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Weak mixing matrix under permutation symmetry breaking

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The two-Higgs-doublet extension of the standard electroweak model is considered. A permutation symmetry-breaking scheme is proposed and used to calculate the weak mixing matrix up to second order. The *CP*-violation factor J and the correction to Bjorken's approximation are then given. A special case is considered.

I. INTRODUCTION

The generation of the mass spectrum of leptons and quarks and the related weak mixing matrix is a fundamental and unsolved problem in particle physics. One way to attack this problem is to impose a certain discrete symmetry on the electroweak model such that no new gauge bosons appear. It has been considered by various authors[1] to extend the permutation symmetry to the Higgs sector of the standard electroweak model. This assumption is plausible since the standard electroweak model possesses S_n symmetry except the Higgs sector. Furthermore, under exact S_n symmetry the mass spectra[2] for both quarks and leptons consists of only two levels. One is $(n-1)$-fold degenerate, the other nondegenerate. For three generations of quarks and leptons, the above spectra give a rather good approximate description. The recent results[3] from CERN LEP and SLAC seem to rule out the fourth generation of the neutrino. Therefore in this article only three generations with S_3 symmetry are considered. The generalization to n generations is straightforward.

In order to remove the above degeneracy and produce a plausible form for the weak mixing matrix in a rather general way, we break the permutation symmetry in two steps. First, we add one more Higgs doublet to the standard model and let it break the permutation symmetry spontaneously. Then it is followed by adding a rather general small perturbation. In the first step, a satisfactory mass pattern[4] is obtained. In the second step, the perturbation is calculated up to second order and a reasonable form for the weak mixing matrix is derived. A general feature comes out naturally, such as $1 \geq V_{tb} \geq V_{ud} \geq V_{cs}$. The *CP*-violation factor[5] J is then expressed in a compact form which is valid up to third order. The corrections to Bjorken's approximation[6] for the mixing matrix are also given.

In Sec. II, the two-Higgs-doublet extension of the standard electroweak model is presented, and spontaneous symmetry breaking is carried out. In Sec. III explicitly symmetry breaking is introduced, and the perturbative calculation is worked out up to second order. In Sec. IV the *CP*-violation factor is considered. In Sec. V a comparison to Bjorken's approximation is made. A special case is considered in Sec. VI. Finally, we conclude our work in the last section.

II. SPONTANEOUS BREAKING OF S_3 SYMMETRY

The two-Higgs-doublet extension of the standard electroweak model has been used by many authors.[7] In this article, the two doublets are chosen to have the form

$$\phi_i = \begin{pmatrix} \phi_i^{+} \\ \phi_i \end{pmatrix}, \quad i=1,2 \ .$$

The transformations under S_3 symmetry are chosen to be

$$\begin{aligned} &\phi_i \sim \Gamma^i, \quad i=1,2 \ , \\ &q_j \sim \Gamma^6 \ , \end{aligned} \tag{1}$$

where Γ^1 (Γ^2) is the one-dimensional totally symmetric (antisymmetric) irreducible representation and Γ^6 the three-dimensional reducible representation which is just the usual permutation between the quark generations. The $SU(2)\times U(1)\times S_3$-invariant Lagrangian density then takes the form

$$\begin{aligned} L_{q\phi} = &\left[a \sum_i \bar{q}_{iL} q_{idR} + b \sum_{i \neq j} \bar{q}_{iL} q_{jdR} \right] \phi_1 + c \sum_{ijK} \epsilon_{ijk} \bar{q}_{iL} q_{jdR} \phi_2 \\ &+ \left[a' \sum_i \bar{q}_{iL} q_{iuR} + b' \sum_{i \neq j} q_{iL} q_{juR} \right] \tilde{\phi}_1 + c' \sum_{ijK} \epsilon_{ijK} q_{iL} q_{juR} \tilde{\phi}_2 + \text{H.c.} \end{aligned} \tag{2}$$

Under spontaneous symmetry breaking, we have[7]

$$\langle \phi_1 \rangle = \begin{pmatrix} 0 \\ \rho_1 \end{pmatrix}, \quad \langle \phi_2 \rangle = \begin{pmatrix} 0 \\ \rho_2 \end{pmatrix} e^{i\theta} \ . \tag{3}$$

The nonvanishing vacuum expectation value of ϕ_2 also represents the breaking of S_3 symmetry and time-reversal invariance. Under the Hermitian basis of quark fields, θ is shown to be $\pm\pi/2$, and the mass matrix for down quarks is given by

$$M_d=\begin{bmatrix} A & B^* & B \\ B & A & B^* \\ B^* & B & A \end{bmatrix} , \qquad A=a\rho_1, \quad B=b\rho_1 \pm ic\rho_2 . \tag{4}$$

The mass matrix for up quarks can be obtained by simply replacing a, b, and c in (4) by a', b', and c'. The mass eigenvalues are found to be

$$\begin{aligned} m_d &= A-\mathrm{Re}B-\sqrt{3}\,\mathrm{Im}B , \\ m_s &= A-\mathrm{Re}B+\sqrt{3}\,\mathrm{Im}B , \\ m_b &= A+2\,\mathrm{Re}B , \end{aligned} \tag{5}$$

with similar expressions for up quarks. The correct mass pattern is clearly shown in (5). We note that the nonvanishing ImB comes from the second Higgs doublet. The corresponding weak mixing matrix is just the unit matrix which may be considered as the zeroth-order approximation in a certain perturbation scheme.

