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Direct tests of *CPT* and *T* invariance in the neutral-kaon mass matrix

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I suggest a possible way of measuring the *T*- and *CPT*-violating parameters in the neutral-kaon mass matrix, which makes use of their influence on, respectively, the difference of normalization and the difference of shape of the decay curves, to an arbitrary channel *c*, of kaons and antikaons. This method is alternative to the direct study of the neutral-kaon oscillations. Both require kaon beams with well-defined strangeness.

CP violation in the neutral-kaon system[1] is experimentally well established. On the other hand, there is a strong theoretical prejudice against the possibility that *CPT* is violated. This prejudice is based on the so-called "*CPT* theorem,"[2] which proves, from general premises concerning what are considered to be "reasonable" field theories, that they will not violate *CPT*. As a consequence of this theorem and that experimental fact, it is widely accepted that *CPT* is a good symmetry, that *CP* violation and *T* violation are equivalent, and have the same strength and origin (whatever it be). However, this point of view is not well founded experimentally. It is true that a *CPT*-invariant phenomenology of the neutral-kaon system[3] is in quite good agreement with all the known experimental facts—a long-standing problem, the too large[4] phase difference $(\Phi_{00}-\Phi_{+-})$,[5] appearing to be eliminated by recent and more precise experiments.[6] It is also true that a detailed study of that phenomenology leads to the conclusion that *T* is indeed violated,[7] and allows us to put rather strong upper bounds on some *CPT*-violating parameters.[8] Nevertheless, direct measurements of directly *CPT*- or *T*-violating parameters are lacking. Taking into account the fundamental character that the discrete symmetries nowadays have in our view of physics, this situation is unsatisfactory.

The present theoretical view of *CP* violation[3] attributes it mainly to *T* violation in the neutral-kaon mass matrix, "direct" *T* violation in the decay amplitudes being tiny or nonexistent.[9,10] Therefore, it is reasonable to look first of all to direct tests and/or measurements of *T* and *CPT* violation in the mass matrix. This is the purpose of this Rapid Communication. I suggest a new class of tests of *T* and *CPT* in the neutral-kaon decays to an *arbitrary* channel *c*, which give us direct access to clearly *T*- and *CPT*-violating quantities in the mass matrix. These tests will hopefully be made possible[11] by the availability, at the CERN Low Energy Antiproton Ring, of high-intensity, well-defined strangeness, kaon beams.

An important point in this Rapid Communication is the comparison of similar experiments performed with beams which at the production moment are in one case pure K^0, in the other case pure $\bar{K}^0$. The importance of such a comparison lies in the fact that, once we admit that maybe *CPT* is violated, there is no *C*-type symmetry remaining to guarantee that such experiments will yield equivalent results.

The time evolution of a beam of neutral kaons is described by a 2×2 complex and non-Hermitian mass matrix, which I denote by R. *T* invariance of R is equivalent to the vanishing of a *real* parameter $(|R_{12}|-|R_{21}|)$. *CPT* invariance of R is equivalent to the vanishing of a *complex* parameter $(R_{22}-R_{11})$. *CP* invariance of R is equivalent to the simultaneous existence in R of *T* and *CPT* invariance.

The physical states $|K_S\rangle$ and $|K_L\rangle$ are related to $|K^0\rangle$ and $|\bar{K}^0\rangle$ by linear equations

$$|K_S\rangle=\frac{a_S|K^0\rangle+\bar{a}_S|\bar{K}^0\rangle}{\sqrt{|a_S|^2+|\bar{a}_S|^2}}\,,\qquad |K_L\rangle=\frac{a_L|K^0\rangle+\bar{a}_L|\bar{K}^0\rangle}{\sqrt{|a_L|^2+|\bar{a}_L|^2}}\,,\tag{1}$$

or equivalently

$$|K^0\rangle=\frac{\bar{a}_L\sqrt{|a_S|^2+|\bar{a}_S|^2}\,|K_S\rangle-\bar{a}_S\sqrt{|a_L|^2+|\bar{a}_L|^2}\,|K_L\rangle}{a_S\bar{a}_L-\bar{a}_Sa_L}\,,\qquad |\bar{K}^0\rangle=\frac{-a_L\sqrt{|a_S|^2+|\bar{a}_S|^2}\,|K_S\rangle+a_S\sqrt{|a_L|^2+|\bar{a}_L|^2}\,|K_L\rangle}{a_S\bar{a}_L-\bar{a}_Sa_L}\,.\tag{2}$$

