

## Rapid Communications

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## Predictions from three-generation Calabi-Yau string theory

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Superstring models are considered where there is  $E_6$  flux breaking to  $[SU(3)]^3$  at compactification and dynamical breaking of  $[SU(3)]^3$  to  $SU(3)_C \times SU(2)_L \times U(1)_Y$  at an intermediate scale  $M_I$ . If the intermediate scale breaking preserves matter parity (to stabilize the proton) and is triggered by supersymmetry breaking, then it is shown that there will always be at least two and often four new light  $SU(3)_C \times SU(2)_L \times U(1)_Y$  neutral chiral multiplets (in addition to the usual standard-model states). The interactions of these new particles with the standard-model particles are deduced and are expected to produce  $\mu \rightarrow e\gamma$  decay and in some cases neutrino masses and oscillations in a range that may be accessible to experiment.

In this Rapid Communication we consider properties of the heterotic string<sup>1</sup> compactified on a three-generation Calabi-Yau manifold. At compactification, these theories possess an  $E_6$  internal symmetry with massless particles in the  $\mathbf{27}$ ,  $\overline{\mathbf{27}}$ , and singlet representations. Two such manifolds that have been well studied are (i) the manifold of Tian and Yau<sup>2</sup> which is defined as the submanifold of  $CP^3 \times CP^3$  given by the intersection of three polynomials suitably modded by  $Z_3$ , and (ii) the manifold of Schimmrigk<sup>3</sup> defined as the submanifold of  $CP^3 \times CP^2$  given by the intersection of two polynomials and suitably modded by  $Z_3 \times Z'_3$ . (For a full definition of these manifolds see Refs. 2-6.) One of the unique features of string theory is the fact that in principle the theory determines all Yukawa couplings. In practice, however, Yukawa couplings can be calculated only for the simplest symmetric manifolds, and then only partially. In particular, the nonrenormalizable  $(\mathbf{27} \mathbf{27})^n$  couplings, which play an important role in determining the low-energy predictions of the theory are still mostly inaccessible.

In spite of these difficulties, one may learn a great deal about these models by imposing the simple requirement that the string model reduces to the standard model at low energies<sup>4,5</sup> (clearly a necessary requirement for any viable theory). The Tian-Yau and Schimmrigk manifolds are nonsimply connected allowing for flux breaking of  $E_6$  at the compactification scale  $M_c \approx M_{Pl}$ . If we embed one  $Z_3$  nontrivially into  $E_6$ , then the only possible flux breakings that preserve the standard model are<sup>6</sup>  $E_6 \rightarrow [SU(3)]^3$ ,  $SU(6) \times U(1)$ , and  $E_6$  (i.e., no flux breaking). The  $[SU(3)]^3$  content of the  $i$ th  $\mathbf{27}$  generation can be written

as  $L_i^l(1,3,\overline{3}) \oplus Q_i^a(3,\overline{3},1) \oplus (Q^c)_{ai}^r(\overline{3},1,3)$ , where  $a, l, r = 1, 2, 3$  label the  $SU(3)_{C,L,R}$  states. The lepton nonet is  $L = [l = (\nu, e); e^c; H; H'; \nu^c; N]$ , while the quark and conjugate quark nonets are  $Q = [q^a = (u, d)^a; H_3 = D^a]$  and  $Q^c = (u_a^c; d_a^c; H_3^c = D_a^c)$ . Here  $l, H, H'$ , and  $q^a$  are  $SU(2)_L$  doublets,  $D^a, D_a^c$  are color triplets, and  $\nu^c, N$  are  $SU(5)$  singlets [ $N$  is also an  $O(10)$  singlet]. Further breaking of  $E_6$  which preserves the standard model can arise only dynamically at some intermediate scale  $M_I$  from vacuum-expectation-value (VEV) growth of  $\nu^c$  and  $N$ . (One generally expects  $\langle N \rangle, \langle \nu^c \rangle \gtrsim 10^{15}$  GeV.) This shows that the second two flux-breaking possibilities above are unsatisfactory as one would be left with a residual  $SU(5)$  symmetry at low energy, since there are no massless adjoint representations in the string spectrum to produce further breaking.<sup>7</sup> However, since<sup>4</sup>  $N = L_3^3$ ,  $\nu^c = L_2^3$ , it is clear the VEV's  $\langle N_i \rangle, \langle \nu_i^c \rangle$  precisely break  $[SU(3)]^3$  to  $SU(3)_C \times SU(2)_L \times U(1)_Y$ . Thus we assume from now on that flux breaking implies  $E_6 \rightarrow [SU(3)]^3$ .

