# Effect of the final-state phases on *CP* violation in the $B_d^0$-$\bar{B}_d^0$ system

Morimitsu Tanimoto
*Science Education Laboratory, Ehime University, 790 Matsuyama, Japan*

Kazunori Goda, Keichi Hirayama, Kei Senba, and Tamon Shinmoto
*Department of Physics, Ehime University 790 Matsuyama, Japan*
(Received 22 December 1989)

We have studied the *CP*-violating asymmetry, taking account of the final-state phase shifts, for the decays $B_d^0 \to D^+D^-$, $B_d^0 \to \pi^+\pi^-$, and $B_d^0 \to K^+\pi^-$, which have tree-level and penguin amplitudes. The final-state phase shifts lead to significant direct *CP* violation for these decays. The numerical predictions of the *CP*-violating asymmetries are given against final-state phase shifts in the framework of the standard model.

The large $B_d^0$-$\bar{B}_d^0$ mixing observed by ARGUS and CLEO[1] has stimulated the study of *CP* violation in the neutral-*B*-meson system. The search for the *CP*-violation effect of the *B* meson[2-4] makes an important step forward in electroweak theory. It is remarked that the standard Kobayashi-Maskawa (KM) model[5] predicts a large indirect *CP* violation due to the $B_d^0$-$\bar{B}_d^0$ mixing in the neutral-*B*-meson decays. On the other hand, direct *CP* violation in the nonleptonic *B* decays occurs in the absence of mixing whenever there are at least two weak-decay amplitudes with different KM factors, which have different final-state-interaction phases.[4,6] In general the *B*-meson decay into a *CP* eigenstate has two different amplitudes: the tree-level amplitude and the loop (penguin) one;[7] hence we cannot neglect the final-state phases in order to test the standard model.[6,8] In this paper, we investigate quantitatively the effect of the final-state phases on *CP* violation of the nonleptonic neutral-*B*-meson decays. A qualitative study has already been given by Gronau,[8] who suggested that the *CP*-violating effect due to two interfering amplitudes becomes large in the KM-suppressed decays.

We study numerically the time-integrated *CP*-violating asymmetry for the typical three decay modes $B_d^0 \to D^+D^-$, $B_d^0 \to \pi^+\pi^-$, and $B_d^0 \to K^+\pi^-$. Those quark subprocesses are $\bar{b} \to \bar{c}c\bar{d}$, $\bar{b} \to \bar{u}u\bar{d}$, and $\bar{b} \to \bar{u}u\bar{s}$, respectively, each of which has both tree and penguin amplitudes.

The time-integrated asymmetry parameter $A(f)$ is defined by[2,4]

$$A(f)=\frac{\Gamma(B^0_{\rm phys}\to f)-\Gamma(\bar{B}^0_{\rm phys}\to \bar{f})}{\Gamma(B^0_{\rm phys}\to f)+\Gamma(\bar{B}^0_{\rm phys}\to \bar{f})} . \quad (1)$$

Then, $A(f)$ is given in terms of the decay amplitudes as

$$A(f)=\frac{(2+x_{B_d}^2)(|T|^2-|\bar{T}|^2)+x_{B_d}^2(|T\lambda|^2-|\bar{T}\bar{\lambda}|^2)-2x_{B_d}(|T|^2{\rm Im}\lambda-|\bar{T}|^2{\rm Im}\bar{\lambda})}{(2+x_{B_d}^2)(|T|^2+|\bar{T}|^2)+x_{B_d}^2(|T\lambda|^2+|\bar{T}\bar{\lambda}|^2)-2x_{B_d}(|T|^2{\rm Im}\lambda+|\bar{T}|^2{\rm Im}\bar{\lambda})} , \quad (2)$$

