$V^0 \to P^0P^0\gamma$ decay rates

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The radiative decay processes of the type $V^0 \to P^0P^0\gamma$ are described by the gauged Wess-Zumino terms in a low-energy effective Lagrangian, there being no bremsstrahlung contributions. Using such an effective Lagrangian, describing pseudoscalar and vector mesons, we have calculated the branching ratios for the decays $\omega \to \pi^0\pi^0\gamma$, $\omega \to \pi^0\eta\gamma$, $\rho \to \pi^0\pi^0\gamma$, $\rho \to \pi^0\eta\gamma$, $\phi \to \pi^0\pi^0\gamma$, $\phi \to \pi^0\eta\gamma$, and $\phi \to K^0\bar{K}^0\gamma$. Since scalar mesons have been neglected, these rates provide estimates of the expected backgrounds in searches for $J^\pi = 0^+$ resonances, particularly the possible four-quark states in ϕ decays.

I. INTRODUCTION

The decays of vector mesons which proceed through anomalies in the chiral-Lagrangian framework have been discussed extensively.[1–6] These calculations can be regarded as an attempt to obtain an approximate low-energy effective Lagrangian. Essentially, the standard nonlinear effective Lagrangian describing pseudoscalar mesons is augmented by a multiplet of linearly transforming vector and axial-vector mesons to include their interactions, which become important at intermediate energies. The experimental data for the anomalous vector-meson decays, for example, $V \to P\gamma$, are reasonably well reproduced in this approach. Here we extend these calculations to the decay modes of the form $V^0 \to P^0P^0\gamma$, where there are no competing bremsstrahlung contributions.

Experimentally,[7,8] only upper bounds on the branching ratios for $\omega \to \pi^0\pi^0\gamma$ and $\phi \to \pi^0\pi^0\gamma$ are known at present. However, these decay rates can be predicted using the anomalous term in a chiral Lagrangian where the relevant coupling constant g_{VVP} has been previously determined.[1–4] Decays involving charged mesons, such as $\omega \to \pi^+\pi^-\gamma$ and $\rho^0 \to \pi^+\pi^-\gamma$, are dominated by bremsstrahlung processes and we shall only be concerned with processes where there are no bremsstrahlung contributions in the present discussion. Interest in studying the decays $\phi \to \pi^0\pi^0\gamma$, $\phi \to \pi^0\eta\gamma$, and $\phi \to \bar{K}^0K^0\gamma$ has been stimulated[8,9] by the fact that they possibly proceed primarily through processes involving scalar resonances[10,11] such as $\phi \to f_0(975)\gamma$ and $\phi \to a_0(980)\gamma$, with subsequent decays into $\pi\pi\gamma$ and $\pi\eta\gamma$. It has been shown[10] that if the $f_0(975)$ and $a_0(980)$ resonances are four-quark $(q^2\bar{q}^2)$ states the processes $\phi \to f_0(975)\gamma$ and $\phi \to a_0(980)\gamma$ are dominant and enhance the decays $\phi \to \pi^0\pi^0\gamma$ and $\phi \to \pi^0\eta\gamma$ by at least an order of magnitude over the results predicted by the Wess-Zumino terms. We calculate these latter contributions, in which virtual vector-meson intermediate states dominate, because they are possibly important backgrounds for searches for scalar resonances. If these scalar mesons are indeed four-quark structures, knowing the nonresonance background, which we calculate here, is particularly important, since the branching ratios are then enhanced by an order of magnitude.[10]

In the next section we present the general framework for our calculations and then we proceed to compute the decay widths and discuss our numerical results.

II. EFFECTIVE LAGRANGIAN

Some time ago Witten[12] revived interest in the Wess-Zumino term[13] in the effective chiral Lagrangian, which is related to the non-Abelian anomaly, and which leads to pseudoscalar vertices proportional to the Levi-Civita symbol $\epsilon_{\mu\nu\alpha\beta}$. The simplest purely hadronic reactions of this type are somewhat complicated, for example, $K\bar{K} \to 3\pi$. The extension of the pseudoscalar chiral Lagrangian by introduction of vector and axial-vector fields enables one to study other anomalous decays. It also extends the range of validity of the effective Lagrangian to the intermediate energy range of the order of vector-meson masses.

