Reformulation of finite-temperature dilepton production

H. Arthur Weldon
Department of Physics, West Virginia University, Morgantown, West Virginia 26506
(Received 21 May 1990)

The multiplicity of e^+e^- or $\mu^+\mu^-$ pairs produced at finite temperature is expressed in terms of the photon proper self-energy at finite temperature. The production rate is strongly dependent on the four-momentum q^μ of the lepton pair and has a resonance at $q^2=\mathrm{Re}\Pi$.

I. INTRODUCTION

The collision of heavy nuclei at ultrarelativistic energies produces such a high energy density that for a short time the system is in local thermal equilibrium. The temperature should be high enough that the quark-gluon plasma predicted by lattice QCD will be produced.[1,2] One of the best signals will be the emission of opposite-sign dileptons[3] because the leptons escape the collision region without reinteracting. They can therefore convey information about the hot interior of the quark-gluon plasma.

High-energy lepton pairs are produced by a single virtual γ that is created by hadronic collisions. The total lepton energy q^0 and momentum $\mathbf{q}$ are the energy-momentum q^μ of the virtual γ. There have been a number of calculations[4-10] of the dilepton multiplicity N for large q^2 [typically $0.2<(q^2)^{1/2}<5.0$ GeV]. In that regime the probability amplitude for the virtual γ to propagate through the plasma is just the free propagator $1/q^2$. However the long range of the electromagnetic interaction can modify the propagation. Such effects are common in nonrelativistic plasmas or in dielectrics, where the propagation of radiation is strongly dependent on frequency. For a particular wave vector $|\mathbf{q}|$, resonant propagation usually occurs at some $q^0\neq|\mathbf{q}|$.

The present paper calculates the dilepton multiplicity N from first principles in order to display the possibility of resonant propagation in a finite-temperature plasma. At large T the plasma is composed of quarks and gluons; at smaller T, of hadrons.

II. DILEPTON MULTIPLICITY

Finite-temperature nuclear matter is produced experimentally by colliding two heavy nuclei. Long before the collision the two incoming nuclei are in an asymptotic initial state $|I\rangle$. If the free lepton states are normalized in a box of volume V, then the inclusive differential probability for emission of leptons into dimensionless cells $V\,d^3p/(2\pi)^3$ of phase space, from the reaction $I\rightarrow l_1\bar{l}_2+$anything, is

$$\sum_F |\langle F l_1\bar{l}_2|S|I\rangle|^2\frac{V\,d^3p_1}{(2\pi)^3}\frac{V\,d^3p_2}{(2\pi)^3} \ . \tag{2.1}$$

Inclusive probabilities are not normalized to unity. Elementary counting shows that integrating over the two-particle inclusive probability yields the two-particle multiplicity.[11] The multiplicity calculated from (2.1) will depend on the specific initial state $|I\rangle$ as, for example, in Drell-Yan calculations which always depend on the quark structure functions of the projectiles.

By contrast, in ultrarelativistic heavy-ion collisions the reactions will be so rapid that within a short time (0.5–1.5 fm/c) the particles will thermalize.[2,12] Thermalization erases all information about a specific initial state $|I\rangle$ and replaces it with an ensemble average over all states $|I\rangle$ each weighted by a Boltzmann factor. Consequently the thermally averaged dilepton lepton multiplicity is

$$N=\sum_I\sum_F|\langle Fl_1\bar{l}_2|S|I\rangle|^2\frac{e^{-\beta E_I}}{Z}\frac{V\,d^3p_1}{(2\pi)^3}\frac{V\,d^3p_2}{(2\pi)^3} \ , \tag{2.2}$$

where $Z=\mathrm{Tr}[\exp(-\beta H)]$ is the canonical partition function. This formula gives the multiplicity in the local rest frame of the plasma. If the plasma has four-velocity u_μ in the lab, then E_I is replaced by $P_I\cdot u$ and Z by $\mathrm{Tr}[\exp(-\beta P\cdot u)]$.