where

$$\lambda=\frac{q}{p}x,\quad x=\frac{\langle f|\bar{B}_d^0\rangle}{\langle f|B_d^0\rangle},\quad \bar{\lambda}=\frac{p}{q}\bar{x},\quad \bar{x}=\frac{\langle \bar{f}|B_d^0\rangle}{\langle \bar{f}|\bar{B}_d^0\rangle} ,$$
$$\frac{p}{q}=\left[\frac{M_{12}^*-i\Gamma_{12}^*/2}{M_{12}-i\Gamma_{12}/2}\right]^{1/2} , \quad (3)$$

where the notations, $T=\langle f|B_d^0\rangle$ and $\bar{T}=\langle \bar{f}|\bar{B}_d^0\rangle$ are used. We take $x_{B_d}=\Delta m/\Gamma=0.7$ given by ARGUS and CLEO.[1]

We calculate the decay amplitudes including the phases via the final-state interaction together with the KM phase, but we neglect the phases by rescattering processes,[6] such as $B_d^0 \to D^+D^- \to \pi^+\pi^-$, in which the final-state phase is difficult to calculate. We study three typical decay modes with the *CP*-conjugate ones: $B_d^0 \to D^+D^-$, $B_d^0 \to \pi^+\pi^-$, and $B_d^0 \to K^+\pi^-$. The relevant quark subprocesses are $\bar{b} \to \bar{c}c\bar{d}$, $\bar{u}u\bar{d}$, $\bar{u}u\bar{s}$, and their *CP*-conjugate processes. These decays occur via the tree and penguin processes.

The relations $|\langle f|B_d^0\rangle|=|\langle \bar{f}|\bar{B}_d^0\rangle|$ and $|\langle \bar{f}|B_d^0\rangle|=|\langle f|\bar{B}_d^0\rangle|$ are no more preserved because two different amplitudes with different KM factors are expressed as[6]

$$\langle f|B_d^0\rangle=\langle f|H_{\rm tree}|B_d^0\rangle+\langle f|H_{\rm penguin}|B_d^0\rangle$$
$$=G_1\exp(i\alpha_1)+G_2\exp(i\alpha_2) , \quad (4)$$

where $G_1, G_2$ are the weak amplitudes and $\alpha_1,\alpha_2$ are the strong phase shifts. The weak amplitudes $G_1$ and $G_2$ have different complex phases due to different KM factors, and furthermore nontrivial phase shifts $\alpha_1 \neq \alpha_2$ are generated from the strong (or electromagnetic) interaction because the two amplitudes in general differ in their isospin structure. The effective Hamiltonian of the tree-level process is given as

$$H_{\text{tree}}=\frac{4}{\sqrt{2}}G_F V_{ib}V_{jk}^*[C_1(\bar{q}_{kL}\gamma_\mu q_{jL})(\bar{q}_{iL}\gamma_\mu b_L)+C_2(\bar{q}_{iL}\gamma_\mu q_{jL})(\bar{q}_{kL}\gamma_\mu b_L)]\ , \quad (5)$$

where $i=u$ or $c$, $j=u$ or $c$, and $k=d$ or $s$, and $V_{ib}$ and $V_{jk}$ are the KM matrix elements. The scale-dependent coefficients $C_1=1.1$ and $C_2=-0.24$ are the QCD coefficients at the scale $\mu\approx m_b$.[9]

The effective penguin Hamiltonian is given by[7,10]

$$H_{\text{penguin}}=\sqrt{2}G_F\frac{\alpha_s}{\pi}\left[\sum V_{ib}V_{ij}^*I_i\right]\left[-\frac{1}{N}(\bar{q}\,{}^\alpha_{jL}\gamma_\mu q_L^\beta)(\bar{q}\,{}^\beta_L\gamma_\mu b_L^\alpha)+(\bar{q}\,{}^\alpha_{jL}\gamma_\mu q_L^\alpha)(\bar{q}\,{}^\beta_L\gamma_\mu b_L^\beta)+\frac{2}{N}(\bar{q}\,{}^\alpha_{jL}q_R^\beta)(\bar{q}\,{}^\beta_R b_L^\alpha)-2(\bar{q}\,{}^\alpha_{jL}q_R^\alpha)(\bar{q}\,{}^\beta_R b_L^\beta)\right]+\text{H.c.}\ , \quad (6)$$

where $N$ is the number of colors, $i$ runs on $u$, $c$, and $t$, $j$ is $d$ or $s$, and greek letters denote the color label. The analytic form of the loop integral function $I_i$ is presented in Refs. 11 and 12. In the $B$-meson system, $I_i$ has the imaginary component being derived from the logarithmic integral.[13] But we neglect its imaginary component because the effect of the final-state phase dominates the $CP$-violating asymmetry $A(f)$.

