Is the Weinberg model of *CP* violation really excluded?

Hai-Yang Cheng
Institute of Physics, Academia Sinica, Taipei, Taiwan 11529, Republic of China
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We give an updated analysis on various *CP*-violating effects in the Weinberg three-Higgs-doublet model of *CP* violation. Because of the improved estimate of the η-η' mixing and of the K-η_0 transition, the sign of ϵ'/ϵ is predicted to be the same as that of the chiral suppression of *CP*-odd $K\to 2\pi$ amplitudes (owing to the presence of tadpole contributions) and is most likely to be positive, contrary to previous calculations. The neutron electric dipole moment d_n due to neutral-Higgs-boson exchange at the one-loop level is reexamined and is found to be below the present experimental limit for reasonable Higgs-boson mass. However, the Weinberg's three-gluon operator arising from charged-Higgs-boson exchange will produce an excessive d_n even if the charged Higgs bosons are uncomfortably light. We conclude that the Weinberg *CP*-violation model is *not* necessarily inconsistent with experiment of measuring ϵ'/ϵ, but it tends to give too large a value of d_n.

I. INTRODUCTION

All the recent interest in the Weinberg three-Higgs-doublet model of *CP* violation[1] seemed to focus on the calculation of the neutron electric dipole moment (EDM) d_n. Bigi and Sanda[2] have reexamined the prediction on d_n due to charged-Higgs-boson exchange and concluded that the lower bound is comparable to the existing experimental upper limit. A possible relationship between d_n and ϵ' due to the charged Higgs boson was derived by Booth, Briere, and Sachs.[3] Pal and Pham[4] investigated the contribution from charged-Higgs-boson exchange between two quarks in a neutron and found that it is much smaller than the usual single-quark EDM. Similar contributions but with neutral-Higgs-boson exchange were studied by Valencia,[5] motivated by the observation made by Anselm *et al.*[6] that neutral-Higgs-boson-induced d_n at the one-loop level can be substantially large due to the fact that the scalar coupling of the Higgs boson to the nucleon is proportional to the nucleon mass rather than to the light-quark mass. The calculation of Anselm *et al.* was reconsidered by Cheng and Li[7] in the light of experimental new information on the scalar and pseudoscalar couplings of the Higgs boson.

Very recently, Weinberg[8] has discovered a three-gluon operator which could potentially make a very large contribution to d_n through neutral- or charged-Higgs-boson exchange and concluded that *CP* violation in the Higgs sector is unnaturally small. However, the anomalous dimension of this three-gluon operator was found in Ref. 9 to have the same magnitude but opposite sign to the previous calculation.[10] This changes a QCD renormalization enhancement factor of 740 into a suppression by 740, significantly relaxing the constraints on *CP* violation in the Higgs sector. Nevertheless, as we shall see in the present paper, the Weinberg's gluonic operator arising from charged-Higgs-boson exchange can still threaten to produce an excessive d_n.

It has been argued from time to time that the predictions of ϵ'/ϵ and d_n in the Weinberg *CP*-violation model are not compatible with experiment. First, ϵ'/ϵ was predicted to be negative with magnitude of order 0.007 (plus large errors),[11,12] which is inconsistent with either the NA31 result[13] $(3.3\pm1.1)\times10^{-3}$ or the E731 preliminary result[14] $(-0.4\pm1.4\pm0.6)\times10^{-3}$. Second, the one-loop contribution to the neutron EDM from neutral-Higgs-boson exchange can easily produce a too large d_n, as first noticed by Anselm *et al.* The purpose of the present paper is to give an updated analysis of the above-mentioned *CP*-violating effects.

This paper is organized as follows. After a brief overview of the model in Sec. II, we study in Sec. III the constraints on the *CP*-violating parameters derived from ϵ. Section IV is devoted to the calculation of ϵ'/ϵ. In Sec. V we reexamine the neutron EDM. Discussion and conclusions are given in Sec. VI.

