PHYSICAL REVIEW D VOLUME 42, NUMBER 7 1 OCTOBER 1990

Analysis of 2γ and $\pi^+\pi^-\gamma$ decays of η and η' using chiral anomalies

Pyungwon Ko
Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637

Tran N. Truong
Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637
*and Centre de Physique Théorique de L'Ecole Polytechnique, 91128 Palaiseau, France**

(Received 18 June 1990)

Using chiral anomalies for $\eta,\eta'\to 2\gamma$ and $\eta\to\pi^+\pi^-\gamma$ amplitudes, we determine that the η,η' mixing angle $\theta_P=-20.8°\pm1.6°$, $f_8/f_\pi=1.23\pm0.06$, and $f_0/f_\pi=1.04\pm0.07$. Predictions are made for the rate and dipion spectra for $\eta,\eta'\to\pi^+\pi^-\gamma$.

There has been interest in studying the η and η' mixing problem and the determination of their decay constants as a result of the recent accurate measurements of $\eta,\eta'\to\gamma\gamma$ widths. Most of the theoretical studies[1] make use of the process $\eta,\eta'\to\gamma\gamma$,[2] and hence use only the data on $\eta,\eta'\to\gamma\gamma$. In an extensive study of the η,η' mixing-angle problem, Gilman and Kauffman[3] found that $\eta,\eta'\to\gamma\gamma$ could be reasonably well understood by assuming $f_8/f_\pi=1.25$. They obtained

$$\theta_P=-23°\pm3°\pm1°\ ,$$

$$\frac{f_0}{f_\pi}=1.04\pm0.04\pm0.05\ .$$

The first error is statistical, and the second takes into account the 5% theoretical uncertainty in $f_8/f_\pi=1.25$. Using the chiral anomaly and vector-meson dominance, they also studied $\eta\to\pi^+\pi^-\gamma$ with the above values of θ_P, f_8, and f_0. Their result on the $\eta\to\pi^+\pi^-\gamma$ width, however, was about a factor of 2 smaller than the experimental rate. This was interpreted as our lacking the knowledge of how to extrapolate the amplitude from the chiral limit to the physical region for η decays. Since they used two experiments, $\eta,\eta'\to\gamma\gamma$, to determine three parameters θ_P, f_8, and f_0, they had to assume the value of $f_8/f_\pi=1.25$ to determine the other two parameters.

The purpose of the present note is to reanalyze $\eta,\eta'\to\pi^+\pi^-\gamma$ using the chiral-anomaly theorems.[4] The problem encountered by Gilman and Kauffman was not new. In fact, a few years ago, it was pointed out by Freund and Zee[5] that there was a problem of reconciling pure vector-meson dominance (VMD) with the low-energy chiral-anomaly theorems.[6] A correct solution to this problem was first given by Fujiwara *et al.*[7] who added a contact term to the VMD $P\to\pi^+\pi^-\gamma$ amplitudes. (Here, $P=\pi$, η, and η'.) The strength of the contact term is such that the low-energy theorems are recovered in the chiral limit. The resulting $\eta,\eta'\to\pi^+\pi^-\gamma$ amplitudes satisfy the low-energy theorems, but violate unitarity.

Recently, one of us (T.N.T.) (Ref. 8) showed that it is possible to construct $P\to\pi^+\pi^-\gamma$ amplitudes satisfying the chiral-anomaly theorems and unitarity by demanding the self-consistency in the calculation of $P\to\gamma\gamma$ using the $P\to\pi^+\pi^-\gamma$ anomaly. Dispersion relations and unitarity, or unitarized chiral perturbation theory, was used for this purpose. In the approximation where the imaginary part of the $P\to\pi^+\pi^-\gamma$ amplitude is neglected, the result of Fujiwara *et al.*[7] is recovered.

Here we use the decay rates of $P\to\gamma\gamma$ and $\eta\to\pi^+\pi^-\gamma$ to determine the η,η' mixing angle θ_P and the decay constants f_8 and f_0. We then make predictions for the rate of $\eta'\to\pi^+\pi^-\gamma$ and for the dipion spectrum in $\eta\to\pi^+\pi^-\gamma$. It should be emphasized that we do not make use of any theoretical calculations of f_s and/or f_0, which is in contrast with previous analyses. Since we have three independent data and three unknown parameters to be fixed, our determination of three parameters θ_P, f_8, and f_0 is free of theoretical uncertainty in assuming certain values for f_8 and/or f_0. Therefore, the analysis presented here is not the same as the earlier studies, even though we get essentially the same numerical values for those parameters within the error.

