Extra U(1) with charges of known fermions proportional to weak hypercharges

K. T. Mahanthappa and P. K. Mohapatra*
Department of Physics, C.B. 390, University of Colorado, Boulder, Colorado 80309
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We study the possibility of an extra $U(1)_X$ whose charges for the fifteen known fermions of each generation are proportional to the corresponding weak hypercharges. Its neutral-current effects are analyzed and compared with those of popular extra U(1)'s from SO(10) and E_6 models.

There has been a lot of activity[1] on the extra U(1) beyond the standard model (SM), which entails an extra neutral boson that mixes with the standard Z boson. Most of these works have been based on the grand unification groups SO(10) and E_6. Very recently Glashow and Sarid[2] have considered the problem from a different angle. They have taken a bottom-up approach rather than the top-down approach considered so far in the literature. They have taken as their starting point the principle of minimality and unifiability. With the addition of an extra neutral fermion to the standard set of fifteen fermions they arrive at the same extra U(1) as coming from SO(10) except for its Higgs structure. Instead let us first consider the possible family-independent extra $U(1)_X$ without having any extra fermions. We observe that, in order to cancel all the anomalies, the X charges of the fermions have to be proportional to the Y charges of $U(1)_Y$ of the SM. Does this mean that the symmetry is the same as $U(1)_Y$? Not necessarily. The symmetry is determined by the whole theory including all Higgs bosons and fermions. If there exist extra fermions (Higgs) with X and Y charges not having the same proportionality constant as that of the canonical fermions (Higgs bosons), we have a $U(1)_X$ distinct from $U(1)_Y$. Such an observation has been made earlier by Barr, Bednarz, and Benesh[3] who arrived at such an extra $U(1)_X$ using SU(7) as a grand unification group (see Appendix A of Ref. 3). They have

$$SU(7)\to SU(5)\otimes SU(2)\to SU(3)_C\otimes SU(2)_L\otimes U(1)_5\otimes U(1)_2 \to SU(3)_C\otimes SU(2)_L\otimes U(1)_Y \to SU(3)_C\otimes U(1)_{em}\ . \quad (1)$$

For the seven-dimensional representation we choose the generators of $U(1)_5$ and $U(1)_2$ to be

$$Y'=\mathrm{diag}(2,2,2,-3,-3,0,0)/6$$

and

$$Y''=\mathrm{diag}(0,0,0,0,0,-1,1)\ ,$$

respectively. Here the first five elements refer to $(\bar d_1,\bar d_2,\bar d_3,e^-,\nu_e)_L$ and the last two to the additional new fermions. Symmetry breaking in (1) are due to Higgs bosons in various representations. $U(1)_5\otimes U(1)_2$ can be written as $U(1)_Y\otimes U(1)_X$ where Y and X are two orthogonal combinations of Y' and Y''. For example, we can choose $Y=Y'-Y''$ and the orthogonal combination $X\propto 12Y'+5Y''=12Y+17Y''$. If we take the fifteen canonical fermions to be in $\psi_L^{\alpha\beta}$ and ψ_L^{α} $(\alpha,\beta=1,\ldots,5)$, then Y gives the canonical weak hypercharges and their X charges are proportional to Y charges. Note that though $\mathrm{tr}(XY)$ is not zero if one confines oneself to the canonical fermions, it is zero for all the fermions in any representation of SU(7). [SU(7) is anomaly-free when 7-, 21-, and 35-dimensional fermionic representations are taken together.] The Z' gauge boson corresponding to $U(1)_X$ become massive in the third stage of symmetry breaking in (1). Such a $U(1)_X$ can also arise from other grand unification groups. The purpose of this Brief Report is to study the neutral-current effects due to this $U(1)_X$. We assume that fermions other than the canonical ones are super heavy.