II. MODEL: AN OVERVIEW

It is well known that if a discrete symmetry is imposed on the Lagrangian to ensure natural flavor conservation (NFC), spontanous *CP* violation cannot occur in the scalar sector which has only two Higgs doublets ϕ_1 and ϕ_2.[15] It is perhaps less known that intrinsic or soft *CP* nonconservation is also absent in the two-Higgs-doublet model if NFC is imposed from the outset. Therefore, in order to break *CP* one needs either a third Higgs doublet ϕ_3 or any number of scalar singlets mixing with ϕ_1 and ϕ_2. *CP* nonconservation in the Weinberg three-Higgs-doublet model stems from the nontrivial phase differences of the vacuum expectation values and/or from the complex quartic terms in the Higgs potential. If *CP* is broken spontaneously, the Kobayashi-Maskawa (KM) quark mixing matrix is real[15] and *CP* violation arises solely from Higgs-boson exchange.

The Yukawa interaction of the charged and neutral Higgs bosons with quarks in the mass eigenstates reads

$$\mathcal{L}_Y^+=(2\sqrt{2}G_F)^{1/2}\sum_{i=1}^{2}(\alpha_k\bar{U}_L K M_D D_R+\beta_i \bar{U}_R M_U K D_L)H_i^+ + \text{H.c.} \quad (1)$$

and

$$\mathcal{L}_Y^0=(2\sqrt{2}G_F)^{1/2}\sum_{i=1}^{5}(g_{1i}\bar{D}M_D D+g_{2i}\bar{D}M_D i\gamma_5 D+g_{3i}\bar{U}M_U U+g_{4i}\bar{U}M_U i\gamma_5 U)H_i^0 , \quad (2)$$

where K is a real Kobayashi-Maskawa (KM) matrix, M_U and M_D are diagonal mass matrices for the up- and down-type quarks, respectively. The coupling constants α_i and β_i are in general complex, whereas g_i's are real.

CP violation in the charged-Higgs-boson sector comes from the imaginary part of the off-diagonal Higgs-boson mass-matrix elements and is sometimes characterized by the imaginary part of the transition propagator:

$$\text{Im}\,A_{12}(q)\equiv\text{Im}\left[\frac{\langle T\{\phi_1^{+*},\phi_2^+\}\rangle_q}{v_1^* v_2}\right]=\sqrt{2}G_F\frac{\text{Im}Z_{12}}{q^2-m_{12}^2} , \quad (3)$$

which is related to the couplings α_i and β_i in Eq. (1) via

$$\text{Im}\,A_{12}(q)=2\sqrt{2}G_F\sum_{i=1}^{2}\frac{\text{Im}\alpha_i\beta_i^*}{q^2-m_{H_i}^2} . \quad (4)$$

On the other hand, the breakdown of *CP* in the propagator of the neutral Higgs boson arises from the mixing of the scalar and pseudoscalar fields:

$$\text{Im}\,A_{11}(q)\equiv\frac{\text{Im}\langle T\{\phi_1^0,\phi_1^0\}\rangle_q}{|v_1|^2}=\sqrt{2}G_F\frac{\text{Im}Z_{11}}{q^2-m_1^2}=2\sqrt{2}G_F\sum_{i=1}^{5}\frac{g_{1i}g_{2i}}{q^2-m_{H_i}^2} ,$$
$$\text{Im}\,A_{22}(q)\equiv\frac{\text{Im}\langle T\{\phi_2^0,\phi_2^0\}\rangle_q}{|v_2|^2}=\sqrt{2}G_F\frac{\text{Im}Z_{22}}{q^2-m_2^2}=2\sqrt{2}G_F\sum_{i=1}^{5}\frac{g_{3i}g_{4i}}{q^2-m_{H_i}^2} , \quad (5)$$

where we have assigned ϕ_1 to couple to d_R. For maximum *CP* violation in the Higgs sector, the quantities $\text{Im}Z_{ij}$ are generally of order unity. For example, assuming one of the charged Higgs bosons (say ϕ_2) is much heavier than the other, we then have[12]

$$\text{Im}Z_{12}\simeq 2\,\text{Im}(\alpha_1\beta_1^*)=-2\,\text{Im}(\alpha_2\beta_2^*)=-2\cot\tilde{\theta}_1\tan\tilde{\theta}_2\sin 2\tilde{\theta}_3\sin\delta_H , \quad (6)$$

with $\tilde{\theta}_i$ and δ_H being the Higgs-boson mixing angles and phase defined in complete analogy to the KM quark mixing matrix. Recall that the analogous quantity in the KM model is the rephasing invariant

$$\text{Im}J=c_1c_2c_3s_1^2s_2s_3s_\delta , \quad (7)$$

using the KM parametrization for the quark mixing matrix. Maximum *CP* violation requires[16] $c_1=1/\sqrt{3}$, $c_2=1/\sqrt{2}$, $c_3=1/\sqrt{2}$, and $s_\delta=1$. This corresponds to $\text{Im}J_{\max}=1/(6\sqrt{3})$ and is not realized in nature.

