The key element in the new approach is the tunneling transducer, which is a quantum-limited electromechanical amplifier even at room temperature. We propose a design for a 2000-kg room-temperature, three-mode detector with a bandwidth of 200 Hz and a noise temperature of 100 mK, which corresponds to an rms strain sensitivity of $h \sim 4\times10^{-18}$.

I. INTRODUCTION

The invention of the tunneling transducer[1] has opened new vistas in the search for gravitational waves. The tunneling transducer holds great promise as an amplifier for gravitational-wave detectors because its performance at room temperature is limited only by zero-point fluctuations of the vacuum electromagnetic field. Unlike Josephson-effect amplifiers, which approach quantum-limited performance in a cryogenic environment, the tunneling transducer is quantum-limited even at room temperature. This new device offers unprecedented opportunities to improve the sensitivity of gravitational-wave detectors.

In this Rapid Communication we show that the unique properties of the tunneling transducer make it possible to take a radically different strategy in the design of massive, resonant-bar gravitational-wave detectors. By this approach, it should be possible to build a *room-temperature*, multimode detector with a burst noise temperature of 100 mK. For a 2000-kg detector, 2 m in length, this corresponds to an rms strain sensitivity of 4×10^{-18} which may be sufficient to detect bursts of gravitational radiation from galactic supernovae.

The key element in our approach is the low noise of the tunneling transducer. The additive noise of the tunneling transducer is extremely small and its gain makes the noise contributed by the following amplifier negligible. Furthermore, the fluctuating back action force of the probe on the antenna is insignificant. Thus the antenna is Brownian-motion limited. The Brownian motion may be minimized by building a wideband, multimode detector. In our proposed design for a room-temperature antenna, the bandwidth is about 200 Hz, which corresponds to a time resolution of 5 msec, or a travel distance of only 1000 km for a pulse of gravitational wave. So with such a short sampling time the direction of propagation and the velocity of gravitational radiation could be studied with a world-wide array of such detectors.

In the next section we present our model and the method used in the calculation of the sensitivity. This is followed by a number of design examples for both room-temperature and cryogenic detectors. In the final section we discuss some practical problems of this new approach and discuss its potential.

II. DETECTOR MODEL AND ANALYSIS METHOD

Multimode transducers have been under development for some time.[2] Those that are being developed are meant for capacitive or inductive transducers which both require, for strong electromechanical coupling, a large area and a light mass. It is difficult to make a small mass with a large area without unwanted low-frequency internal resonances. On the other hand, because a tunneling probe is coupled to a microscopic area, we need a small, compact mass, which is easily made. We note that, although the tunneling probe observes the oscillation of only a few atoms at the kilohertz operating frequency, these atoms are constrained to move collectively as part of the fundamental normal mode of the antenna.

The bandwidth of a multimode antenna is just the frequency difference between the extremal eigenmodes. In the continuous limit of an infinite number of modes the bandwidth is limited by the speed of propagation of an acoustic wave in the antenna material. The bandwidth in this limit is the frequency of the antenna itself.

As a compromise we consider a three-mode antenna consisting of a massive bar and a two-stage oscillator. This can give a reasonable bandwidth, 200 Hz in our room-temperature example, but is still practical to construct. The tuning requirement is not stringent: in practice the eigenfrequencies of the three separate resonators need only be frequency matched with a precision comparable to the bandwidth of the detector.

We assume a generic antenna of aluminum alloy with an inertial mass of 2000 kg, a fundamental frequency of 1 kHz and a length of 2 m. Attached to the end face of this antenna is a two-mode resonator with sequentially smaller masses m_2 and m_3 which we will determine in an optimization procedure. In our design a tunneling probe measures the relative separation between the third and first resonators, see Fig. 1. The mass of the first resonator m_1 is taken to be one half the inertial mass of the bar. The resonant frequencies of the individual resonators are $\omega_n=(k_n/m_n)^{1/2}$ and the resonator quality factors are $Q_n=\omega_n m_n/b_n$ $(n=1,2,3)$. The Langevin forces which are responsible for the Brownian motion of the three resonators are denoted by f_n, which have single-sided spectral densities, $S_{f_n}^{1/2}=2k_BTm_n\omega_n/Q_n$, where k_B is Boltzmann's constant and T is the thermodynamic temperature. The force of the gravitational wave, f_{gw}, acts on the first resonator; we will assume that it is an impulse of strength p_0 which arrives at $t=0$, i.e., $f_{gw}=p_0\delta(t)$. We write the equations of motion in terms of inertial coordinates:

$$m_1\ddot{x}_1=-b_1\dot{x}_1-k_1x_1+k_2(x_2-x_1)+b_2(\dot{x}_2-\dot{x}_1)+f_1+f_{gw}\ , \tag{1a}$$

$$m_2\ddot{x}_2=-k_2(x_2-x_1)-b_2(\dot{x}_2-\dot{x}_1)+k_3(x_3-x_2)+b_3(\dot{x}_3-\dot{x}_2)+f_2\ , \tag{1b}$$

