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Statistical quantization of GUT models and phase diagrams of W condensation for the Universe with finite fermion density

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The problems of statistical quantization for grand-unified-theory models are studied using as an example the Weinberg-Salam model with finite fermion density under the conditions of neutral and electric charge conservation. The relativistic R_γ gauge with an arbitrary parameter is used and the one-loop effective potential together with its extremum equations are found. We demonstrate (and this is our main result) that the thermodynamic potential obtained from the effective one, after the mass shell for ξ is used, remains gauge dependent if all temperature ranges (not only the leading high-temperature terms) are considered. The contradiction detected within the calculational scheme is eliminated after the redefinition of the model studied is made with the aid of the terms which are proportional to the "non-Abelian" chemical potential and equal to zero identically when the unitary gauge is fixed. The phase diagrams of the W condensation are established and all their peculiarities are displayed. We found for the universe with a zero neutral charge density that the W condensate occurs at any small fermion density ρ and appears at first near the point of symmetry restoration. For all $\rho\neq 0$ this condensate exists only in the finite-temperature domain and evaporates completely or partially when T goes to zero.

I. INTRODUCTION

The study of phase transitions connected with W condensation for realistic models of grand unified theories (GUT) belongs to a very interesting field of activity in modern theoretical physics. This kind of condensation for the electrically uncharged universe with a finite fermion density was found first[1] for the case $T=0$. This scenario was then extended to finite temperature by the present authors.[2] When $T\neq 0$ the possibility of W condensation is strongly connected with the Higgs mechanism which always takes place in realistic GUT's and also changes as influenced by temperature[3] and density. This link leads to many peculiarities for the W condensate phase diagram which can show up in cosmology where appropriate external conditions might be realized. Today these peculiarities have been investigated by many authors both at the one-loop level[4–6] and taking into account some specific features of the high-loop corrections.[7] Many parallels between W condensation for the finite external density case (which is the subject of this paper) and for the case when the external magnetic field is applied[8,9] are now found as well. Such parallels are very interesting and open the possibility to give in the future a new status for the W-boson condensation phenomenon.

However, many aspects of GUT, after W condensation occurs, are not yet completely clear. In particular, many difficulties arise from incorporating into the non-Abelian models the chemical potential which supplies the neutral charge conservation within the spontaneous-symmetry-breaking phase. We demonstrate below that the usual ansatz of embedding the chemical potentials into the non-Abelian models, which is found to be completely correct in many papers (see, e.g., Refs. 4 and 5) must be refined when the neutral charge conservation takes place. This modification of the model studied (if the usual quantization rules are kept) is necessary because the gauge invariance of the physical modes (which are some part of spectra on the mass shell) is lost and it is not possible to give the gauge-independent thermodynamical potential for the models studied. As a consequence, the calculations of many physical properties for such models (for example, the W-condensate phase diagram) do not have a solid basis and the results obtained, in particular, in the unitary gauge[2,4] and within the relativistic ones[5,6] are not the same.

The goal of the present paper is to build (following the Weinberg-Salam model) the appropriate formalism for the quantum-statistical description of the GUT models with a finite fermion density under the conditions of neutral and electric charge conservation. The maximum possible number of chemical potentials (here it is three) are embedded in the model and we consider the rather broad kind of boundary condition when the W-condensate phase diagram is investigated. The results obtained have many model-independent features which seem inherent to any realistic model of GUT but they are also useful for the Weinberg-Salam (WS) model to be considered alone. This model is a realistic part of a GUT and has many reasons to be useful for practice. In detail we investigate only the leptonic sector of the WS model which is related to the electroweak interaction of the electron with the neutrino and completely omit its hadronic part. All questions connected with the renormalizability of this model are ignored and we do not take into account its possible anomalies (see, e.g., Ref. 10).

II. THE MODEL AND QUANTIZATION FORMALISM

The Lagrangian has the standard form

$$L=-\tfrac{1}{4}(G^i_{\mu\nu})^2-\tfrac{1}{4}(F_{\mu\nu})^2-\bar{\psi}_L\gamma_\mu\left[\left[\partial_\mu+i\frac{\tilde{g}}{2}B_\mu\right]I-ig\frac{\tau^i}{2}W^i_\mu\right]\psi_L-\bar{e}_R\gamma_\mu(\partial_\mu+i\tilde{g}B_\mu)e_R$$
$$-\left|\left[\left[\partial_\mu-i\frac{\tilde{g}}{2}B_\mu\right]I-i\frac{g}{2}\tau^iW^i_\mu\right]\phi\right|^2-h(\bar{\psi}_L\phi e_R+\bar{e}_R\phi^\dagger\psi_L)-\frac{\lambda^2}{2}(\phi^\dagger\phi-a^2/2\lambda^2)^2\ , \qquad (2.1)$$

where all abbreviations are the usual ones and many details connected with this model may be found in Ref. 4 and 10. The complex doublet of scalar fields is given by

$$\phi=\frac{1}{\sqrt{2}}\left[\begin{bmatrix}0\\ \zeta\end{bmatrix}+\begin{bmatrix}ih_1+h_2\\ \sigma-ih_3\end{bmatrix}\right]\ , \qquad (2.2)$$

where the four real scalar fields are introduced and ζ is the symmetry-breakdown parameter. All vector fields are defined as usual,

$$W^\pm_\mu=\frac{1}{\sqrt{2}}(W^1_\mu\mp iW^2_\mu),\quad A_\mu=W^3_\mu\sin\theta+B_\mu\cos\theta\ ,$$
$$Z_\mu=-W^3_\mu\cos\theta+B_\mu\sin\theta \qquad (2.3)$$

with $\tan\theta=\tilde{g}/g$. After spontaneous symmetry breaking the electron and all the vector fields (except the electromagnetic) acquire mass and the chosen charge operator leads to the usual definition of the electromagnetic coupling constant $e=g\tilde{g}/\sqrt{g^2+\tilde{g}^2}$.

We introduce the three chemical potentials into the model studied. Two potentials are necessary for conservation of the electrical and neutral charges (here μ_1 and μ_3), whereas the third (μ_2) is introduced directly to the Lagrangian with the aid of the lepton density operator

$$N_2=\bar{e}\gamma_4e+\tfrac{1}{2}\bar{\nu}\gamma_4(1+\gamma_5)\nu \qquad (2.4)$$

because we also wish to support the conservation of this quantity. The simplest way to incorporate the chemical potentials μ_1 and μ_3 into the model is to require that the appropriate gauge fields have the nonzero vacuum expectation values

$$\langle W^3_\mu\rangle=\frac{i}{g}\left[\mu_1-\mu_3\frac{2\cos^2\theta}{\cos2\theta}\right]u_\mu\ ,$$
$$\langle B_\mu\rangle=\frac{i}{\tilde{g}}\left[\mu_1+\mu_3\frac{2\sin^2\theta}{\cos2\theta}\right]u_\mu\ , \qquad (2.5)$$

and then to consider these values on the same footing with ζ. Here the four-velocity vector of the medium has the form $u_i=0$ and $u_4=1$ in the system at rest.