We begin by calculating the $B_d^0\to D^+D^-$ decay amplitude. The tree-level amplitude is given by

$$\langle D^+D^-|H_{\text{tree}}|B_d^0\rangle=\frac{4}{\sqrt{2}}G_F V_{cb}V_{cd}^*[C_1\langle D^+D^-|(\bar{d}_L\gamma_\mu c_L)(\bar{c}_L\gamma_\mu b_L)|B_d^0\rangle+C_2\langle D^+D^-|(\bar{c}_L\gamma_\mu c_L)(\bar{d}_L\gamma_\mu b_L)|B_d^0\rangle]\ . \quad (7)$$

The evaluation of the hadronic matrix element depends on the hadronic model used. The simplest way is the one based on the factorization approximation, which expresses the four-quark operators in terms of the factorized color-singlet current matrix elements.[9] Note that the $1/N$ expansion argument provides some theoretical justification for the factorization approximation since the factorization follows to leading order in a $1/N$ expansion.[14] This approximation may be good for the heavy meson decays. Thus, we get

$$\langle D^+D^-|H_{\text{tree}}|B_d^0\rangle=\frac{1}{\sqrt{2}}G_F V_{cb}V_{cd}^*\left[\left(C_1+\frac{1}{N}C_2\right)\langle D^+|-\bar{d}\gamma_\mu\gamma_5 c|0\rangle\langle D^-|\bar{c}\gamma_\mu b|B_d^0\rangle+\left(C_2+\frac{1}{N}C_1\right)\langle D^+D^-|\bar{c}\gamma_\mu c|0\rangle\langle 0|-\bar{d}\gamma_\mu\gamma_5 b|B_d^0\rangle\right]\ , \quad (8)$$

where the $1/N$ factor arises from the color mismatch in forming a color singlet after Fierz transformation. On the other hand, the penguin amplitude is classified into the timelike and spacelike gluon emission amplitudes.[11,12] We obtain the timelike amplitudes taking $q_j=d$ and $q=c$ in Eq. (6) as follows:

$$\langle D^+D^-|H^{\text{time}}_{\text{penguin}}|B_d^0\rangle=\sqrt{2}G_F\frac{\alpha_s}{\pi}\left[\sum V_{ib}V_{id}^*I_i\right]\left[\left(1-\frac{1}{N^2}\right)\tfrac{1}{4}\langle D^+|-\bar{d}\gamma_\mu\gamma_5 c|0\rangle\langle D^-|\bar{c}\gamma_\mu b|B_d^0\rangle+2\left(\frac{1}{N^2}-1\right)\langle D^+|\bar{d}_L c_R|0\rangle\langle D^-|\bar{c}_R b_L|B_d^0\rangle\right]\ , \quad (9)$$

where the first hadronic matrix element is the same one in Eq. (8), but the second one is a new matrix element. This matrix element is reduced by using the equations of motion of the quarks under the factorization approximation as follows:[10]

$$\langle D^+|\bar{d}_L c_R|0\rangle\langle D^-|\bar{c}_R b_L|B_d^0\rangle=-\frac{1}{4}\frac{M_D^2}{(m_d+m_c)(m_b-m_c)}\langle D^+|-\bar{d}\gamma_\mu\gamma_5 c|0\rangle\langle D^-|\bar{c}\gamma_\mu b|B_d^0\rangle\ . \quad (10)$$