In the previous work,[1–4] which we shall follow, the chiral Lagrangian contains 0^-, 1^-, and 1^+ nonets of mesons, while scalar mesons are not included. The chiral Lagrangian for pseudoscalar mesons at low energies takes the form

$$\mathcal{L} = \frac{f_\pi^2}{8} \mathrm{Tr}(\partial_\mu U \, \partial_\mu U^\dagger) , \tag{2.1}$$

where $U = \exp[(2i/f_\pi)\Pi]$ and $f_\pi = 135$ MeV. A mass term proportional to

$$MU + U^\dagger M ,$$

where $M = \mathrm{diag}\,(m_u, m_d, m_s)$ is the usual 3×3 quark mass, is added to break exact chiral symmetry.

The pseudoscalar Lagrangian is further enlarged by adding terms for spin-one mesons and electromagnetic interactions are introduced assuming vector-meson dominance.[14] The left- and right-handed spin-one mesons $A_{L\mu}$ and $A_{R\mu}$ are related to the vector and axial-vector mesons by

$$A_{L\mu}=\tfrac{1}{2}(V_\mu+A_\mu) \text{ and } A_{R\mu}=\tfrac{1}{2}(V_\mu-A_\mu) . \quad (2.2)$$

The coupling of these spin-one mesons to the pseudoscalar mesons is assumed to be given by the standard prescription

$$\partial_\mu U \to \partial_\mu U - igA_{\mu L}U^\dagger + igUA_{\mu R} . \quad (2.3)$$

Here g is a phenomenological "gauge" coupling constant. Its magnitude is equal to $\sqrt{2}f_{\rho\pi\pi}$ where $f_{\rho\pi\pi}$ is related to the experimental $\rho\to 2\pi$ decay width by

$$\Gamma(\rho\to 2\pi)=\tfrac{2}{3}(f_{\rho\pi\pi})^2\frac{|\mathbf{q}_\pi|^3}{m_\rho^2} , \quad (2.4)$$

giving $(f_{\rho\pi\pi})^2/4\pi=3.0$. This phenomenological prescription must also be augmented by mass terms for the vector and axial-vector fields. Thus, one also adds to the pseudoscalar Lagrangian the term,

$$\mathcal{L}'=-\tfrac{1}{2}(F^L_{\mu\nu}F^L_{\mu\nu}+F^R_{\mu\nu}F^R_{\mu\nu}) - m_\rho^2\mathrm{Tr}(A_{\mu L}A_{\mu L}+A_{\mu R}A_{\mu R}) , \quad (2.5)$$

where

$$F^L_{\mu\nu}=\partial_\mu A_{\nu L}-\partial_\nu A_{\mu L}-ig[A_{\mu L},A_{\nu L}] \quad (2.6)$$

with a similar definition for $F^R_{\mu\nu}$. The pseudoscalar- and axial-vector-meson fields mix and the following physical quantities are obtained:

$$A_\mu=\tilde{A}_\mu+\frac{g\tilde{f}_\pi}{2m_\rho^2}\partial_\mu\Pi , \quad (2.7a)$$

$$\tilde{\Pi}=Z^{-1}\Pi , \quad (2.7b)$$

$$Z=\left[1+\frac{g^2f_\pi^2}{4m_\rho^2}\right]^{-1/2} , \quad (2.7c)$$

$$\tilde{f}_\pi=Zf_\pi . \quad (2.7d)$$

Gauging of the Wess-Zumino term in the chiral Lagrangian results in the following anomalous terms:[1-3]

$$\mathcal{L}_{VVP}=-g_{VVP}\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}(\partial^\mu V^\nu\partial^\alpha V^\beta\Pi) \quad (2.8)$$

and

$$\mathcal{L}_{V_{PPP}}=ih\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}(V^\mu\partial^\nu\Pi\partial^\alpha\Pi\partial^\beta\Pi) . \quad (2.9)$$

Here g_{VVP} is given by

$$g_{VVP}=\frac{3g^2}{16\pi^2f_\pi} , \quad (2.10)$$

and h is

$$h=-\frac{g}{2\pi^2f_\pi^3}\left[1-\frac{3}{4}\left[\frac{g^2f_\pi^2}{m_\rho^2}\right]+\frac{3}{32}\left[\frac{g^2f_\pi^2}{m_\rho^2}\right]^2\right] . \quad (2.11)$$