The *effective* generic model we consider consists of the canonical fifteen fermions transforming under the gauge group $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y\otimes U(1)_X$ as given in Table I. We have taken the X charges of these fermions to be the same as the Y charges. We assume the coupling constants of the two U(1)'s to be different but of the same order, even though they are related due to the underlying

TABLE I. Quantum numbers of the fermions and Higgs bosons.

Fermion	$SU(3)_C$	T_3	Y	X
ν_L	1	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
e_L	1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
u_L	3	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$
d_L	3	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$
e_R	1	0	-1	-1
u_R	$\bar{3}$	0	$\frac{2}{3}$	$\frac{2}{3}$
d_R	$\bar{3}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
Higgs bosons				
ϕ	1	$\pm\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
η	1	0	0	a

unification group. The Higgs field ϕ is a doublet of $SU(2)_L$. It is needed to break the SM and give masses to the quarks and charged leptons. The neutrino remains massless in this model. The Higgs field η is needed to break the extra $U(1)_X$. It remains invariant under the standard-model group. Its X charge is arbitrary. Any nonzero charge a will suffice for our purpose. All our analyses are independent of this charge which we take to be order of unity. We assume that the scale of $\langle\eta\rangle$ is much higher than that of $\langle\phi\rangle$.

There are three neutral vector bosons, B, W_3, and B', for which the mass-square matrix is

$$\frac{1}{4}\begin{bmatrix} g_1^2 v_1^2 & -g g_1 v_1^2 & g_1 g_2 v_1^2 \\ -g g_1 v_2^2 & g^2 v_1^2 & -g g_2 v_1^2 \\ g_1 g_2 v_1^2 & -g g_2 v_1^2 & g_2^2(v_1^2+\frac{4}{9}a^2 v_2^2) \end{bmatrix}, \tag{2}$$

where g, g_1, and g_2 are the $SU(2)_L$, $U(1)_Y$, and $U(1)_X$ coupling constants. Here $v_1=\sqrt{2}\langle\phi\rangle$ and $v_2=\sqrt{2}\langle\eta\rangle$. The first two rows and the first two columns are proportional to each other, and hence we can diagonalize the upper left-hand 2×2 matrix. This is exactly same as in the SM.[4] After the first stage of diagonalization the mass matrix takes the form

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & m_{Z^{(0)}}^2 & -p \\ 0 & -p & M^2 \end{bmatrix}, \tag{3}$$

where

$$\tan\theta_W=\frac{g_1}{g}, \quad p=\frac{1}{4}\frac{g g_2 v_1^2}{\cos\theta_W}, \quad \text{and } M^2=\frac{g_2^2}{4}(v_1^2+\tfrac{4}{9}a^2 v_2^2).$$

The second stage mixes $Z_\mu^{(0)}$ with B'_μ giving Z_μ and a new neutral boson Z'_μ:

$$\begin{bmatrix} Z \\ Z' \end{bmatrix}=\begin{bmatrix} \cos\chi & \sin\chi \\ -\sin\chi & \cos\chi \end{bmatrix}\begin{bmatrix} Z^{(0)} \\ B' \end{bmatrix}, \tag{4}$$

where

$$\tan(2\chi)=2p/(M^2-M_{Z^{(0)}}^2).$$

Then the neutral-current interaction for any fermion ψ is given by

$$\mathcal{L}_{NC}=-i\bar{\psi}\gamma^\mu\left[eQA_\mu-\frac{e}{\sin\theta_W\cos\theta_W}Z_\mu\left[(I_3-Q\sin^2\theta_W)\cos\chi+\frac{\cos\theta_W g_2}{g}X\sin\chi\right]\right.$$
$$\left.-\frac{e}{\sin\theta_W\cos\theta_W}Z'_\mu\left[-(I_3-Q\sin^2\theta_W)\sin\chi+\frac{\cos\theta_W g_2}{g}X\cos\chi\right]\right]\psi. \tag{5}$$