On the other hand, taking $q_j=d$ and $q=d$ in Eq. (6), the spacelike penguin amplitude is obtained as

$$\langle D^+D^-|H^{\text{space}}_{\text{penguin}}|B_d^0\rangle=\sqrt{2}G_F\frac{\alpha_s}{\pi}\left[\sum V_{ib}V_{id}^*I_i\right]\left[\left(1-\frac{1}{N^2}\right)\tfrac{1}{4}\langle D^+D^-|\bar{d}\gamma_\mu d|0\rangle\langle 0|-\bar{d}\gamma_\mu\gamma_5 b|B_d^0\rangle+2\left(\frac{1}{N^2}-1\right)\langle D^+D^-|\bar{d}_L d_R|0\rangle\langle 0|\bar{d}_R b_L|B_d^0\rangle\right]\ . \quad (11)$$

Since the hadronic matrix elements $\langle D^+D^-|\bar{d}\gamma_\mu d|0\rangle$ and $\langle D^+D^-|\bar{d}_L d_R|0\rangle$ vanish in the factorization approximation,[10] the effect of the spacelike penguin process disappears. Then, the $B_d^0\to D^+D^-$ decay amplitude is given in terms of one hadronic matrix element as

$$\langle D^+D^-|B_d^0\rangle=\langle D^+D^-|H_{\rm tree}|B_d^0\rangle+\langle D^+D^-|H_{\rm penguin}|B_d^0\rangle$$
$$=\frac{1}{\sqrt{2}}G_F\left\{V_{cb}V_{cd}^*\left[C_1+\frac{1}{N}C_2\right]\exp(i\alpha_1)+\frac{\alpha_s}{2\pi}\left[\sum V_{ib}V_{id}^*I_i\right]\exp(i\alpha_2)\left[\left[1-\frac{1}{N^2}\right]+2\left[1-\frac{1}{N^2}\right]\frac{M_D^2}{(m_d+m_c)(m_b-m_c)}\right]\right\}$$
$$\times\langle D^+|-\bar{d}\gamma_\mu\gamma_5 c|0\rangle\langle D^-|\bar{c}\gamma_\mu b|B_d^0\rangle\ . \qquad (12)$$

For the $CP$-conjugate process $\bar{B}_d^0\to D^+D^-$, the decay amplitude is easily given by replacing the KM matrix elements in Eq. (12) such as $V_{ij}\to V_{ij}^*$ and $V_{ij}^*\to V_{ij}$. In the same way, the decay amplitudes of the other decays $B_d^0\to\pi^+\pi^-$ and $B_d^0\to K^+\pi^-$ are calculated. The results are summarized as follows:

$$\langle\pi^+\pi^-|B_d^0\rangle=\frac{1}{\sqrt{2}}G_F\left\{V_{ub}V_{ud}^*\left[C_1+\frac{1}{N}C_2\right]\exp(i\alpha_1)+\frac{\alpha_s}{2\pi}\left[\sum V_{ib}V_{id}^*I_i\right]\exp(i\alpha_2)\left[\left[1-\frac{1}{N^2}\right]+2\left[1-\frac{1}{N^2}\right]\frac{M_\pi^2}{(m_d+m_u)(m_b-m_u)}\right]\right\}$$
$$\times\langle\pi^+|-\bar{d}\gamma_\mu\gamma_5 u|0\rangle\langle\pi^-|\bar{u}\gamma_\mu b|B_d^0\rangle\ ,$$
$$\langle K^+\pi^-|B_d^0\rangle=\frac{1}{\sqrt{2}}G_F\left\{V_{ub}V_{us}^*\left[C_1+\frac{1}{N}C_2\right]\exp(i\alpha_1)+\frac{\alpha_s}{2\pi}\left[\sum V_{ib}V_{is}^*I_i\right]\exp(i\alpha_2)\left[\left[1-\frac{1}{N^2}\right]+2\left[1-\frac{1}{N^2}\right]\frac{M_K^2}{(m_s+m_u)(m_b-m_u)}\right]\right\}$$
$$\times\langle K^+|-\bar{s}\gamma_\mu\gamma_5 u|0\rangle\langle\pi^-|\bar{u}\gamma_\mu b|B_d^0\rangle\ , \qquad (13)$$

where $\alpha_1$ and $\alpha_2$ are unknown final-state phases and are generally different for each process. For the $K^+\pi^-$ decay amplitude in Eq. (13), the annihilation form-factor term is neglected.