III. EXPLICIT S_3 SYMMETRY BREAKING

In order to derive a reasonable weak mixing matrix, we introduce a perturbative scheme in this section. We write the mass matrices in the form

$$\begin{aligned} M_u &= M_u^0 + P' , \\ M_d &= M_d^0 , \end{aligned} \tag{6}$$

where M^0 is obtained from the spontaneous symmetry breaking of SU(2)×U(1)×S(3) as given by expression (4) and P' is a small perturbation which represents an explicit breaking of S_3 symmetry. We first diagonalize M^0 by transformation U^0 as before

$$\begin{aligned} U^0 M_u U^{0\dagger} &= D_u^0 + P , \\ U^0 M_d U^{0\dagger} &= D_d^0 , \end{aligned} \tag{7}$$

where D^0 is the diagonal matrix. Since the diagonal elements of P can always be absorbed into D^0 by removing from P' the part which is similar to M^0 and is also diagonalized by U^0, therefore the most general form for P is

$$\begin{aligned} & P_{11}=P_{22}=P_{33}=0 , \\ & P_{ij}=P_{ji}^* , \end{aligned} \tag{8}$$

with

$$|P_{ij}| \ll m_c - m_u .$$

The zeroth-order eigenstates are then chosen to be the eigenstates of M_u^0 with eigenvalues m_u, m_c, and m_t, respectively, and are denoted by u^0, c^0, and t^0. The properly normalized eigenstates corrected up to the nth order are related to the zeroth-order ones as follows:

$$\begin{bmatrix} u \\ c \\ t \end{bmatrix} = U^{(n)} \begin{bmatrix} u^0 \\ c^0 \\ t^0 \end{bmatrix} . \tag{9}$$

The weak mixing matrix V calculated up to the nth order of P is then given by

$$V^{(n)} = U^{(n)} U_u^0 U_d^{0\dagger} = U^{(n)} , \tag{10}$$

where $U_u^0 = U_d^0 = U^0$.

The perturbative calculations are straightforward. The first-order result is

$$V^{(1)} = \begin{bmatrix} 1 & -D^* & -E^* \\ D & 1 & -F^* \\ E & F & 1 \end{bmatrix} , \tag{11}$$

with

$$D=\frac{P_{12}}{m_c-m_u}, \quad E=\frac{P_{13}}{m_t-m_u}, \quad F=\frac{P_{23}}{m_t-m_c} . \tag{12}$$

Up to second order, the result is

$$V^{(2)} = \begin{bmatrix} 1-(|D|^2+|E|^2)/2 & -D^*+E^*F/R & -E^*+F^*D^*/(1+R) \\ D-EF^*(1+R)/R & 1-(|D|^2+|F|^2)/2 & -F^*-E^*D(1+R) \\ E+DFR/(1+R) & F+D^*ER & 1-(|E|^2+|F|^2)/2 \end{bmatrix} , \tag{13}$$

where

$$R=\frac{m_c-m_u}{m_t-m_c} . \tag{14}$$

We note that the above $V^{(1)}$ and $V^{(2)}$ satisfy the unitarity condition up to the first and the second orders, respectively. If P_{12}, P_{13}, and P_{23} are about the same order of magnitude, then from expressions (12) we have

$$|D| > |F| > |E| , \tag{15}$$

owing to $m_t > m_c > m_u$. This then implies

$$1 > V_{tb} > V_{ud} > V_{cs} , \tag{16}$$

as can be seen from expression (13). This result is in good agreement with the experimental data.[8]

IV. THE *CP*-VIOLATION FACTOR

The *CP*-violation factor J defined[5] by

$$\mathrm{Im}(V_{mi} V_{nj} V_{mj} V_{ni}) = J \sum_{pk} \epsilon_{mnp} \epsilon_{ijk} \tag{17}$$

can be expressed in various forms. For example, in Maskawa-Kobayashi[9] form for the weak mixing matrix, J takes the expression

$$J = S_1^2 S_2 S_3 C_1 C_2 C_3 S_\delta ,$$

where $S_1=\sin\theta_1$, etc. In our second-order expression (13) for the weak mixing matrix, it can be easily shown to have the form

$$J=\mathrm{Im}(DE^*F)\ . \quad (18)$$

We note that since the third-order state shift contributes only to the forth-order term in J, the expression (18) for the CP-violation factor is still valid even when the third-order state shift is considered.