Since $\tan\chi$ is expected to be small, $|\tan\chi|\le 0.08$, we can write the neutral current in the form

$$\mathcal{L}_{NC}=-i\bar{\psi}\gamma^\mu\left[eQA_\mu-\frac{e}{\sin\theta_W\cos\theta_W}Z_\mu[(I_3-Q\sin^2\theta_W)+\lambda X]-\frac{e}{\sin\theta_W\cos\theta_W}Z'_\mu\left[\frac{\cos\theta_W g_2}{g}X\right]\right]\psi, \tag{6}$$

where $\lambda=(\cos\theta_W g_2/g)\sin\chi$. This is the generic neutral current with one extra U(1). In Table II we have given the g_A and g_V of the standard fermions with Z and Z'.

As there exist discussions of the mass of Z', which yield a lower limit of $\sim$300 to 500 GeV, we forego such a discussion. Here we want to focus on the effects which distinguish $U(1)_X$ from U(1)'s coming from SO(10) and E_6.[2,5] In the latter case we choose the popular ones: (i) the one in which E_6 breaks to a rank-6 group via the Hosotani mechanism and then breaks to a rank-5 group by the Higgs mechanism and (ii) the one in which E_6 directly breaks to a rank-5 through the Hosotani mechanism. These models are designated as χ, I, and η, respectively, by Hewett and Rizzo.[5] In Table III we have compiled various experimentally observable parameters for the four models. The quantities $r_{1,2,3,4}$ are defined as

$$(\Gamma_e/\Gamma_h)=r_1(\Gamma_e/\Gamma_h)^{(0)}, \quad (\Gamma_\nu/\Gamma_e)=r_2(\Gamma_\nu/\Gamma_e)^{(0)},$$
$$(\Gamma_e\Gamma_h/\Gamma_{all}^2)=r_3(\Gamma_e\Gamma_h/\Gamma_{all}^2)^{(0)}$$

and

$$(\Gamma_{all})=r_4(\Gamma_{all}^{(0)}).$$

The Γ's are the respective decay widths of Z. The superscript (0) denotes the SM value. In the first three ratios the effect of the change in the ρ parameter cancels out. The way to compare different models is to make a global fit to the experimental data with the one unknown parameter λ. A pair of new variables have been introduced

TABLE II. g_A and g_V for the fermions with Z and Z'.

Fermion	$\frac{g_{1A}c_W s_W}{e}$	$\frac{g_{1A}c_W s_W}{e}$	$\frac{2g_{2V}}{g_2}$	$\frac{2g_{2a}}{g_2}$
ν	$\frac{1}{4}-\frac{1}{4}\lambda$	$\frac{1}{4}-\frac{1}{4}\lambda$	$-\frac{1}{2}$	$-\frac{1}{2}$
e	$-\frac{1}{4}+s_W^2-\frac{3}{4}\lambda$	$-\frac{1}{4}+\frac{1}{4}\lambda$	$-\frac{3}{2}$	$\frac{1}{2}$
u	$\frac{1}{4}-\frac{2}{3}s_W^2+\frac{5}{12}\lambda$	$\frac{1}{4}-\frac{1}{4}\lambda$	$\frac{5}{6}$	$-\frac{1}{2}$
d	$-\frac{1}{4}-\frac{1}{3}s_W^2-\frac{1}{12}\lambda$	$-\frac{1}{4}+\frac{1}{4}\lambda$	$-\frac{1}{6}$	$\frac{1}{2}$

TABLE III. Comparison of some extra U(1) models.