For high-energy dileptons produced by a single virtual γ with energy $q^0=E_1+E_2$ and momentum $\mathbf{q}=\mathbf{p}_1+\mathbf{p}_2$, the emission amplitude is

$$\langle Fl_1\bar{l}_2|S|I\rangle=\frac{e_0\bar{u}_1\gamma_\mu v_2}{V\sqrt{2E_1 2E_2}}\int d^4x\, e^{iq\cdot x}\langle F|A^\mu(x)|I\rangle \ , \tag{2.3}$$

where A^μ is the exact Heisenberg field, e_0 is the unrenormalized charge, and lepton spinors are normalized to $\bar{u}u=2m, \bar{v}v=-2m$. Substituting this into (2.2) gives

$$N=e_0^2L_{\mu\nu}\mathcal{M}^{\mu\nu}\frac{d^3p_1}{(2\pi)^3E_1}\frac{d^3p_2}{(2\pi)^3E_2} \ , \tag{2.4}$$

where the lepton and photon tensors are given by

$$L_{\mu\nu}=\frac{1}{4}\sum_{\text{spins}}\bar{u}_1\gamma_\mu v_2\bar{v}_2\gamma_\nu u_1 =[p_{1\mu}p_{2\nu}+p_{2\mu}p_{1\nu}-(p_1\cdot p_2+m^2)g_{\mu\nu}] \ , \tag{2.5}$$

$$\mathcal{M}^{\mu\nu}\equiv\sum_F\sum_I\int d^4x\, d^4y\, e^{iq\cdot(x-y)}\langle F|A^\mu(x)|I\rangle \times\langle I|A^\nu(y)|F\rangle\frac{e^{-\beta E_I}}{Z} \ . \tag{2.6}$$

The photon tensor can be simplified by using $E_I = E_F + q^0$ to write the Boltzmann factor in terms of E_F. Then the sum on I can be performed by completeness to give

$$\mathcal{M}^{\mu\nu} = e^{-\beta q^0} \sum_F \int d^3x\, d^4y\, e^{iq\cdot(x-y)} \times \langle F| A^\mu(x) A^\nu(y) |F\rangle \frac{e^{-\beta E_F}}{Z} . \tag{2.7}$$

Because of translation invariance, the matrix element depends only on the difference $x-y$. The four-dimensional integration over $x+y$ gives the total space-time volume Ω. Thus

$$\mathcal{M}^{\mu\nu} = \Omega e^{-\beta q^0} 2\pi \rho^{\mu\nu}(q) , \tag{2.8}$$

$$\rho^{\mu\nu}(q) \equiv \int \frac{d^4x}{2\pi} e^{iq\cdot x} \sum_F \langle F| A^\mu(x) A^\nu(0) |F\rangle \frac{e^{-\beta E_F}}{Z} . \tag{2.9}$$

The tensor $\rho^{\mu\nu}$ (including the single factor of 2π) is the photon spectral function at finite temperature. At $T=0$ it reduces to the usual spectral function because the only state $|F\rangle$ that survives $\beta \to \infty$ is the vacuum. For later purposes it is important to note that

$$e^{-\beta q^0} \rho^{\mu\nu}(q^0, \mathbf{q}) = \rho^{\nu\mu}(-q^0, -\mathbf{q}) \tag{2.10}$$

and that $\rho^{\mu\nu}$ is symmetric in μ and ν.

Since N is the total multiplicity in the entire four-volume Ω, the quotient is the multiplicity per unit space-time volume: $N/\Omega = dN/d^4x$. Combining (2.4) and (2.8) gives

$$\frac{dN}{d^4x} = 2\pi e_0^2 L_{\mu\nu} \rho^{\mu\nu}(q) e^{-\beta q^0} \frac{d^3p_1}{(2\pi)^3 E_1} \frac{d^3p_2}{(2\pi)^3 E_2} . \tag{2.11}$$

This is the fundamental result. From here on the principal issue will be how to calculate the spectral function.

The same result can also be expressed in terms of matrix elements of the current. McLerran and Toimela[4] define the tensor

$$W^{\mu\nu}(q) = \int d^4x\, e^{-iq\cdot x} \sum_F \langle F| J^\mu(x) J^\nu(0) |F\rangle \frac{e^{-\beta E_F}}{Z} . \tag{2.12}$$

This is related to $\rho^{\mu\nu}(-q)$ by Maxwell's equations:

$$e_0^2 W^{\mu\nu}(q) = 2\pi (q^2 g^{\mu\alpha} - q^\mu q^\alpha) \rho_{\alpha\beta}(-q) (q^2 g^{\beta\nu} - q^\beta q^\nu) . \tag{2.13}$$

Therefore (2.11) can also be written

$$\frac{dN}{d^4x} = e_0^4 L_{\mu\nu} \frac{W^{\mu\nu}(q)}{q^4} \frac{d^3p_1}{(2\pi)^3 E_1} \frac{d^3p_2}{(2\pi)^3 E_2} . \tag{2.14}$$