The intermediate-scale spontaneous breaking is a dynamical phenomenon. In general, this requires a (mass)<sup>2</sup> to turn negative. The only mass present after compactification for the  $\mathbf{27}$  and  $\overline{\mathbf{27}}$  states are the soft breaking masses  $m \sim 1$  TeV arising from supersymmetry breaking. These (mass)<sup>2</sup> are expected to turn negative at<sup>8</sup>  $M_I \approx 10^{16}$  GeV, leading to VEV growth from the  $(\mathbf{27} \mathbf{27})^n$  nonrenormalizable interactions.<sup>4-6</sup>

One can always make a linear transformation on the generation index so that only  $\langle N_1 \rangle \neq 0$  and all other  $\langle N_i \rangle$  vanish. If, in addition  $\langle \nu_i^c \rangle = 0$ , then one can also arrange that  $\langle \nu_2^c \rangle \neq 0$  and all other  $\langle \nu_i^c \rangle$  vanish. This will occur nat-

usually if  $N_i$  and  $v_f^i$  fall in different symmetry classes. In fact, the requirement of proton stability imposes matter-parity ( $M_2$ ) invariance on the manifold.<sup>9</sup> Here  $M_2 = CU_z$ , where  $U_z$  is an element of  $[SU(3)]^3$ ,  $\text{diag } U_z = (1, 1, 1) \times (-1, -1, 1) \times (-1, -1, 1)$ , and  $C$  operates on the generation index ( $C^2 = 1$ ). Since  $U_z N_i = N_i$ ,  $U_z v_f^i = -v_f^i$  we see that  $M_2$  invariance requires  $i = 1$  to be  $C$  even and  $i = 2$  to be  $C$  odd.

One can now prove the following theorem.

For any Calabi-Yau string model with (i) breaking of  $E_6 \rightarrow [SU(3)]^3$  at the compactification scale  $M_c$  (e.g., from flux breaking), (ii) intermediate-scale breaking of  $[SU(3)]^3 \rightarrow SU(3) \times SU(2) \times U(1)$  at the scale  $M_{PI} > M_I \gtrsim 10^{15}$  GeV triggered by a mass  $m \lesssim 1$  TeV (which is true if triggered by supersymmetry breaking), and (iii) VEV's of  $N_i, v_f^i$  obeying  $\sum \langle N_i \rangle \langle v_f^i \rangle = 0$  (as is true with  $M_2$  invariance), then there are always at least two new non- $E_6$ -singlet exotic light chiral multiplets (in addition to the three generations of light states of the standard model), and in some cases four new non- $E_6$ -singlet light chiral multiplets. In addition there may also be light  $E_6$ -singlet fields.

In the frame where only  $\langle N_1 \rangle$  and  $\langle v_2^i \rangle$  are nonzero, these new non- $E_6$ -singlet states are

$$n_1 = (N_1 + \bar{N}_1)/\sqrt{2}, \quad \hat{v}_2^c = (v_2^c + \bar{v}_2^c)/\sqrt{2}, \quad (1a)$$

$$n_2 = \cos\theta N_2 + \sin\theta \bar{v}_1^c, \quad \bar{n}_2 = \cos\theta \bar{N}_2 + \sin\theta v_1^c, \quad (1b)$$

where  $\tan\theta = \langle v_2^c \rangle / \langle N_1 \rangle$ . Here Eq. (1a) represents the two new states that are always present while Eq. (1b) may also be present in some cases.

