PHYSICAL REVIEW D VOLUME 42, NUMBER 7 1 OCTOBER 1990

Relationship between form factors in semileptonic $\bar{B}$ and D decays and exclusive rare $\bar{B}$-meson decays

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(Received 3 May 1990)

We show that existing data on the semileptonic decays $D \to K e^+ \nu_e$ and $D \to K^* e^+ \nu_e$ provide information on matrix elements relevant for the rare $\bar{B}$-meson decays $\bar{B} \to K e^+ e^-$, $\bar{B} \to K^* e^+ e^-$, and $\bar{B} \to K^* \gamma$ and that future data on $\bar{B} \to \rho e \bar{\nu}_e$ and $\bar{B} \to \pi e \bar{\nu}_e$ will further constrain them. A discussion of how the form factors for these latter decays are in turn determined by semileptonic D decays at the corresponding recoil momenta is also presented.

The rare $\bar{B}$-meson decays provide a sensitive probe of the Higgs sector of the standard model. The minimal standard model requires only a single Higgs doublet to spontaneously break the gauge group $SU(3)\times SU(2)\times U(1)$ to the low-energy gauge group $SU(3)_{\text{color}}\times U(1)_{\text{e.m.}}$. However, a more complicated Higgs sector typically occurs in extensions of the standard model designed to solve the hierarchy puzzle (e.g., low-energy supersymmetry).

The inclusive rates for the rare decays $\bar{B} \to X_s \gamma$ and $\bar{B} \to X_s e^+ e^-$, where X_s is a strange hadronic final state, are likely to be dominated by short-distance physics. Assuming that these inclusive decays can be modeled by b-quark decay, their rates have been computed in the minimal standard model and in models with two Higgs doublets.[1-3] In this Brief Report we relate $\bar{B}$- and D-meson *semileptonic* decay form factors to the hadronic matrix elements that determine the short-distance contributions to the exclusive decays $\bar{B} \to K e^+ e^-$, $\bar{B} \to K^* e^+ e^-$, and $\bar{B} \to K^* \gamma$. The matrix elements that determine the short-distance contribution to these processes (which might also dominate these exclusive decays) have been estimated using various phenomenological models.[4,5] However, our main results, while less complete than a calculation of the relevant matrix elements, will be systematic consequences of QCD: the corrections to our results are suppressed by powers of Λ_{QCD} divided by the heavy-quark mass or by the strong-interaction fine-structure constant evaluated at a heavy-quark-mass scale.

Two of the hadronic matrix elements required for the rare $\bar{B}$-meson decays mentioned above are

$$\langle K(\mathbf{p}')|\bar{s}\sigma_{\mu\nu}b|\bar{B}(\mathbf{p})\rangle = is[(p+p')_\mu(p-p')_\nu-(p-p')_\mu(p+p')_\nu] \tag{1a}$$

and

$$\langle K^*(\mathbf{p}',\epsilon)|\bar{s}\sigma_{\mu\nu}b|\bar{B}(\mathbf{p})\rangle = g_+\epsilon_{\mu\nu\lambda\sigma}\epsilon^{*\lambda}(p+p')^\sigma + g_-\epsilon_{\mu\nu\lambda\sigma}\epsilon^{*\lambda}(p-p')^\sigma + h\epsilon_{\mu\nu\lambda\sigma}(p+p')^\lambda(p-p')^\sigma(\epsilon^*\cdot p) \ . \tag{1b}$$

Since

$$\sigma^{\mu\nu} = \frac{i}{2}\epsilon^{\mu\nu\lambda\sigma}\sigma_{\lambda\sigma}\gamma_5 \tag{2}$$

we can express two others,

$$\langle K(\mathbf{p}')|\bar{s}\sigma_{\mu\nu}\gamma_5 b|\bar{B}(\mathbf{p})\rangle = -s\epsilon_{\mu\nu\lambda\sigma}(p+p')^\lambda(p-p')^\sigma \tag{3a}$$

and

$$\begin{aligned}\langle K^*(\mathbf{p}',\epsilon)|\bar{s}\sigma_{\mu\nu}\gamma_5 b|\bar{B}(\mathbf{p})\rangle = {} & ig_+[\epsilon^*_\nu(p+p')_\mu-\epsilon^*_\mu(p+p')_\nu]+ig_-[\epsilon^*_\nu(p-p')_\mu-\epsilon^*_\mu(p-p')_\nu] \\ & +ih[(p+p')_\nu(p-p')_\mu-(p-p')_\nu(p+p')_\mu](\epsilon^*\cdot p) \ ,\end{aligned} \tag{3b}$$

in terms of the same form factors g_+, g_-, h, and s, which are Lorentz-invariant functions of $t=(p-p')^2$. Since the operators $\bar{s}\sigma_{\mu\nu}\gamma_5 b$ and $\bar{s}\sigma_{\mu\nu}b$ require renormalization, they also depend on the subtraction point μ.