III. CONSTRAINT ON Im A

The parameter $\text{Im}\,A_{12}$ or $\text{Im}(\alpha\beta^*)$ is constrained by *CP* violation observed in the kaon system:

$$\epsilon=\frac{1}{2\sqrt{2}}(\epsilon_m+2\xi_0)e^{i\pi/4} ,$$
$$\epsilon_m\equiv\text{Im}M_{12}/\text{Re}M_{12},\quad \xi_0\equiv\text{Im}a_0/\text{Re}a_0 , \quad (8)$$

where a_0 is the isospin-zero amplitude of $K\to 2\pi$. Since $\text{Im}M_{12}$ in the Weinberg model is known to be dominated by the long-distance contribution,[17] it suffices to just consider the $\Delta S=1$ *CP*-odd Lagrangian induced by the charged-Higgs-boson penguin diagram[18]

$$\mathcal{L}_-=\tilde{f}\bar{d}\sigma^{\mu\nu}(1+\gamma_5)\lambda^a s G_{\mu\nu}^a \quad (9)$$

(in our convention $\mathcal{L}=\mathcal{L}_+ + i\mathcal{L}_-$) where[19]

$$\tilde{f}=-\frac{G_F}{\sqrt{2}}\frac{g_s}{32\pi^2}m_s\sum_{i,j}\text{Im}(\alpha_i\beta_i^*)\lambda_i x_{ij}\times\left[\frac{1}{2(1-x_{ij})}+\frac{1}{(1-x_{ij})^2}-\frac{1}{(1-x_{ij})^3}\ln\frac{1}{x_{ij}}\right] , \quad (10)$$

with $\lambda_i=K_{id}K_{is}, x_{ij}=m_j^2/m_{H_i}^2$. We shall see shortly that one of the charged Higgs bosons must be light enough in order to get ϵ large enough. Assuming $m_{H_2}\gg m_{H_1}$ and $m_t\gg m_{H_1}$, it is easily seen that the top-quark contribution to $\tilde{f}$ is negligible because $\lambda_c\gg\lambda_t$. Hence,

$$\tilde{f}\simeq\frac{G_F}{\sqrt{2}}\frac{g_s}{32\pi^2}m_s m_c^2\sin\theta_C\cos\theta_C\text{Im}(\alpha_1\beta_1^*)\left[\ln\frac{m_{H_1}^2}{m_c^2}-\frac{3}{2}\right] . \quad (11)$$

It should be stressed that there exists a crucial difference between the W- and Higgs-boson penguin diagrams: the latter does not correspond to a local four-quark operator as the loop integral does not give a factor of k^2 canceling the pole in the gluon operator.

The dispersive effects on the imaginary part of the K^0-$\bar{K}^0$ mass matrix arise from the π, η, and η' poles:

$$4m_K(\mathrm{Im}M_{12})_{\mathrm{LD}} \cong \sum_i^{\pi,\eta,\eta'} \frac{\mathrm{Im}(\langle K^0|\mathcal{L}|i\rangle\langle i|\mathcal{L}|\bar{K}^0\rangle)}{m_K^2-m_i^2}$$
$$= \frac{2\kappa}{m_K^2-m_\pi^2}\langle K^0|\mathcal{L}_-|\pi^0\rangle\langle\pi^0|\mathcal{L}_+|\bar{K}^0\rangle\ , \tag{12}$$

with

$$\kappa = 1+\frac{m_K^2-m_\pi^2}{m_K^2-m_\eta^2}[(\tfrac{1}{3})^{1/2}(1+\delta)\cos\theta+2(\tfrac{2}{3})^{1/2}\rho\sin\theta]^2$$
$$+\frac{m_K^2-m_\pi^2}{m_K^2-m_{\eta'}^2}[(\tfrac{1}{3})^{1/2}(1+\delta)\sin\theta-2(\tfrac{2}{3})^{1/2}\rho\cos\theta]^2\ , \tag{13}$$