Let us define $F_{P\gamma\gamma}(p^2)$ by

$$\mathcal{M}(P(p)\to\gamma(\epsilon_1,k_1)+\gamma(\epsilon_2,k_2)) = -\epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu k_1^\nu\epsilon_2^\alpha k_2^\beta F_{P\gamma\gamma}(p^2)\ .$$

The axial chiral anomaly predicts

$$F_{\pi\gamma\gamma}(0)=\frac{e^2}{4\pi^2 f_\pi}\ , \tag{1a}$$

$$F_{\eta_8\gamma\gamma}(0)=\frac{1}{\sqrt{3}}\frac{f_\pi}{f_8}F_{\pi\gamma\gamma}(0)\ , \tag{1b}$$

$$F_{\eta_0\gamma\gamma}(0)=2\left(\frac{2}{3}\right)^{1/2}\frac{f_\pi}{f_0}F_{\pi\gamma\gamma}(0)\ , \tag{1c}$$

where $f_\pi=93$ MeV. Equation (1a) gives $\Gamma(\pi^0\to\gamma\gamma)=7.6$ eV, compared with the experimental value 7.57 ± 0.32 eV.

The $P\to\pi^+\pi^-\gamma$ amplitudes are also given by the chiral anomalies. Define $F_{P\pi\pi}$ by

$$\mathcal{M}(P(p)\to\pi^+(q_+)\pi^-(q_-)\gamma(\epsilon,k)) = -i\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu k^\nu q_+^\alpha q_-^\beta F_{P\pi\pi}(s,t,u)\ ,$$

where $s=(q_+ + q_-)^2$, etc. The chiral-anomaly relations are

$$F_{\pi\pi\pi}(0,0,0)=\lambda\,, \tag{2a}$$

$$F_{\eta_8\pi\pi}(0,0,0)=\frac{\lambda}{\sqrt{3}}\frac{f_\pi}{f_8}\,, \tag{2b}$$

$$F_{\eta_0\pi\pi}(0,0,0)=\lambda\left[\frac{2}{3}\right]^{1/2}\frac{f_\pi}{f_0}\,, \tag{2c}$$

where $\lambda=F_{\pi\gamma\gamma}(0)/ef_\pi^2=9.45\times10^{-9}\ \text{MeV}^{-3}$. In order to calculate the physical amplitudes where $m_\pi^2\neq0$ and $s\geq 4m_\pi^2$, Gilman and Kauffman used ρ VMD to extrapolate Eqs. (2b) and (2c) to the physical region.[3,9] The $\eta_8\to\pi^+\pi^-\gamma$ amplitude is

$$F_{\eta_8\pi\pi}(s,t,u)=\frac{\lambda}{\sqrt{3}}\frac{m_\rho^2}{m_\rho^2-s}\,, \tag{3}$$

with a similar expression for $F_{\eta_0\pi\pi}(s,t,u)$. With $f_8/f_\pi=1.25$, $f_0/f_\pi=1.04$, and the mixing angle $\theta_P=-20°$, they found that $\Gamma(\eta\to\pi^+\pi^-\gamma)=37$ eV, which is almost a factor of 2 smaller than the experimental value 64.3 eV. It can be shown that a prescription similar to Eq. (3) would lead to $\Gamma(\rho\to\pi\gamma)=35$ keV. [There are three Feynman diagrams associated with Eq. (2a) (Ref. 8).]