In the nonlinear variation of this model the spin-one mesons are introduced as gauge particles and the model is reformulated[3] in terms of nonlinearly transforming vector multiplets by imposing a suitable chiral-invariant constraint. That is, the axial-vector matrix of fields is eliminated by a special gauge transformation.[3] One then finds the Gell-Mann–Sharp–Wagner value of the coupling constant g_{VVP}:

$$g_{VVP}=\frac{3m_\rho^4}{8\pi^2f_{\rho\pi\pi}^2f_\pi^5} . \quad (2.12)$$

In the first (linear) approach one finds[1,2] $g_{VVP}=10.6$ GeV^{-1} while in the second (nonlinear) variation g_{VVP} has the value[3] $g_{VVP}=7.95$ GeV^{-1}.

In addition to these effective strong-interaction terms, one introduces electromagnetic interactions, assuming the vector-meson-dominance approximation at these intermediate energies. The photon coupling to vector mesons is then given by the additional term[14]

$$\mathcal{L}_{\text{em}}=\frac{\sqrt{2}e}{g}A_\mu\left[m_\rho^2\rho_\mu^0+\frac{1}{3}m_\omega^2\omega_\mu-\frac{\sqrt{2}}{3}m_\phi^2\phi_\mu\right] , \quad (2.13)$$

where the nonet ansatz, or ideal mixing, is understood. That is, in terms of quarks, the neutral vector mesons are

$$\rho^0=\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d}) ,$$

$$\omega=\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d}) ,$$

and

$$\phi=s\bar{s} . \quad (2.14)$$

We note in passing that "hidden symmetry" is a closely related alternative for introducing vector and axial-vector mesons in the effective Lagrangian.[15] Here we shall follow Refs. 1–3, using the same values of the parameters, since they have been determined by fitting the $V\to P\gamma$ and $P\to V\gamma$ decay data.

III. $V^0\to P^0P^0\gamma$ DECAY WIDTHS

The effective Lagrangians relevant for the calculation of the $V^0\to P^0P^0\gamma$ decay widths are

$$\mathcal{L}_{\omega\omega\eta}=-\frac{g_{VVP}}{\sqrt{2}}\epsilon_{\mu\nu\alpha\beta}\partial^\mu\omega^\nu\partial^\alpha\omega^\beta\left[\frac{1}{\sqrt{3}}\eta_8+\sqrt{2/3}\eta_0\right] , \quad (3.1)$$

$$\mathcal{L}_{\omega\rho\pi^0}=-\sqrt{2}g_{VVP}\epsilon_{\mu\nu\alpha\beta}\partial^\mu\rho^\nu\partial^\alpha\omega^\beta\pi^0 , \quad (3.2)$$

$$\mathcal{L}_{\rho\rho\eta}=-\frac{g_{VVP}}{\sqrt{2}}\epsilon_{\mu\nu\alpha\beta}\partial^\mu\rho^\nu\partial^\alpha\rho^\beta\left[\frac{1}{\sqrt{3}}\eta_8+\sqrt{2/3}\eta_0\right] , \quad (3.3)$$

$$\mathcal{L}_{VK^{0*}K^0} = -g_{VVP}\epsilon_{\mu\nu\alpha\beta}\left[\partial^\mu K^{0*\nu}\partial^\alpha\phi^\beta \bar{K}^0 + \frac{1}{\sqrt{2}}\partial^\mu(\omega-\rho^0)^\nu\partial^\alpha K^{0*\beta}\bar{K}^0 + \text{H.c.}\right] . \tag{3.4}$$

