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Gluon color-electric dipole moment and its anomalous dimension

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The purely gluonic dimension-6 CP-violating operator recently discovered by Weinberg is identified as a color-electric dipole moment for the gluon. It can be represented in a manifestly Bose-symmetric form using Dirac algebra. This simplifies the calculation of its anomalous dimension to first order in the QCD coupling constant.

It was recently pointed out by Weinberg[1] that there is a CP-violating operator of dimension 6 that is constructed out of gluon fields only. It can be induced as a term in the low-energy effective Hamiltonian by exchange of heavy Higgs bosons,[1,2] by the exchange of gluinos in supersymmetric models,[3] or by exchange of gauge bosons in left-right-symmetric models.[4] It can also arise from the color-electric dipole moment of a heavy quark at scales below the heavy-quark threshold.[5] It gives a contribution to the neutron electric dipole moment that is not suppressed by any light-quark masses or mixing angles, and so can tighten constraints on certain models for CP violation.

The gluonic operator $\mathcal{O}_G$ is generated at the mass scale of some heavy particle. To compute the electric dipole moment of the neutron, it must be evolved down to the hadronic scale using the renormalization group. This evolution can change the prediction for the neutron electric dipole moment by orders of magnitude. In this paper, we present some details of our recent calculation of the anomalous dimension of $\mathcal{O}_G$ to first order in the QCD coupling constant.[6] The calculation was greatly simplified by using Dirac algebra to represent the operator in a Bose-symmetric form. This representation may prove valuable in other investigations of the operator $\mathcal{O}_G$. It also reveals that this operator has a simple physical interpretation as a color-electric dipole moment operator for the gluon.

The purely gluonic dimension-6 CP-violating operator discovered by Weinberg[1] is

$$\mathcal{O}_G(\mu) = -\tfrac{1}{3} f^{abc} g_{\alpha\beta} \tilde{G}^a_{\mu\nu} G^{b\mu\alpha} G^{c\nu\beta}, \tag{1}$$

where $\tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho} G^{a\lambda\rho}$, μ is the renormalization scale, and our convention is $\epsilon^{0123} = +1$. This expression for $\mathcal{O}_G$ is not manifestly Bose symmetric. It can be written in a compact form which is also Bose symmetric by using Dirac algebra:

$$\mathcal{O}_G(\mu) = \frac{i}{768} f^{abc} G^a_{\mu\nu} G^b_{\lambda\rho} G^c_{\sigma\tau} \times \mathrm{Tr}([\gamma^\mu,\gamma^\nu][\gamma^\lambda,\gamma^\rho][\gamma^\sigma,\gamma^\tau]\gamma_5). \tag{2}$$

The Bose symmetry follows from the antisymmetry of f^{abc} together with the Dirac trace identity

$$\mathrm{Tr}([\gamma^\mu,\gamma^\nu][\gamma^\lambda,\gamma^\rho][\gamma^\sigma,\gamma^\tau]\gamma_5) = -\mathrm{Tr}([\gamma^\lambda,\gamma^\rho][\gamma^\mu,\gamma^\nu][\gamma^\sigma,\gamma^\tau]\gamma_5). \tag{3}$$

Under any permutation of $[\gamma^\mu,\gamma^\nu]$, $[\gamma^\lambda,\gamma^\rho]$, $[\gamma^\sigma,\gamma^\tau]$, the trace in (3) changes by the sign of the permutation. In explicit calculations, the representation (2) has the additional advantage of replacing cumbersome algebraic manipulations of the Levi-Civita tensor $\epsilon^{\mu\nu\lambda\rho}$ with much simpler Dirac algebra.

Representation (2) also allows the operator $\mathcal{O}_G$ to be given a simple physical interpretation: it is the color-electric dipole moment for the gluon field strength. It is well known that a quark color-electric dipole moment violates CP and will contribute to the neutron electric dipole moment. It should then come as no surprise that a gluon color-electric dipole moment also violates CP and also contributes to the neutron electric dipole moment. The quark color-electric dipole moment operator is

$$\mathcal{O}_q = \tilde{G}^a_{\mu\nu} \bar{q}(\tfrac{1}{2}\sigma^{\mu\nu}) T^a q, \tag{4}$$