	Our model	χ model	I model	η model
r_1	$1-0.62\lambda$	$1+1.30\lambda$	$1-1.66\lambda$	$1-0.30\lambda$
r_2	$1-0.5\lambda$	$1-3.32\lambda$	$1+3.20\lambda$	$1-0.13\lambda$
r_3	$1-0.03\lambda$	$1+1.86\lambda$	$1-1.95\lambda$	$1-0.07\lambda$
r_4	$1-1.17\lambda$	$1-0.13\lambda$	$1-0.13\lambda$	$1-0.21\lambda$
p	0.63λ	-1.33λ	1.70λ	0.30λ
n	1.43λ	-2.10λ	2.73λ	0.51λ
p/n	0.44	0.63	0.62	0.59
s_1	55.0	65.4	70.0	86.7
s_2	37.5	18.2	16.7	11.1
s_3	7.5	16.4	13.3	2.2

by Renard and Verzegnassi[6] which are more sensitive in differentiating the different models. Those are the parameters p and n defined as

$$n=(R_Z-\tfrac{1}{2}\gamma)-(R_Z-\tfrac{1}{2}\gamma)^{(0)} ,$$

$$p=(R_Z-\tfrac{1}{3}\zeta)-(R_Z-\tfrac{1}{3}\zeta)^{(0)} ,$$

where

$$R_Z=\frac{3}{59}\frac{\Gamma(Z\to\text{hadrons})}{\Gamma(Z\to e^+e^-)} ,$$

$$\gamma=\frac{9}{\alpha}\frac{\Gamma(Z\to e^+e^-)}{M_Z} ,$$

and

$$\zeta=2M_W^2/(M_Z^2\{1+[1-4\alpha\pi/(\sqrt{2}G_FM_Z^2)]^{1/2}\}) .$$

These particular combinations of the experimentally observable quantities have an advantage of canceling all the possible nonmixing effects as due to the Higgs boson, the top-quark contributions and QCD corrections which are equivalent to the change in the ρ parameter. These parameters are sensitive only to the pure mixing effects. We have compiled the value of p, n, and p/n for the four models. It is very striking that the values of the p/n ratio for all the other three models are very close to each other and are very much different from that of our model. We have plotted the p vs n in Fig. 1. The square box in the middle represents the dead region, because of 1% error in theoretical calculations of the QCD corrections. If the experimental observations fall within this square,

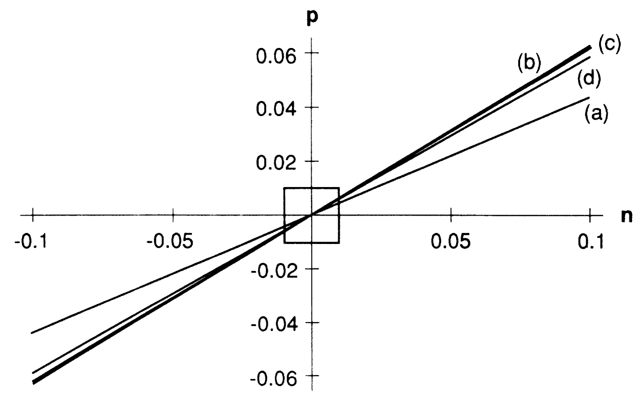

FIG. 1. Comparison of the four models described in the text on the n-p plane. The variables are proportional to the Z-Z' mixing. The square in the middle represents the expected 1% experimental resolution. (a) our model, (b) χ model, (c) I model, and (d) η model.

then the effect of any extra U(1) is indistinguishable from the SM. Even if the results fall outside this square, it will be very difficult to distinguish the other three models whereas our model has a clear distinguishing feature. At the moment all the experimental data are consistent with p and n being zero.[7] But in the immediate future we might get a clear signal of the deviation from the SM. Similar analyses with all the known extra U(1) models and other noncanonical ones is currently under way and will be presented elsewhere.[8]

Finally in the last three rows of Table III we give the branching ratios of $Z'\to$hadrons(s_1), $Z'\to l^+l^-(s_2)$ and $Z'\to$neutrinos(s_3). Here s_1 includes only u, d, c, s, t, and b quarks, s_2 includes only e, μ, and τ leptons and s_3 includes only the left-handed neutrinos for the three generations. The numbers are given as a percentage with only consideration to these three. We have not included the right-handed neutrino along with the exotic fermions in E_6, the Higgs and gauge bosons.

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*Electronic address: Bitnet pramoda@COLOPHYS.

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