# Repulsive gravitational effects of global monopoles

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A monopole formed as a consequence of the spontaneous breakdown of a global symmetry should have a mass that grows linearly with the distance off its core. It was recently shown by Barriola and Vilenkin that the gravitational effect of this configuration is equivalent to that of a deficit solid angle in the metric, plus that of a relatively tiny mass at the origin. Here we show that this small effective mass is negative. Global monopoles thus share with other topological defects, such as domain walls and global strings, a repulsive gravitational potential. We solve numerically the coupled equations for the metric and the scalar field, to precisely determine this repulsive potential and in order to analyze the solution when gravitational effects are already significant close to the monopole core. We study the motion of test particles in a monopole background, and discuss the possible implications of a negative effective mass.

## I. INTRODUCTION

A quantum field theory with spontaneous symmetry breaking such that the manifold of equivalent vacua after the breakdown is not shrinkable to a point contains monopole solutions.[1,2] If the symmetry that is broken is a gauge symmetry, the monopole configuration has finite energy, and its mass is concentrated in a very tiny core. On the contrary, a monopole formed after the breakdown of a global symmetry would have a linearly divergent mass, due to the long-range Nambu-Goldstone field. It would thus be impossible to form an isolated monopole in flat space. But if the global symmetry in question broke down while the early Universe was expanding and cooling down, one expects the horizon size to be the typical distance between monopoles and antimonopoles, thus becoming a natural cutoff on their energy density. The situation is analogous to that of the logarithmically divergent energy per unit length of straight global strings.[3]

Most grand unified theories predict the existence of gauge monopoles. There is less theoretical motivation, instead, to consider global monopoles. Their existence is, nevertheless, a possibility that does not seem to conflict with standard cosmology, even for very heavy monopoles, because their annihilation rate would be large enough to avoid monopole dominance of the energy density, due to the long-range interaction between monopoles and antimonopoles.[4] If global monopoles existed, they would share with other topological defects, such as domain walls and strings, curious and rather unconventional gravitational effects. It was recently shown by Barriola and Vilenkin[4] that when gravity is taken into account, the linearly divergent mass of a global monopole has an effect analogous to that of a deficit solid angle plus that of a tiny mass at the origin, of the order of the mass in the core. In the present article we show that this small gravitational potential is actually repulsive. In Sec. II we present a simplified model for the global monopole in order to discuss the main features of its metric in a simple manner. Then we solve numerically the coupled Einstein and scalar field equations for a specific global monopole model to rigorously determine the properties of the monopole solution. We also discuss the case of large energy density, when gravity is already significant close to the core of the monopole. In Sec. III we analyze the motion of test particles around the monopole, and finally in Sec. IV some conclusions are drawn with regard to possible implications of an effective negative mass.

## II. FIELD EQUATIONS FOR A GLOBAL MONOPOLE

### A. The model

To be specific, we shall work within a particular field-theoretical model, where a global O(3) symmetry is broken down to U(1), as in Ref. 4. The Lagrangian is (we work in units such that $\hbar=c=1$)

$$\mathcal{L}=\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^a-\frac{\lambda}{4}(\phi^a\phi^a-\eta^2)^2\ , \tag{1}$$

with $a=1,2,3$ the internal O(3) index. The ansatz for a monopole configuration is

$$\phi^a=\eta f(r)\frac{x^a}{r}\quad\text{with } x^a x^a=r^2\ , \tag{2}$$

so that we will actually have a monopole solution if $f\to 1$ at spatial infinity. The metric around a monopole should be static and spherically symmetric. It should then be possible to find coordinates such that the metric element reads

$$ds^2=-B(r)dt^2+A(r)dr^2+r^2(d\theta^2+\sin^2\theta\, d\varphi^2)\ . \tag{3}$$

In this background, the equation for the scalar field in the monopole configuration (2) is