To prove the above result, we first examine the states in the  $i = 1, 2$  generations which become superheavy due to intermediate-scale breaking when  $[SU(3)]^3$  breaks to  $SU(3) \times SU(2) \times U(1)$ , and 12 vector bosons become massive by absorbing 12 Goldstone bosons. The supersymmetric (SUSY) massive vector multiplets that form have also 12 massive scalar bosons as well as 12 massive Dirac

spinor  $\Psi$ . The latter arise by combining the gaugino partners  $\lambda$  of the massive vector bosons with the Weyl spinor partners  $\chi_L$  of the absorbed Goldstone bosons. Thus  $\Psi = \chi_L + iP_R \lambda \equiv (\lambda; \chi_L)$ , where  $P_R = (1 + \gamma^5)/2$ . To pick out the fermion states that become massive, we note that the  $SU(3)_L$  gaugino interactions lead to the mass term

$$L_{SU(3)_L} = -i\sqrt{2}g_L \bar{\lambda}_L^a [(T^a) \chi_{ir}^{\prime\prime} \langle (\Phi_i^{\dagger})_r \rangle - \bar{\chi}_{ir}^{\prime\prime} (T^a) \langle (\bar{\Phi}_i^{\dagger})_r \rangle] + \text{H.c.}, \quad (2)$$

where  $\lambda_L^a, g_L$  are the  $SU(3)_L$  gaugino and gauge coupling constant,  $2T^a$  are the  $SU(3)$  Gell-Mann matrices,  $(\chi_i, \Phi_i)$  are the chiral multiplets ( $\Phi_{3i}^3 \equiv N_i, \Phi_{2i}^3 \equiv v_f^i$ ), and  $(\bar{\chi}_i, \bar{\Phi}_i)$  are the corresponding mirror generation multiplets. [A similar formula holds for the  $SU(3)_R$  gauge interactions.] Direct inspection allows one easily to pick out the 12 massive Dirac states (the spontaneous breaking must be approximately  $D$  flat<sup>4,5</sup> implying  $\langle N_1 \rangle \equiv \langle \bar{N}_1 \rangle$ , etc.):

$$(\lambda_L^{(+)}; c l_1 + s H_2^c), (\lambda_L^{(-)}; c \bar{l}_1 + s \bar{H}_2^c), \\ (-\lambda_{1+}^R; e_2^c), (\lambda_{1-}^R; e_2^c), (-\lambda_{4+}^R; e_1^c), (\lambda_{4-}^R; e_1^c), \quad (3)$$

$$(\lambda_{6-}^R; c v_f^i - s \bar{N}_2), (\lambda_{6+}^R; c \bar{v}_f^i - s N_2),$$

$$(\hat{g}_R \lambda_8^R - \hat{g}_L \lambda_8^L; (N_1 - \bar{N}_1)/\sqrt{2}), \quad (4)$$

$$(\frac{1}{2} \hat{g}_R (\sqrt{3} \lambda_3^R - \lambda_8^R) - \hat{g}_L \lambda_8^L; (v_2^c - \bar{v}_2^c)/\sqrt{2}),$$

where  $\lambda_L^{(\pm)} = \pm (\lambda_{4\pm}^L, \lambda_{6\pm}^L)$ ,  $\lambda_{4\pm} = (\lambda_4 \pm i\lambda_5)/\sqrt{2}$ , etc.,  $c = \cos\theta, s = \sin\theta$ , and  $\hat{g}_{R,L} = g_{R,L}/(g_R^2 + g_L^2)^{1/2}$ .

We see from Eq. (4) that the four states of Eq. (1) are those that are orthogonal to the states involving  $N_i, v_f^i$  that become massive by the gauge interactions. Thus the gauge interactions give them zero mass. To verify that they are indeed light, we must check that they do not grow superheavy masses from the superpotential  $W$ . The general form of the renormalizable (27)<sup>3</sup> interactions read<sup>4</sup>

$$W_{(27)^3} = (\lambda_{ijk}^1 d_i u_j D_k + \lambda_{ijk}^3 u_i^c d_j^c D_k^c) + \lambda_{ijk}^3 (-H_i H_j N_k - H_i v_j^c l_k + H_i^c e_j^c l_k) \\ + \lambda_{ijk}^4 (-D_i N_j D_k^c + D_i e_j^c u_k^c - D_i v_j^c d_k^c - q_i l_j D_k^c + q_i H_j u_k^c + q_i H_j^c d_k^c), \quad (5)$$

