Brief Reports

Brief Reports are accounts of completed research which do not warrant regular articles or the priority handling given to Rapid Communications; however, the same standards of scientific quality apply. (Addenda are included in Brief Reports.) A Brief Report may be no longer than four printed pages and must be accompanied by an abstract.

Poles of the πN P_{11} partial-wave amplitude

R. E. Cutkosky and S. Wang
Physics Department, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 27 December 1989)

We have carried out fits to the energy dependence of the P_{11} partial-wave amplitude as obtained in two different single-energy analyses of π-N scattering data. Data on inelastic cross sections were also used. The fits used an analytic approximation to the coupled-channel amplitudes suggested by the Dyson equations. From each set of partial-wave data, two resonances were obtained, at roughly 1470 and 1700 MeV. Each of these resonances is associated with a cluster of poles on the Riemann surface.

During the past few years, a large amount of very accurate new experimental data on π-N scattering below about 700 MeV/c has become available. These new data have been incorporated into a recent partial-wave analysis by Arndt, Ford, and Roper[1] (the single-energy amplitudes are referred to here as VPI, the energy-dependent parametrization as AFR). An interesting feature of the AFR analysis is that they found the $P_{11}(1470)$ (known as the Roper resonance) to be associated with two poles on the Riemann surface of the partial-wave amplitude. An older study by Cutkosky *et al.*[2] (CFHK) of the resonance structure from their partial-wave amplitudes[3] (LBCM) had reported only one pole at this energy. The two AFR poles have been interpreted as evidence for the existence of two nearly degenerate resonances. On the other hand, a three-star P_{11} resonance at 1700 MeV was not seen by the AFR analysis. These newer AFR results have led to speculation about the adequacy of simple constituent quark models of the nucleon resonances.

However, it is known to be quite natural for a multichannel resonance to be associated with auxiliary poles on other sheets.[4] In the CFHK analysis, only the pole reached most directly by analytic continuation from the real axis had been looked for. Also, there were weaknesses in the subthreshold analytic structure of the original energy-dependent parametrization used by AFR which have been pointed out by Höhler.[5] In view of the controversies which still exist about the interpretation of the two AFR poles, we have carried out parallel fits to the energy dependence of the LBCM amplitudes and the VPI amplitudes, to examine in detail by the CFHK method the pole structure of the P_{11} partial wave.

Our fits to the LBCM data differ from those originally reported in three ways. First, we limited our fit to the data below 1855 MeV, which is the region of the VPI data. Second, the original CFHK fits used data on inelastic cross sections, but enlarged the errors artificially, by a factor of 3, so that these data would have only a qualita-

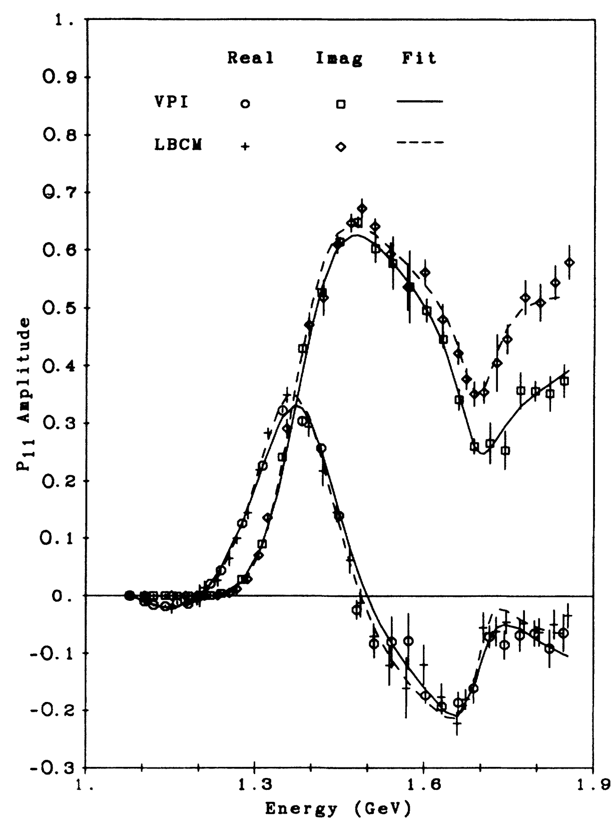

FIG. 1. Fits to the πN elastic-scattering amplitudes.

TABLE I. Resonance poles (MeV) on various sheets.