where the parameters δ and ρ are introduced via

$$\langle\eta_8|\mathcal{L}|K^0\rangle = (\tfrac{1}{3})^{1/2}(1+\delta)\langle\pi^0|\mathcal{L}|K^0\rangle\ ,$$
$$\langle\eta_0|\mathcal{L}|K^0\rangle = -2(\tfrac{2}{3})^{1/2}\rho\langle\pi^0|\mathcal{L}|K^0\rangle\ , \tag{14}$$

so that the deviation of δ from zero and ρ from unity implies the breakdown of SU(3)-flavor symmetry in the K-η_8 transition, and nonet symmetry in K-η_0, respectively. Both the $1/N_c$ approach[20] and the theoretical analysis of the experimental measurement[21] of $\eta,\eta'\to\gamma\gamma$ rates indicate an η-η' mixing angle $\theta\approx-20°$.

The SU(3)-breaking parameter δ is estimated to be 0.17 in Ref. 22. As for the nonet-symmetry-breaking parameter ρ, we have shown recently[23] that it can be uniquely determined by the radiative decay $K_L\to\gamma\gamma$ in conjunction with the direct emission of $K_L\to\pi^+\pi^-\gamma$. We found ρ to be 0.78 ± 0.05.[24] As a result,

$$4m_K(\mathrm{Im}M_{12})_{\mathrm{LD}} = \frac{0.31}{m_K^2-m_\pi^2}\langle K^0|\mathcal{L}_-|\pi^0\rangle\langle\pi^0|\mathcal{L}_+|\bar{K}^0\rangle\ . \tag{15}$$

If $\theta=-10°$ and $\rho=1$ are employed as done previously in Ref. 12, the coefficient in Eq. (15) will become $-0.7/(m_K^2-m_\pi^2)$. We thus see that the sign of $(\mathrm{Im}M_{12})_{\mathrm{LD}}$ flips for realistic values of θ and ρ. As will be seen later, it will affect the prediction on the sign of ϵ'/ϵ.

To evaluate $(\mathrm{Im}M_{12})_{\mathrm{LD}}$ numerically we need to know the CP-odd matrix element $\langle\pi^0|\mathcal{L}_-|K^0\rangle\equiv\tilde{f}A_{K\pi}$. It has been computed in the MIT bag model and was found to be[19] $A_{K\pi}=0.4\ \mathrm{GeV}^3$ for $\alpha_s=1$.[25,26] This together with the experimental value[27] $\langle\pi^0|\mathcal{L}_+|K^0\rangle=2.578\times10^{-7}$ GeV yields

$$4m_K(\mathrm{Im}M_{12})_{\mathrm{LD}}$$
$$=-9.7\times10^{-16}\frac{\mathrm{Im}(\alpha_1\beta_1^*)}{m_{H_1}^2}\left[\ln\frac{m_{H_1}^2}{m_c^2}-\frac{3}{2}\right]\mathrm{GeV}^2\ . \tag{16}$$

Using the bag-model result[19] for $\langle K^0|\mathcal{L}_-|\bar{K}^0\rangle$ we have checked that the dispersive contribution to $\mathrm{Im}M_{12}$ is indeed quite large compared to the short-distance one, $(\mathrm{Im}M_{12})_{\mathrm{LD}}\gg(\mathrm{Im}M_{12})_{\mathrm{box}}$. Since ξ_0/ϵ_m will be shown in the next section to be of order $-0.01\sim-0.07$, we find from Eqs. (8) and (16) that

$$\frac{\mathrm{Im}(\alpha_1\beta_1^*)}{m_{H_1}^2}\left[\ln\frac{m_{H_1}^2}{m_c^2}-\frac{3}{2}\right]=0.024\text{–}0.027\ \mathrm{GeV}^{-2}\ , \tag{17}$$

where use of $|\epsilon|=2.27\times10^{-3}$ has been made.

It is evident from Eqs. (6) and (17) that the existence of a light charged Higgs boson is generally required in the Weinberg model in order to accommodate ϵ unless v_3 is unexpectedly high. The present experimental low limit on the charged-Higgs-boson mass[28] $m_{H^+}>19$ GeV implies

$$\mathrm{Im}(\alpha_1\beta_1^*)>2.4\ . \tag{18}$$

If charged Higgs bosons are not seen in Z^0 decay at the CERN e^+e^- collider LEP, the lower bound of m_{H^+} will be pushed to 45 GeV and $\mathrm{Im}(\alpha_1\beta_1^*)>9.2$, which is uncomfortably large.