This is essentially Eq. (15) of McLerran and Toimela.[4]

III. THE PHOTON SPECTRAL FUNCTION

There is an advantage to expressing the dilepton cross section in terms of the photon spectral function, viz., the spectral function can be directly related to the photon proper self-energy. To do that, first express the real-time, thermal propagator in coordinate space:

$$D^{\mu\nu}(x) = -2\pi i \theta(t) \rho^{\mu\nu}(x) - 2\pi i \theta(-t) \rho^{\nu\mu}(-x) . \tag{3.1}$$

The Fourier transform gives the dispersion representation

$$D^{\mu\nu}(q^0, \mathbf{q}) = \int_{-\infty}^{\infty} d\sigma \left[\frac{\rho^{\mu\nu}(\sigma, \mathbf{q})}{q^0 - \sigma + i\eta} - \frac{\rho^{\nu\mu}(\sigma, -\mathbf{q})}{q^0 + \sigma - i\eta} \right] . \tag{3.2}$$

Using (2.10), the imaginary part is

$$\mathrm{Im} D^{\mu\nu}(q^0, \mathbf{q}) = -\pi (1 + e^{-\beta q^0}) \rho^{\mu\nu}(q^0, \mathbf{q}) . \tag{3.3}$$

The next task is to relate the propagator $D^{\mu\nu}$ to the proper self-energy. At $T=0$ this is trivial, but for $T>0$ a bit more effort is necessary because (3.2) has singularities in all four quadrants of the complex q^0 plane. That part of the propagator with singularities only in the second and fourth quadrants obeys the Feynman prescription. These are related by[13]

$$D^{\mu\nu}(q) = D_F^{\mu\nu}(q)(1+n) - D_F^{\mu\nu *}(q) n , \tag{3.4}$$

where $n = 1/[\exp(q^0/T) - 1]$. The imaginary part of this gives

$$\mathrm{Im} D^{\mu\nu}(q) = (1+2n) \mathrm{Im} D_F^{\mu\nu}(q) . \tag{3.5}$$

Comparing (3.3) and (3.5) gives for the spectral function

$$\rho^{\mu\nu}(q^0, \mathbf{q}) = \frac{-1}{\pi} \frac{e^{\beta q^0}}{e^{\beta q^0} - 1} \mathrm{Im} D_F^{\mu\nu}(q^0, \mathbf{q}) . \tag{3.6}$$

To apply (3.6) one needs to relate D_F to the proper (i.e., one-photon-irreducible) self-energy $\Pi^{\mu\nu}$ using the Schwinger-Dyson equation

$$(q^2 g^\mu_\lambda - \Pi^\mu_\lambda) D_F^{\lambda\nu} = -g^{\mu\nu} + \alpha q^\mu q^\nu / q^2 , \tag{3.7}$$

where α is a gauge parameter. Because of current conservation $q_\mu \Pi^{\mu\nu} = 0$. At $T=0$ one subtraction renders the self-energy finite and the $T>0$ contributions do not change this. Thus

$$\Pi^{\mu\nu}(q) = \left[1 - \frac{1}{Z_3} \right] (q^2 g^{\mu\nu} - q^\mu q^\nu) + \frac{1}{Z_3} \Pi_f^{\mu\nu} , \tag{3.8}$$

where Z_3 is the $T=0$ wave-function renormalization constant and $\Pi_f^{\mu\nu}$ is a finite function of q^μ, T, and the renormalized charge e. Current conservation requires that it be a linear combination of two conserved tensors:[13,14]

$$\Pi_f^{\mu\nu} = \Pi_T P_T^{\mu\nu} + \Pi_L P_L^{\mu\nu} . \tag{3.9}$$

In the rest frame of the fluid the tensors are

$$P_T^{00}=0,\quad P_T^{0s}=0,\quad P_T^{rs}=-\delta^{rs}+\frac{q^r q^s}{|\mathbf{q}|^2}\;,$$
$$P_L^{00}=-\frac{|\mathbf{q}|^2}{q^2},\quad P_L^{0s}=-\frac{q^0 q^s}{q^2},\quad P_L^{rs}=-\frac{q^r q^s}{q^2}\left[\frac{q^0}{|\mathbf{q}|}\right]^2 . \tag{3.10}$$