Our ansatz of dealing with the chemical potentials (see also Refs. 4 and 5) gives the same result as the usual quantization procedure which follows the statistical density matrix

$$\rho=\exp\left[-\int_0^\beta dx_4\int d^3x\,(H-\mu_iN_i)\right]\ , \qquad (2.6)$$

with the effective Hamiltonian $\tilde{H}=H-\mu_iN_i$. For the model studied one can build three Noether conserved currents j^k_μ;

$$j^k_\mu=gG^i_{\mu\nu}\epsilon^{ijk}W^j_\nu-ig\bar{\psi}_L\gamma_\mu\frac{1}{2}\tau^k\psi_L$$
$$+[(\partial_\mu-\tfrac{1}{2}ig\tau^iW^i_\mu-\tfrac{1}{2}i\tilde{g}B_\mu)\phi](\tfrac{1}{2}ig\tau^k\phi^\dagger)$$
$$-(\tfrac{1}{2}ig\tau^k\phi)[(\partial_\mu+\tfrac{1}{2}ig\tau^iW^i_\mu+\tfrac{1}{2}i\tilde{g}B_\mu)\phi^\dagger]\ , \qquad (2.7)$$

which arise from the global SU(2) invariance, and some additional current j^0_μ:

$$j^0_\mu=\bar{\psi}_L\gamma_\mu(\tfrac{1}{2}\tilde{g})\psi_L+\bar{e}_R\gamma_\mu(i\tilde{g})e_R$$
$$+[(\partial_\mu-\tfrac{1}{2}ig\tau^iW^i_\mu-\tfrac{1}{2}i\tilde{g}B_\mu)\phi](\tfrac{1}{2}i\tilde{g}\phi^\dagger)$$
$$-(\tfrac{1}{2}i\tilde{g}\phi)[(\partial_\mu+\tfrac{1}{2}ig\tau^iW^i_\mu+\tfrac{1}{2}i\tilde{g}B_\mu)\phi^\dagger]\ , \qquad (2.8)$$

which corresponds to the U(1) group. Now the electromagnetic and neutral weak currents are

$$j^{\rm em}_\mu=j^3_\mu\sin\theta+j^0_\mu\cos\theta,\quad j^N_\mu=-j^3_\mu\cos\theta+j^0_\mu\sin\theta\ , \qquad (2.9)$$

and the corresponding densities

$$N_1=j^{\rm em}_4/e,\quad N_3=j^N_4/e\cot2\theta\ , \qquad (2.10)$$

we use to define the effective Hamiltonian. One more extra current may be obtained due to the invariance of (1.1) under the independent U(1) global transformations of leptons:

$$\psi_L\to e^{i\alpha}\psi_L,\quad e_R\to e^{i\alpha}e_R\ , \qquad (2.11)$$

which gives the conserved lepton current

$$j^l_\mu=i\bar{\psi}_L\gamma_\mu\psi_L+i\bar{e}_R\gamma_\mu e_R\ . \qquad (2.12)$$

The latter is also introduced into our scheme through the quantity $N_2=j^l_4$ and no other chemical potentials are possible to define for the model studied.

After the functional integration over the all canonical momenta is performed, the effective Lagrangian with the shifted fields arises and completely coincides with the ansatz used above. All chemical potentials (except μ_2) are nonlinearly embedded into the Lagrangian found and their appearance means that some kind of external sources are applied to the theory studied. No problems exist if only such sources are introduced to the Abelian degrees of freedom but this is not the case for the non-Abelian ones. Although these sources do not manifest themselves directly, the appropriate gauge fields acquire nonzero vacuum expectation values, and if these gauge fields are non-Abelian, no possibility exists to give the gauge-invariant thermodynamical potential without redefinition of the model studied. This nontrivial fact we shall demonstrate once more when the one-loop effective potential is calculated.

III. THE ONE-LOOP EFFECTIVE POTENTIAL

The effective potential $V[\zeta,\mu]$ will be calculated within the relativistic R_γ gauge which contains two arbitrary parameters. After all fields are shifted the effective potential $V[\zeta,\mu]$ has the form

$$V[\zeta,\mu]=V^{\mathrm{tr}}[\zeta,\mu]+V^{(1)}[\zeta,\mu]+\cdots\ , \tag{3.1}$$

where

$$V^{\mathrm{tr}}[\zeta,\mu]=\lambda^2\zeta^4/8-a^2\zeta^2/4-\mu_3^2\zeta^2/2\cos^2 2\theta \tag{3.2}$$

and $V^{(1)}[\zeta,\mu]$ is the one-loop potential which is calculated in what follows. In the approximation adopted the one-loop potential has two independent parts:

$$V^{(1)}[\zeta,\mu]=V_N^{(1)}[\zeta,\mu]+V_C^{(1)}[\zeta,\mu]\ , \tag{3.3}$$

where we consider separately the contributions of the neutral and charged particles, respectively.

The neutral block of our model contains three interacting fields (Z_μ, σ, and h_3) and in addition the ghost. The latter is embedded in the model after the gauge condition is applied,

$$\partial_\mu Z_\mu+\gamma h_3=0\ , \tag{3.4}$$

and within the standard prescription of quantization[10] its one-loop Green's-function has a very simple form:

$$G_{c\bar{c}}=\frac{1}{p^2+\gamma M_Z}\ , \tag{3.5}$$

where $M_Z=\zeta\sqrt{g^2+\bar{g}^2}/2$. The Green's functions for the other fields are combined within the single matrix

$$D^{-1}(p)=\begin{vmatrix} (p^2+M_Z^2)\delta_{\mu\nu} & \omega^2 u_\nu & i(M_Z-\gamma)p_\nu \\ \omega^2 u_\mu & p^2+m_\sigma^2 & i\omega^2\dfrac{u\cdot p}{M_Z} \\ -i(M_Z-\gamma)p_\mu & -i\omega^2\dfrac{u\cdot p}{M_Z} & p^2+m_{h_3}^2 \end{vmatrix} \tag{3.6}$$

whose determinant is easily calculated:

$$\begin{aligned} \det D^{-1}(p)&=(p^2+M_Z^2)^2K(p)\ ,\\ K(p)&=(p^2+m_\sigma^2)R(p)(p^2+M_Z^2)\\ &\quad-\omega^4\left[R(p)+(p^2+\gamma M_Z)^2\frac{(u\cdot p)^2}{M_Z^2}\right]\ , \end{aligned} \tag{3.7}$$

where

$$\begin{gathered} R(p)=(p^2+M_Z^2)(p^2+m_{h_3}^2)-(M_Z-\gamma)^2p^2\ ,\\ \omega^2=i\mu_3\sqrt{g^2+\bar{g}^2}\zeta/\cos 2\theta,\quad m_{h_3}^2=(m_\sigma^2-\lambda^2\zeta^2)+\gamma^2\ ,\\ m_\sigma^2=3\lambda^2\zeta^2/2-a^2/2-\mu_3^2/\cos^2 2\theta\ . \end{gathered} \tag{3.8}$$