The $CP$-conjugate decay amplitudes are given by replacing the KM matrix elements in Eq. (13) such as $V_{ij}\to V_{ij}^*$ and $V_{ij}^*\to V_{ij}$. Note that the final states $D^+D^-$ and $\pi^+\pi^-$ are $CP$ eigenstates but $K^+\pi^-$ are not. We have no decay amplitudes such as $\bar{B}_d^0\to K^+\pi^-$ and their $CP$ conjugate decays.

We present the numerical result for time-integrated and time-differential $CP$ violation. We begin by showing the physical parameters used in our calculation. The quark masses are taken so that $(m_d,m_u,m_s,m_c,m_b)=(0.009,0.005,0.175,1.4,4.95)$ in GeV units,[15] and the top-quark mass is a free parameter. Also, the value of $\alpha_s$ is fixed as 0.23. Although the color number $N$ is 3, we take the $1/N=0$ limit in the decay amplitudes due to two reasons: The analysis of the nonleptonic decays of heavy mesons based on $1/N=0$ is very successful phenomenologically as shown by Bauer, Stech, and Wirbel,[9] and the factorization approximation used in our calculations follows from leading order in a $1/N$ expansion.[14] We use the KM matrix parametrized by Chau and Keung,[16] where we take $s_y=0.046$ and $s_z/s_y=0.09$ following from the recent ARGUS and CLEO results.[17] Although the KM phase $\phi$ is unknown, the value of $\phi=150°$ is taken typically.

We present the numerical results of the asymmetries $A(f)$. The relevant parameters $\lambda$ and $\bar{\lambda}$ are easily calculated by use of Eqs. (12) and (13). When a final state is the $CP$ eigenstate, we get $\bar{\lambda}=1/\lambda$. But, we obtain $\lambda=\bar{\lambda}=0$ for $f=K^+\pi^-$ because $\langle\bar{f}|B_d^0\rangle=\langle f|\bar{B}_d^0\rangle=0$. On the other hand, since the value of $\alpha_1$-$\alpha_2$ is unknown, we investigate the dependence of the $CP$-violating asymmetry on $\alpha_1$-$\alpha_2$ in the region from $-180°$ to $180°$. We show the asymmetry parameter $A(f)$ versus $\alpha_1$-$\alpha_2$ for $B_d^0\to D^+D^-$, $\pi^+\pi^-$, and $K^+\pi^-$ in the case of $m_t=100$ GeV and $\phi=150°$ in Fig. 1. We have found that the asymmetry parameters depend remarkably on $\alpha_1$-$\alpha_2$ for

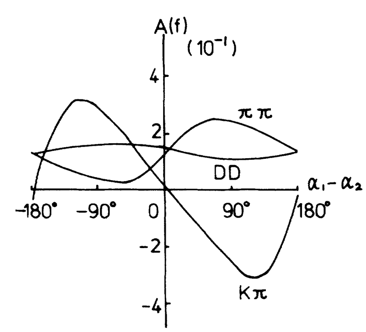


FIG. 1. The asymmetry parameter $A(f)$ vs $\alpha_1$-$\alpha_2$ for $B_d^0\to D^+D^-$, $B_d^0\to\pi^+\pi^-$, and $B_d^0\to K^+\pi^-$, where $m_t=100$ GeV, $s_z/s_y=0.09$, and $\phi=150°$.

the $\pi^+\pi^-$ and $K^+\pi^-$ final states, but rather mildly for $D^+D^-$. It is concluded that the asymmetry for $B_d^0 \to D^+D^-$ depends mildly on the final-state phase shifts, and so the previous prediction by the standard model[4] is not so changed; however, the previous predictions of the other two processes[4] are not reliable unless the final-state phase shifts $\alpha_1$-$\alpha_2$ are known.