At a subtraction point $\mu \le m_b$ it is appropriate to go over to an effective theory where the b quark couples to the gluon degrees of freedom in a manner that is described by a Wilson line and is independent of the heavy-quark mass and spin. In the effective theory the mass of the b quark is taken to infinity in such a way that p_b^μ/m_b is held fixed, but the four-momentum of the light degrees of freedom are neglected compared with m_b.

It is possible to relate the form factors in Eqs. (1) and (3) to those that occur in the analogous matrix elements of the vector and axial-vector current (which are also required for the rare decays we are considering)

$$\langle K(\mathbf{p}')|\bar{s}\gamma_\mu b|\bar{B}(\mathbf{p})\rangle = f_+(p+p')_\mu + f_-(p-p')_\mu \ , \tag{4a}$$

$$\langle K^*(\mathbf{p}',\epsilon)|\bar{s}\gamma_\mu\gamma_5 b|\bar{B}(\mathbf{p})\rangle = f\epsilon_\mu^* + a_+(\epsilon^*\cdot p)(p+p')_\mu + a_-(\epsilon^*\cdot p)(p-p')_\mu \ , \tag{4b}$$

$$\langle K^*(\mathbf{p}',\epsilon)|\bar{s}\gamma_\mu b|\bar{B}(\mathbf{p})\rangle = ig\epsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu}(p+p')^\lambda(p-p')^\sigma \ . \tag{4c}$$

In the effective theory in the rest frame of the $\bar{B}$ meson, we can set

$$\gamma^0 b = b \ , \tag{5}$$

since the $\bar{B}$ meson contains a b quark (and not a $\bar{b}$ quark). In a general frame this becomes $b = \not{v} b$ where v is the bottom-quark four-velocity. Thus we have, "in the effective theory,"

$$\langle K^*(\mathbf{p}',\epsilon)|\bar{s}\sigma_{0i} b|\bar{B}(\mathbf{0})\rangle = -\frac{i}{2}\langle K^*(\mathbf{p}',\epsilon)|\bar{s}\gamma_i b|\bar{B}(\mathbf{0})\rangle \ , \tag{6a}$$

$$\langle K^*(\mathbf{p}',\epsilon)|\bar{s}\sigma_{0i}\gamma_5 b|\bar{B}(\mathbf{0})\rangle = \frac{i}{2}\langle K^*(\mathbf{p}',\epsilon)|\bar{s}\gamma_i\gamma_5 b|\bar{B}(\mathbf{0})\rangle \ , \tag{6b}$$

$$\langle K(\mathbf{p}')|\bar{s}\sigma_{0i} b|\bar{B}(\mathbf{0})\rangle = -\frac{i}{2}\langle K(\mathbf{p}')|\bar{s}\gamma_i b|\bar{B}(\mathbf{0})\rangle \ . \tag{6c}$$

Using Eqs. (6) and the fact that to leading order in $\alpha_s(m_b)/\pi$ the matching at $\mu = m_b$ between the complete theory and the effective theory is trivial, gives the relations