 

$$\frac{1}{A}\ddot{f}+\left[\frac{2}{Ar}+\frac{1}{2B}\left[\frac{B}{A}\right]^{\cdot}\right]\dot{f}-\frac{2}{r^2}f-\lambda\eta^2(f^2-1)f=0\ ,\tag{4}$$

where the dot indicates derivatives with respect to $r$. The energy-momentum tensor of the global monopole configuration reads

$$\begin{aligned}T_t^t&=-\eta^2\left[\frac{f^2}{r^2}+\frac{\dot{f}^2}{2A}+\frac{\lambda}{4}\eta^2(f^2-1)^2\right],\\ T_r^r&=-\eta^2\left[\frac{f^2}{r^2}-\frac{\dot{f}^2}{2A}+\frac{\lambda}{4}\eta^2(f^2-1)^2\right],\\ T_\theta^\theta=T_\varphi^\varphi&=-\eta^2\left[\frac{\dot{f}^2}{2A}+\frac{\lambda}{4}\eta^2(f^2-1)^2\right].\end{aligned}\tag{5}$$

### B. A simplified version

We start our analysis with an extremely simplified model for the monopole configuration, just to display the main features of the exact solution in a simple manner. Rather than solving the coupled equations for the metric and the scalar field, let us assume the following configuration [which is not a solution of Eq. (4)]:

$$f=\begin{cases}0 & \text{if } r<\delta,\\ 1 & \text{if } r>\delta\ .\end{cases}\tag{6}$$

In other words, we are modeling the monopole by a pure false vacuum inside the core, and an exactly true vacuum at the exterior. Einstein equations inside the core are solved by a de Sitter metric:

$$ds^2=-(1-H^2r^2)dt^2+\frac{dr^2}{1-H^2r^2}+r^2d\Omega^2\ ,\tag{7}$$

with $H^2=(8\pi G/3)(\lambda\eta^4/4)$ ($G$ is Newton's constant). The exterior solution reads[4]

$$ds^2=-\left[1-8\pi G\eta^2-\frac{2GM}{r}\right]dt^2+\frac{dr^2}{1-8\pi G\eta^2-\frac{2GM}{r}}+r^2d\Omega^2\ ,\tag{8}$$

where $M$ is an arbitrary integration constant. Both $M$ as well as the core radius $\delta$ are determined by Einstein equations at the boundary between the interior and exterior regions, which are tantamount (assuming there is no surface layer with its own energy density at the boundary) to the continuity of the metric and its first derivatives with respect to radial proper distance.[5] The result is

$$\delta=\frac{2}{\sqrt{\lambda}\eta},\quad M=-\frac{16\pi}{3}\frac{\eta}{\sqrt{\lambda}}\ .\tag{9}$$

We conclude that it is possible to match an interior de Sitter solution to an exterior global monopole solution, thanks to the deficit solid angle of the latter, but only with a negative mass. Let us mention, by the way, that an interior de Sitter solution does not match to an ordinary Schwarzchild exterior solution for any value of the mass, be it positive or negative, without a surface layer at the boundary, in which case Einstein equations determine the motion of this layer.[6]

The model we just presented is not an exact solution for the metric around a global monopole. It is, however, an exact solution of Einstein equations that shares most features of the real thing, as we shall confirm in the next section, and that already contains the striking aspects we want to discuss right away. First of all, one knows that Birkoff's theorem states that the only static, spherically symmetric vacuum solution of Einstein equations is the Schwarzschild metric, and that the parameter $M$ in the solution is determined by the integral of $T_t^t$ along the source, independent of the equation of state of the source. Is the solution (8) with the value (9) for $M$ in conflict with this theorem? One would think that up to a distance $r$ away from the core only the energy density inside a sphere of radius $r$ should contribute, and that Biroff's theorem should apply in that sense, even though we are dealing neither with a vacuum solution nor with an isolated source. Hence, at first glance a negative mass looks surprising, since $T_t^t$ is positive definite all the way from the origin. There is, however, no contradiction. Indeed, in our simplified model (6) we have, when $r>\delta$,

$$4\pi\int_0^r r^2T_t^t(r)dr=4\pi\eta^2r-\frac{16\pi}{3}\frac{\eta}{\sqrt{\lambda}}\ .\tag{10}$$