where T^a is the color matrix for the fundamental representation and $\frac{1}{2}\sigma^{\lambda\rho} = (i/4)[\gamma^\lambda,\gamma^\rho]$ is the spin matrix for the Dirac spinor representation of the Lorentz group. We can identify the operator $\mathcal{O}_G$ as the color-electric dipole moment for the gluonic field strength $G^a_{\mu\nu}$ by expressing it in an analogous form,

$$\mathcal{O}_G = -\tfrac{1}{12} \tilde{G}^a_{\mu\nu} G^b_{\alpha\beta} (\mathcal{S}^{\mu\nu})^{\alpha\beta,\gamma\delta} (F^a)_{bc} G^c_{\gamma\delta}, \tag{5}$$

where $(F^a)_{bc} = i f^{bac}$ is the color matrix for the adjoint representation and $\mathcal{S}^{\lambda\rho}$ is the spin matrix for the antisymmetric tensor representation of the Lorentz group. This matrix can in fact be represented compactly using Dirac algebra

$$(\mathcal{S}^{\lambda\rho})^{\alpha\beta,\gamma\delta} = -\frac{i}{64}\mathrm{Tr}([\gamma^\alpha,\gamma^\beta][\gamma^\lambda,\gamma^\rho][\gamma^\gamma,\gamma^\delta]). \tag{6}$$

This matrix does indeed satisfy the algebra of generators of the Lorentz group:

$$[\mathcal{S}^{\mu\nu},\mathcal{S}^{\lambda\rho}] = i(g^{\nu\lambda}\mathcal{S}^{\mu\rho} + g^{\mu\rho}\mathcal{S}^{\nu\lambda} - g^{\nu\rho}\mathcal{S}^{\mu\lambda} - g^{\mu\lambda}\mathcal{S}^{\nu\rho}). \tag{7}$$

That the expression (5) is equivalent to (2) follows from the identity $\epsilon^{\mu\nu\lambda\rho}[\gamma_\lambda,\gamma_\rho] = -2i[\gamma^\mu,\gamma^\nu]\gamma_5$. Thus the operator $\mathcal{O}_G$ is proportional to the color-electric dipole moment of the gluonic field strength as claimed.

We now show how the representation (2) for $\mathcal{O}_G$ can be used to simplify the calculation of its anomalous dimension. The term $\int d^4x\, \mathcal{O}_G$ can appear as a CP-violating term in the low-energy effective action obtained by integrating out heavy particles. Its renormalization-group equation to order α_s has the form

$$\mu\frac{\partial}{\partial\mu}\int d^4x\,\mathcal{O}_G(\mu)=\gamma_{GG}\frac{\alpha_s(\mu)}{4\pi}\int d^4x\,\mathcal{O}_G(\mu)+\cdots\,, \tag{8}$$

where the ellipsis represents other CP-violating interactions, such as $\int d^4x\,\mathcal{O}_q$. To lowest order in α_s, these other operators do not feed back into the evolution of $\mathcal{O}_G$, so the coefficient $\gamma_{GG}\alpha_s/4\pi$ is an eigenvalue of the anomalous-dimension matrix.

The determination of the coefficient γ_{GG} requires the calculation of some gauge-invariant quantity to one-loop order. One possibility is to calculate the CP-violating part of the effective action to one-loop order using the background-field method.[7] Another possibility is to use ordinary field-theory techniques to calculate a physical quantity such as the scattering amplitude between on-shell gluons. The simplest such scattering amplitude is the CP-violating part of the four-gluon scattering amplitude. There are seven diagrams that contribute to this scattering amplitude at tree level. Six of them involve a CP-violating three-gluon vertex and an ordinary three-gluon vertex joined by a gluon propagator, and the seventh consists of just a CP-violating four-gluon vertex. The sum of the seven diagrams is gauge invariant.