Now the one-loop effective potential for the neutral block is presented in the standard form

$$\begin{aligned} \beta V_N^{(1)}[\zeta,\mu]&=2\int\frac{d^3p}{(2\pi)^3}\ln[1-\exp(-\beta\sqrt{p^2+M_Z^2})]\\ &\quad+\sum_{i=1}^{4}\int\frac{d^3p}{(2\pi)^3}\ln\{1-\exp[-\beta\epsilon_i(p)]\}\\ &\quad-2\int\frac{d^3p}{(2\pi)^3}\ln[1-\exp(-\beta\sqrt{p^2+\gamma M_Z})]\ , \end{aligned} \tag{3.9}$$

where all spectra $\epsilon_i(p)$ are defined as

$$\prod_{i=1}^{4}[p_4^2+\epsilon_i^2(p)]=K(p) \tag{3.10}$$

and introduce dependence on γ into $V_N^{(1)}[\zeta,\mu]$. However, on the tree mass shell of ζ, where

$$\zeta^2=a^2/\lambda^2+2\mu_3^2/\lambda^2\cos^2 2\theta\ , \tag{3.11}$$

the expression found for $V_N^{(1)}[\zeta,\mu]$ does not depend on γ and reduces to the thermodynamic potential, which has the form

$$\begin{aligned} \beta\Omega_N^{(1)}[\mu]&=2\int\frac{d^3p}{(2\pi)^3}\ln[1-\exp(-\beta\sqrt{p^2+M_Z^2}]\\ &\quad+\sum_{i=1}^{2}\int\frac{d^3p}{(2\pi)^3}\ln\{1-\exp[-\beta\epsilon_i(p)]\}\ . \end{aligned} \tag{3.12}$$

Here the physical spectra $\epsilon_i(p)$ are found to be

$$\begin{aligned} \epsilon_{1,2}^2(\mathbf{p})&=\mathbf{p}^2+\frac{1}{2}\left[m_\sigma^2+M_Z^2+\frac{4\mu_3^2}{\cos^2 2\theta}\right]\\ &\quad\pm\left[\frac{1}{4}\left[M_Z^2-m_\sigma^2-4\frac{\mu_3^2}{\cos^2 2\theta}\right]^2+4\mathbf{p}^2\frac{\mu_3^2}{\cos^2 2\theta}\right]^{1/2} \end{aligned} \tag{3.13}$$

and together with the other two spectra

$$\epsilon^2(\mathbf{p})=\mathbf{p}^2+\gamma M_Z \tag{3.14}$$

these functions are the solutions of the equation $K(p)=0$ when the tree mass shell for ζ is exploited. The last fact is very important since only for this choice of the mass shell for ξ do the nonphysical spectra cancel each other and the thermodynamic potential is not a function of γ for all temperature ranges. However, such a cancellation is absent if any other mass shell for ξ is accepted (see, e.g., Ref. 6) and for this case the undesirable gauge dependence of all results is generated and the additional dependence of T is introduced in all high temperature expansions.

The charge block of our model contains fermions and the $W^{\pm}$ gauge bosons connected with the $h^{\pm}$ fields. The ghost fields are also necessary here when the gauge conditions are being applied,

$$\tilde{\partial}^{\pm}_{\mu} W^{\pm}_{\mu} - \alpha h^{\pm} = 0 \ , \tag{3.15}$$

but their Green's function again separates and has the standard simple form

$$G_{c\bar{c}} = \frac{1}{\tilde{p}^{\,2}_{W} + \alpha M_W} \ , \tag{3.16}$$

where $\tilde{p}^{\,W}_{\nu} = p_\nu - iu_\nu \mu_W$ and $M_W = g\zeta/2$. The Green's function of the other fields has the matrix form

$$D_c^{-1}(p) = \begin{bmatrix} (\tilde{p}^{\,2}_{W} + M_W^2)\delta_{\mu\nu} & i(M_W \tilde{p}_\nu - \alpha \tilde{p}^{\,W}_{\nu}) \\ -i(M_W \tilde{p}_\mu - \alpha \tilde{p}^{\,W}_{\mu}) & (\tilde{p}^{\,2}_{h} + m_h^2) \end{bmatrix} \tag{3.17}$$

with the abbreviations

$$\begin{aligned} &\tilde{p}^{\,h}_{\nu} = p_\nu - iu_\nu(\mu_1 - \mu_3), \quad \tilde{p}^{\,W}_{\nu} = p_\nu - iu_\nu(\mu_1 - \mu_3 - x) \ , \\ &x = \mu_3/\cos 2\theta, \quad \tilde{p}_\nu = p_\nu - iu_\nu(\mu_1 - \mu_3 + x) \ , \\ &m_h^2 = [m_\sigma^2 - (\lambda\zeta)^2] + (\alpha^2 + x^2) \ . \end{aligned} \tag{3.18}$$

The determinant of the matrix $[D_c^{-1}(p)]$ is easily calculated,

$$\det[D_c^{-1}(p)] = (\tilde{p}^{\,2}_{W} + M_W^2)^3 [(\tilde{p}^{\,2}_{W} + M_W^2)(\tilde{p}^{\,2}_{h} + m_h^2) - (M_W \tilde{p}_\nu - \alpha \tilde{p}^{\,W}_{\nu})^2] \ , \tag{3.19}$$

and for the gauge used it contains contributions from both the physical and nonphysical degrees of freedom. However, on the chosen mass shell ζ, the ratio of two determinants which defines the thermodynamic potential for the model studied,

$$\frac{\det[D_c^{-1}(p)]}{\det[G_{c\bar{c}}^{-1}(p)]^2} = (\tilde{p}^{\,2}_{W} + M_W^2)^3 [(\tilde{p}^{\,2}_{W} + M_W^2)(\tilde{p}^{\,2}_{h} + m_h^2) - (M_W \tilde{p}_\nu - \alpha \tilde{p}^{\,W}_{\nu})^2] / (\tilde{p}^{\,2}_{W} + \alpha M_W)^2 \ , \tag{3.20}$$

must be gauge invariant for all momenta. Unfortunately, this is not the case here and the ratio of determinants is dependent on α when $\mu_3 \neq 0$. The contradiction arisen within the calculational scheme points out the quantum anomaly in the model studied and this fact is an actual difficulty for the formalism presented since now one loses the chance to find the gauge-independent thermodynamic potential. Nevertheless, for our case, since all physical modes are separated, the solution of the problem displayed is completely obvious and for restoring the gauge invariance some terms must be subtracted from the Lagrangian studied. On the one-loop level these terms are proportional to the scalar unphysical fields

$$L_A = 2\frac{\mu_3}{\cos 2\theta}[h^+(i\tilde{p}^{\,W}\cdot u)h^- + M_W u_\mu (h^+ W^-_\mu - W^+_\mu h^-)] \tag{3.21}$$

and arise from the expression $Z_\mu J^N_\mu$ after all fields have been shifted. The new Lagrangian has an appropriate matrix Green's function with the determinant as

$$\det[\tilde{D}_c^{-1}(p)] = (\tilde{p}^{\,2}_{W} + M_W^2)^3 \left\{ (\tilde{p}^{\,2}_{W} + M_W^2) \left[\left[\tilde{p}^{\,2}_{W} - \frac{\mu_3^2}{\cos^2 2\theta} \right] + m_h^2 \right] - (M_W - \alpha)^2 \tilde{p}^{\,2}_{W} \right\} \ , \tag{3.22}$$

where all physical spectra are kept without modification. The nonphysical degrees of freedom have the form

$$[\epsilon^g(p) - \mu_W]^2_{1,2} = \mathbf{p}^2 + (\alpha M_W + k/2) \pm \sqrt{k M_W(\alpha - M_W) + (k/2)^2} \ , \tag{3.23}$$

where $k = m_\sigma^2 - (\lambda\zeta)^2$ (and it is equal to zero on the mass shell of ζ) and depends on α essentially, but when $k=0$, these spectra are the same as the ghost ones and cancel each other. Thus the gauge independence of all physical results is restored here and we insist that the redefinition of any non-Abelian gauge model (where the neutral charge conservation must take place) is an obligatory step. Unfortunately the ansatz proposed is heuristically found and our redefinition is the choice of a new version of the model studied in which all basic properties of the initial model are kept without a problem with the gauge independence for all physical quantities in any gauge chosen (see Appendixes A and B). Both versions of the model (initial and new) exactly coincide in the unitary gauge which is a more physical gauge and all calculations within it are reliable too (see, for example, Ref. 11, and Appendix C).