From the diagonalization of R we find that

$$r_S\equiv\frac{a_S}{\bar{a}_S}=\frac{2R_{12}}{-\Delta\lambda+R_{22}-R_{11}}=\frac{-\Delta\lambda-R_{22}+R_{11}}{2R_{21}}\,,\qquad r_L\equiv\frac{a_L}{\bar{a}_L}=\frac{2R_{12}}{\Delta\lambda+R_{22}-R_{11}}=\frac{\Delta\lambda-R_{22}+R_{11}}{2R_{21}}\,,\tag{3}$$

where $\Delta\lambda \equiv \lambda_L - \lambda_S$ is the difference of the time-evolution factors (Hamiltonian eigenvalues) corresponding to the eigenvectors $|K_L\rangle$ and $|K_S\rangle$, respectively.

I now define the parameters that signal T and CPT violation in R: the real T-violating parameter

$$\chi \equiv \frac{|R_{12}|^2 - |R_{21}|^2}{|R_{12}|^2 + |R_{21}|^2} = \frac{|r_S r_L|^2 - 1}{|r_S r_L|^2 + 1}, \tag{4}$$

and the complex CPT-violating parameter

$$\theta \equiv \frac{R_{22} - R_{11}}{\Delta\lambda} = \frac{r_S + r_L}{r_S - r_L}. \tag{5}$$

I used Eqs. (3). We look for distinctive experimental signs of θ and χ. The current view of CP violation holds that θ is zero, while χ is approximately equal to twice the leptonic asymmetry, i.e., 6.6×10^{-3}.[12]

The most obvious way to measure the mixing parameters[13] is the observation of the mixing of the neutral kaons as a function of time. In particular, we easily find that

$$\chi = \frac{\Gamma(\bar{K}^0(t) \to K^0) - \Gamma(K^0(t) \to \bar{K}^0)}{\Gamma(\bar{K}^0(t) \to K^0) + \Gamma(K^0(t) \to \bar{K}^0)}. \tag{6}$$

T violation is equivalent to a difference of the probabilities of an antikaon being found a time t later as a kaon, and of a kaon being found a time t later as an antikaon. This was the observation made in Ref. 14. Indeed, it is easy to demonstrate directly that the asymmetry in the right-hand side of Eq. (6) is T violating. Notice that the asymmetry is time independent; therefore, this way of observing T violation has the advantage that, in order to obtain higher statistics, we may use the time-integrated quantities. Unfortunately, this asymmetry is probably difficult to measure with the required accuracy, for such a measurement requires (1) accurate knowledge of the relative intensities of the beams of K^0 and $\bar{K}^0$, and (2) accurate knowledge of the relative cross sections of the reactions by which the kaons and antikaons are identified. The second requirement is particularly awkward, as was emphasized in Ref. 11: identification reactions of strong origin ($\bar{K}^0 p \to \Lambda\pi^+$, $K^0 p \to nK^+$) have relatively low (and badly known) cross sections; identification reactions of weak origin (semileptonic decays of the kaons) may include themselves some CPT violation, which would[11] effectively mimic the effect that we are trying to disentangle, T violation in R.

On the other hand,

$$\Gamma(K^0(t) \to K^0) = Ae^{-\gamma_S t} + Be^{-\gamma_L t} + \exp\left(-\frac{\gamma_S + \gamma_L}{2} t\right)[C\cos(\Delta m\, t) + D\sin(\Delta m\, t)], \tag{7}$$

with $\Delta m \equiv m_L - m_S$. The expression for $\Gamma(\bar{K}^0(t) \to \bar{K}^0)$ is similar, only with the changes $A \leftrightarrow B$ and $D \to -D$. The expressions for the coefficients in Eq. (7) are such that

$$\theta = \frac{2A - C - iD}{2A + C + iD} = \frac{C - 2B - iD}{C + 2B - iD}. \tag{8}$$

Thus, the observation of the evolution, as a function of time, of the transition probability from K^0 to K^0, or, equivalently, from $\bar{K}^0$ to $\bar{K}^0$, allows us to measure the CPT-violation parameter in R. For this end, we do not need to compare experiments performed with kaons with experiments performed with antikaons; we also do not need to know accurately either the beam intensity or the cross sections of the reactions which allow us to identify the kaons and antikaons for, as is seen in Eq. (8), the absolute normalization of the decay curves is irrelevant. On the other hand, a precise fit to the decay curve by the expression of Eq. (7) is needed, and may be difficult to obtain.