This quantity is indeed positive. The negative constant term is precisely the same negative effective mass of Eq. (9). If this were a Newtonian source, the linear term would produce a logarithmic potential. But the global monopole is an ultrarelativistic source, with tension along the radial direction as large as the energy density. The correct Newtonian limit of Einstein equations,[3] $\nabla^2\phi(r)=8\pi G(\rho+p_r)=0$, leads to a constant potential. This corresponds to the constant term of $g_{tt}$ in Eq. (8). Its constancy does not mean absence of gravitational effects. On the contrary, it will usually have far more significant consequences than the repulsive term. As shown in Ref. 4, it can be understood as a kind of deficit solid angle, much like the case of gauge cosmic strings.[7,8]

Although in most situations the repulsive potential of the gravitational field of a global monopole will be more than compensated by an opposite effect due to the solid deficit angle, it seems interesting to us how a repulsive gravitational potential appears out of a static spherically symmetric system with positive-definite energy density. Moreover, we see that global monopoles share repulsive gravitational effects with other topological defects, such as domain walls[7,9] and global strings.[10] Now we turn to the analysis of the exact solution for the system, to rigorously confirm the features advanced in this simplified model.

### C. Coupled Einstein and scalar field equations

Einstein equations for the static spherically symmetric metric (3) are formally solved by[11]

$$\left[\frac{r}{A}\right]' = 1 + 8\pi G r^2 T_t^t(r) \,, \qquad B = \frac{1}{A}\exp\left[8\pi G \int_\infty^r (T_r^r - T_t^t) A r \, dr\right] , \tag{11}$$

where the time coordinate has been scaled so that $B = A^{-1}$ as $r \to \infty$. Let us now introduce the dimensionless quantities

$$x \equiv \sqrt{\lambda}\eta r, \quad \Delta \equiv 8\pi G \eta^2 \,. \tag{12}$$

In terms of the variable $x$, the equation for the scalar field around the global monopole can be written, after use of the Einstein equations, as

$$\ddot{f}(x) = \frac{A}{x}\left[-\dot{f}\left[1 - \Delta f^2 - \frac{\Delta}{4}x^2(f^2-1)^2\right] + 2\frac{f}{x} + x f(f^2-1)\right] - \frac{\dot{f}}{x} \,, \tag{13}$$

while the equation for the metric coefficient $A(x)$ reads

$$\left[\frac{x}{A}\right]^{\cdot} = -\frac{\Delta}{2}x\dot{f}^2\left[\frac{x}{A}\right] + 1 - \Delta f^2 - \frac{\Delta}{4}x^2(f^2-1)^2 \,. \tag{14}$$

The equation for $A(x)$ can be formally integrated and written as

$$A^{-1}(x) = 1 - \Delta - \frac{2M_A(x)}{x} \,, \tag{15}$$

with

$$M_A(x) = \frac{\Delta}{2}\exp\left[-\frac{\Delta}{2}\int_0^x dy\, \dot{f}^2(y) y\right]\int_0^x dy \left[f^2 - 1 + \frac{y^2}{4}(f^2-1)^2 + (1-\Delta)\frac{y^2}{2}\dot{f}^2\right]\exp\left[\frac{\Delta}{2}\int_0^y dz\, \dot{f}^2 z\right] . \tag{16}$$

Analogously

$$B(x) = 1 - \Delta - \frac{2M_B(x)}{x} \,, \tag{17}$$

with

$$M_B(x) = M_A(x)\exp\left[\Delta\int_\infty^x dy\, \dot{f}^2 y\right] + (1-\Delta)\frac{x}{2}\left[1 - \exp\left[\Delta\int_\infty^x dy\, \dot{f}^2 y\right]\right] . \tag{18}$$