Both four-tensors are orthogonal to q_μ. The three-tensors P_T^{rs} and P_L^{rs} are transverse and longitudinal to the three-vector q^s, respectively. These tensors are idempotent ($P_TP_T=P_T, P_LP_L=P_L$), orthogonal ($P_TP_L=P_LP_T=0$), and sum to $g^{\mu\nu}-q^\mu q^\nu/q^2$. The solution to (3.7) is

$$D_F^{\mu\nu}(q)=-\frac{Z_3 P_T^{\mu\nu}}{q^2-\Pi_T}-\frac{Z_3 P_L^{\mu\nu}}{q^2-\Pi_L}+q^\mu q^\nu \text{ terms} . \tag{3.11}$$

Applying (3.6) to this gives for the spectral function

$$\rho^{\mu\nu}(q)=\frac{Z_3}{\pi}\frac{e^{\beta q^0}}{e^{\beta q^0}-1}(\rho_T P_T^{\mu\nu}+\rho_L P_L^{\mu\nu})+q^\mu q^\nu \text{ terms} , \tag{3.12a}$$

$$\rho_j=\frac{\text{Im}\Pi_j}{(q^2-\text{Re}\Pi_j)^2+(\text{Im}\Pi_j)^2} , \tag{3.12b}$$

for $j=T$ or L. This automatically satisfies (2.10) because $\text{Im}\Pi$ is an odd function of q^0.

IV. RESULTS AND CONCLUSIONS

We may now substitute the spectral function (3.12) into the dilepton multiplicity (2.11). The terms in (3.12) proportional to $q^\mu q^\nu$ do not contribute when contracted with the lepton tensor (2.5). The factor Z_3 renormalizes the bare charge, $Z_3 e_0^2=e^2$. Thus

$$\frac{dN}{d^4x}=2e^2 L_{\mu\nu}(P_T^{\mu\nu}\rho_T+P_L^{\mu\nu}\rho_L)\frac{d^3p_1}{(2\pi)^3E_1}\frac{d^2p_2}{(2\pi)^3E_2}\Big/(e^{\beta q^0}-1) . \tag{4.1}$$

This is the general result and it depends on the directions of $\mathbf{p}_1$ and $\mathbf{p}_2$. If the data are binned only by the total q^μ, then integration gives

$$\int\frac{d^3p_1}{E_1}\int\frac{d^3p_2}{E_2}\delta^4(p_1+p_2-q)L_{\mu\nu}(p_1,p_2)=\frac{2\pi}{3}B(q_\mu q_\nu-q^2 g_{\mu\nu}) , \tag{4.2a}$$

$$B=\left[1+\frac{2m^2}{q^2}\right]\left[1-\frac{4m^2}{q^2}\right]^{1/2} . \tag{4.2b}$$

Since the lepton mass m is typically much smaller than q^2, usually $B\approx 1$. Contracting (4.2a) with $P_T^{\mu\nu}$ and $P_L^{\mu\nu}$ gives, for the differential multiplicity,

$$\frac{dN}{d^4x\,d^4q}=\frac{\alpha}{12\pi^4}B(2\mathcal{R}_T+\mathcal{R}_L)/(e^{\beta q^0}-1) , \tag{4.3a}$$

$$\mathcal{R}_j=\frac{-q^2\text{Im}\Pi_j}{(q^2-\text{Re}\Pi_j)^2+(\text{Im}\Pi_j)^2} . \tag{4.3b}$$

For $q^0>0$, $\text{Im}\Pi_j$ is negative.[15] The combination $2\mathcal{R}_T+\mathcal{R}_L$ enters because there are two transverse modes and only one longitudinal mode. In a Lorentz frame where the plasma is not at rest, q^0 in (4.3) should be replaced by $q\cdot u$, where u^μ is the four-velocity of the plasma. The self-energies are functions of the Lorentz-invariant variables T, q^2, and $q\cdot u$. The validity of (4.3) depends on the wavelength being smaller than the system size. The mean free path of the γ can be much larger than the system size, which is necessary if it is to carry information about the hot interior. The larger the mean free path ($\approx|\mathbf{q}|/|\text{Im}\Pi|$) becomes, the more nearly $\mathcal{R}_j$ approaches $\delta(q^2-\text{Re}\Pi_j)$. This is the case in geometrical optics, where $q^0=|\mathbf{q}|/n$ is the only frequency that can propagate in a media with index of refraction n.