Let us now consider the kaon decays to some channel c. Such decays also provide a method, which up to now was unnoticed, of measuring the T- and CPT-violating quantities in R. I define the parameters $A_K, \ldots, D_K$ by

$$|\langle c | K^0(t)\rangle|^2 = A_K e^{-\gamma_S t} + B_K e^{-\gamma_L t} + \exp\left(-\frac{\gamma_S + \gamma_L}{2} t\right)[C_K\cos(\Delta m\, t) + D_K\sin(\Delta m\, t)]. \tag{9}$$

I define $A_{\bar{K}}, \ldots, D_{\bar{K}}$ by a similar fit to the decay curve of a beam which at the production instant is pure $\bar{K}^0$. Now, it follows from Eqs. (2) that

$$\frac{A_{\bar{K}}}{A_K} = |r_L|^2, \quad \frac{B_{\bar{K}}}{B_K} = |r_S|^2, \quad \frac{C_{\bar{K}} + iD_{\bar{K}}}{C_K + iD_K} = r_S r_L^*. \tag{10}$$

From these equations we find that

$$\chi = \frac{A_{\bar{K}}B_{\bar{K}} - A_K B_K}{A_{\bar{K}}B_{\bar{K}} + A_K B_K} = \frac{C_{\bar{K}}^2 + D_{\bar{K}}^2 - C_K^2 - D_K^2}{C_{\bar{K}}^2 + D_{\bar{K}}^2 + C_K^2 + D_K^2}, \tag{11}$$

$$\theta = \frac{A_K(C_{\bar{K}} + iD_{\bar{K}}) + A_{\bar{K}}(C_K + iD_K)}{A_K(C_{\bar{K}} + iD_{\bar{K}}) - A_{\bar{K}}(C_K + iD_K)} = \frac{B_{\bar{K}}(C_K - iD_K) + B_K(C_{\bar{K}} - iD_{\bar{K}})}{B_{\bar{K}}(C_K - iD_K) - B_K(C_{\bar{K}} - iD_{\bar{K}})}. \tag{12}$$

Thus, we can measure the CPT- and T-violating parameters in the mass matrix without having recourse to $K^0 \leftrightarrow \bar{K}^0$ oscillations, just by comparing, respectively, the shapes and normalizations of the decay curves, to an *arbitrary* channel c, of kaons and antikaons. The fact that c is arbitrary is important, for it allows us to make the measurements with the channel that turns out to be experimentally more comfortable ($\pi^+\pi^-$ is the obvious candidate; the semileptonic channels are[11] promising). We might also compare the results of similar experiments performed using different channels.

Notice that, for the measurement of the T-violating parameter χ, the knowledge of the intensities of the K^0 and $\bar{K}^0$ beams used (the normalization of the decay curves) is fundamental. The same does not happen, however, in what concerns the measurement of θ. For that measurement, it is the *shapes* of the decay curves that are relevant. Let us define, as usual, $\eta_c \equiv \langle c|K_L\rangle/\langle c|K_S\rangle$. It is not really η_c that we measure when we observe the oscillations in the K^0 and $\bar{K}^0$ decays to the channel c. (Indeed, η_c is not invariant under a rephasing of $|K_L\rangle$ and $|K_S\rangle$; therefore, we must do, at least, one phase convention before we have in hand a definite quantity to be measured. However, once there is *CPT* violation in R there ceases to exist a unique natural phase convention which offsets the rephasing noninvariance of η_c.) Rather, in the $\bar{K}^0$ decays we measure $\eta_{\bar{K}} \equiv -(C_{\bar{K}}+iD_{\bar{K}})/(2A_{\bar{K}})$, and in the K^0 decays we measure $\eta_K \equiv (C_K+iD_K)/(2A_K)$. Equation (12) may be rewritten as

$$\theta = \frac{\eta_{\bar{K}}-\eta_K}{\eta_{\bar{K}}+\eta_K} . \tag{13}$$

What I am pointing out here is that, once there is *CPT* violation in the mass matrix, the "η_c" parameters cease to be well defined, and "their" measurements in beams with different compositions should yield different values. $\eta_{\bar{K}}$ and η_K will then have both different moduli (due to the effect of Reθ) and different phases (due to the effect of Imθ), and their moduli will both be different from the modulus of η_c, which is measured by comparing the partial widths to the channel c of K_L and K_S.

The proposal to measure χ that I make here does not suffer from the problems with comparison of identification-reaction cross sections that were raised in Ref. 11 as a criticism to the suggestion in Ref. 14 for measuring χ, Eq. (6). The measurement of T violation is likely to be more difficult than the measurement of *CPT* violation, for the former requires a knowledge of the relative normalization of the decay curves, while the latter only requires the observation of the difference of their shapes, when we work with beams with different compositions.

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