Let us first discuss the asymptotic behavior of these functions. A global monopole solution should have $\lim_{r\to\infty} f = 1$. If this convergence is fast enough, as in flat space, then $M_A(x)$ and $M_B(x)$ will also quickly converge to finite values. In that case, defining $M_A \equiv M_A(\infty)$, one finds the asymptotic expansions

$$\begin{aligned} f(x) &= 1 - \frac{1}{x^2} - \frac{\frac{3}{2} - \Delta}{x^4} + O(x^{-6}) \,, \\ M_A(x) &= M_A + \frac{\Delta}{2x} + O(x^{-3}) \,, \\ M_B(x) &= M_A(x)\left[1 - \frac{\Delta}{x^4}\right] + \frac{1-\Delta}{2}\frac{\Delta}{x^3} + O(x^{-7}) \,. \end{aligned} \tag{19}$$

Notice that the dependence on $\Delta$ of the asymptotic expansion for $f$ is very weak. It appears that the asymptotic behavior of the monopole solution is, in terms of the dimensionless variable $x = \sqrt{\lambda}\eta$, quite independent of the scale of symmetry breakdown $\eta$ up to values as large as the Planck scale [$\Delta = 1$ when $\eta^2 = (8\pi G)^{-1}$]. However, in order to confirm the existence of monopole solutions up to $\Delta = 1$ as well as to determine the value of $M_A(x)$, we must turn to a numerical analysis of the coupled system of equations (13) and (14). We do that through a fourth-order Runge-Kutta routine for the quantities $f(x)$ and $g(x) \equiv x/A(x)$. We impose the boundary conditions at the origin $f(0) = g(0) = 0$ and $\dot{g}(0) = 1$. The value of $\dot{f}(0)$ is adjusted so that $f \to 1$ for large $x$. In Fig. 1 we display $f(x)$ for different values of $\Delta$. Its shape is quite insensitive to $\Delta$ in the range $0 \le \Delta \le 1$ not only asymptotically, but also close to the origin. This is best seen in Fig. 2, where we plot the value of $\dot{f}$ at the origin as a function of $\Delta$. Only very close to $\Delta = 1$ a significant dependence of $\dot{f}$ on $\Delta$ builds up. Thus, very strong gravitational effects already at the monopole core do not preclude the existence of a regular monopole solution. Figure 3 shows $M_A(x)/\Delta$, which again is not very sensitive to $\Delta$. $M_A$ is indeed negative all the way from the origin, and quickly approaches an asymptotic value of order $M_A \approx -0.75\Delta$. Returning to ordinary mass units, this means

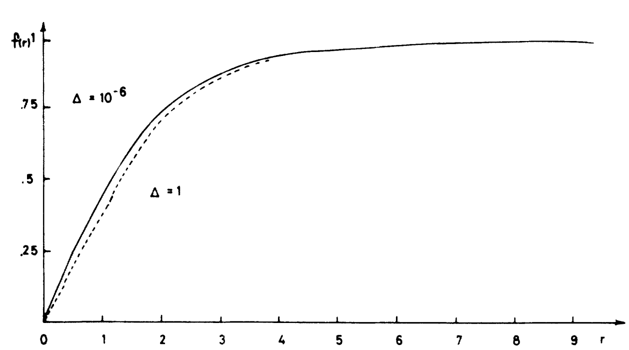


FIG. 1. The function $f(r)$, which modulates the monopole configuration as defined in Eq. (2), is plotted vs the dimensionless coordinate $x = \sqrt{\lambda}\eta r$ for different values of $\Delta = 8\pi G\eta^2$. The shape of the curve is quite insensitive to the value of $\Delta$ in the interval $0 \le \Delta \le 1$.