IV. CALCULATION OF ϵ'/ϵ (Ref. 29)

In order to reduce theoretical uncertainties in calculations we will use the formula

$$\frac{\epsilon'}{\epsilon}=-\frac{1}{22}\left[\frac{2\xi_0}{\epsilon_m+2\xi_0}\right] \tag{19}$$

to compute ϵ'/ϵ. The reason is that the result of ξ_0/ϵ_m and hence ϵ'/ϵ does not depend on the detail of the CP-odd matrix element $\langle\pi|\mathcal{L}_-|K\rangle$. Previous estimates indicated an ϵ'/ϵ to be either ≈-0.007 (Refs. 11 and 12) or $\approx-\frac{1}{22}$ (Ref. 30), depending on whether or not dispersive contributions to $\mathrm{Im}M_{12}$ are included.

It was first pointed out in Ref. 11 that the CP-odd amplitude $K\to2\pi$ induced by $\mathcal{L}_-$ involves an additional pole contribution arising from the strong-interaction scattering $K\pi\to K\pi$ followed by a $K\to$vacuum weak transition, viz.,

$$\langle\pi^+\pi^-|\mathcal{L}_-|K^0\rangle=\langle\pi^+\pi^-|\mathcal{L}_-|K^0\rangle_{\mathrm{direct}}$$
$$+\text{pole term}\ , \tag{20}$$

with

$$\text{pole term}=S\frac{1}{m_K^2}\langle0|\mathcal{L}_-|K^0\rangle \tag{21}$$

and S being a $K\pi$ strong vertex. This tadpole contribution was not considered in the original analysis of Ref. 30. To see the importance of the pole term, we write down the lowest-order chiral Lagrangian for strong interactions:

$$\mathcal{L}_s^{(2)}=\frac{f_\pi^2}{8}\mathrm{Tr}[(\partial_\mu U\,\partial^\mu U^\dagger)+(M^\dagger U+U^\dagger M)]\ , \tag{22}$$

where M is the meson matrix, and the meson matrix U has the general expression[31]

$$U=1+2i\frac{\phi}{f_\pi}-2\frac{\phi^2}{f_\pi^2}-ia_3\frac{\phi^3}{f_\pi^3}+2(a_3-1)\frac{\phi^4}{f_\pi^4}+\cdots\ ,$$
$$\phi\equiv\phi^a\lambda^a,\quad \mathrm{Tr}(\lambda^a\lambda^b)=\delta^{ab}\ . \tag{23}$$

A different realization of chiral symmetry corresponds to a different value of a_3. From Eq. (22) we find $S=(a_3m_K^2/2f_\pi^2)$ and hence the tadpole contribution is a_3 dependent. Since $\mathcal{L}_-$ transforms as $(\bar{3}_L,3_R)$ under chiral transformations, the lowest-order chiral representation for $\mathcal{L}_-$ must be of the form $\mathrm{Tr}(\lambda_6U)$. It is then obvious from Eq. (23) that the direct $K\to2\pi$ weak amplitude is proportional to a_3. More precisely,[12]

$$\langle\pi^+\pi^-|\mathcal{L}_-|K^0\rangle_{\text{direct}}=-i\frac{\sqrt{2}}{f_\pi}\left[\frac{a_3}{2}\right]\langle\pi^0|\mathcal{L}_-|K^0\rangle\ . \tag{24}$$

Since the physical amplitude should be independent the value of a_3, the pole term and the direct *CP*-odd amplitude must compensate each other, as also indicated by a realistic calculation;[12] that is, the $\mathcal{L}_-$-induced $K\to2\pi$ transition vanishes to lowest order in chiral symmetry.[11]