Sheet	Position				Residue			
	VPI		LBCM		VPI		LBCM	
I	1384	$-119i$	1370	$-114i$	19	$-68i$	8	$-74i$
II	1382	$-138i$	1360	$-120i$	-6	$-105i$	-23	$-77i$
III	1513	$-54i$	1435	$-66i$	-15	$-i$	-30	$-36i$
IV	1514	$-66i$	1427	$-73i$	-38	$-5i$	-51	$-29i$
I	1689	$-54i$	1698	$-44i$	-10	$+3i$	-9	$-2i$
II	1676	$-83i$	1670	$-60i$	-54	$+21i$	-29	$+29i$
III	1690	$-61i$	1696	$-52i$	-8	$+0i$		$-3i$
IV	1683	$-83i$	1678	$-67i$	-39	$+12i$	-20	$+17i$

tive influence on the fits. In the fits reported here, we have used the errors quoted by the experimental groups.[6,7] Third, some information about the fixed and variable parameters used in the CFHK fits has been lost, and we have not been able to reproduce the fitting functions exactly. However, we have checked that we also still obtain compatible fits in other partial waves, such as P_{33}.

CFHK used data at nine energies below 1310 MeV from Bugg *et al.*[8] (QMC) to supplement the LBCM data at the higher energies. These data have larger errors than the new VPI data. For this work, we added two additional near-threshold points from Höhler and Koch[9] (HK), which were arbitrarily assigned errors intermediate between the QMC errors and the VPI errors. It should be noted that the data of LBCM and VPI are qualitatively similar to each other and to the HK data, although they do differ in some details, and the VPI data have much smaller errors.

In our fits, we considered $\pi\pi N$ final states which can be characterized either as the quasi-two-body "isobar" states $\pi\Delta$ or ρN, or as a nonresonant $\pi\pi N$ background in which both pions are in S waves. This is a somewhat simplified model, but it allows us to make a direct comparison with the available inelastic data. We therefore have, effectively, a four-channel parametrization. The t matrix (as a function of the squared energy s) is parametrized as

$$t_{ab}=\gamma_{ia}f_a(s)G_{ij}(s)\gamma_{jb}f_b(s)\ , \tag{1}$$

where $a,b=1,\ldots,4$ are channel indices and $i,j=1,\ldots,N$ are internal "resonance" indices (implicitly summed). The γ_{ia} are energy-independent coupling factors for the vertex between channel a and resonance i, and the f_a are form factors. The factor G_{ij} is the dressed propagator matrix, which can be written in terms of a bare propagator G_{ij}^0 and a self-energy matrix Σ_{ij} using the Dyson equation

$$G_{ij}=G_{ij}^0+G_{il}^0\Sigma_{lk}G_{kj}\ , \tag{2}$$

where $G_{ij}^0=\pm\delta_{ij}/(s_i-s)$, and

$$\Sigma_{lk}=\sum_a \gamma_{la}\Phi_a(s)\gamma_{ka}\ . \tag{3}$$

Here Φ_a is a "channel propagator" which satisfies $\mathrm{Im}\Phi_a(s)=f_a(s)^2\rho_a(s)$, where ρ_a is the effective phase-space density for channel a. The analyticity and unitarity properties of this approximation were discussed by CFHK. The adjustable parameters are the γ_{ia} and the s_i. Four resonance terms were used in fitting each set of data. Two of these terms were needed to describe the nucleon pole and other interaction effects which govern the near-threshold behavior. CFHK originally used fewer adjustable parameters.

The fits to the elastic amplitudes are shown in Fig. 1. For both sets of data, the χ^2 values are high in the energy region 1320–1500 MeV, which is the rising side of the 1470-MeV resonance. It had already been noted by

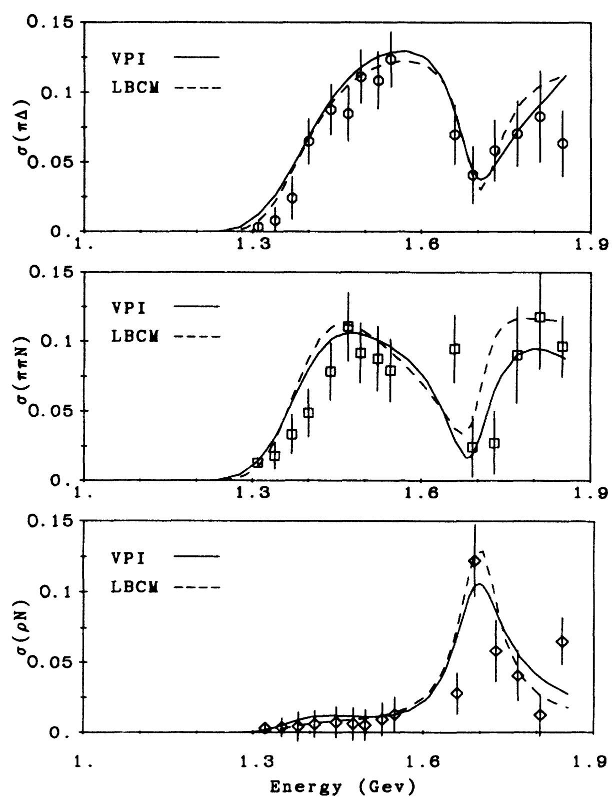

FIG. 2. Cross sections for $\pi\Delta$, $\pi\pi N$ (S wave), and ρN production, normalized to the unitarity circle.