$$s = \frac{f_+ - f_-}{4m_b} \ , \tag{7a}$$

$$h = -\frac{g}{2m_b} + \frac{a_+ - a_-}{4m_b} \ , \tag{7b}$$

$$g_+ - g_- = -m_b g \ , \tag{7c}$$

$$g_+ + g_- = \frac{f}{2m_b} + \frac{p\cdot p'}{m_b} g \ . \tag{7d}$$

[Since $i\partial_\mu(\bar{s}\gamma^\mu b) = m_b(\bar{s}b)$ and $i\partial_\mu(\bar{s}\gamma^\mu\gamma_5 b) = -m_b(\bar{s}\gamma_5 b)$, matrix elements of the scalar and pseudoscalar bilinears are also determined in terms of the vector and axial-vector form factors.] In Eqs. (7) it is understood that s, h, and $g_\pm$ are evaluated at a subtraction point $\mu = m_b$. Since the vector and axial-vector currents are partially conserved, they do not require renormalization (in the complete theory) and so $f_\pm$, $a_\pm$, f, and g are independent of the subtraction point μ. These relations certainly hold in the kinematic region near "zero recoil" where $|\mathbf{p}'| \ll m_b$, in the rest frame of the $\bar{B}$ meson (i.e., $\mathbf{p} = 0$) since in this region all the light degrees of freedom have four-momenta that are negligible compared with m_b. Also in this kinematic region it is easy to see that

$$(f_+ + f_-) \sim m_b^{-1/2}, \quad (f_+ - f_-) \sim m_b^{1/2} \ , \tag{8a}$$

$$(a_+ + a_-) \sim m_b^{-3/2}, \quad (a_+ - a_-) \sim m_b^{-1/2} \ , \tag{8b}$$

$$g \sim m_b^{-1/2}, \quad f \sim m_b^{1/2} \ , \tag{8c}$$

$$(g_+ + g_-) \sim m_b^{-1/2}, \quad (g_+ - g_-) \sim m_b^{1/2} \ , \tag{8d}$$

$$h \sim m_b^{-3/2}, \quad s \sim m_b^{-1/2} \ . \tag{8e}$$

(Here we display only the dependence on the heavy-quark mass; powers of the strong-interaction scale Λ_{QCD} must be inserted for dimensional consistency.) Hence the two terms on the right-hand side of Eqs. (7b) and (7d) are of equal importance. Of course there is also the logarithmic dependence of the form factors on m_b which arises from the anomalous scaling of $\bar{s}\Gamma b$ (where Γ is any collection of gamma matrices) in the effective theory. This is computed by moving the subtraction point μ down to the QCD scale and is given by[6-8]

$$\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}\left[\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right]^{-6/27} \ . \tag{9}$$

Note that relations (8a) and (8b) imply that in (7a) we can (to leading order in $\Lambda_{\mathrm{QCD}}/m_b$) set $(f_+ - f_-) \simeq 2f_+$ and in (7b) we can set $(a_+ - a_-) \simeq 2a_+$.

Experimentally, the semileptonic D-meson decays $D \to K\bar{e}\nu_e$ and $D \to K^*\bar{e}\nu_e$ have been studied extensively.[9] In an effective theory where both the charm and bottom quarks are treated as heavy there is an SU(2)-flavor symmetry[10,11]

$$\begin{bmatrix} c \\ b \end{bmatrix} \to U \begin{bmatrix} c \\ b \end{bmatrix}, \quad U \in \mathrm{SU}(2) \ . \tag{10}$$

This flavor symmetry relates heavy quarks of the same four-velocity v but different mass (and hence different four-momentum). From this flavor symmetry it follows [see Eqs. (8) and (9)] that

$$(f_+ + f_-)^{(\bar{B}\to K)} = \left[\frac{m_c}{m_b}\right]^{1/2} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} (f_+ + f_-)^{(D\to K)} , \tag{11a}$$

$$(f_+ - f_-)^{(\bar{B}\to K)} = \left[\frac{m_b}{m_c}\right]^{1/2} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} (f_+ - f_-)^{(D\to K)} , \tag{11b}$$

$$(a_+ + a_-)^{(\bar{B}\to K^*)} = \left[\frac{m_c}{m_b}\right]^{3/2} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} (a_+ + a_-)^{(D\to K^*)} , \tag{11c}$$

$$(a_+ - a_-)^{(\bar{B}\to K^*)} = \left[\frac{m_c}{m_b}\right]^{1/2} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} (a_+ - a_-)^{(D\to K^*)} , \tag{11d}$$

$$g^{(\bar{B}\to K^*)} = \left[\frac{m_c}{m_b}\right]^{1/2} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} g^{(D\to K^*)} , \tag{11e}$$

$$f^{(\bar{B}\to K^*)} = \left[\frac{m_b}{m_c}\right]^{1/2} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} f^{(D\to K^*)} . \tag{11f}$$

In Eqs. (11), the form factors are evaluated at the same *four-velocity* transfer $v \to v'$. Thus in Eqs. (11) $D\to K$ and $D\to K^*$ form factors at $t_D=(m_c v - p')^2$ are being related to $\bar{B}\to K$ and $\bar{B}\to K^*$ form factors at $t_B=(m_b v - p')^2$. The range in t for which Eqs. (11) are valid is discussed below.