The one-loop effective potential for the model studied within the charge block divides into two parts,

$$V_C^{(1)}[\mu,\zeta] = \tilde{V}_C^{(1)}[\zeta,\mu] + V_C^{(1)}[\zeta,\mu|\alpha] \ , \tag{3.24}$$

where the first term does not depend on the gauge used and the parameter α defines only the last one:

$$\beta V_C^{(1)}[\zeta,\mu|\alpha]=\sum_{i=1}^{2}\int\frac{d^3p}{(2\pi)^3}\ln(\{1-\exp[-\beta(\epsilon_i^g-\mu^W)]\}\{1-\exp[-\beta(\epsilon_i^g+\mu^W)]\})$$
$$-2\int\frac{d^3p}{(2\pi)^3}\ln(\{1-\exp[-\beta(\sqrt{\mathbf{p}^2+\alpha M_W}-\mu^W)]\}\{1-\exp[-\beta(\sqrt{\mathbf{p}^2+\alpha M_W}+\mu^W)]\}) \qquad (3.25)$$

which is written here for the model after the terms (3.21) have been taken into account. On the tree mass shell for ξ this part of the effective potential is equal to zero but this is not the case for Refs. 5 and 6. The last fact is the reason why in these papers the thermodynamic potential is gauge dependent. The potential $\tilde{V}_C^{(1)}[\zeta,\mu]$ for the charged particles and neutrino ($i=e_L,e_R,\nu,W^{\pm}$) has the standard form

$$\tilde{V}_c^{(1)}[\zeta,\mu]=n_i\int\frac{d^3p}{(2\pi)^3}\ln(\{1+\sigma_i\exp[-\beta(\epsilon_i-\mu_i)]\}\{1+\sigma_i\exp[-\beta(\epsilon_i+\mu_i)]\}) , \qquad (3.26)$$

where $\eta_i=(-1,-1,-1,3)$ and $\sigma_i(1,1,1,-1)$ are the numerical coefficients, $\epsilon_i(=\sqrt{p^2+m_i^2})$ are the excitation spectra (with $m_e=\lambda_e\zeta/\sqrt{2}$ and $M_W=g\zeta/2$). The chemical potential of each particle is expressed through the initial quantities μ_i with the aid of the algebraic equations

$$\begin{aligned}
&\mu_{e_L}=\mu_1+\mu_2-\mu_3=-\mu_{\bar{e}_L} ,\\
&\mu_{e_R}=\mu_1+\mu_2+(2\sin^2\theta/\cos2\theta)\mu_3=-\mu_{\bar{e}_R} ,\\
&\mu_\nu=\mu_2+\mu_3/\cos2\theta=-\mu_{\bar{\nu}} , \qquad (3.27)\\
&\mu_W=\mu_1-(2\cos^2\theta/\cos2\theta)\mu_3=-\mu_{W^+} ,\\
&\mu_h=\mu_1-\mu_3=-\mu_{h^+} .
\end{aligned}$$

The expressions obtained for $V_N^{(1)}[\zeta,\mu]$ and $V_C^{(1)}[\zeta,\mu]$ are the final result of our calculation aimed to find the one-loop effective potential of the WS model.

IV. THE HIGH-TEMPERATURE LIMIT

Now we are able to define the high-temperature asymptotic behavior of the one-loop effective potential and investigate its properties. In the limit $T\gg m_i,\mu_i$ [where the expressions (3.2), (3.9), and (3.26) must be calculated] only the large momenta in the integrals are essential and all spectra are simplified for this momentum region. However, since the gauge parameters are arbitrary we must fix the ratio γ/T (and α/T also) before all calculations have been made. Two different values for this ratio (zero or infinity) correspond to the different gauge classes and the expression found for the effective potential is not the same. Nevertheless the thermodynamic potential is the same for both cases (see Appendix C), of course, if the gauge invariance takes place for the model studied [in our case the terms (3.21) must be added to the Lagrangian]. Below we consider only the gauge-invariant version of our model and choose the covariant gauge for what follows (the case of the unitary gauge is elaborated in Ref. 4 and is briefly discussed in Appendix C).

In the case under consideration (when $T\gg\gamma$ also) all formula are simple enough to be calculated analytically. The needed limit for V is found with the aid of the standard formula[12] where only the leading terms are taken into account and the electron mass is omitted for simplicity. The result obtained has the simple form

$$V=\frac{\lambda^2\zeta^4}{8}-\frac{1}{2}\left[\frac{a^2}{2}+x^2\right]\zeta^2+\left[\frac{T^2}{24}\left[\frac{\alpha\zeta^2}{2}-6x^2\right]-\frac{T^2}{12}[(\mu_{e_L}^2+\mu_{e_R}^2)+\mu_\nu^2+6\mu_W^2]\right] , \qquad (4.1)$$

where $\alpha=6\lambda^2+6e^2(1+2\cos^2\theta)/\sin^2 2\theta$ and $x=\mu_3/\cos2\theta$ are the new abbreviations. Equation (4.1) is an essential point here and in what follows the W-condensate phase diagram will be found on its basis. But the expression (4.1) is not the same as the one in Refs. 5 and 6 since in these papers the gauge-noninvariant version of the model is considered.

The extremum equations for the effect action (4.1) are easily calculated and we arrange them in a form which is convenient for the further transformations:

$$\begin{aligned}
&\frac{\partial V}{\partial\mu_1}=-\frac{T^2}{6}(\mu_{e_L}+\mu_{e_R}-6\mu_{W^+})=0 ,\\
&\frac{\partial V}{\partial\mu_2}=-\frac{T^2}{6}(\mu_{e_L}+\mu_{e_R}+\mu_\nu)=-\rho ,\\
&\frac{\partial V}{\partial\mu_3}=-x(\zeta^2+\underline{T^2/2})/\cos2\theta-\frac{T^2}{6}\left[-\mu_{e_L}+\mu_{e_R}\frac{2\sin^2\theta}{\cos2\theta}+\frac{\mu_\nu}{\cos2\theta}+6\mu_{W^+}\frac{2\cos^2\theta}{\cos2\theta}\right]=0 ,\\
&\frac{\partial V}{\partial\zeta}=\lambda^2\zeta^3/2-a^2\zeta/2-x^2\zeta+\underline{\alpha T^2\zeta/24}=0 .
\end{aligned} \qquad (4.2)$$

The equations found (in spite of the fact that only the high-temperature limit is studied) are dependent on the gauge chosen. When the other classes of gauges are considered (for example, the unitary one) some coefficients (before the terms which are underlined) in the last two equations of (4.2) are changed but if the quantity ζ is eliminated from these two equations, the equation obtained [along with the other two equations in (4.2)] is completely gauge invariant. This important fact one must bear in mind in what follows.