One of us (T.N.T.) has recently shown that the type of correction which is used in Eq. (3) would lead to an erroneous relation between $P\to\gamma\gamma$ anomalies and $P\to\pi^+\pi^-\gamma$ anomalies. The correct results are [8]

$$F_{\eta_8\pi\pi}(s)=\frac{\lambda}{\sqrt{3}}\frac{f_\pi}{f_8}F(s)\left[1+\frac{1}{2}\frac{s}{s_\rho}\right], \tag{4a}$$

$$F_{\eta_0\pi\pi}(s)=\lambda\left[\frac{2}{3}\right]^{1/2}\frac{f_\pi}{f_0}F(s)\left[1+\frac{1}{2}\frac{s}{s_\rho}\right], \tag{4b}$$

where

$$F(s)=\frac{s_R-8\gamma m_\pi^2/\pi}{s_R-s+\gamma(s-4m_\pi^2)[h(s)-i\rho(s)]}\,,$$

$$h(s)=\frac{2}{\pi}\left[\frac{s-4m_\pi^2}{s}\right]^{1/2}\ln\left[\frac{\sqrt{s}+(s-4m_\pi^2)^{1/2}}{2m_\pi}\right],$$

with $F(0)=1$. These amplitudes are constructed in such a way that the following conditions are met. (i) At $s=0$, the chiral anomalies, Eqs. (2b) and (2c), are recovered. (ii) Their phases satisfy the final-state theorem (unitarity of the S matrix) which requires that they have the correct P-wave $\pi\pi$ phase shift (up to 1 GeV). (iii) At $s=s_\rho$, these amplitudes satisfy the ρ vector-meson-dominance relations. s_R is determined by the requirement that $F_{P\pi\pi}$ has the correct P-wave $\pi\pi$ phase shift due to the unitarity relation. γ is fixed by the experimental ρ width, $\Gamma_\rho\simeq152$ MeV. We find $\sqrt{s_R}=710$ MeV and $\gamma=0.185$. In the narrow-width approximation, $F(s)\simeq s_\rho/(s_\rho-s)$, and Eq. (4a) becomes

$$F_{\eta_8\pi\pi}(s)=\frac{\lambda}{\sqrt{3}}\frac{s_\rho}{s_\rho-s}\left[1+\frac{1}{2}\frac{s}{s_\rho}\right],$$

which is the same as that obtained from the prescription of Fujiwara *et al.*:

$$F_{\eta_8\pi\pi}(s)=\frac{\lambda}{\sqrt{3}}\frac{f_\pi}{f_8}\left[1+\frac{3}{2}\frac{s}{s_\rho-s}\right]. \tag{5}$$

Using the usual prescription of introducing an imaginary part in s_ρ of Eq. (5) would result in violation of the final-state phase theorem of 13° at $s=s_\rho$. From now on, we use Eq. (4a) and (4b) in our phenomenological analysis.

Because $F_{\eta_8\pi\pi}$ and $F_{\eta_0\pi\pi}$ have the same s dependence, we can write

$$F_{\eta\pi\pi}(s)=F_{\eta\pi\pi}(0)F(s)\left[1+\frac{1}{2}\frac{s}{s_\rho}\right], \tag{6a}$$

$$F_{\eta'\pi\pi}(s)=F_{\eta'\pi\pi}(0)F(s)\left[1+\frac{1}{2}\frac{s}{s_\rho}\right]. \tag{6b}$$

Using $\Gamma(\eta\to\pi^+\pi^-\gamma)=0.064\pm0.006$ keV using Table I, we have

$$|F_{\eta\pi\pi}(0)|=(6.47\pm0.25)\times10^{-9}\ \text{MeV}^{-3}\,, \tag{7}$$

where we have made use of the experimental fact that $B(\eta\to\pi^+\pi^-\gamma)$ and $B(\eta\to\gamma\gamma)$ are very accurately measured but not the total η width. From the experimental information[10] $\Gamma(\eta\to\gamma\gamma)=0.51\pm0.02\pm0.04$ keV and $\Gamma(\eta'\to\gamma\gamma)=4.7\pm0.5\pm0.5$ keV we have

$$|F_{\eta\gamma\gamma}(0)|=(2.49\pm0.10)\times10^{-5}\ \text{MeV}^{-1}\,, \tag{8a}$$

$$|F_{\eta'\gamma\gamma}(0)|=(3.28\pm0.24)\times10^{-5}\ \text{MeV}^{-1}\,. \tag{8b}$$