Going beyond the lowest-order chiral expansion, both the direct weak amplitude and the pole term will receive a_3-independent contributions from the higher-derivative chiral Lagrangians. Unfortunately, we do not know how to compute higher-order contributions to the matrix elements $\langle\pi\pi|\mathcal{L}_-|K\rangle_{\text{direct}}$ and $\langle0|\mathcal{L}_-|K\rangle$ in terms of present techniques. Nevertheless, defining a suppression factor D,

$$\langle\pi\pi|\mathcal{L}_-|K\rangle=\langle\pi\pi|\mathcal{L}_-|K\rangle_{\text{direct}}D\ , \tag{25}$$

we anticipate that D is of $O(m_K^2,m_\pi^2)/\Lambda_\chi^2$ because the four derivative contributions are suppressed by factors of p^2/Λ_χ^2, where $\Lambda_\chi=2\pi f_\pi=830$ MeV (Ref. 32) is a chiral-symmetry-breaking scale. It should be stressed that $D=1$ in the earlier calculation of Sanda and Deshpande[30] as the pole term was not considered by them. Because of the nonlocality of the Higgs-boson penguin diagram due to the gluon field, the vacuum-insertion method is not suitable for evaluating the $\mathcal{L}_-$induced transition amplitudes. Following Ref. 11 to choose $a_3=1$ in Eq. (24) [this simply amounts to a redefinition of D in Eq. (25)] we obtain

$$\langle\pi^+\pi^-|\mathcal{L}_-|K^0\rangle=-i\frac{1}{\sqrt{2}f_\pi}\langle\pi^0|\mathcal{L}_-|K^0\rangle D\ . \tag{26}$$

From Eqs. (26) and (15) it is evident that ξ_0/ϵ_m is independent of the detail of the matrix element $\langle\pi|\mathcal{L}_-|K\rangle$, as we promised before. Using the experimental value of $\langle\pi^+\pi^-|\mathcal{L}_+|K^0\rangle=2.745\times10^{-7}$ GeV,[27] we find

$$\frac{\xi_0}{\epsilon_m}=-0.196D\ , \tag{27}$$

which is of order -0.01 to -0.07. Since $\xi_0/\epsilon_m\ll1$, it follows from Eq. (19) that

$$\frac{\epsilon'}{\epsilon}\cong0.017D\sim(0.4-6.0)\times10^{-3}\ . \tag{28}$$

The central value of NA31 measurement[13] $(3.3\pm1.1)\times10^{-3}$ is fitted provided that

$$D=(0.37\text{ GeV}/\Lambda_\chi)^2\ . \tag{29}$$

This is indeed what we expected before, namely, D is of $O(m_K^2,m_\pi^2)/\Lambda_\chi^2$.

We notice that the sign of ϵ'/ϵ in previous calculations[30,11,12] is predicted to be opposite to that of D because of the negativity of the matrix element $(\mathrm{Im}M_{12})_{\mathrm{LD}}$. Since D is most likely to be positive (unless the tadpole contribution dominates over the direct amplitude), the Weinberg model of *CP* violation seems to be consistent with the NA31 measurements of ϵ'/ϵ.

V. NEUTRON ELECTRIC DIPOLE MOMENT

In this section we review the calculations of the neutron EDM in the Weinberg model of *CP* violation. Especially, we wish to reexamine the potentially hazardous contribution to d_n from neutral-Higgs-boson exchange. The present experimental bound is[33]

$$d_n^{\text{expt}}<1.2\times10^{-25}\ e\text{ cm}\ . \tag{30}$$

A. Quark electric dipole moment

The quark EDM due to charged-Higgs-boson exchange is given by[34]

$$d_q=\frac{\sqrt{2}G_F}{12\pi^2}m_q\mathrm{Im}(\alpha_1\beta_1^*)\times\sum_i\frac{x_i}{(1-x_i)^2}\left[\frac{3}{4}-\frac{5}{4}x_i+\frac{1-\frac{3}{2}x_i}{1-x_i}\ln x_i\right]K_{iq}^2 \tag{31a}$$

for charge $-\frac{1}{3}$ quarks, and

$$d_q=\frac{\sqrt{2}G_F}{12\pi^2}m_q\mathrm{Im}(\alpha_1\beta_1^*)\times\sum_i\frac{x_i}{(1-x_i)^2}\left[x_i-\frac{1}{2}\frac{1-3x_i}{1-x_i}\ln x_i\right]K_{qi}^2 \tag{31b}$$

for charge $\frac{2}{3}$ quarks, where $x_i=m_i^2/m_{H_1}^2$. It is easily seen that $d_d\gg d_u$ and the dominant contribution to d_d is due to the c quark. Numerically,

$$\tfrac{4}{3}d_d=-9\times10^{-26}\ e\text{ cm}\ . \tag{32}$$

It should be stressed that this is the EDM for the *current* down quark, whereas the quark-model relation $d_n=\frac{4}{3}d_d-\frac{1}{3}d_u$ is valid only for *constituent* quarks. It seems to us that the prediction $-9\times10^{-26}\ e$ cm should be viewed as a *lower bound* on d_n

