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Semiempirical formulas for the color-hyperfine mass splitting in hadrons

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We slightly modify a semiempirical formula of Song for the color-hyperfine mass splitting in mesons and generalize the formula to baryons. We compare the results of these formulas with experiment and extrapolate them to obtain predictions of the masses of as-yet-undiscovered hadrons.

Recently, Song[1] proposed a semiempirical formula for the mass difference between a vector and a pseudoscalar meson containing a given quark and antiquark. Song's formula, which is motivated from QCD, is an approximation to the expectation value of the color-hyperfine interaction between a quark-antiquark pair. We refer the reader to Song's paper[1] for details of how he obtained his formula.

In this Comment we slightly modify Song's formula into a form which we find more suitable for applying to baryons. We next generalize the modified meson formula to obtain a new semiempirical formula for the color-hyperfine splitting in baryons. We compare the predictions of our formula for mesons with those of Song and with the available data. We then compare the results of our baryon formula with the data and make predictions of the masses of as yet unobserved baryons.

Song's meson formula is given by

$$M_V - M_P = p\alpha_s(2\mu)\mu^q/\overline{M} \ , \tag{1}$$

where M_V is the mass of an S-wave vector meson containing a quark-antiquark pair with masses m_1 and m_2, M_P is the mass of the corresponding pseudoscalar meson, μ is the reduced mass, $\overline{M}$ is the spin-weighted average of M_V and M_P, α_s is the strong-interaction running coupling constant of QCD, and p and q are positive flavor-independent parameters. Song chooses α_s to be

$$\alpha_s(Q) = \frac{4\pi}{\beta_0 t + (\beta_1/\beta_0)\ln t} \ , \tag{2}$$

where

$$t = \ln(Q^2/\Lambda^2), \quad \beta_0 = 11 - 2n_f/3 \ , \quad \beta_1 = 102 - 38n_f/3 \ , \tag{3}$$

with Λ being the QCD scale parameter and n_f the effective number of quark flavors.

Because Eq. (1) contains the quantity $\overline{M}$, which refers specifically to mesons, it is not convenient to apply this equation to baryons. We therefore slightly modify Eq. (1) as follows:

$$M_V - M_P = p\alpha_s(2\mu)\mu^q/(m_1 + m_2) \ , \tag{4}$$

The change of replacing $\overline{M}$ by $m_1 + m_2$ is of course trivial, and we do so for the primary purpose of generalizing to baryons. Our Eq. (4) has a drawback compared to Eq. (1) because it applies only to ground-state mesons, whereas Eq. (1) can be applied also to excited states with no orbital angular momentum. In order to overcome this difficulty we multiply Eq. (4) by a factor γ^n, where n is the number of nodes in the radial wave function. Then we get

$$M_V - M_P = p\alpha_s(2\mu)\mu^q\gamma^n/(m_1 + m_2) \ , \tag{5}$$

where γ is a positive constant. The ground state has $n = 0$ and thus is unchanged from Eq. (4). Our motivation for including the factor γ^n comes from detailed calculations[2] with potential models. We note that Eq. (5) has an advantage compared to Eq. (1) in that it enables one to calculate the vector-pseudoscalar mass splitting even when neither M_V nor M_P is known experimentally.

The quantity $M_V - M_P$ can be written

$$M_V - M_P = 4R_{12} \ , \tag{6}$$

where R_{12} is a matrix element of the color-hyperfine interaction for mesons in the Fermi-Breit approximation.

In the Fermi-Breit approximation to the one-gluon-exchange interaction of QCD, the operator that describes the color-hyperfine interaction in baryons is a sum of two-quark operators. Nevertheless, it is a poor approximation to let the color-hyperfine splitting in baryons be a sum of three terms similar in form to the meson formula given in Eq. (5). The reason is that a baryon matrix element of a two-quark operator depends on the third or spectator quark through its effect on the baryon wave function. The interaction of QCD has the property that, for any two quarks of a given flavor, the larger the mass of the third quark, the smaller will be the baryon wave function. Then, because the QCD color-hyperfine interaction between two quarks is singular, the matrix element of this operator will increase as the mass of the spectator quark increases. This effect was first pointed out by Cohen and Lipkin[3] for the case in which the first two quarks are light, and also considered by other authors in more general cases.[4-6]