There are some general properties of (4.3) that are noteworthy. (i) The imaginary parts of the self-energies describe both the absorption of the virtual γ into the plasma and the creation of the virtual γ by the plasma. If Γ_T and Γ_L are the rates for creation of the transverse and longitudinal modes, they are related to the self-energy by[15]

$$\Gamma_T=|\text{Im}\Pi_T|/(e^{\beta q^0}-1) ,$$
$$\Gamma_L=|\text{Im}\Pi_L|/(e^{\beta q^0}-1) , \tag{4.4}$$

where the absolute value bars take account of the fact that $\text{Im}\Pi$ is negative. (ii) The real and imaginary parts of Π are proportional to the electromagnetic coupling α. Therefore, for large q^2 the denominators of (4.3b) are negligible and

$$\mathcal{R}_j\approx|\text{Im}\Pi_j|/q^2 . \tag{4.5}$$

Consequently for large q^2 the dilepton multiplicity (4.3) becomes

$$\frac{dN}{d^4x\,d^4q}\approx\frac{\alpha}{12\pi^4}B\frac{2\Gamma_T+\Gamma_L}{q^2} , \tag{4.6}$$

which is the starting point for many calculations.[4–10,16–18] (iii) The most interesting possibility is that (4.3) can have a resonance at $q^2=\text{Re}\Pi_j$. Precisely at the resonance

$$\mathcal{R}_j|_{\text{resonance}}=q^2/|\text{Im}\Pi_j| \ . \tag{4.7}$$

This is automatically of order $1/\alpha$, whereas (4.5) is of order α. The signal this resonance creates for the quark-gluon plasma is explored in a separate paper.[19]

ACKNOWLEDGMENTS

This work was supported in part by the U.S. National Science Foundation under Grant No. PHY-9008431.

[1] L. McLerran, Rev. Mod. Phys. **58**, 1021 (1986); J. Cleymans, R. V. Gavai, and E. Suhonen, Phys. Rep. **130C**, 218 (1986); F. Karsch, Z. Phys. C **38**, 147 (1988).

[2] K. Kajantie and L. McLerran, Annu. Rev. Nucl. Part. Sci. **37**, 293 (1987).

[3] E. V. Shuryak, Phys. Lett. **78B**, 150 (1978); Phys. Rep. **61**, 71 (1979).

[4] L. D. McLerran and T. Toimela, Phys. Rev. D **31**, 545 (1985).

[5] R. C. Hwa and K. Kajantie, Phys. Rev. D **32**, 1109 (1985).

[6] K. Kajantie, J. Kapusta, L. McLerran, and A. Mekjian, Phys. Rev. D **34**, 2746 (1986).

[7] K. Kajantie, M. Kataja, L. McLerran, and P. V. Ruuskanen, Phys. Rev. D **34**, 811 (1986).

[8] H. von Gersdorff, L. McLerran, K. Kataja, and P. V. Russkanen, Phys. Rev. D **34**, 794 (1986); **34**, 2755 (1986).

[9] J. Cleymans, J. Fingberg, and K. Redlich, Phys. Rev. D **35**, 2153 (1987).

[10] C. Gale and J. Kapusta, Phys. Rev. C **35**, 2107 (1987); Nucl. Phys. **A495**, 423c (1989).

[11] A. H. Mueller, Phys. Rev. D **4**, 150 (1971); C. E. DeTar, D. Z. Freedman, and G. Veneziano, *ibid.* **4**, 906 (1971).

[12] K. J. Eskola, K. Kajantie, and J. Lindfors, Phys. Lett. B **214**, 613 (1988).

[13] N. P. Landsman and Ch.G. van Weert, Phys. Rep. **145C**, 142 (1987).

[14] H. A. Weldon, Phys. Rev. D **26**, 1394 (1982).

[15] H. A. Weldon, Phys. Rev. D **28**, 2007 (1983); R. L. Kobes and G. W. Semenoff, Nucl. Phys. **B260**, 714 (1985); **B272**, 329 (1986).

[16] R. Baier, B. Pire, and D. Schiff, Phys. Rev. D **38**, 2814 (1988).

[17] T. Altherr, P. Aurenche, and T. Becherraway, Nucl. Phys. **B315**, 436 (1989).

[18] E. Braaten, R. D. Pisarski, and Tzu Chiang Yuan, Brookhaven National Laboratory Report No. TH-90-1, 1990 (unpublished).

[19] H. A. Weldon, West Virginia University report, 1990 (unpublished).