Eventually Eqs. (11) may play a role in determining the magnitude of the V_{ub} element of the Kobayashi-Maskawa matrix.[10,12] Since the ρ and K^* (and the π and K) are in the same SU(3)-flavor octet, and there is a unique way to combine the weak currents [an SU(3)-antitriplet operator] with the $\bar{B}$ mesons [an SU(3) triplet] into an octet, we have in the limit of SU(3)-flavor symmetry that the axial-vector and vector form factors for $\bar{B}\to K^*$ ($\bar{B}\to K$) are related by Clebsch-Gordan coefficients to those for $\bar{B}\to\rho$ ($\bar{B}\to\pi$) which arise from the weak $b\to u$ transition. If measurements of the form factors for the Cabibbo-suppressed semileptonic D decays $D\to\rho e^+\nu_e$ and $D\to\pi e^+\nu_e$ are eventually made, then the $\bar{B}\to\pi e\bar{\nu}_e$ and $\bar{B}\to\rho e\bar{\nu}_e$ form factors will follow directly from the analogs of Eqs. (11) (with the replacements $K\to\pi, K^*\to\rho$) and isospin symmetry.[13]

Equations (11), (8), and (7) show that all of the form factors we are considering can be related and that, as we have emphasized already, these relations will certainly hold near the zero recoil point where $t=(p-p')^2$ is near its maximum value t_m. This region of applicability is sufficient to allow the determination of V_{ub} and the prediction of the $\bar{B}\to Ke^+e^-$ and $\bar{B}\to K^*e^+e^-$ form factors for high mass e^+e^- pairs. Eventually, experimental data on the semileptonic $\bar{B}$ decay $\bar{B}\to\rho e\bar{\nu}_e$ [combined with SU(3)-flavor symmetry] will help with the extrapolation of the form factors $a_\pm$, f, and g [determined near $t_m=(m_B-m_{K^*})^2$ by the data on the semileptonic decay $D\to K^*\bar{e}\nu_e$] to low t. However, $\bar{B}\to K^*\gamma$ occurs at $t=0$ and its amplitude cannot be related to the semileptonic form factors if Eqs. (7) are not valid at this point. Even in comparing $\bar{B}$ to D semileptonic decays, it would be inconvenient to be restricted kinematically to $|\mathbf{p}'| \ll m_c$. It is therefore important to understand the region in recoil momentum for which Eqs. (11), (8), and (7) hold.

The delineation of this region of applicability is currently under investigation,[14] but we can make some remarks here. One would certainly expect the heavy-quark symmetry as well as Eq. (5) to break down if the process under consideration probes the light degrees of freedom at the heavy-quark momentum scale m_Q. However, in heavy-quark decays there is a suppression of the momentum transfer Q_l experienced by the light degrees of freedom. In particular, for a heavy-to-light transition

$$Q_l^2 \sim (m_l/m_Q)(t_m - t) \le m_l m_Q ,$$

where $m_l \sim \Lambda_{\rm QCD}$ is some mass scale characterizing the light degrees of freedom. Thus even $\bar{B}\to\pi$ transitions only probe $Q_l^2 \lesssim 2$ GeV^2. At such momentum transfers one might expect that perturbative processes (which could violate the conditions under which our relations are derived) would dominate.[15] However, it has been argued[16] that the pion form factor is still dominated by soft processes for $Q_l^2 \le 10$ GeV^2. Phenomenological models for the soft contributions to heavy-to-light transitions suggest that at high Q_l^2 they are dominated by the low-Bjorken-x end-point regions of the spectator quark's momentum distributions.[14] In this case the heavy quark stays almost on its mass shell, and the relations derived here remain applicable, over the full Dalitz plots for their respective decays as well as the photon point in $\bar{B}\to K^*\gamma$.

This work was supported in part by the U.S. Department of Energy under Contract No. DE-AC0381-ER40050 and by a grant from the Natural Sciences and Engineering Research Council of Canada.

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