$$m_3\ddot{x}_3=-k_3(x_3-x_2)-b_3(\dot{x}_3-\dot{x}_2)+f_3\ . \tag{1c}$$

We explicitly ignore the fluctuating back-action force of the tunneling transducer.[3] It would appear as a force between m_1 and m_3 with a white spectral density of $S_{f_{ba}}=\hbar\kappa(I_{dc}/e)^{1/2}$. For $\kappa=10^{10}\ \mathrm{m}^{-1}$ and $I_{dc}=10^{-5}$ A; $S_{f_{ba}}^{1/2}=8.3\times10^{-19}$ N/$\sqrt{\mathrm{Hz}}$. Comparing this to the Langevin force in an extreme case of $T=10$ mK and $Q=10^9$ for a transducer of mass 10^{-4} kg, which is $S_{f_1}^{1/2}=1.3\times10^{-16}$ N/$\sqrt{\mathrm{Hz}}$, we see that the back-action force will always be negligible.

The tunneling probe has an effective resistance which depends exponentially upon the gap (x_3-x_1), $R=R_0\times\exp[-2\kappa(x_3-x_1)]$, where R_0 is the nominal probe resistance, usually $10^6\ \Omega$ or greater and κ defines the distance scale over which tunneling takes place. A typical value of κ is $10^{10}\ \mathrm{m}^{-1}$. We assume that the displacements to be measured are very small so we can make a Taylor expansion of the exponential. The small signal current in excess of the nominal dc tunneling current is thus given by

$$i_{out}=2\kappa I_{dc}(x_3-x_1)+i_{shot}\ , \tag{2}$$

where i_{shot} is the current shot noise associated with the dc tunneling current I_{dc}. It has a single-sided spectral density $S_{i_{shot}}=eI_{dc}$, where e is the charge of the electron.

Tunneling currents up to 10 μA have been measured at room temperature under high vacuum for a gap of 10 Å with a potential difference of 1 V across the conductors.[4] Excess low-frequency noise, with a $1/f$-type spectrum, was observed in the tunneling current. This could be a major limitation in sensitivity and the source of this noise must be found and eliminated.

The figure of merit which is usually adopted for resonant-bar gravitational-radiation detectors is the minimum detectable or the noise equivalent impulse, i.e., the impulse strength $p_{0\min}$ which gives a signal-to-noise ratio in the filtered output of unity. This is usually expressed as a noise temperature $T_n=p_{0\min}^2/2m_1k_B$. In order to calculate the signal-to-noise ratio one must calculate the noise spectrum S_{i_n} of the measured quantity, which is i_{out} in this case, and the Fourier transform of the output current in response to the assumed signal; we denote this as $i_{sig}(\omega)$. The maximum signal-to-noise ratio is given by

$$\left[\frac{S}{N}\right]_{\max}^2=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{|i_{sig}(\omega)|^2}{S_{i_n}}d\omega\ . \tag{3}$$

We set $(S/N)_{\max}^2$ equal to unity and solve the above equation for $p_{0\min}$ from which we calculate the burst noise temperature of the antenna. Equations (1) and the integral of Eq. (3) are solved numerically in our analysis. In the next section we give some results of these calculations.

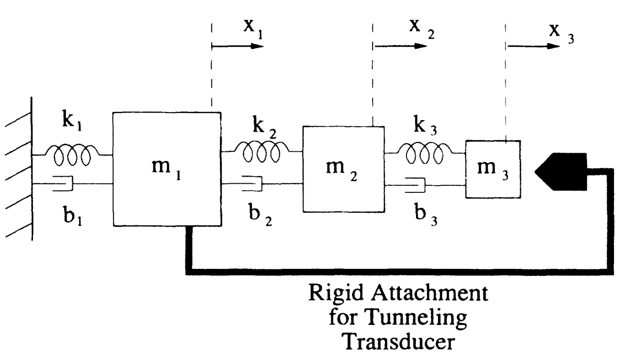

FIG. 1. Schematic diagram of antenna and two-stage resonator. Effective masses are denoted by m_i; k_i and b_i are the equivalent spring and damping constants. Dynamic displacements are shown as x_i; the tunnel transducer is sensitive to x_3-x_1.

III. RESULTS

In Table I we present the following four cases: (1) room-temperature aluminum antenna and transducer, (2) liquid-nitrogen temperature aluminum antenna and transducer, (3) liquid-nitrogen temperature silicon antenna and transducer, and (4) 50-mK temperature silicon antenna and transducer. We assume the aluminum antennae are tuned to 1 kHz; this corresponds to a length of about 2.4 m. Since the velocity of sound in silicon is approximately 9000 m/sec, as compared to 5500 m/sec for aluminum, the silocon antennae frequencies would necessarily be a bit higher. The longest silicon single crystals which can be made at present are approximately 3 m in length which would give such an antenna a resonant frequency of 1.5 kHz. Table I lists the masses, mechanical Q's, and the assumed tunneling current. The computed noise temperature is given in the table. The impulse strain sensitivity for a gravitational pulse of duration τ_g is calculated from

TABLE I. Four types of transducers.