In the nonet scheme, we have

$$F_{\eta\gamma\gamma}(0)=F_{\pi\gamma\gamma}\left[\frac{f_\pi}{f_8}\frac{\cos\theta_P}{\sqrt{3}}-\left[\frac{8}{3}\right]^{1/2}\frac{f_\pi}{f_0}\sin\theta_P\right], \tag{9a}$$

$$F_{\eta'\gamma\gamma}(0)=F_{\pi\gamma\gamma}\left[\frac{f_\pi}{f_8}\frac{\sin\theta_P}{\sqrt{3}}+\left[\frac{8}{3}\right]^{1/2}\frac{f_\pi}{f_0}\cos\theta_P\right], \tag{9b}$$

$$F_{\eta\pi\pi}(0)=\lambda\left[\frac{f_\pi}{f_8}\frac{\cos\theta_P}{\sqrt{3}}-\left[\frac{2}{3}\right]^{1/2}\frac{f_\pi}{f_0}\sin\theta_P\right], \tag{9c}$$

$$F_{\eta'\pi\pi}(0)=\lambda\left[\frac{f_\pi}{f_8}\frac{\sin\theta_P}{\sqrt{3}}+\left[\frac{2}{3}\right]^{1/2}\frac{f_\pi}{f_0}\cos\theta_P\right]. \tag{9d}$$

TABLE I. Rate (in keV) for $\eta\to\pi^+\pi^-\gamma$ and $\eta'\to\pi^+\pi^-\gamma$ using Eqs. (6) and (11) when $|F_{\eta,\eta'\pi\pi}(0)|=1.0\times10^{-9}$ MeV. The experimental data are shown in the last column.

	Rate by Eq. (6)	Rate by Eq. (11) $s_{\rho'}=2s_\rho$	Rate by Eq. (11) $s_{\rho'}=\infty$	Expt. data
$\Gamma(\eta\to\pi^+\pi^-\gamma)$	1.53×10^{-3}	1.38×10^{-3}	1.18×10^{-3}	$(64\pm6)\times10^{-3}$
$\Gamma(\eta'\to\pi^+\pi^-\gamma)$	2.20	2.10	1.11	62 ± 6

Using Eqs. (7), (8a), and (8b) in Eqs. (9a)–(9c) with their signs determined from the chiral anomalies, we have

$$\theta_P = -23.6° \pm 1.8° ,$$
$$\frac{f_8}{f_\pi} = 1.47 \pm 0.07 , \tag{10}$$
$$\frac{f_0}{f_\pi} = 1.02 \pm 0.07 .$$

Using Eq. (10), we can determine $F_{\eta'\pi\pi}(0) = (5.45 \pm 0.34) \times 10^{-9}$ MeV^{-3}, or using Eq. (4b), we get $\Gamma(\eta' \to \pi^+\pi^-\gamma) = 65 \pm 8$ keV. This value is, however, insensitive to the variation of parameters given by Eq. (10).

The dipion spectrum in $\eta \to \pi^+\pi^-\gamma$ is given by Eq. (6a), and is approximately proportional to $|\mathcal{M}|^2 \sim |1 + \frac{3}{2}(s/s_\rho)|^2$. [See Fig. 1(a), the solid curve.] These same relations also hold in the prescription of Fujiwara *et al.* The experimental data by Layter *et al.*[11] is, however, dominated by the pure ρ propagator $|\mathcal{M}|^2 \sim |1 + s/s_\rho|^2$. We suspect the experimental data has a problem with the systematic errors, because at *larger* values of s, the data has a *smaller* slope than that given by the pure ρ propagator.

Let us now discuss the possibility that future experiments could give a smaller dipion slope than that given by Eqs. (4) and (5). Could we modify and if so how would it affect our analysis for θ_P, f_s, and f_0? The solution to this question was previously given.[8] In the method of Fujiwara *et al.*, the modification of the VMD amplitude was made by a contact term.[7] In the approach of one of us (T.N.T.),[8] a contact term was also introduced, but was unitarized. (It corresponds to the polynomial ambiguity in the Muskhelishvili-Omnes equations.[12]) They both represent some uncalculable high-energy (or short distance) effects. Instead of Eqs. (4), we could represent them by the phenomenological parametrization

$$F_{\eta\pi\pi}(s) = \tilde{F}_{\eta\pi\pi}(0) F(s) \left[1 + \frac{1}{2} \frac{s}{s_{\rho'} - s} \right] , \tag{11}$$

where $s_{\rho'} \simeq 2s_\rho$, and could represent some ρ' resonance at 1.1 GeV found in the pion form-factor measurement, or some inelastic effect. (We have to give $s_{\rho'}$ an imaginary part for $s > 1$ GeV^2.) The calculated rates for $\eta, \eta' \to \pi^+\pi^-\gamma$ are given in Table I. It is seen that the rates are about 10% lower. This leads to a new determination