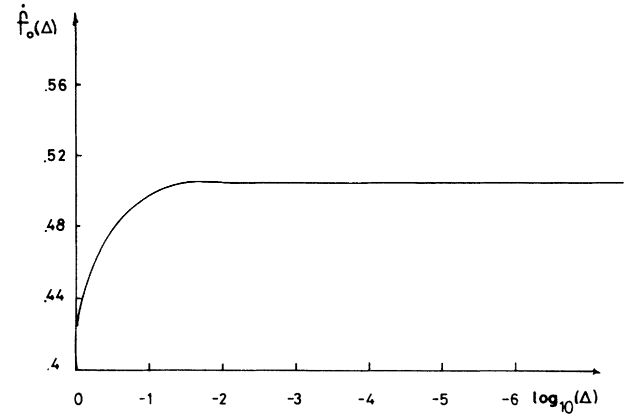


FIG. 2. The derivative at the origin of the function $f(x)$ as a function of $\Delta$ is plotted in a logarithmic scale. No significant departure from the flat space value occurs until $\Delta$ is close to 1.

$$M_A \approx -6\pi \frac{\eta}{\sqrt{\lambda}} \ . \tag{20}$$

Notice that the mass $M_A$ being negative, there is no event horizon around a global monopole (at least when $\Delta \leq 1$). Indeed, in the Schwarzschild metric with negative mass, a light ray can reach the origin from infinity.[12] Of course, there would be a horizon around a black hole of mass $M > M_A$ that swallowed a global monopole. Now, if we allow the parameter $\Delta$ to be larger than 1 (notwithstanding that a classical treatment of such a case is most certainly inadequate), it seems that we will still have monopole configurations. The metric around the monopole with $\Delta > 1$ would have a horizon, which would be located closer to the monopole core as larger is $\Delta$. Indeed, the asymptotic expansion (19) for $f(r)$ appears to be valid for arbitrary $\Delta$. Moreover, we have checked numerically that when $\Delta > 1$ there are regular solutions for $f$ that behave not so differently from the $\Delta \leq 1$ case, at least in the range between $x=0$ and the point where $A(x)$ vanishes, where the coordinate system we were using becomes singular. If these solutions have the appropriate asymptotic behavior at infinity, we would have rather curious monopole configurations, since the coordinate $r$ would be timelike at large distances off the core. The metric would read asymptotically, after an appropriate redefinition of the coordinates $t$, $r$ into $R$, $\tau$,

$$ds^2 = -d\tau^2 + dR^2 + (\Delta - 1)\tau^2 d\Omega^2 \ , \tag{21}$$

and we would have $f=f(\tau)$. It would not be a static metric. Observers outside the horizon would be forced to move away from the origin. In that regard, it is similar to the situation outside the de Sitter horizon, rather than to the interior of a Schwarzschild one. Confront this situation with that of other topological defects when the scales involved approach the Planck scale. In the case of a gauge string, the exterior solution can only be either flat space with a conical deficit angle or a Kasner-type metric.[13,14] There is numerical evidence that for small deficit angle the exterior metric is indeed flat, but that as the deficit angle would approach $2\pi$ the metric becomes singular, with an approach to the singularity similar to that in a Kasner regime.[15] In the case of a global string with scale of symmetry breaking $\eta$, the mass per unit length of string becomes $\mu \approx \eta^2 \ln(\rho/\delta)$, where $\rho$ is the radial proper distance to the core and $\delta$ the core radius. When $\rho$ is such that $\mu$ approaches the Planck scale, the metric develops a curvature singularity,[16] which is again as that of a Kasner-type metric.[17] For some nonstandard equation of state at the core of the global string, though, there is a horizon rather than a singularity at a finite proper distance off the core.[17] The case of the global monopole metric when $\Delta > 1$ is similar to the latter. In conclusion, although the mass of the global monopole grows linearly with energy, this divergence does not lead to a singularity in the metric,[4] and it seems that the same happens even for $\Delta > 1$, in which case the qualitative change in the metric would be the appearance of a horizon that moves closer to the monopole core as larger is $\Delta$, such that the metric is no longer static asymptotically.