Because of the Cohen-Lipkin effect, we include a baryon color-hyperfine matrix element a factor $F_{ij,k}$ which increases as the mass m_k of the spectator quark increases. This factor simulates the effect of the shrinking of the wave function with increasing mass of the spectator quark. Such a factor appears naturally in potential models inspired by QCD. We obtain the desired property with an expression for $F_{ij,k}$ which has the simple form

$$F_{ij,k}=(m_i+m_j+rm_k)/(m_i+m_j+m_k) , \tag{7}$$

where r is a parameter. In order to ensure that $F_{ij,k}$ increases as the mass m_k increases, we must take $r>1$. The functional form of $F_{ij,k}$ has no special theoretical significance, and other functional forms with similar properties will do as well.

We take a baryon color-hyperfine matrix element $R_{ij,k}$ to be a product of $F_{ij,k}$ and a term similar to that on the right-hand side of Eq. (5). We thus consider baryon color-hyperfine matrix elements of the form

$$8R_{ij,k}=F_{ij,k}p\alpha_s(2\mu_{ij})\mu_{ij}^q\gamma^n/(m_i+m_j) , \tag{8}$$

where again p and q are positive flavor-independent parameters. The factor 8 multiplying $R_{ij,k}$ in Eq. (7) is included so that we have the same normalization for color-hyperfine matrix elements in baryons as in mesons, taking into account the fact that according to QCD the color-hyperfine interaction is only half as strong in baryons as in mesons: see Eq. (6).

In order to write baryon mass differences in terms of color-hyperfine matrix elements, we need expressions for baryon wave functions. We use the wave functions of Franklin *et al.*[7] In the description of these authors, if all quarks have the same flavor, the wave function (excluding color coordinates) is completely symmetric; if only two quarks have the same flavor, they are ordered as the first two, and if all three quarks have different flavors, the two lightest are the first two. Then the expressions for the ground-state baryon mass differences in terms of the color-hyperfine matrix elements are[6]

$$M_{123}^*-M_{123}=3R_{13,2}+3R_{23,1} , \tag{9}$$

$$M_{123}-M_{123}^0=4R_{12,3}-2R_{13,2}-2R_{23,1} , \tag{10}$$

where M^* denotes the mass of a baryon of spin $\frac{3}{2}$, M denotes the mass of a baryon of spin $\frac{1}{2}$ with the first two quarks having spin 1, and M^0 denotes the mass of a baryon of spin $\frac{1}{2}$ with the first two quarks having spin 0. At least two quarks must have different flavors for M to exist, and all three quarks must have different flavors for M^0 to exist.

Before presenting our results, we briefly discuss the values of the parameters and quark masses. Song[1] has given arguments that q should have the value $q=0.6\pm0.2$, and we restrict ourselves to values in this range. We also restrict q to have the same value for mesons and baryons. For the QCD scale parameter Λ, Song has chosen the value $\Lambda=100$ MeV. Although this value may be somewhat small, we use it. We present our results with the effective number of quark flavors $n_f=4$, but we have also made calculations with $n_f=3$ and $n_f=5$, obtaining very similar answers. Song has made calculations with three different sets of quark masses. However, all of his sets violate inequalities among the quark consitute masses which were shown[8] to hold under rather general assumptions. We use quark masses that are consistent with the inequalities of Ref. 8, except that we neglect the small mass difference between m_u and m_d. Our quark masses are (in MeV)

$$m_u=m_d=300, \quad m_s=500, \quad m_c=1800, \quad m_b=5200 . \tag{11}$$

In Table I we compare our calculated values of M_V-M_P with those calculated by Song and with experimental values (when known) from the Particle Data Group.[9] Our values were obtained with

$$q=0.7, \quad p=2.129\ (\text{GeV})^{2-q} , \quad n_f=4, \quad \Lambda=100\ \text{MeV} . \tag{12}$$

We find similar results, not shown, with values of q differing from the above by about 10%, provided that we make compensating adjustments in p. Song's values shown in Table I are those of his set 1, and are quite close to the values given in his sets 2 and 3.