where  $\lambda_{ijk}^1$ , etc. are coupling constants (to be determined from the complex structure). A similar expression holds for the  $(\bar{27})^3$  couplings. One sees that when  $\langle N_i \rangle, \langle v_f^i \rangle$  become nonzero, this does not produce any mass growth in  $N_i$  or  $v_f^i$ , preserving the massless nature of the states of Eq. (1). The nonrenormalizable contributions  $W_{nr} = (27 \bar{27})^n / M_c^{2n-3}$  produce masses for states in the  $\mathbf{27}$  and  $\bar{\mathbf{27}}$  of size  $M_{nr} \approx \langle N \rangle^{2n-2} / M_c^{2n-3}$ . Minimizing the effective potential  $V_{\text{eff}}$  shows that<sup>4,5</sup>  $\langle N \rangle^{4n-4} \approx m^2 M_c^{4n-6}$ , where  $m \lesssim 1$  TeV is the mass triggering the spontaneous breaking, e.g., the SUSY soft-breaking mass.<sup>10</sup> [This can be seen from dimensional analysis since, e.g., supersymmetry breaking produces a contribution to  $V_{\text{eff}}$  of size  $(-m^2)N^2$ .] Thus  $M_{nr} \approx m$  and hence the nonrenormalizable  $F$  terms produce only electroweak size masses to any state.

In addition to the  $\mathbf{27}_i$  and  $\bar{\mathbf{27}}_i$  states, there are  $E_6$  sing-

lets  $\Phi_a$  of  $H^1[\text{End}(T)]$ , whose couplings are

$$W_{\Phi} = M_{ab} \Phi_a \Phi_b + \lambda_{abc} \Phi_a \Phi_b \Phi_c + \lambda_{aij} \Phi_a \mathbf{27}_i \bar{\mathbf{27}}_j. \quad (6)$$

At an arbitrary point in moduli space, instanton corrections can grow masses for some of the  $\Phi_a$  of size<sup>11</sup>  $M_{ab} M_c [\exp(-R/R_P)]$ , where  $R$  is the "radius" of the Calabi-Yau manifold and  $R_P = 1/M_{PI}$ . Those  $\Phi_a$  which become superheavy can be eliminated in a power series in  $\mathbf{27} \bar{\mathbf{27}}$ , i.e., their effects have already been included in  $W_{nr}$ . The remaining massless or light  $E_6$  singlets will contribute to intermediate-scale symmetry breaking from the terms

$$\lambda_{a11} \Phi_a N_1 \bar{N}_1 + \lambda_{a22} \Phi_a v_2^c \bar{v}_2^c, \quad (7a)$$

$$\lambda_{a12} \Phi_a (N_1 \bar{N}_2 + v_1^c \bar{v}_2^c) + \lambda_{a21} \Phi_a (N_2 \bar{N}_1 + v_2^c \bar{v}_1^c). \quad (7b)$$

[For  $M_2$  matter parity, only the  $C$ -even  $\Phi_a$  enter in Eq. (7a) and only the  $C$ -odd  $\Phi_a$  in Eq. (7b).] If couplings of

Eq. (7a) are present, these cubic terms dominate the symmetry breaking leading, in the usual fashion, to an intermediate scale  $M_I \approx m \sim 1$  TeV. This violates hypothesis (ii) of the theorem which thus requires that acceptable manifolds have vanishing  $\lambda_{a11}$  and  $\lambda_{a22}$ . [Manifolds with  $M_I \approx m$  are, of course, phenomenologically unacceptable, leading, e.g., to very rapid proton decay, which is why hypothesis (ii) is imposed in the theorem.] On the other hand, using an analysis similar to that of Ref. 4, it is easy to show that by minimizing the effective potential the terms of Eq. (7b) are flat at the true minimum of the effective potential since  $\lambda_{a12}\Phi_a$ ,  $\lambda_{a21}\Phi_a$ ,  $N_2$ ,  $\bar{N}_2$ ,  $v_1^c$ ,  $\bar{v}_1^c$  all have vanishing VEV's. (Other extrema, where these fields grow VEV's, do exist but lie considerably higher. They would correspond to spontaneous breaking of matter parity.) Thus the Eq. (7b)  $E_6$ -singlet interactions do not contribute to the intermediate-scale symmetry breaking. They can, however, contribute to the mass matrix, i.e., replacing  $N_1$ ,  $\bar{N}_1$ ,  $v_1^c$ ,  $\bar{v}_1^c$  by their VEV's, the mass terms of Eq. (7b) are  $M(\lambda_{a12}\Phi_a\bar{n}_2 + \lambda_{a21}\Phi_a n_2)$ , where  $M \equiv (\langle N_1 \rangle^2 + \langle v_1^c \rangle^2)^{1/2}$ , which would make  $n_2$  and  $\bar{n}_2$  superheavy. Hence we conclude that the fermion states of Eq. (1a) are light with mass  $O(m) \lesssim 1$  TeV, and by softly broken supersymmetry, their scalar particles must also be light. If the couplings  $\lambda_{a21}$  and/or  $\lambda_{a12}$  vanish, then  $n_2$  and/or  $\bar{n}_2$  will also be light. This result is unchanged when one includes nonrenormalizable terms involving  $E_6$  singlets in Eq. (6).