After some simple algebra is performed and Eqs. (3.27) are taken into account, the system (4.2) is simplified to

$$
\begin{aligned}
&\tfrac{4}{3}\mu_1+\tfrac{1}{3}\mu_2-\mu_3\left[\frac{5\cos^2\theta}{3\cos 2\theta}+\frac{1}{2}\right]=0\ ,\\
&(\mu_1+2x\sin^2\theta)/3+\mu_2/2=\rho/T^2\ ,\\
&\xi^2 x+\frac{T^2}{6}\left[3\mu_1+4\mu_2+3\mu_3\left[\frac{2}{\cos 2\theta}-1\right]\right]=0\ ,\\
&\lambda^2\zeta^2-(a^2+2x^2-\alpha T^2/12)=0\ ,
\end{aligned}
\tag{4.3}
$$

and in such form one can solve it step by step. The first two equations of (4.3) allow one to express μ_1 and μ_2 through μ_3,

$$
\begin{aligned}
&\mu_1=-\frac{3}{5}\frac{\rho}{T^2}+\frac{3}{10}\mu_3\left[\frac{4\sin^2\theta+15\cos^2\theta}{3\cos 2\theta}+\frac{3}{2}\right]\ ,\\
&\mu_2=12\rho/5T^2-13x/10\ ,
\end{aligned}
\tag{4.4}
$$

and we propose to substitute the solution (4.4) and the one for ξ found from the fourth equation into the third equation of the set (4.3). The result is a closed equation for μ_3 only:

$$1.3\rho+\tfrac{73}{120}xT^2+\frac{x}{\lambda^2}(2x^2+a^2-\alpha T^2/12)=0 \tag{4.5}$$

which is solved approximately for T near T_c. The appropriate solution has the form

$$x=-13\rho/(\gamma T^2) \tag{4.6}$$

and the two other ones are not considered since they have a temperature behavior different from (4.6). Here the simple abbreviation $\gamma=\frac{73}{12}$ is used. Within Eq. (4.5) for the phase $\zeta^2\neq 0$ no other appropriate solutions exist and this fact is very important for what follows. The fourth equation of the set (4.3) is investigated after the solution found for μ_3 is substituted into it. Near the T_0 point [here $T_0=a(12/\alpha)^{1/2}$] this equation has the form

$$\lambda^2\zeta^2=2(13)^2\rho^2/(\gamma T^2)^2-\alpha(T^2-T_0^2)/12\ , \tag{4.7}$$

and one sees that the point $\zeta^2=0$ shifts to the right from T_0 and this shift depends on ρ (Ref. 3):

$$\rho^2=(\gamma/13)^2\alpha T_c^4(T_c^2-T_0^2)/24\ . \tag{4.8}$$

The curve defined (see Fig. 1) separates the phase with $\zeta\neq 0$ from the unbroken phase and all our calculations are valid to the left of the curve b. For the region $\zeta=0$ also we later give some estimations but they are beyond the formalism presented.

The condition for W condensation has the usual form

$$M_{W^\pm}=\mu_{W^\pm} \tag{4.9}$$

and after Eq. (3.27) for μ_W is used it follows that

$$g\zeta/2=\mp\left[\mu_1-\frac{2\cos^2\theta}{\cos 2\theta}\mu_3\right]\ . \tag{4.10}$$

Equation (4.9) gives the left boundary of the W-condensate phase which determines the evaporation line of a charge condensate when T goes to zero. To eliminate μ_1, Eq. (4.4) is exploited and then the simple condition takes place:

$$g\zeta/2=\pm(3\rho/5T^2+x/20)\ , \tag{4.11}$$

which is studied near the point T_c where the asymptotic behavior (4.6) for x can be used. After some simple algebra is performed the final equation for the transition line has the form

$$3\rho=\pm\gamma g T^2\zeta\ , \tag{4.12}$$

where $\zeta(T)$ is defined by Eq. (4.7). Now one can see that the curve $\rho(T)$, when T is near T_c, strictly follows $\zeta(T)$ and [according to Eqs. (4.7) and (4.12)] must be equal to zero at the point $T=T_0$. The asymptotic behavior near T_0 for this curve is found with the aid of Eq. (4.7) after some algebraic manipulations. The result obtained (see the curves d in Fig. 2) is presented as

$$\rho^2=(\alpha/24)\gamma^2T^4(T^2-T_0^2)/[(13)^2-9\lambda^2/2g^2] \tag{4.13}$$

and demonstrates the complicated behavior of $\rho(T)$ in the critical temperature region. This behavior is sensitive to the ratio $2\lambda/g$ (that is the same as the parameter $K=m_\sigma/M_W$ which was used for the WS model in other papers[3,9] also) and it is qualitatively different (see Fig. 2) according to its value. In particular, if $3m_\sigma/M_W\gg 26\sqrt{2}$ we go back to the case previously considered by us [see Fig. 2(a) and Ref. 4] and no essential changes of the W-condensate phases appear. The other limit $3m_\sigma/M_W\ll 26\sqrt{2}$ is more important because it

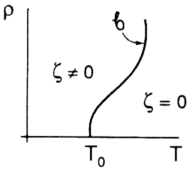

FIG. 1. The dependence of the critical Higgs temperature on the external density. The curve b is the right boundary of the Higgs condensate. The points $T_0=a(12/\alpha)^{1/2}$.

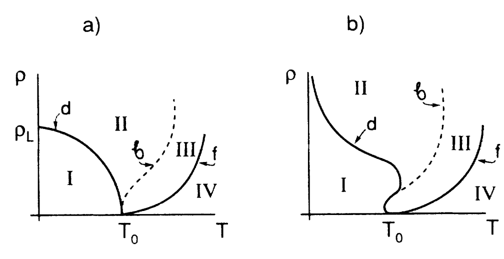

FIG. 2. The W^+ condensate phase diagrams for the universe with zero neutral charge density. The phases I and II are with $\zeta\neq 0$, whereas the phases III and IV are unbroken ones. The charge W^+ condensate occupies the II and III regions. $K\gg 1$ for the case a, and $K\ll 1$ for the case b. Here the parameter $K=m_\sigma/M_W$ and $\rho L=M_W^3/6\pi^2$.

leads to a new version of W condensation [see Fig. 2(b)] which has a novel behavior near the $T=0$ point.

V. THE CASE $T=0$

Our formalism in the zero-temperature limit completely follows the expressions obtained above. Under the ansatz accepted for renormalizability, the one-loop effective potential is defined by Eq. (3.26) only and has a very simple form:

$$V=\lambda^2\zeta^4/8-\tfrac{1}{2}(a^2/2+x^2)\zeta^2-(\mu_{e_L}^4+\mu_{e_R}^4+\mu_\nu^4)/24\pi^2 . \tag{5.1}$$

All its extremum equations are easily found:

$$\begin{aligned}&(\mu_{e_L}^3+\mu_{e_R}^3)/6\pi^2=0 ,\\ &(\mu_{e_L}^3+\mu_{e_R}^3+\mu_\nu^3)/6\pi^2=0 ,\\ &x\zeta^2/\cos2\theta+\left[-\mu_{e_L}^3+\mu_{e_R}^3\frac{2\sin^2\theta}{\cos2\theta}+\frac{\mu_\nu^3}{\cos2\theta}\right]\Big/6\pi^2=0 ,\\ &\tfrac{1}{2}(\lambda^2\zeta^2-a^2)\zeta-\zeta x^2=0\end{aligned} \tag{5.2}$$

and below we discuss its solution.