Our meson formula like that of Song, cannot simultaneously give good fits to the mass splittings in heavy and light mesons. Like Song, we have chosen to get better fits to the heavy mesons rather than to the light ones. We have two reasons for this choice: (1) Perturbation theory, on which the formula is based, works better for heavy mesons than for light ones, and (2) we want to

TABLE I. A comparison of our values for the vector-pseudoscalar meson mass differences with those of Song (Ref. 1) and with the experimental values, where known, from the particle Data Group (Ref. 9). All masses and energies are in MeV.

Quark content	Mesons	Data	Song (set 1)	This work
$\bar{q}q$	$\rho(770)-\pi(138)$	630 ± 3	530	510
$\bar{s}q$	$K^*(894)-K(496)$	398 ± 1	370	370
$\bar{q}c$	$D^*(2009)-D(1867)$	142 ± 2	143	142
$\bar{s}c$	$D_s^*(2213)-D_s(1969)$	143 ± 3	139	140
$\bar{c}c$	$J/\psi(3097)-\eta_c(2980)$	117 ± 2	115	117
$\bar{b}q$	$B^*(5330)-B(5278)$	52 ± 6	53	55
$\bar{b}s$	$B_s^*-B_s$		52	59
$\bar{b}c$	$B_c^*-B_c$		58	71
$\bar{b}b$	$\Upsilon(9460)-\eta_b$		55	64

extrapolate to get predictions for still heavier mesons.

Note that, although Song's formula and ours give results which differ by less than 3 MeV for mesons containing one or two c quarks, our predictions differ from Song's by as much as 13 MeV for mesons containing one or two b quarks. We estimate from these differences that predictions of vector-pseudoscalar mass splittings for mesons containing b quarks may be in error by around 15 MeV or possibly even more.

If we try to apply our formula to radial excited states, we come up against a problem in the $c\bar{c}$ system. The value of our parameter γ is given by

$$\gamma=(\psi'-\eta_c')/(J/\psi-\eta_c) .$$

Empirically, using the data of the Particle Data Group,[9] we find the value $\gamma=0.79$, but from potential models,[2] we find $\gamma\simeq 0.6$. The η_c' is not listed as an established resonance, and so we prefer to wait for more experimental information on this state before presenting numerical results for the masses of radial excitations. In the remainder of this Comment, we confine our attention to ground states.

In Table II we compare our calculated mass splittings of baryons with experimental values from the tables of the Particle Data Group.[9] We also show in Table II our predictions for mass splittings of as yet undiscovered baryons. We see from Table II that our generalized formula works rather well for both light and heavy baryons. (The largest disagreement, 15 MeV, occurs in the case of the Σ-Λ masses difference, because this mass difference is a small quantity which arises from taking the difference between two large matrix elements.) We believe that the reason our results are better for light baryons than for light mesons is that the color-hyperfine interaction between two quarks is only half as large as the color-hyperfine interaction between a quark and an antiquark.

TABLE II. A comparison of our calculated values of baryon mass differences with the experimental values, where known. Experimental results are from the Particle Data Group (Ref. 9) except for the $\Xi_c(2470)$, which is from Avery *et al.* (Ref. 10). All masses are in MeV. Our notation is that Ξ_c' and Ξ_b' denote baryons of spin $\frac{1}{2}$ with the two light quarks having spin 1.

Quark content	Baryon	Data	This work
qqq	$\Delta(1232)-N(939)$	293 ± 2	292
qqs	$\Sigma(1193)-\Lambda(1116)$	77 ± 1	62
qqs	$\Sigma^*(1385)-\Lambda(1116)$	269 ± 2	271
ssq	$\Xi^*(1533)-\Xi(1318)$	215 ± 2	215
qqc	$\Sigma_c(2452)-\Lambda_c(2285)$	167 ± 3	165
qqc	$\Sigma_c^*-\Lambda_c(2285)$		242
qsc	$\Xi_c'-\Xi_c(2470)$		103
qsc	$\Xi_c^*-\Xi_c(2470)$		180
ssc	$\Omega_c^*-\Omega_c$		77
qqb	$\Sigma_b-\Lambda_b$		204
qqb	$\Sigma_b^*-\Lambda_b$		233
qsb	$\Xi_b'-\Xi_b$		141
qsb	$\Xi_b^*-\Xi_b$		171
ssb	$\Omega_b^*-\Omega_b$		32

As a consequence, perturbation theory is a better approximation for baryons than for mesons.