The anomalous-dimension coefficient γ_{GG} could be determined by calculating the divergent parts of all one-loop corrections to the four-gluon scattering amplitude. This calculation would be extremely tedious, but fortunately the same information can be obtained from a small subset of the diagrams. If the incoming gluon momenta are labeled p and q, only two of the seven tree-level diagrams contain a pole in their invariant mass $(p+q)^2$. Since the entire scattering amplitude is gauge invariant, the residue of this pole is also gauge invariant. It factorizes into a product of two three-gluon vertices, one of which is CP-violating. Similarly, only for a small subset of the one-loop diagrams does the divergent part have a pole in the invariant mass $(p+q)^2$ of two external gluon legs. The residue of this pole is gauge invariant and the relevant term in the residue factorizes into the product of perturbative corrections to the CP-violating three-gluon vertex and an ordinary tree-level three-gluon vertex. Thus we can reduce the calculation of the anomalous dimension γ_{GG} to the calculation of the divergent part of the matrix element of $\int d^4x\,\mathcal{O}_G$ between two on-shell gluons with the third gluon leg off its mass shell: $r^2\neq 0$, where $r=-(p+q)$. Setting the first two gluons on-shell amounts to setting $p^2=q^2=0$ and dropping terms proportional to p^μ and q^ν. A further simplification is that the third leg can be treated as if it was on its mass shell. If it has momentum r and Lorentz index λ, terms proportional to r^2 can be dropped because they do not contribute to the residue of the pole in $(p+q)^2$. Terms proportional to r^λ can also be dropped because they vanish after contracting with the three-gluon vertex.

To avoid complicated algebraic manipulations of the Levi-Civita tensor $\epsilon^{\mu\nu\lambda\rho}$, we extract the Feynman rules for the CP-violating vertex directly from expression (2). The Feynman rule for the CP-violating vertex with three external gluon legs with incoming momenta p,q,r, Lorentz indices μ,ν,λ, and color indices a,b,c has the elegant form

$$-\frac{i}{16}f^{abc}\mathrm{Tr}([p,\gamma^\mu][q,\gamma^\nu][r,\gamma^\lambda]\gamma_5)\,, \tag{9}$$

where inside the trace, $p=p_\mu\gamma^\mu$, etc. The Feynman rule for four gluons with incoming momenta p,q,r,s, Lorentz indices μ,ν,λ,ρ, and color indices a,b,c,d is

$$\tfrac{1}{16}f^{abe}f^{ecd}\mathrm{Tr}([p,\gamma^\mu][q,\gamma^\nu][\gamma^\lambda,\gamma^\rho]\gamma_5)+5\text{ similar terms}\,. \tag{10}$$

The term shown explicitly is proportional to the momenta p and q of two of the legs, and the sum is over the six pairs of legs. Expressions (9) and (10) are much simpler than the Feynman rules that follow directly from (1).

Aside from wave-function renormalization on the external lines, there are four topologically distinct diagrams that contribute to this amplitude and they are shown in Fig. 1. For each of the diagrams shown, there are actually three diagrams that correspond to the cyclic permutations of the external lines. We calculated the diagrams in Feynman gauge using an ultraviolet momentum cutoff μ. Each diagram contains quadratic divergences, but they cancel after summing over the three diagrams in each topological class. This follows from the simple identity

$$\mathrm{Tr}(p\gamma^\mu\gamma^\nu\gamma^\lambda\gamma_5)+\text{c.p.'s}=0\,, \tag{11}$$

where the expression is summed over the three cyclic permutations (c.p.'s) of (p,μ), (q,ν), (r,λ).

With the quadratic divergences having canceled, the remaining divergences are logarithmic and should sum up to an expression proportional to (9). The straightforward calculation of the diagrams does not produce the desired form automatically. One must put the expression into that form using trace identities that are consequences of the Levi-Civita identity $\epsilon^{[\mu\nu\lambda\rho}g^{\alpha]\beta}=0$. We define a tensor function of three four-vectors as

$$T^{\mu\nu\lambda}(p_1,p_2,p_3)=\mathrm{Tr}([p_1,\gamma^\mu][p_2,\gamma^\nu][p_3,\gamma^\lambda]\gamma_5)\,. \tag{12}$$

From (3), this tensor changes sign under the simultaneous interchange of any pair of arguments and the corresponding pair of indices. The CP-violating three-gluon vertex (9) is proportional to $T^{\mu\nu\lambda}(p,q,r)$. Each of the diagrams in Fig. 1 can be reduced to combinations of the functions $T^{\mu\nu\lambda}$, with the momentum arguments being various combinations of p, q, and r. Using momentum conservation

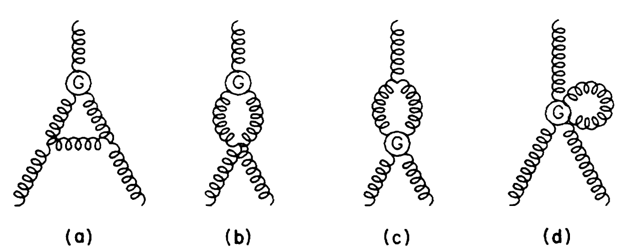

FIG. 1. Diagrams that determine the anomalous-dimension coefficient γ_{GG}. The circle with G inside represents the gluonic CP-violating operator $\mathcal{O}_G$.