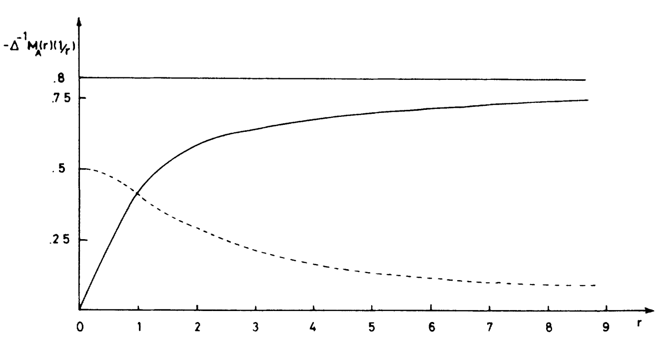


FIG. 3. The solid line represents $-M_A(x)/\Delta$, the effective monopole mass that effects the repulsive gravitational force. This ratio approaches an asymptotic value of order 0.75. The dashed line represents the gravitational potential term $-2M_A(x)/x$. Again, the shape of these curves is very insensitive to $\Delta$ in the interval $0 \leq \Delta \leq 1$.

## III. GRAVITATIONAL EFFECTS OF GLOBAL MONOPOLES

We now study the motion of test particles around a global monopole. Since the effective mass $M_A(x)$ approaches very quickly its asymptotic value, it is a good approximation to take it as the constant $M_A$, unless we were interested in a test particle moving right into the monopole core. So let us consider the geodesic equations in the metric (3) with $B = A^{-1} = (1 - \Delta - 2GM/r)$, where $M$ is an arbitrary constant. It could be just the negative asymptotic monopole mass $M_A$, as well as the mass of an object with a global monopole inside. The geodesic equations for a particle moving in the plane $\theta = \pi/2$ read

$$\frac{dt}{dp} = \frac{1-\Delta}{B} \ ,$$

$$r^2 \frac{d\varphi}{dp} = J \ , \tag{22}$$

$$A(r) \left[ \frac{dr}{dp} \right]^2 = \frac{(1-\Delta)^2}{B} - \frac{J^2}{r^2} - E^2 \ ,$$

where we have chosen the affine parameter $p$ to coincide with the coordinate $t$ asymptotically, and where the integration constant $E$ is such that $ds^2=-E^2dp^2$. $J$ is also a constant of motion, the angular momentum.

In terms of rescaled quantities we can make the equations (22) look the same as in the ordinary Schwarzschild metric. This is achieved through

$$M=\bar{M}(1-\Delta),\quad \varphi=\bar{\varphi}(1-\Delta)^{-1/2},\quad J=\bar{J}(1-\Delta)^{1/2}\,,$$
$$E^2=\bar{E}^2(1-\Delta),\quad t=\bar{t}(1-\Delta)^{-1},\quad p=\bar{p}(1-\Delta)^{-1}\,. \tag{23}$$

Known results for the Schwarzschild metric in the barred quantities can then be translated into those for the monopole case. Remember, though, that if $M$ is the mass of the monopole alone, there would not exist bound orbits, since this mass is negative.

Consider for instance light propagation. The change in the angular coordinate for a light ray scattered in a Schwarzschild metric is $\delta\bar{\varphi}\approx\pi+4G\bar{M}/r_0$, with $r_0$ the distance of closest approach. The angle by which light is deflected in a monopole background is thus $\epsilon=\delta\bar{\varphi}(1-\Delta)^{-1/2}-\pi$. For small $\Delta$,

$$\epsilon=\pi\frac{\Delta}{2}+\frac{4GM}{r_0}\,. \tag{24}$$

The first term is the effect of the deficit solid angle. Indeed, as pointed out in Ref. 4, if we neglect its small effective mass $M_A$ the metric around the monopole can be written in appropriate coordinates as $ds^2=-dt^2+dr^2+(1-\Delta)r^2d\Omega^2$. Thus, it is obvious that the change in the $\varphi$ coordinate of a light ray scattered in a plane $\theta=\pi/2$ is $\pi(1-\Delta)^{-1/2}$.