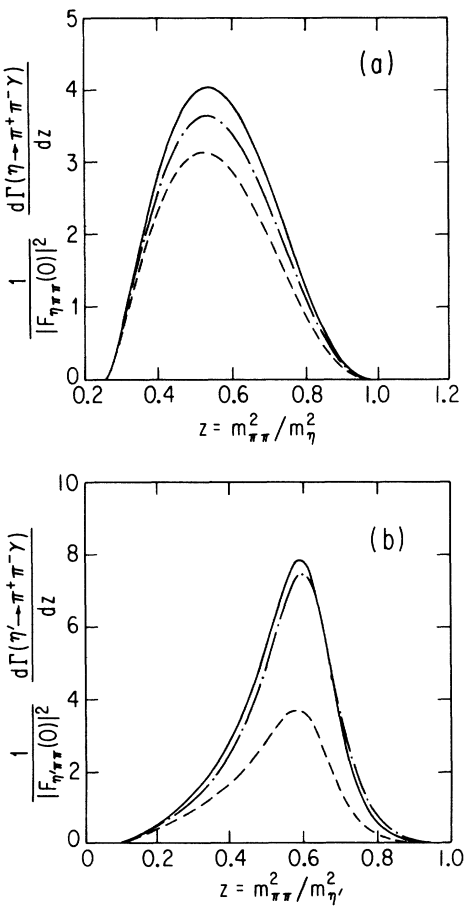

FIG. 1. Dipion spectra for (a) $\eta \to \pi^+\pi^-\gamma$, and (b) $\eta' \to \pi^+\pi^-\gamma$ in units of $|F_{\eta,\eta'\pi\pi}(0)|^2 \times \text{MeV}^7$: Solid curves for Eq. (6); dash-dotted curves for Eq. (11) with $s_{\rho'} = 2s_\rho$; and dotted curves for Eq. (11) with $s_{\rho'} = \infty$.

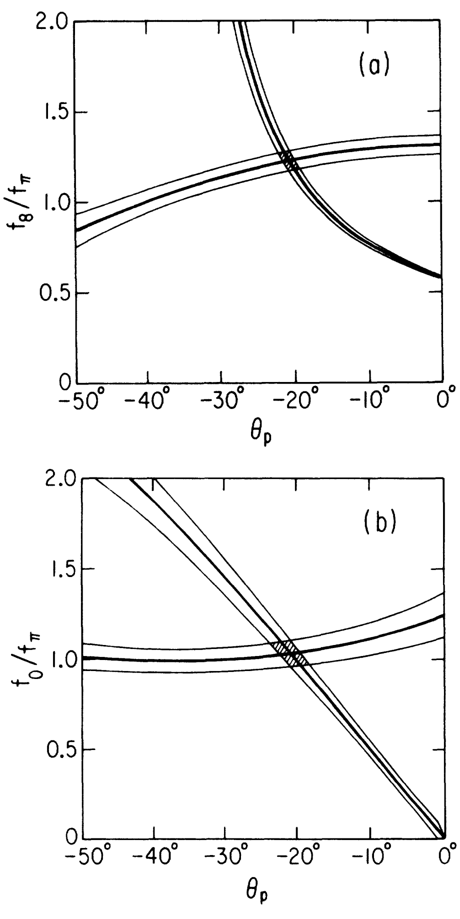

FIG. 2. (a) f_8/f_π and (b) f_0/f_π vs θ_P, using Eq. (11) with $s_{\rho'} = 2s_\rho$. Within one standard deviation, the allowed regime of parameters is denoted by the shaded region. This is determined by using Eqs. (7)–(9). The intersection point is the central value quoted in Eq. (12).

of the mixing angle and the decay constants

$$\theta_P = -20.8° \pm 1.6° ,$$
$$\frac{f_8}{f_\pi} = 1.23 \pm 0.06 , \qquad (12)$$
$$\frac{f_0}{f_\pi} = 1.04 \pm 0.07 .$$

The allowed regions for these three parameters are shown in Fig. 2. We note that this new set of parameters is more consistent with the study of the mass matrix for η and η' system, and also with the study of the SU(3)-breaking effect [the value of f_8/f_π given by Eq. (10) is too large].