$p+q+r=0$ and the symmetries of the tensor (12), one can easily derive the trace identities

$$T^{\mu\nu\lambda}(p,q,p)+T^{\mu\nu\lambda}(p,p,r)+\text{c.p.'s}=-3T^{\mu\nu\lambda}(p,q,r)\,,$$
$$T^{\mu\nu\lambda}(p,q,q)+T^{\mu\nu\lambda}(p,r,r)+\text{c.p.'s}=-3T^{\mu\nu\lambda}(p,q,r)\,. \tag{13}$$

The terms $T^{\mu\nu\lambda}(p,p,p)$ and its cyclic permutations can be dropped because of the identity

$$T^{\mu\nu\lambda}(p,p,p)=-8p^2\mathrm{Tr}(p\gamma^\mu\gamma^\nu\gamma^\lambda\gamma_5)\,. \tag{14}$$

This term is either zero because p is on the mass shell ($p^2=0$), or else the factor of p^2 cancels a pole so that the term does not contribute to the residue of that pole. Dropping terms of the form (14), we can derive additional trace identities

$$T^{\mu\nu\lambda}(p,p,q)+T^{\mu\nu\lambda}(p,r,p)+\text{c.p.'s}=3T^{\mu\nu\lambda}(p,q,r)\,,$$
$$T^{\mu\nu\lambda}(p,r,q)+\text{c.p.'s}=0\,. \tag{15}$$

After summing over the three cyclic permutations corresponding to each of the diagrams in Fig. 1, the identities in (13) and (15) allow the diagram to be reduced to a multiple of $T^{\mu\nu\lambda}(p,q,r)$ as desired. The divergences from wave-function renormalization of the three external gluon lines automatically have the desired form. Thus the sum of the logarithmic divergences is proportional to the tree-level expression (9). From the coefficient, we read off the diagonal anomalous-dimension coefficient γ_{GG} defined in (5),

$$\gamma_{GG}=-C_A-2N_f\,. \tag{16}$$

The contributions in Feynman gauge from the individual diagrams are $11C_A/2$ from diagram 1(a), 0 from diagram 1(b), $-23C_A/2$ from diagram 1(c), and 0 from diagram 1(d). The remaining factor of $5C_A-2N_f$ comes from wave-function renormalization on the three external gluon lines. Here $C_A=3$ is the Casimir for the adjoint representations of SU(3) and N_f is the number of light quarks at the scale μ.

If we use the alternative operator $\mathcal{O}_1(\mu)=g_s(\mu)^3\times\mathcal{O}_G(\mu)$, the anomalous dimension coefficient in (8) is changed in an obvious way. To leading order in $\alpha_s=g_s^2/4\pi$, the running coupling constant $g_s(\mu)$ satisfies $\mu(\partial/\partial_\mu)g_s(\mu)=-\beta(\alpha_s/4\pi)g_s(\mu)$, where $\beta=(11C_A-2N_f)/3$. The coefficient analogous to γ_{GG} in (8) is then $\gamma_{11}=\gamma_{GG}-3\beta=-12C_A$. This result, which was first given in Refs. 6 and 8, has now been verified by three other groups.[7]

We have shown that the purely gluonic dimension-6 *CP*-violation operator discovered by Weinberg has a simple physical interpretation as the color-electric dipole moment for the gluonic field strength. Dirac algebra can be used to express this operator in a Bose-symmetric form. This form simplifies the calculation of its anomalous dimension, and we believe it could be equally useful in other investigations of this new *CP*-violation operator.

Note added in proof. After this work was accepted for publication, it was brought to our attention that the anomalous dimension of the gluon color-electric dipole moment was first calculated by Morozov.[8]

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