The above result is quite general, depending mainly on the supersymmetric gauge interactions, a stable proton, and the role of supersymmetry breaking in producing intermediate-scale breaking. *The existence of these additional light states represents, therefore, an important low-energy consequence of this class of string theories.*

To see what signals these new particles might produce, we examine next their interaction structure. To do this, we must now be more specific about the properties of the manifolds. We assume explicitly now that  $M_2$  is conserved. For convenience, let us denote the generation index as  $i = (n, r)$ , where  $n$  labels the  $C$ -even states and  $r$  the

$C$ -odd states. One may divide the general mass matrix below  $M_I$  into the  $M_2$ -even part and the  $M_2$ -odd part. The former has its rows labeled by  $(H'_n, \bar{H}'_n, l_r) \equiv \Phi'_a$  and columns by  $(H_n, \bar{H}_n, \bar{l}_r) \equiv \Phi_a$ . For the string model to be in accord with the standard model, there must be one pair of light Higgs doublets  $H$  and  $H'$  to effect electroweak breaking. (More than one light Higgs boson will, in general, produce flavor-changing neutral-current interactions as well as make the fit to  $\sin^2\theta_W$  more difficult.) How these light Higgs bosons arise is discussed elsewhere.<sup>12</sup> We use here only the fact that the three light generations of quarks and leptons lie in the  $M_2$ -odd sector with  $C$ -even doublets  $l_n, q_n$  (as is the case for the Tian-Yau and Schimrigk manifolds). Then the light Higgs bosons must lie in the  $M_2$ -even sector if these Higgs bosons are to give quarks and leptons mass after electroweak breaking (when  $\langle H \rangle, \langle H' \rangle$  become nonzero). In this situation  $H$  is a linear combination of  $\{\Phi_a\}$  and  $H'$  of  $\{\Phi'_a\}$ . One can write, therefore,  $\Phi_a = V_{aH}H + V_{aa}\chi_a$  and  $\Phi'_a = V'_{aH'}H' + V'_{aa}\chi'_a$ , where the unitary matrices  $\{V_{aH}, V_{aa}\}$  and  $\{V'_{aH'}, V'_{aa}\}$  diagonalize  $M_{ab}^{(e)}$ , the  $M_2$ -even mass matrix (i.e.,  $\Phi'_a M_{ab}^{(e)} \Phi_b$  with  $\bar{V}M^{(e)}V = \text{diag}$ ). Here the  $\{\chi_a, \chi'_a\}$  fields remain superheavy. The  $M_2$ -odd mass matrix  $\xi_a M_{ab}^{(o)} \xi_b$  is labeled (for the fermionic sector) by  $\xi_a = (\lambda_L^{(-)}, l_n, \bar{H}_r, H'_r)$  and  $\xi_b = (\lambda_L^{(+)}, \bar{l}_n, H_r, \bar{H}'_r)$ , where  $a = 1, \dots, n_a$ ,  $b = 1, \dots, n_b$ . By hypothesis,  $n_a = n_b + 3$ , i.e., three of the  $\xi_a$  are massless after intermediate-scale breaking (and prior to electroweak breaking). These are the three light  $SU(2)_L$ -doublet lepton generations,  $l_p$ ,  $p = 1, 2, 3$ , of the standard model. Thus one may write  $\xi_a = l_p U_{pa}^\dagger + \eta_a U_{aa}^\dagger$ , where  $\eta_a$  and  $\xi_b$  remain superheavy. One similarly has that the light quarks  $q_p, u_p^c, d_p^c$  and conjugate leptons  $e_p^c$ ,  $p = 1, 2, 3$  lie in the  $C$ -even sector,  $q_n, u_n^c, d_n^c, e_n^c$ , with unitary transformations similar to Eq. (7) relating these  $C$ -even fields to the standard model states  $q_p, u_p^c, d_p^c, e_p^c$ ,  $p = 1, 2, 3$ .