From the first equation, the simplest important condition immediately follows,

$$\mu_{e_L}=-\mu_{e_R} , \tag{5.3}$$

which indicates that our ground state has equal Fermi spheres; for example, for the left-handed electrons and for the second equation of the set (5.2), after the condition (5.3) is taken into account, it is also solved at once,

$$\rho=\mu_\nu^3/6\pi^2 , \tag{5.4}$$

and defines the Fermi sphere for neutrinos. After some algebra has been performed the usual condition of W condensation is written in a simple form,

$$g\zeta_0/2=\pm[(6\pi^2\rho)^{1/3}+x/2] , \tag{5.5}$$

where the quantity $x=\mu_3/\cos2\theta$ must obey the cubic equation

$$x^3/48\pi^2+x(a^2+2x^2)/\lambda^2=-\rho . \tag{5.6}$$

If the chemical potential μ_3 is not embedded into the system (see, for example, Ref. 2) the quantity $x=0$ and Eq. (5.5) along with the last equation of the set (5.2) is easily solved. The solution found fixes the finite critical density

$$\rho_L=a^3(g/2\lambda)^3/6\pi^2 \tag{5.7}$$

the same as obtained (at first) in Ref. 1. The existence of $\rho_L\neq0$ means that for $\rho\geq\rho_L$ the W condensate does not evaporate up to the point $T=0$ [see, for example, Fig. 2(a)]. But if $x\neq0$, the solution of Eq. (5.5) is more complicated and we are able to demonstrate that in some cases the critical density disappears.

The mentioned case concerns a universe with a zero neutral charge density and is realized if the parameter $K=m_\sigma/M_W$ is very small [see Fig. 2(b)]. For proving this fact we use Eq. (5.6) and the last one of the set (5.2) and eliminate with their aid the quantities ζ_0^2 and ρ from Eq. (5.5) squared. The resulting equation gets the form

$$(g/2\lambda)^2(a^2+2x^2)=(\{6\pi^2[x^3/48\pi^2+x(a^2+2x^2)/\lambda^2]\}^{1/3}-x/2)^2 \tag{5.8}$$

and we may see that each side of (5.8) depends on different parameters. The parameter $K^2=(2\lambda/g)^2$ defines only the left-hand side of Eq. (5.8) and if $K\ll1$, all solutions of this equation are lost, because as a function of x each side of Eq. (5.8) has an opposite analytic behavior. The result of losing a solution of Eq. (5.8) is a crucial circumstance for W condensation since now the critical density point [see point ρ_L in Fig. 2(a)] disappears and the W condensate phase evaporates near the point $T=0$ for any ρ [see Fig. 2(b)]. We see, in this case, the complete analogy between the entering of a quantity μ_3 (which controls the neutral charge conservation) and the embedding of any external magnetic field into the model. For the last case—for example, see Ref. 9—the same parameter K appears and the W condensate structure for the WS model in a magnetic field is sensitive to its value as well. The "nonstandard" W-condensate phase diagram obtained here is an essential point for cosmology since a universe with a zero neutral charge density with $(m_\sigma/M_W)^2\ll1$ seems to be a more preferable scenario.

The right boundary for W condensation (the curves f in Fig. 2) is only estimated here. When the phase $\zeta=0$ is considered we exploit the radiation mass of the W bosons $M_W\sim gT$ (as was found, for example, in Ref. 13) and the high-temperature behavior for $\mu\sim\rho/T^2$. After the condition of W condensation ($gT\sim\rho/T^2$) is solved one finds that $\rho\sim T^3$ on the right boundary and the W-condensate domain occupies the finite region of T for each ρ.

VI. CONCLUSIONS

Here the main properties of the W-condensate phase are briefly outlined and its peculiarities are discussed in its connections with other phenomena. The WS model used within this paper is an inessential fact since there exist many reasons to consider all results found as model independent. We embedded the maximum possible number of chemical potentials into our model (here, three potentials) to support the electrical and neutral charge conservation under the condition of nonzero external fermion density. In principle, the set of chemical potentials adopted must be practically the same for any gauge theory, since it follows the standard set of the observable forces in low-energy physics. If an incomplete set of potentials is incorporated into the theory some quantity always exists which is not conserved.

We found also that many peculiarities of the W-condensation phenomena are strongly connected with the Higgs mechanism properties and they are able to give unambiguous information about this. In particular, for a universe with a zero neutral charge density the W condensation at first occurs near the symmetry-restoration point for any small value of the external fermion density and then evaporates completely (or partially) when the temperature decreases. Here we do not agree with the results of Ref. 5 where the boundary found for the W-condensate phase has no peculiarities near the Higgs critical point and therefore such types of condensation are not possible for small $\rho < \rho_L$. The W-condensate phase diagram obtained in our paper possesses the opposite tendency because it is based on the fact that the effective mass of gauge particles has a noncanonical behavior near the Higgs critical point. Moreover, the analytical behavior found for the boundary of the condensate near the critical point T_0 is very sensitive to the parameter $K = m_\sigma / M_W$ which defines the Higgs mechanism properties within the model adopted. The same parameter also influences the behavior of the boundary curves near the point $T=0$, where two qualitatively different possibilities for the W condensation (according to the value of K) are obtained. Here it is very important to notice that the parameter K appears in a qualitatively different situation: for example, if the WS model is affected by an external magnetic field[9] where it also defines the different structure of the W-condensation phase. We see in this analogy something deeper than a simple coincidence.

Unfortunately, the quantization problem for the GUT model with the Higgs mechanism after all chemical potentials are introduced is solved in the present paper only heuristically. We proposed a new gauge-invariant version of the model studied and proved that the initial model [the model without term (3.21)] cannot reproduce a gauge-independent thermodynamical potential when neutral charge conservation has taken place. The complexity of this task seems to be an essential problem for realistic models of GUT.

To summarize all the results obtained, we stress once more that the phase transitions connected with W condensation seem to take place for any realistic model of GUT with the Higgs mechanism. These phenomena are also essential for astrophysics and cosmology where our predictions are able to give a good chance to distinguish some global properties of the Universe. For example, we find that a universe with the zero neutral charge density has exactly the same phase diagram for W^+ and W^- condensation, but this symmetry does not exist if the universe has a nonzero neutral one. The other qualitative peculiarities of the phase diagram are obtained near the point $T=0$. These peculiarities promise to be interesting as well because they indicate what the value for the parameter $K = m_\sigma / M_W$ must be. In particular, if we know that the W condensate does not evaporate down to the point $T=0$ this means that the universe has either a finite neutral charge density or the parameter $K \gg 1$. The strong correlation of the W-condensation phenomenon with the Higgs mechanism is also important because many peculiarities of the phase diagram for the W-condensation are completely due to this fact. Near the Higgs critical point the condensation always takes place for any small ρ and this feature qualitatively separates GUT's with the Higgs mechanism from the other possible models without it.