The contributions to the decays $\omega\to\pi^0\pi^0\gamma$ and $\omega\to\pi^0\eta\gamma$ are shown in Fig. 1. The decay of the ρ^0 is described by very similar diagrams. In Fig. 2 the diagrams contributing to the decay $\phi\to\bar{K}^0K^0\gamma$ are shown. The decays $\phi\to\pi^0\pi^0\gamma$ and $\phi\to\eta\pi^0\gamma$ are given by the same diagrams (Fig. 1) as $\omega\to\pi^0\pi^0\gamma$ and $\omega\to\eta\pi^0\gamma$, respectively. The physical mass eigenstates $\omega_{\mu p}$ and $\phi_{\mu p}$ are related to the ideally mixed nonet states ω_μ and ϕ_μ [Eq. (2.14)] by

$$\omega_{\mu p} = \omega_\mu + \epsilon\phi_\mu \tag{3.5a}$$

and

$$\phi_{\mu p} = \phi_\mu - \epsilon\omega_\mu , \tag{3.5b}$$

where ϵ has been determined from $\phi\to\rho\pi$ decay data to have the small magnitude: $|\epsilon|=0.076$. In the vector-meson propagators in Figs. 1 and 2 the mass of the vector meson has also been replaced by $M_V - i\Gamma_V/2$ where Γ_V is total measured decay width of the appropriate meson state. The physical η and η' pseudoscalar-meson states are given by the combinations $\eta=\cos\theta\eta_8-\sin\theta\eta_0$ and $\eta'=\sin\theta\eta_8+\cos\theta\eta_0$, as usual.

As an example we discuss the amplitude for $\omega\to\pi^0\pi^0\gamma$ in some detail. From the diagrams in Fig. 1, using the effective Lagrangian, the decay amplitude is

$$A = g_{VVP}^2\frac{e\sqrt{2}}{3g}\{(k\cdot q)[(\epsilon^\omega\cdot\epsilon)l_1^2-(\epsilon\cdot l_1)(\epsilon^\omega\cdot l_1)]-(k\cdot\epsilon)[(\epsilon^\omega\cdot q)l_1^2-(q\cdot l_1)(\epsilon^\omega\cdot l_1)]$$
$$+(k\cdot l_1)[(\epsilon^\omega\cdot q)(\epsilon\cdot l_1)-(l_1\cdot q)(\epsilon^\omega\cdot\epsilon)]\}\frac{1}{l_1^2-m_\rho^2+im_\rho\Gamma_\rho}+(l_1\rightleftarrows l_2) , \tag{3.6}$$

where k is the four-momentum of the ω, l_1 and l_2 are internal vector-meson momenta, q is the photon four-momentum, and ϵ^ω and ϵ are the ω and γ polarization vectors. The amplitudes for the other decays can be found in a similar manner.

The partial decay width is given by

$$\Gamma(V\to PP\gamma) = \frac{1}{192\pi^3 M}\int\sum_{\text{pol}}|A(V\to PP\gamma)|^2 dE\, d\omega . \tag{3.7}$$

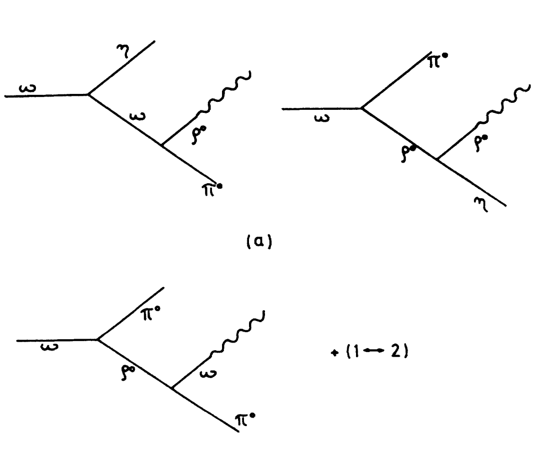

FIG. 1. Diagrams contributing to the decays (a) $\omega\to\eta\pi^0\gamma$ and (b) $\omega\to\pi^0\pi^0\gamma$.

The phase-space integrations were done numerically. The values $g_{VVP}=7.95$ GeV^{-1} (nonlinear realization) and $g=8.66$ were used and the mixing angle θ was taken to be $-22°$ in the numerical calculations. Results for all the processes considered are given in Table I, where we have also included the data available for comparison. All results would increase by a factor of 3.16 if the linear realization of the effective Lagrangian ($g_{VVP}=10.6$ GeV^{-1}) were used.