The effect of the mass of the monopole has the opposite direction, but it is certainly negligible. It is given by $\epsilon=-4G|M_A|/r_0\approx-\frac{3}{2}\Delta(\delta/r_0)$, so it will be smaller as the distance of closest approach compared to the core radius $\delta$ increases. The effect of the deficit angle, by the way, is independent of the impact parameter of the trajectory, and it does not depend on the speed of the particle scattered by the monopole either. Slowly moving particles are effected a bit more by the small repulsive mass. The Newtonian deflection can be written as

$$\epsilon\approx-2G|M_A|/r_0v^2=-\tfrac{3}{4}\Delta\frac{\delta}{r_0}\frac{1}{v^2}\,.$$

Anyhow, the effect of the mass $M_A$ is indeed insignificant.

The effect of the deficit angle could actually be appreciable. Consider the case of the Sun: We can write the deflection by a global monopole in terms of the observed bending of light by the Sun $\epsilon_s$ as $\Delta/2\approx\epsilon_s(\eta/10^{17}\ \mathrm{GeV})^2$. One could use this effect to place a bound on the abundance of global monopoles in the Sun.[18]

There is a totally different effect of the gravitational field of a global monopole we want to mention. During the formation of the monopoles, as the phase transition develops, the gravitational field changes very quickly, until the deficit solid angle grows to its final value. This time-dependent gravitational field produces pairs of particles by quantum effects. The analogous process was first considered for the case of cosmic-string formation.[19,20] The energy density in particles produced by the gravitational field during the formation of cosmic strings of energy per unit length $\eta^2$ is estimated to be $\rho\approx(G\eta^2)^2/\tau^4$, where $\tau$ is the time at which the phase transition occurs. We have evaluated the effect of the case of global monopoles formation. Assuming the formation of one monopole per horizon volume, the result is actually of the same order of magnitude as in the case for gauge strings. This is probably not so surprising. Compare, for instance, the energy density contributed by one string per horizon at time $t$. Since it has energy per unit length $\eta^2$ and length $t$, and we have one in a volume $t^3$, we get $\rho_{\mathrm{string}}\approx\eta^2/t^2$. With one monopole per horizon, with mass $\eta^2t$ inside it, we get again $\rho_{\mathrm{monopole}}\approx\eta^2/t^2$. Thus, the energy density in either strings or monopoles (with just one per horizon) has the same time dependence as the energy density $\rho$ that drives the early Universe evolution, be it matter or radiation dominated. The ratio $\rho_{\mathrm{string}}/\rho$ being constant is partly what makes strings a good candidate to the seeding mechanism for galaxy formation.

## IV. CONCLUSIONS

We have seen that a global monopole has gravitational effects of nature similar to other topological defects. Indeed, its main effect, due to its mass that grows linearly with the distance off its core, can be understood in terms of a deficit solid angle,[4] rather than a Newtonian gravitational potential, much like the case of a straight gauge string. From our analysis of the coupled Einstein and scalar field equations we conclude that there is also a relatively small gravitational potential of repulsive nature, corresponding to a mass at the origin of order the mass in the core, $\eta/\sqrt{\lambda}$, but with opposite sign. The appearance of repulsive gravitational effects does not come out as a surprise when one deals with highly nonrelativistic sources, with tensions as large as the energy density, as is the case here. The chief example is de Sitter space, the solution to Einstein equations with positive cosmological constant. It has a Newtonian repulsive potential proportional to $r^2$. It follows domain walls, with a linear repulsive potential, then global strings, with logarithmic repulsive effects, and finally the global monopole, with the usual Newtonian potential for a point particle but with negative mass. These effects can be understood[3] from the Newtonian limit of general relativity, which would imply the Poisson equation for the gravitational potential $\nabla\varphi=4\pi G(\rho+p_x+p_y+p_z)$. For nonrelativistic sources $p_i\ll\rho$. But when there is cosmological constant or for the topological defects in question, some $p_i$ are as large as $\rho$ and with opposite sign. In the exterior of the global monopole, for instance, we have $p=-\rho$ only in the radial direction, so nothing but a constant contribution to the Newtonian potential is expected. Inside the core, however, there are tensions in other directions too, and that is the origin of the repulsive effects.