Using the above results, one may eliminate the  $\Phi_a, \Phi'_a, \xi_a, \eta_a$ , etc. in terms of the mass eigenstate below  $M_I$ . Keeping only the terms with light fields yields the low-energy effective theory. From the superpotential  $W_{(27)^3}, W_{(\bar{27})^3}$  of Eq. (5) one finds

$$W_{\text{eff}} = (\lambda_{pp'}^{(l)} H' e_p^c l_{p'} + \lambda_{pp'}^{(u)} H q_p u_{p'}^c + \lambda_{pp'}^{(d)} H' q_p d_{p'}^c) + \{[\lambda_p H l_p n_2 + \bar{\lambda}_p H l_p \bar{n}_2 + (\lambda_1 n_1 + \hat{\lambda}_2 \hat{v}_2^c) H H'] + (\lambda_{ap}^{(0)} \Phi_a^{(0)} H l_p + \lambda_{ap}^{(e)} \Phi_a^{(e)} H' H) + (m_1 n_2 \bar{n}_2 + m_2 n_2 n_2 + m_3 \bar{n}_2 \bar{n}_2) + (m_4 n_1 \hat{v}_2^c + m_5 n_1 n_1 + m_6 \hat{v}_2^c \hat{v}_2^c) + m_{ab} \Phi_a \Phi_b\} + W_{\text{seesaw}}, \quad (8)$$

and from the gaugino interactions (of  $\lambda_L^{(-)}$ )

$$\mathcal{L}_{\text{gaugino}} = g_L U_{p\lambda}^\dagger l_p \gamma^0 [\lambda_{pp'}^{(g)} e_p^c H^\dagger + 2^{-1/2} (s n_1 - c \hat{v}_2^c) \bar{l}_{p-1}^\dagger + (\lambda' n_2 + \bar{\lambda}' \bar{n}_2) H'^\dagger + H (\lambda n_2^\dagger + \bar{\lambda} \bar{n}_2^\dagger)] + \text{H.c.} \quad (9)$$

In Eqs. (8) and (9), the couplings  $\lambda_{pp'}^{(l)}, \lambda_{pp'}^{(u)}$ , etc., are just the elementary coupling constants, e.g.,  $\lambda_{ijk}^3$  multiplied by the various unitary transformations projecting to the light sector, e.g.,  $V_{aH}, U_{pa}^\dagger$ , etc.  $\Phi_a = (\Phi_a^{(0)}, \Phi_a^{(e)})$  are the  $C$ -odd,  $C$ -even  $E_6$ -singlet fields that remain light. [If  $n_2$  and/or  $\bar{n}_2$  become superheavy, they are to be deleted in Eqs. (8) and (9).] The  $H^\dagger, H'^\dagger, \bar{l}^\dagger$ , etc., are scalar fields, e.g.,  $\bar{l}_{p-1}$  is the slepton partner of the  $p = 1$  lepton.

The first set of parentheses in Eq. (8) just represents the low-energy standard-model superpotential. *Equation (9) and the quantity in the curly brackets in Eq. (8) are the*