Our results for baryons were obtained with the same value of q, Λ, and n_f as for mesons, shown in Eq. (12) above. The values of p and r were taken as adjustable parameters and found to be

$$p=1.488\ (\mathrm{GeV})^{2-q},\quad r=1.285 . \tag{13}$$

The value of p for baryons turns out to be only 70% as large as for mesons. The reason is that in QCD the color-hyperfine interaction is short ranged and so is sensitive to the behavior of the wave function at small distances. But because the quark-quark potential at small distances is only half as strong as the quark-antiquark potential, the quark-quark wave function is smaller than the quark-antiquark wave function at small separations, and so the effect of the color-hyperfine interaction is further reduced in baryons compared to mesons. The fact that r turns out to be 28% larger than unity shows that the Cohen-Lipkin effect is appreciable. (The value $r=1$ corresponds to no effect.) This fact can again be understood in potential models, as the wave function shrinks with increasing mass of the spectator quark. This has the effect of increasing the expectation value of color-hyperfine operator, which is singular at the origin in the Fermi-Breit approximation. Again, we have verified that our results are relatively insensitive to the value of n_f and to variations of q of about 10%, provided we make compensating changes in other parameters.

If we know from experiment the mass of one baryon containing quarks of given flavors, then we can use our formula to calculate the masses of all other ground-state baryons with the same quark content. Because we know the mass of the $\Lambda_c(2285)$ from experiment,[9] we calculate that the as-yet-unobserved Σ_c^* has mass

$$M^*(\Sigma_c^*)=2527\ \mathrm{MeV} . \tag{14}$$

Likewise, because we know[9] the mass of the $\Xi_c(2470)$ (M^0), we calculate that the as-yet-unobserved Ξ_c^* and Ξ_c' have masses

$$M^*(\Xi_c^*)=2650\ \mathrm{MeV},\quad M(\Xi_c')=2573\ \mathrm{MeV} . \tag{15}$$

Based on the stability of the baryon formula to small changes in parameters and on the goodness of the fit to the known baryon masses, we expect that the predicted masses of baryons containing a c quark will turn out to be within 10 to 15 MeV of the actual masses.

In conclusion, we have slightly modified Song's semiempirical formula for the color-hyperfine splitting in mesons and have obtained a generalized formula for baryons. The form of our formulas is motivated by QCD in the Fermi-Breit approximation, but the parameters of the formula are empirical. Using our formulas, we have obtained results in rather good agreement with experiment for heavy mesons and both heavy and light baryons. By choosing a formula for mesons that is a little different from that of Song, we obtained predictions that varied from his by as much as 13 MeV. We regard this difference as an indication of how much in error these formulas may be in predicting the masses of as-yet-undiscovered mesons. We also have made predictions for the mass splittings and masses of some as-yet-unobserved baryons: namely, the $\Sigma_c^*(2527)$, the $\Xi_c^*(2650)$, and the $\Xi_c'(2573)$.

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[1]Xiaotong Song, Phys. Rev. D **40**, 3655 (1989).
[2]D. B. Lichtenberg, E. Predazzi, R. Roncaglia, M. Rosso, and J. G. Wills (unpublished).
[3]I. Cohen and H. Lipkin, Phys. Lett. **106B**, 119 (1981).
[4]J. M. Richard and P. Taxil, Ann. Phys. (N.Y.) **150**, 267 (1983).
[5]D. B. Lichtenberg, Phys. Rev. D **35**, 2183 (1987).
[6]M. Anselmino, D. B. Lichtenberg, and E. Predazzi, Z. Phys. C (to be published).
[7]J. Franklin, D. B. Lichtenberg, W. Namgung, and D. Carydas, Phys. Rev. D **24**, 2910 (1981).
[8]D. B. Lichtenberg, Phys. Rev. D **40**, 3675 (1989).
[9]Particle Data Group, G. P. Yost *et al.*, Phys. Lett. B **204**, 1 (1988).
[10]P. Avery *et al.*, Phys. Rev. Lett. **62**, 868 (1989).