V. BJORKEN'S APPROXIMATION

It is well known that all weak mixing matrix elements can be expressed in terms of their independent magnitudes up to certain ambiguities.[10] Bjorken proposed an approximate scheme[6] to determine all phases of generalized Kobayashi-Maskawa matrix elements from their moduli. The basic assumption in this scheme is that matrix elements decrease rapidly with increasing generation change. For three generations, it gives the approximate relations

$$V_{ud}\sim V_{cs}\sim V_{tb}\sim 1\ , \quad (19)$$

$$V_{cd}\sim -V_{us}^*,\quad V_{ts}\sim -V_{cb}^*\ , \quad (20)$$

$$V_{ub}+V_{td}^*\sim -V_{us}V_{ts}^*\ . \quad (21)$$

We see that our first-order result (11) satisfies the above relations automatically except the term $V_{us}V_{ts}$ which amounts to second order in the present perturbative scheme. Our second-order result (13) reproduces the relation (21) but gives corrections to expressions (19) and (20). These corrections can be read out directly from expression (13) as follows:

$$\begin{aligned} V_{ud}&=1-(|D|^2+|E|^2)/2\ ,\\ V_{cs}&=1-(|D|^2+|F|^2)/2\ ,\\ V_{tb}&=1-(|E|^2+|F|^2)/2\ ,\\ V_{cd}&=-V_{us}^*-EF^*\ ,\\ V_{ts}&=-V_{cb}^*-ED^*\ . \end{aligned} \quad (21')$$

We note that the correction terms to the Bjorken approximate relations are all second order in our scheme. Since D, E, and F correspond to the interactions between different generations, these correction terms in (21′) are just the neglected higher-generation-changed ones in Bjorken's approximation. Therefore the present perturbative scheme can automatically produce the higher-generation-changed corrections to Bjorken's approximation.

VI. A SPECIAL CASE

In this section, we consider a special case in which we assume that the matrix elements of the perturbation P are equal to each other if they have the same generation change. That is, $P_{12}=P_{23}$. Then from expression (12) we have $F=RD$. This in turn implies

$$R=\frac{V_{12}+V_{21}^*}{V_{23}+V_{32}^*}\ . \quad (22)$$

The above expression (22) is valid only in the Hermitian basis for the quark mass matrix. Since this relation is obtained from the ratio of two unitarity relations expressed up to second order in our scheme, the corresponding rephasing-invariant form can be readily obtained:

$$R=\left|\frac{V_{12}V_{22}^*+V_{11}V_{21}^*}{V_{23}V_{33}^*+V_{22}+V_{32}^*}\right|=\left|\frac{V_{13}V_{23}^*}{V_{21}V_{31}^*}\right|\ . \quad (23)$$

The above expression can then be used in all phase conventions. Since R is just the ratio of the two level distances in the $\frac{2}{3}$ charged quark mass spectrum as defined in (14), we then have

$$m_t\sim m_c\left|\frac{V_{21}V_{31}^*}{V_{13}V_{23}^*}\right|\ . \quad (24)$$

This gives the upper bound for the top-quark mass:[8]

$$m_t<57m_c\ . \quad (25)$$

We also note that the CP-violation factor J now takes the form

$$J=R\,\mathrm{Im}D^2E^*\ .$$

VII. CONCLUSIONS AND DISCUSSIONS

In this article, we extend S_3 symmetry to the Higgs sector of the extended standard electroweak model which contains two Higgs doublets. Symmetry is then broken in two steps. The first step is to break symmetry spontaneously, and a reasonably good quark mass pattern which is given by expression (5) is thus obtained. The second stage of explicit symmetry breaking is then followed by introducing a small perturbation which assumes a most general form. The weak mixing matrix is then calculated up to second order which is given in expression (13). The first-order result simply reproduces Bjorken's approximate relations for the weak mixing matrix, while the second-order result gives the corrections to them. The CP-violation factor is also expressed in a simple form (18). A relation which expresses the top-quark mass in terms of the weak mixing matrix elements is derived and is given in (24).

We note that the complex parameters D and F can always be chosen to be real by redefining the phases of the quark fields without changing the form of the weak mixing matrix (13). In this choice, we have at most five parameters and the CP-violation factor becomes

$$-DF\,\mathrm{Im}E\ .$$

A rough estimate of the magnitudes of these parameters can be obtained by comparing them to Wolfenstein's form.[11] It gives

$$D\sim\lambda,\quad F\sim\lambda^2,\quad |E|\sim\lambda^3\ ,$$

with $\lambda=0.22$.

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