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APPENDIX A

For more complete illustration of the problem appearing in the bosonic charged sector we present here the results of the calculation in the Coulomb gauge for which the gauge-fixing term has the form

$$L_{\mathrm{gf}} = -\frac{1}{\rho} |\partial_i W_i^{(\pm)} - \rho M_W h^{(\pm)}|^2 \tag{A1}$$

and the corresponding ghost propagators are found as

$$G^{-1}_{C^{(+)}\bar{C}^{(+)}} = \rho M_W^2 - \nabla^2 , \quad G^{-1}_{C^{(-)}\bar{C}^{(-)}} = \rho M_W^2 - \nabla^2 . \tag{A2}$$

It is convenient to represent the inverse propagator for the bosonic charged sector in matrix form and in momentum space:

$$D^{-1}(p) = \begin{bmatrix} (\tilde{p}_W^{\,2} + M_W^2)\delta_{ji} + (1/\rho - 1) p_j p_i & -p_i \tilde{p}_4^{\,W} & 0 \\ -\tilde{p}_4^{\,W} p_j & (\tilde{p}_W^{\,2} + M_W^2) - \tilde{p}_{4W}^{\,2} & i M_W \tilde{p}_4 \\ 0 & -i M_W \tilde{p}_4 & (\tilde{p}_h^{\,2} + m_h^2) \end{bmatrix} , \tag{A3}$$

where $m_h^2 = \frac{1}{2}(\lambda^2 \zeta^2 - a^2)^6 + \rho M_W^2$ and the momenta $\tilde{p}_\alpha$, $\tilde{p}_\alpha^{\,W}$, $\tilde{p}_\alpha^{\,h}$ have been defined in (3.18). It is easy to calculate the determinant of this matrix and with the ghost contribution we have

$$\begin{aligned} \frac{\det D^{-1}}{(\det G_{c\bar{c}}^{-1})^2} = \frac{1}{\rho} \frac{(\tilde{p}_W^{\,2} + M_W^2)^2}{(\mathbf{p}^2 + \rho M_W^2)^2} \{ & (\tilde{p}_W^{\,2} + M_W^2)(\mathbf{p}^2 + \rho M_W^2)^2 + (\tilde{p}_{4h}^{\,2} - \tilde{p}_{4W}^{\,2}) \mathbf{p}^2 (\mathbf{p}^2 + \rho M_W^2) \\ & + (\tilde{p}_{4h}^{\,2} - \tilde{p}_4^{\,2}) M_W^2 [(\mathbf{p}^2 + \rho M_W^2) + \rho \tilde{p}_{4W}^{\,2}] \\ & + \tfrac{1}{2}(\lambda^2 \zeta^2 - a^2)[(\mathbf{p}^2 + \rho M_W^2)(\tilde{p}_W^{\,2} + M_W^2) - \mathbf{p}^2 \tilde{p}_{4W}^{\,2}] \} . \end{aligned} \tag{A4}$$

One can verify the essential gauge dependence of the ratio (A4) found for two determinants even on the chosen mass shell of ζ. To solve this problem we shall consider (as we have done above) the new Lagrangian $L'=L-L_A$, where L_A is given by (3.21). In this case the inverse propagator has the form

$$D^{-1}(p)=\begin{pmatrix} (\tilde{p}_W^2+M_W^2)\delta_{ji}+(1/\rho-1)p_jp_i & -p_i\tilde{p}_4^W & 0 \\ -\tilde{p}_4^W p_j & (\tilde{p}_W^2+M_W^2)-\tilde{p}_{4W}^2 & iM_W\tilde{p}_4^W \\ 0 & -iM_W\tilde{p}_4^W & (\tilde{p}_W^2+\rho M_W^2+m_\sigma^2-\lambda^2\zeta^2) \end{pmatrix} \tag{A5}$$

and its determinant with the ghost contribution is modified as

$$\frac{\det D^{-1}}{(\det G_{c\bar{c}}^{-1})^2}=\frac{1}{\rho}\frac{(\tilde{p}_W^2+M_W^2)^2}{(\mathbf{p}^2+\rho M_w^2)^2}\{(\tilde{p}_W^2+M_W^2)(\mathbf{p}^2+\rho M_W^2)^2+(m_\sigma^2-\lambda^2\zeta^2)[(\mathbf{p}^2+\rho M_W^2)(\tilde{p}_W^2+M_W^2)-\mathbf{p}^2\tilde{p}_{4W}^2]\}\ . \tag{A6}$$

Here $m_\sigma^2=3\lambda^2\zeta^2/2-a^2/2-x^2$. Now it is easy to show that on the mass shell of ζ (where $\zeta^2=a^2/\lambda^2+2x^2/\lambda^2$ and $m_\sigma^2=\lambda^2\zeta^2$) the ratio of the two determinants is simplified to

$$\left.\frac{\det D^{-1}(p)}{(\det G_{C\bar{C}}^{-1})^2}\right|_{(m_\sigma^2=\lambda^2\zeta^2)}=(\tilde{p}_W^2+M_W^2)^3/\rho \tag{A7}$$

and its gauge dependence is trivial and it does not influence the physical results. Formula (A7) involves the standard spectra where only the physical degrees of freedom are taken into account.

APPENDIX B

Taking into account the wide popularity of the 't Hooft gauge as an example of the relativistic gauge we insert here the results of the calculations for the bosonic charged sector using this gauge which is fixed by the term

$$L_{\text{gf}}=-\frac{1}{\rho}|\tilde{\partial}_\mu^W W_\mu^{(-)}-\rho M_W h^{(-)}|^2\ . \tag{B1}$$

The ghost and boson propagators are bound as follows:

$$G_{C^{(+)}\bar{C}^{(+)}}^{-1}=(\rho M_W^2-\tilde{\partial}_{W^{(+)}}^2),\ G_{C^{(-)}\bar{C}^{(-)}}^{-1}=(\rho M_W^2-\tilde{\partial}_{W^{(-)}}^2)\ , \tag{B2}$$

$$D^{-1}(p)=\begin{pmatrix} (\tilde{p}_W^2+M_W^2)\delta_{\nu\mu}+(1/\rho-1)\tilde{p}_\nu^W\tilde{p}_\mu^W & 2M_Wxu_\mu \\ -2M_Wxu_\nu & (\tilde{p}_h^2+m_h^2) \end{pmatrix} \tag{B3}$$

and the ratio of the appropriate determinants has the form

$$\frac{\det D^{-1}}{(\det G_{c\bar{c}}^{-1})^2}=\frac{1}{\rho}\frac{(\tilde{p}_W^2+M_W^2)^2}{(\tilde{p}_W^2+\rho M_W^2)^2}\{(\tilde{p}_W^2+\rho M_W^2)[(\tilde{p}_h^2+m_h^2)(\tilde{p}_W^2+M_W^2)+4M_W^2x^2]+(\rho-1)(2xM_W\tilde{p}_{4W})^2\} \tag{B4}$$

and this expression is gauge dependent even on the mass shell of ζ. Here we use the definitions $m_h^2=\frac{1}{2}(\lambda^2\zeta^2-a^2)+\rho M_W^2$ and $x=\mu_3/\cos 2\theta$.

After the Lagrangian is changed to $L'=L-L_A$ [using (3.21) as it has been made above] we get the new matrix instead of (B3) which takes the diagonal form

$$D^{-1}(p)=\begin{pmatrix} (\tilde{p}_W^2+M_W^2)\delta_{\nu\mu}+(1/\rho-1)\tilde{p}_\nu^W\tilde{p}_\mu^W & 0 \\ 0 & (\tilde{p}_W^2+\rho M_W^2+m_\sigma^2-\lambda^2\zeta^2) \end{pmatrix} \tag{B5}$$

and its determinant with the ghost one is easily calculated:

$$\frac{\det D^{-1}}{(\det G_{c\bar{c}}^{-1})^2}=\frac{1}{\rho}\frac{(\tilde{p}_W^2+M_W^2)^3}{(\tilde{p}_W^2+\rho M_W^2)^2}(\tilde{p}_W^2+\rho M_W^2)\times(\tilde{p}_W^2+\rho M_W^2+m_\sigma^2-\lambda^2\zeta^2)\ . \tag{B6}$$

Now we show that with the tree-level solution of the equation of motion where $m_\sigma^2=\lambda^2\zeta^2$ the ratio (B6) has the form

$$\left.\frac{\det D^{-1}(p)}{[\det G_{c\bar{c}}^{-1}(p)]^2}\right|_{(m_\sigma^2=\lambda^2\zeta^2)}=(\tilde{p}_W^2+M_W^2)^3/\rho \tag{B7}$$

and its gauge dependence is trivial.