	Case 1	Case 2	Case 3	Case 4
Antenna	Aluminum	Aluminum	Silicon	Silicon
T	300 K	77 K	77 K	0.05 K
L	2.4 m	2.4 m	3.0 m	3.0 m
v	5500 m/sec	5500 m/sec	9000 m/sec	9000 m/sec
m_1	1000 kg	1000 kg	75 kg	75 kg
m_2	8 kg	8 kg	0.05 kg	0.025 kg
m_3	0.2 kg	0.08 kg	5×10^{-4} kg	8.3×10^{-6} kg
Q_1	2.0×10^{5}	4.5×10^{5}	2.0×10^{8}	1.0×10^{9}
Q_2	1.0×10^{5}	2.0×10^{5}	2.0×10^{7}	1.0×10^{8}
Q_3	5.0×10^{4}	2.0×10^{5}	2.0×10^{6}	1.0×10^{7}
I_{dc}	1.0×10^{-5} A	1.0×10^{-5} A	1.0×10^{-5} A	1.0×10^{-5} A
T_n	102 mK	15 mK	0.33 mK	0.32 μK
h	4.2×10^{-18}	1.6×10^{-18}	4.1×10^{-19}	1.3×10^{-20}

the equation[5]

$$h=\left[\frac{\sqrt{2}}{\tau_g}\right]\left[\frac{L}{v^2}\right]\left[\frac{k_B T_n}{m_1}\right]^{1/2} . \tag{4}$$

The speed of sound in the antenna material is v and L is the length of the antenna; τ_g is assumed to be 1 msec. The optimum mass ratios were determined numerically by constructing contour plots of the noise temperature as a function of the two mass ratios m_2/m_1 and m_3/m_1.

The first case in Table I represents the most exciting prospect. The result shows that it ought to be possible to make a detector at room temperature that is sensitive to supernovae events in our Galaxy.

The bandwidth is about 200 Hz which corresponds to a measurement time of about 5 msec. As stated earlier, the detector is Brownian-motion limited so we can estimate the antenna noise temperature from the familiar formula $T_n = T(\tau_{\text{meas}}/\tau_0)$, where τ_{meas} is the measurement averaging time and τ_0 is the $1/e$ relaxation time of the antenna which is given by $\tau_0 = Q/\omega$. In case 1, $\tau_{\text{meas}}/\tau_0$ is $\frac{0.005}{16}$ (for $Q \sim 10^5$) so T_n is estimated to be 94 mK which is close to the number we calculate in the complete analysis.

In case 2 we consider the improvement in sensitivity which is possible by cooling the antenna to 77 K, the temperature of liquid nitrogen. Apart from the factor of 4 reduction in the physical temperature, which gives the same reduction factor in the noise temperature, there is an additional noise improvement from the increase of the mechanical Q's. Relative to a room-temperature antenna, one at 77 K is only a little more difficult to operate continuously.

In case 3 we give a set of parameters which could describe an all-silicon antenna operated at 77 K. It is now possible to grow single crystals of silicon up to 150 kg. Because of the increased velocity of sound in silicon, approximately 9000 m/sec in Si as compared to 5500 m/sec in Al the cross section for gravitational radiation is increased and the factor that is lost due to the decrease in mass is almost compensated by the sound-velocity increase. The foremost advantage of silicon is the potential for achieving much higher Q factors. A Q of 2×10^8 has been measured in a 30-kg silicon crystal at 77 K, which is close to the fundamental limit at that temperature.[6] Moreover, it would be somewhat easier to build a detector with a mass of 150 kg rather than 2000 kg.

The last case is included to show that under extreme cryogenic conditions noise temperatures can be in the nanokelvin range if near state-of-the-art mechanical Q's can be achieved. If it were possible to operate a detector under these conditions the strain sensitivity would be approximately 10^{-20}.

We emphasize that since the detector we propose is at room temperature it should operate continuously. In other words its duty cycle is close to 100% which is of utmost importance in a practical search for gravitational radiation.

Our conclusions are based on the assumptions that (i) a steady tunnel current of 1–10 μA can be maintained continuously. Of special concern is the onset of "popcorn" noise bursts in the tunnel current. (ii) Q's of 50000 can be obtained for small aluminum oscillators at room temperature. Although we have obtained a Q of 250000 at room temperature with a 200-kg aluminum dumbbell antenna, thermoelastic or other losses may limit the Q's of smaller masses to about 50000. On the other hand, Q's of 600000 have been measured at room temperature for small silicon crystal oscillators.[7] These assumptions are quite reasonable, however, they need to be verified.

The question of the rate of expected gravitational-wave events from our Galaxy is not settled by any means. Estimates both of the strength and the rate of occurrence of sources of gravitational radiation are uncertain by several orders of magnitude.[8] Under such circumstances it may be prudent to attempt to measure these quantities using the instrument we have described. With experimental data from an array of sensitive detectors, fully automated and operating continuously at room temperature (or at 77 K) for several years it may be possible not only to establish the existence of gravitational waves but also to determine the rate at which sources emit gravitational radiation within our Galaxy. This is the strategy we propose.

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