In conclusion, we strongly urge experimentalists to remeasure the dipion spectrum in $\eta \to \pi^+\pi^-\gamma$ to test the above analysis. [See Fig. 1(a), the dash-dotted and dotted curves.] Precise measurement of the dipion spectrum in $\eta' \to \pi^+\pi^-\gamma$, especially at low dipion, mass is also needed. [See Fig. 1(b).] It is also very important to remeasure the pion production in the Coulomb field,[13]

$$\pi^\pm + Z \to \pi^\pm \pi^0 + Z ,$$

with a much-higher-energy incident $\pi^\pm$ beam to cover not only the low-energy $\pi^\pm\pi^0$ but also the ρ-resonance region.

We would like to thank Jon Rosner for useful discussions. One of us (T.N.T.) would like to thank the Enrico Fermi Institute and the Department of Physics for financial support and hospitality. Part of the work was supported by Department of Energy Grant No. DE-FG02-90ER-40560.

*Permanent address.

[1]J. Donoghue, B. Holstein, and Y. C. R. Lin, Phys. Rev. Lett. **55**, 2766 (1985); G. Grunberg, Phys. Lett. **168B**, 141 (1986); T. N. Pham, Phys. Rev. D **30**, 234 (1986).

[2]S. L. Adler, Phys. Rev. **117**, 2426 (1966); J. S. Bell and R. Jackiw, Nuovo Cimento **60A**, 47 (1969).

[3]F. J. Gilman and R. Kauffman, Phys. Rev. D **36**, 2761 (1987).

[4]R. Aviv, N. D. Hari Dass, and R. F. Sawyer, Phys. Rev. Lett. **26**, 591 (1971); S. L. Adler, B. W. Lee, S. B. Treiman, and A. Zee, Phys. Rev. D **4**, 3497 (1971); M. Terentev, Pis'ma Zh. Eksp. Teor. Fiz. **14**, 140 (1971) [JETP Lett. **14**, 94 (1971)]; J. Wess and B. Zumino Phys. Lett. **37B**, 95 (1971); W. A. Bardeen, Phys. Rev. **184**, 1848 (1969); R. Aviv and A. Zee, Phys. Rev. D **5**, 2372 (1971).

[5]P. G. O. Freund and A. Zee, Phys. Lett. **132B**, 419 (1983); **144B**, 455E (1984).

[6]O. Kaymakcalan *et al.*, Phys. Rev. D **30**, 594 (1984); S. Rudaz, Phys. Lett. **145B**, 281 (1984).

[7]T. Fujiwara *et al.*, Prog. Theor. Phys. **73**, 926 (1985).

[8]T. N. Truong, in *Low Energy Antiproton Physics,* proceedings of the Ettore Majorana International School (Plenum, New York, to be published); Enrico Fermi Institute Report No. EFI-90-26, A965-0490, 1990 (unpublished); University of California, Santa Barbara Report No. UCSBTH-90-29, 1990 (unpublished).

[9]M. S. Chanowitz, Phys. Rev. Lett. **35**, 977 (1975).

[10]Crystal Ball Collaboration, H. Marsiske *et al.*, in *Proceedings of the Eighth International Workshop on Photon-Photon Collisions, Shoresh, Jerusalem Hills, Israel, 1988,* edited by U. Karshon (World Scientific, Singapore, 1988), p. 15.

[11]J. G. Layter *et al.*, Phys. Rev. D **7**, 2565 (1973).

[12]N. I. Muskhelishvili, *Singular Integral Equations* (P. Noordhoof, Groningen, 1953); R. Omnes, Nuovo Cimento **8**, 316 (1958).

[13]Y. N. Antipov *et al.*, Phys. Rev. D **36**, 21 (1987).