*interactions involving the new low-energy particles of Eq. (1) [ $m_i \lesssim 1$  TeV are their masses]. These represent the new physics predicted by these Calabi-Yau string models that obey the standard model at low energies.* In addition to this there are seesaw masses coming from  $\nu_p$ -Higgs-boson-superheavy interactions, obtained from eliminating the superheavy fields:  $W_{\text{seesaw}} = \frac{1}{2} \nu_p \mu_{pp'} \nu_{p'}$ . One can show that<sup>4,13</sup>  $\mu_{pp'} \approx m_l^2 / M_I$ , where  $m_l$  are charged-lepton masses and  $M_I$  the scale of the superheavy masses. For  $m_l$  one has  $\mu_{pp'} \approx 10^{-6}$  eV. Below the electroweak scale where  $\langle H \rangle \neq 0$ , there are additional neutrino mass terms

due to the neutrino coupling to the new light particles  $n_2$ ,  $\bar{n}_2$ , and  $\Phi_a^{(0)}$ . The off-diagonal couplings of  $\nu_p$  to these fields are  $\mu_p = \lambda_p \langle H \rangle$ ,  $\bar{\mu}_p = \bar{\lambda}_p \langle H \rangle$ ,  $\mu_{ap} = \lambda_{ap}^{(0)} \langle H \rangle$ , etc., where  $\mu_{pp'} \ll \mu_p, \bar{\mu}_p, \mu_{ap} \ll m_i, m_{ab}$ . If none of the  $n_2, \bar{n}_2, \Phi_a^{(0)}$  remain light all three neutrinos have mass of size  $O(\mu_{pp'}) \approx 10^{-6}$  eV. If one of  $n_2, \bar{n}_2, \Phi_a^{(0)}$  is light, then one neutrino becomes heavier with mass  $O(\mu_p^2/m_i)$  (e.g.,  $\sim 1$  eV for  $\mu_p \approx 1$  MeV). In general if  $k$  of the  $n_2, \bar{n}_2, \Phi_a^{(0)}$  are light,  $k$  of the neutrinos become heavier (e.g.,  $\sim 1$  eV for  $k \leq 3$  and all three are heavier for  $k > 3$ ). The neutrino mass matrix also gives rise to neutrino oscillations; the case where  $n_2$  and  $\bar{n}_2$  remain light but no light  $\Phi_a$  survives is treated in Ref. 13.

The theorem proven in this paper guarantees that  $n_1$  and  $\hat{\nu}_2$  of Eq. (1a) are always residual light states irrespective of the mixing with  $E_6$  singlets. Thus the interactions of Eq. (9) leads to the decay  $\mu \rightarrow e + \gamma$ . Analysis using an expansion in the mixing parameter  $\tan\theta \approx \langle \nu_2^c \rangle / \langle N_1 \rangle$  shows that  $U_{\rho\lambda}^{\dagger} \approx (\tan\theta)^3$  and Eq. (9) then leads to a  $\mu \rightarrow e\gamma$  decay mediated by  $\hat{\nu}_2^c$  exchange with an effective interaction  $\mathcal{L}_{\text{eff}} \approx (ea/4m_\mu) F^{\mu\nu} \bar{\mu} \sigma_{\mu\nu} e_R + \text{H.c.}$ , where  $a = (\alpha_2/8\pi)(m_e/m_\tau) \tan^4\theta (m_\mu^2/m_{\hat{\nu}_2^c}^2) L(m_e^2/m_{\hat{\nu}_2^c}^2)$  and  $L$  is the loop function. The current experimental

limit on  $a$  is<sup>14</sup>  $a \leq 2.4 \times 10^{-13}$  and this limit is expected to improve by an order of magnitude at the Los Alamos MEGA experiment. Thus the  $\mu \rightarrow e\gamma$  experiment becomes a probe for the mass of the  $\hat{\nu}_2^c$  particle. With the expected range of  $\tan\theta \lesssim 1.0$ ,  $\hat{\nu}_2^c$  mass  $M_{\hat{\nu}_2^c}$  below 1 TeV becomes accessible.

We have seen that three-generation Calabi-Yau compactifications of the heterotic superstring where  $E_6$  flux breaks to  $[SU(3)]^3$  and  $[SU(3)]^3$  subsequently breaks to the standard model at  $M_I$  will always lead to the two new light chiral multiplets of Eq. (1a) provided only that matter parity is preserved at the intermediate scale (to stabilize the proton) and the intermediate-scale breaking is triggered by soft supersymmetry breaking. With additional (mild) assumptions, the nature of the interactions of these new particles, Eqs. (8) and (9) can be deduced, which leads to neutrino masses and interactions whose effects may be experimentally detectable.

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