We have studied the *CP*-violating asymmetry taking account of the final-state phase shifts. We have found that the final-state phase shifts play an important role for the decays $B_d^0 \to D^+D^-$, $\pi^+\pi^-$, and $K^+\pi^-$, each of which has both a tree-level and penguin amplitude. The existence of the penguin amplitude in addition to the tree-level amplitude leads to direct *CP* violation via the imaginary component from the relative final-state phase shifts. For the $K^+\pi^-$ decays, only direct *CP* violation gives the asymmetry. In the $K^+\pi^-$ decay, the penguin amplitude is larger than the tree-level amplitude, so direct *CP* violation could be large. For the $B_d^0 \to D^+D^-$ and $\pi^+\pi^-$ decays, indirect *CP* violation via $B_d^0$-$\bar{B}_d^0$ mixing significantly contributes to the asymmetry parameter. But the effect of the final-state phase shifts is also important in these decays. Thus, the study of the final-state interaction is significant to give the reliable predictions of the *CP*-violating asymmetries in the framework of the standard model.


[1]ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. B **192**, 245 (1987); CLEO Collaboration, M. Artuso *et al.*, Phys. Rev. Lett. **62**, 2233 (1989).

[2]I. I. Bigi and A. I. Sanda, Nucl. Phys. **B193**, 85 (1981); Phys. Rev. D **29**, 1393 (1984); L. Wolfenstein, Nucl. Phys. **B246**, 45 (1984); J. F. Donoghue, T. Nakada, E. A. Paschos, and D. Wyler, Phys. Lett. B **195**, 285 (1987); D-S. Du and Z-Y. Zhao, Phys. Rev. Lett. **59**, 1072 (1987); I. Dunietz and R. G. Sachs, Phys. Rev. D **37**, 3186 (1988); E. A. Paschos and U. Türke, Phys. Rep. C **178**, 145 (1989).

[3]I. Dunietz and T. Nakada, Z. Phys. C **36**, 503 (1987).

[4]I. Dunietz and J. L. Rosner, Phys. Rev. D **34**, 1404 (1986); D. Du, I. Dunietz, and D-di Wu, *ibid.* **34**, 3414 (1986).

[5]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[6]I. I. Bigi and A. I. Sanda, Nucl. Phys. **B281**, 41 (1987).

[7]A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, Zh. Eksp. Teor. Fiz. **72**, 1275 (1977) [Sov. Phys. JETP **45**, 670 (1977)].

[8]M. Gronau, Phys. Rev. Lett. **63**, 1451 (1989).

[9]M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **29**, 637 (1985); **34**, 103 (1987).

[10]M. B. Gavela *et al.*, Phys. Lett. **154B**, 425 (1985); L-L. Chau and H-Y. Cheng, Phys. Rev. Lett. **53**, 1037 (1984); **59**, 958 (1987); Phys. Lett. **165B**, 429 (1985).

[11]M. Tanimoto, Phys. Lett. B **218**, 481 (1989); Phys. Rev. Lett. **62**, 2797 (1989).

[12]M. Tanimoto, T. Shinmoto, K. Hirayama, and K. Senba, Phys. Rev. D **40**, 2934 (1989).

[13]M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **43**, 242 (1979).

[14]A. J. Buras, J.-M. Gérard, and R. Rückl, Nucl. Phys. **B268**, 16 (1986).

[15]J. Gasser and H. Leutwyler, Phys. Rep. C **87**, 77 (1982); see Paschos and Türke, Ref. 2.

[16]L. L. Chau and W. Y. Keung, Phys. Lett. **53B**, 1804 (1984).

[17]ARGUS Collaboration, Report No. DESY 89-152, 1989 (unpublished); CLEO Collaboration, Phys. Rev. Lett. **64**, 16 (1990).