APPENDIX C

There is an opinion that the unitary gauge reproduces the incorrect results when the perturbative calculations are based on the loop expansion. However, nobody proved this statement explicitly (Ref. 14 we discuss separately) and moreover work has been done (see, e.g., Ref. 11) in which the opposite statement is affirmed by direct calculations. Here we reproduce some points from Ref. 4 which evidently demonstrate that the unitarity gauge gives the correct result for any models where the gauge invariance of the physical quantities is not in doubt.

Let us consider the gauge-invariant version of our model [the model studied with the additional terms (3.21)] for which the effective potential above was found in the R_γ range. Of course, this one-loop potential is not the same as the one in the unitary gauge,

$$V=\frac{\lambda^2\xi^4}{8}-\tfrac{1}{2}(x^2+a^2/2)\xi^2+\left[\frac{T^2}{24}\left[\frac{\alpha_u\xi^2}{2}-3x^2\right]-\frac{T^2}{12}(\mu_{eL}^2+\mu_{eR}^2+\mu_\nu^2+6\mu_W^2)\right], \tag{C1}$$

but one can see that both expressions (4.1) and (C1) give the same thermodynamical potential

$$\Omega=-\frac{1}{8\lambda^2}(2x^2+a^2)^2+\frac{\alpha}{2\lambda^2}(2x^2+a^2)\frac{T^2}{24}-\frac{T^2}{12}(3x^2+6\mu_W^2+\mu_\nu^2+\mu_{eL}^2+\mu_{eR}^2) \tag{C2}$$

when the appropriate mass shell (here the tree one) for $\xi^2=(1/\lambda^2)(2x^2+a^2)$ is used and the connection between α and α_u is taken into account ($\alpha=3\lambda^2+\alpha_u$). So we again stress that on the mass shell of ξ the unitary gauge reproduces the same results as any relativistic one and this correspondence takes place step by step in each order of loop expansion. The extremum equations for the potential (C1) are very close to those for expression (4.1)

$$\begin{aligned}
&\frac{\partial V}{\partial\mu_1}=-\frac{T^2}{6}(\mu_{eL}+\mu_{eR}-6\mu_{W^+})=0\ ,\\
&\frac{\partial V}{\partial\mu_2}=-\frac{T^2}{6}(\mu_{eL}+\mu_{eR}+\mu_\nu)=-\rho\ ,\\
&\frac{\partial V}{\partial\mu_3}=-x(\xi^2+\underline{T^2/4})/\cos2\Theta-\frac{T^2}{6}\left[-\mu_{eL}+\mu_{eR}\frac{2\sin^2\Theta}{\cos2\Theta}+\mu_\nu/\cos2\Theta+\frac{2\cos^2\Theta}{\cos2\Theta}6\mu_{W^+}\right]=0\ ,\\
&\frac{\partial V}{\partial\xi}=\lambda^2\xi^3/2-a^2\xi/2-x^2\xi+\underline{a_uT^2\xi/24}=0\ ,
\end{aligned} \tag{C3}$$

but the last two equations have some coefficients (the underlined ones) which differ from the corresponding coefficients in Eqs. (4.2). Nevertheless the full coincidence of the equations sets (only those which define μ_i) can be found if one excludes the parameter ξ with the aid of the fourth equation in each set. The first two equations in each set remained without change but the third equation now has the more complicated form

$$\left.\frac{\partial V}{\partial\mu_3}\right|_{\frac{\partial V}{\partial\xi}=0}=-\frac{x}{\cos2\Theta}\left[\frac{T^2}{4}+\frac{1}{\lambda^2}(2x^2+a^2-\alpha_uT^2/12)\right]-\frac{T^2}{12}\frac{\partial}{\partial\mu_3}(6\mu_W^2+\mu_\nu^2+\mu_{eR}^2+\mu_{eL}^2) \tag{C4}$$

but it is the same for both gauges. Moreover Eq. (C4) (like the other two equations for μ_i) can be obtained directly from expression (C2) when the extremum equation of the thermodynamic potential is found. The equation for ξ is not gauge invariant indeed (as was mentioned earlier in Ref. 14) and this fact seems to display the essence of all controversies. Nevertheless the tendency found which lead to the changing of some coefficients in Eq. (C3) is more necessary than harmful since it gives the chance to find all equations for μ_i to be gauge independent. Here we must agree that the equation for ξ defines (numerically) different values for the critical point T_c in the unitary and covariant gauge but today there is no evidence that it is better (see, e.g., Ref. 11) to forget the unitary gauge completely or omit this unessential point.

APPENDIX D

Here we compare once again the results found in Refs. 5 and 6 with ours. All these papers give different predictions, and it is very important today to find reasons which lead to these discrepancies. Unfortunately, we do not agree with a number of results of the papers mentioned above, but, of course, this is not the final verdict and further discussion could be very fruitful.

To compare all existing results it is very useful at first to present the effective potential of the model studied (or its thermodynamic one) as the sum of two parts which are responsible for the neutral and charged sectors, respectively. Our main result is that gauge invariance (when neutral charge conservation takes place) is kept on the quantum level only within the neutral sector and it is broken in the charged one. Because of this fact there are many reasons to discuss these sectors of the model studied separately.

In the neutral sector the high-temperature expansion for the effective potential is the same for Ref. 5 and our paper (if $\gamma \ll T$, that is the case of the Feynman or special R_γ gauges). It is very likely that the effective potentials found in both these papers will coincide for the whole range of T if in Ref. 5 formula (45) is corrected. Reference 6 reproduces the same effective potential but when the thermodynamic one is calculated the author is mistaken by using the one-loop mass shell for ξ instead of the tree one. The thermodynamic potential thus found is gauge dependent and the additional dependence on T is introduced in all high-temperature expressions.

The charged sector of the model considered (when neutral charge conservation takes) is really gauge dependent on the quantum level. Without redefining the model [see, e.g., our additional term (3.21)] the correct calculations are not possible and we stress that the high-temperature expansion found for the new gauge-invariant version of this model is not the same as for the old one and these expansions differ on terms of order $T^2\mu_3^2$. In Refs. 5 and 6 the expressions for the effective potential are the same and they correspond to the initial version of the model studied. Although these expressions are not able to give the gauge-independent thermodynamic potential on the quantum level [see the discussion of the formula (3.20) in our paper] the authors investigate their high-temperature expansions and obtain different results. For us the first task was to build the appropriate expression for $V[\xi,\mu]$ and then to exploit it, particularly in the high-temperature region. Of course, we do not insist that our generalization of the model studied [when the terms (3.21) are embedded] is uniquely possible but it is important to understand that the problem of quantization really exists and must be solved in the future.

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