Some consequences of the existence of particles with negative masses have been considered a while ago. In Newtonian physics they would lead to runaway solutions:

a particle of negative mass repels another particle with equal positive mass with the same strength as that with which the second particle attracts the first. So this system, left alone, accelerates itself without any external intervention, up to infinite speeds. This result was extended to general relativity by Bondi,[21] who showed that the system moves with constant proper acceleration, asymptotically approaching the speed of light. These runaway solutions could be seen as a drawback for a theory that allows particles with negative mass. Could global monopoles cause these runaway solutions? They do indeed have negative active gravitational mass (using the distinction made by Bondi between inertial and passive and active gravitational mass of a body). They are not, however, isolated point particles, but extended sources, even though the effective negative mass reaches its asymptotic value very quickly outside the core. One cannot have a global monopole by itself, but rather monopoles and antimonopoles, which probably annihilate themselves very efficiently, so that the motion of a system with a monopole and a particle with equal positive mass will not run away. It is interesting, though, to imagine this self-acceleration taking place for a while. Using particles of negative mass time machines have also been devised.[22] Again, monopoles not being just point particles of negative mass, it does not seem possible to organize them in moving shells of negative mass as those used to build closed timelike geodesics in Ref. 22.

Apart from the academic interest in the effective negative mass of the global monopole configuration, the smallness of a potential $G\eta/r$ seems to make it irrelevant in astronomical situations. Confront this with the repulsive potential of a global string, $-G\eta^2\ln(r/\delta)$. The repulsive potential of a global string would make matter initially at rest at $r=r_0$ around a static string acquire a speed $\sqrt{2G\eta^2\ln(r/r_0)}$. A global string acting for a time $t_0$ on a uniform distribution of matter could blow out "holes" with radius of order $\sqrt{2G\eta^2}t_0$. They could be as big as about 1 Mpc if we take $t_0$ as large as the Hubble time and $G\eta^2\approx 10^{-6}$. Matter would accumulate at the edges, moving away from the string at speeds of order $\sqrt{2G\eta^2}$, apart from logarithmic corrections. This effect could have interesting astronomical consequences. On the other hand an effective negative point mass of order $10^{-8}$ g, as such of a global monopole at a grand-unified-theory (GUT) scale, could hardly be of astronomical significance.

Though astronomically insignificant, there are still many curious processes one can think of when a global monopole is present, related to its effective negative mass. For instance, one could wonder how it would affect an evaporating black hole with a global monopole inside. If the black hole that swallowed the global monopole had a mass $M \gg 1/G\eta$, its Schwarzschild radius would be much larger than the monopole core radius. Neglecting the small effective mass of the monopole, the Schwarzschild radius would be given by $r_s=2GM/(1-\Delta)$. The dependence on $\Delta$, however, disappears from the expression for the surface gravity, and thus from the Hawking temperature of the black hole. Only the effective negative mass could then eventually introduce a very small change in the relation between the original mass of the black hole and its Hawking temperature. This correction, however, is probably insignificant, because when the black hole evaporates down to a mass $M\approx 1/G\eta$ (still much larger than $-M_A$ if $\Delta<1$) its Hawking temperature is already $T\approx\eta$, at which the global symmetry that originated the monopole could be restored. One could conjecture that the black hole would continue its evaporation without heating over this temperature, but rather using its energy to restore the symmetry as much as possible around the monopole core, until the mass of the hole becomes comparable to the effective negative mass of the monopole and the horizon disappears, and only the monopole is left behind. The analysis of such a process, however, requires a much more sophisticated analysis, with the effect of the large curvature around the black hole taken into account to determine the monopole configuration. It seems, though, to deserve further study.

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