IV. CONCLUSION

The branching ratios for the decays $\omega\to\pi^0\pi^0\gamma$, $\omega\to\pi^0\eta\gamma$, $\rho^0\to\pi^0\pi^0\gamma$, and $\rho^0\to\pi^0\eta\gamma$ were calculated using an effective Lagrangian approach[1-3] and are presented in Table I. We note that the branching ratio for $\omega\to\pi^0\pi^0\gamma$ is only slightly below the present experimental limit. Precise measurements of these branching ratios would provide a very sensitive determination of the value of g_{VVP} since these decay rates are proportional to

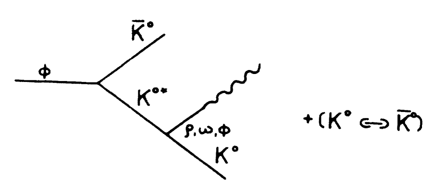

FIG. 2. Diagrams contributing to the decay $\phi\to K^0\bar{K}^0\gamma$.

TABLE I. The branching ratios for $V^0 \to P^0 P^0 \gamma$ decays and the present experimental upper bounds.

Decays	Branching ratios (present calculation)	Branching ratios (experimental data)
$\omega \to \pi^0 \pi^0 \gamma$	8.21×10^{-5}	$< 4 \times 10^{-4}$ (Ref. 7)
$\omega \to \pi^0 \eta \gamma$	6.26×10^{-6}	
$\rho^0 \to \pi^0 \pi^0 \gamma$	2.89×10^{-5}	
$\rho^0 \to \pi^0 \eta \gamma$	3.98×10^{-6}	
$\phi \to \pi^0 \pi^0 \gamma$	3.46×10^{-5}	$< 10^{-3}$ (Ref. 8)
$\phi \to \pi^0 \eta \gamma$	5.18×10^{-5}	
$\phi \to K^0 \bar{K}^0 \gamma$	4.19×10^{-8}	

$(g_{VVP})^4$. Our results for the processes $\phi \to \pi^0 \pi^0 \gamma$, $\phi \to \pi^0 \eta \gamma$, and $\phi \to K^0 \bar{K}^0 \gamma$ are useful in determining the backgrounds for the investigation of the ϕ decays into scalar mesons $\phi \to f_0(975)\gamma$ and $\phi \to a_0(980)\gamma$, particularly since it has been shown[10] that these decay rates are enhanced by more than an order of magnitude if these scalar mesons are four-quark $(q^2\bar{q}^2)$ states.

Note added in proof. The processes $\omega \to \pi^0 \pi^0 \gamma$ and $\rho^0 \to \pi^0 \pi^0 \gamma$ have previously been calculated by Singer[16] using an effective Lagrangian approach. Final-state interactions in $\omega \to \pi\pi\gamma$ decays have also been considered by Levy and Singer.[17] We thank Professor Singer for calling these references to our attention and for pointing out numerical errors in our original manuscript.

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[1]O. Kaymakchalan, S. Rajeev, and J. Schechter, Phys. Rev. D **30**, 594 (1984).
[2]H. Gomm, O. Kaymakchalan, and J. Schechter, Phys. Rev. D **30**, 2345 (1984).
[3]O. Kaymakchalan and J. Schechter, Phys. Rev. D **31**, 1109 (1985).
[4]U. Meissner, Phys. Rep. **161**, 213 (1988).
[5]D. Ebert and H. Reinchardt, Nucl. Phys. **B271**, 188 (1986).
[6]D. Ebert and H. Reinchardt, Phys. Lett. B **173**, 453 (1986).
[7]S. I. Dolinsky *et al.*, Z. Phys. C **42**, 511 (1989).
[8]V. P. Druzhinin *et al.*, Z. Phys. C **37**, 1 (1987).
[9]G. Cosme *et al.*, Phys. Lett. **48B**, 155 (1974).
[10]N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. **B315**, 465 (1989).
[11]S. Nussinov and T. N. Truong, UCLA Report No. UCLA/80/TEP 11 (unpublished).
[12]E. Witten, Nucl. Phys. **B223**, 422 (1983).
[13]J. Wess and B. Zumino, Phys. Lett. **37B**, 95 (1971).
[14]J. J. Sakurai, *Currents and Mesons* (University of Chicago, Chicago, 1969).
[15]M. Bando *et al.*, Phys. Rep. **146**, 217 (1988).
[16]P. Singer, Phys. Rev. **128**, 2789 (1962); **130**, 2441 (1963); **161**, 1694(E) (1967).
[17]N. Levy and P. Singer, Phys. Rev. D **3**, 2134 (1971).