CFHK that this partial wave was especially hard to fit. In the 1700-MeV region, the fits to both sets of data are satisfactory. In this region, the VPI data are more elastic, and the LBCM imaginary part data lie between those of HK and VPI. There have been no new experiments in this region since the analyses by HK and LBCM.

The fits to the inelastic cross sections are shown in Fig. 2. Although there are several discrepant points, these fits are generally satisfactory and quite similar.

We have looked for poles on four different sheets of the Riemann surface: sheet I is the sheet reached most directly from the real axis, sheet II is behind the $\pi\Delta$ branch cut, sheet III is behind the ρN branch cut, while sheet IV is behind both unstable-particle branch cuts. Sheet I has the most physical significance, but sheet II does have some secondary interest for the 1470-MeV resonance, while sheet III has some secondary interest for the 1700-MeV resonance. The resonance poles are listed in Table I. The poles on sheets III and IV which are associated with the lower resonance are unstable; their positions are very sensitive to small changes in the fit. The $\pi\pi N$ branch point is ∞ sheeted. There may also be poles on some of these other sheets, but we have not looked for them.

Conventional resonance parameters obtained by extrapolation from the pole on sheet I, as described by CFHK, are given in Table II. The poles and the resonance parameters derived from the two sets of data are similar. In view of the qualitative similarity of the data, this is not surprising, but we also see that the derived parameter values are very sensitive to the method of analysis. The values of the widths should be considered uncertain by at least 30%. The 1700-MeV resonance is "upside down"—the background phase is close to π. The inelastic data contribute strongly to the evidence for this resonance, and it shows up very clearly in the fits to both sets of elastic data.

TABLE II. Resonance parameters (MeV) and width percentages.

	M	Γ	πN	$\pi\Delta$	$\pi\pi N$	ρN
VPI	1500	661	63%	15%	20%	2%
LBCM	1471	545	69%	13%	17%	1%
VPI	1700	117	23%	17%	16%	44%
LBCM	1706	93	21%	10%	18%	51%

We conclude that the differences in resonance structure reported by CFHK and AFR arise from the different parametrizations used, rather than from differences in the data. Our fits also suggest that there may still be some small residual systematic problems with either the new experimental data or with the derived partial waves, in the energy region 1300–1500 MeV. Shifts in the P_{11} amplitude might be possible if small coordinated shifts were also made in other partial waves. At higher energies, new experiments would clearly be desirable. There are also improvements that could be made in our model of the energy dependence. First, we might require that the s-channel nucleon pole, with the correct residue, also emerge from the dynamical model as a bound state. In the region of the 1470-MeV resonance, it would be useful to have a treatment of three-body interactions which did not rely on the isobar model. This could also give valuable information about π-π interactions.

We thank R. A. Arndt and L. D. Roper for sending us the P_{11} data from SAID solution SM89. We also thank them and D. Bugg, G. Höhler, B. Nefkens, and M. Sadler for discussions. This work was supported by the U.S. Department of Energy under Contract No. DE-AC02-76ER-03066.

[1]R. A. Arndt, J. M. Ford, and L. D. Roper, Phys. Rev. D **32**, 1085 (1984); R. A. Arndt and L. D. Roper, VPI-SU SAID solution SM89 (private communication).

[2]R. E. Cutkosky, C. P. Forsyth, R. E. Hendrick, and R. L. Kelly, Phys. Rev. D **20**, 2839 (1979); in *Baryon 1980,* proceedings of the IV International Conference on Baryon Resonances, Toronto, Canada, 1980, edited by N. Isgur (University of Toronto, Toronto, 1980), p. 19.

[3]R. E. Cutkosky *et al.*, Phys. Rev. D **20**, 2804 (1979).

[4]R. J. Eden and J. R. Taylor, Phys. Rev. **133**, B1575 (1964); M. Kato, Ann. Phys. (N.Y.) **31**, 130 (1965); D. Morgan and M. R. Pennington, Phys. Rev. Lett. **59**, 2818 (1987).

[5]Particle Data Group, G. Höhler, Phys. Lett. B **204**, 366 (1989).

[6]D. J. Herndon *et al.*, Phys. Rev. D **11**, 3165 (1975); R. L. Longacre *et al.*, *ibid.* **17**, 1795 (1978).

[7]K. W. J. Barnham, in *Proceedings of the Topical Conference on Baryon Resonances,* edited by R. T. Ross and D. H. Saxon (Oxford University, New York, 1976), p. 109; (private communication to CFHK).

[8]D. V. Bugg (private communication to CFHK).

[9]G. Höhler *et al.*, *Handbook of Pion-Nucleon Scattering* (Physics Data, No. 12-1) (Fachsinformationzentrum, Karlsruhe, 1979); R. Koch, in *Baryon 1980* (Ref. 2), p. 3.