B. d_n from neutral-Higgs-boson exchange

The quark EDM from neutral-Higgs-boson exchange is proportional to m_q^3 and hence is negligible compared to that due to charged-Higgs-boson exchange. Moreover, we just mentioned an intrinsic difficulty: what is the connection of the EDM or the color dipole moment of the *current* quark to d_n is rather unclear. This problem is circumvented if we let the neutral Higgs boson couple to the neutron directly, that is, we wish to calculate d_n directly without worrying about the quark EDM.

Since the neutron carries no electric charge, the photon vertex must be of the magnetic type. The neutron EMD reads[6]

$$d_n=\mu_n\left[\frac{1}{8\pi^2 m_N^2}\right]\times\sum_i\int_0^\infty k^2dk^2\left[\frac{1+2m_N^2/k^2}{(1+4m_N^2/k^2)^{1/2}}-1\right]\times\langle\sigma_i H_i\rangle_k f_\sigma F_\sigma(k^2)f_H F_H(k^2)\,, \tag{33}$$

where $\langle\sigma_i H_i\rangle=\frac{1}{2}\mathrm{Im}\langle T\{\phi_i^0,\phi_i^0\}\rangle$, $\mu_n=-1.91$ nuclear magneton is the neutron magnetic moment, f_σ and f_H are the scalar and pseudoscalar couplings, respectively, of the Higgs boson with the neutron, and $F_\sigma(k^2)$ as well as $F_H(k^2)$ are the corresponding form factors. More precisely, we need to evaluate the neutron matrix elements

$$\begin{aligned} f_\sigma&=-(\sqrt{2}G_F)^{1/2}\sum_q\langle n|m_q\bar{q}q|n\rangle\,,\\ f_H&=-(\sqrt{2}G_F)^{1/2}\sum_q\langle n|m_q\bar{q}i\gamma_5 q|n\rangle\,. \end{aligned}\tag{34}$$

Under heavy-quark expansion,[35,6]

$$\begin{aligned} m_h\bar{q}_h q_h&\to-\frac{2}{3}\frac{\alpha_s}{8\pi}GG+O\left[\frac{\mu^2}{m_h^2}\right],\\ m_h\bar{q}_h i\gamma_5 q_h&\to-\frac{1}{2}\frac{\alpha_s}{8\pi}G\tilde{G}+O\left[\frac{\mu^2}{m_h^2}\right], \end{aligned}\tag{35}$$

where the subscript h denotes a heavy quark, μ is a typical hadronic mass scale, $GG\equiv G^a_{\mu\nu}G^{\mu\nu}$, $G\tilde{G}\equiv\epsilon_{\mu\nu\alpha\beta}G^{a\mu\nu}G^{a\alpha\beta}$. Consequently,

$$\begin{aligned} f_\sigma&=-(\sqrt{2}G_F)^{1/2}\left[m_uB_u+m_dB_d+m_sB_s-\left\langle n\left|\frac{\alpha_s}{4\pi}GG\right|n\right\rangle\right],\\ f_H&=-(\sqrt{2}G_F)^{1/2}\left[m_uC_u+m_dC_d+m_sC_s-\frac{3}{4}\left\langle n\left|\frac{\alpha_s}{4\pi}G\tilde{G}\right|n\right\rangle\right], \end{aligned}\tag{36}$$

where $B_u=\langle n|\bar{u}u|n\rangle$, $C_u=\langle n|\bar{u}i\gamma_5 u|n\rangle$; . . . .

Since the nucleon mass can be expressed in terms of the nucleon matrix elements of GG and light-quark scalar densities, B_i and since B_u+B_d is fixed by the experimental measurement of the pion-nucleon σ term $\sigma_{\pi N}\sim 55$ MeV, we find (for details of the derivation, see Refs. 36 and 37)

$$\begin{aligned} &m_uB_u=18\ \mathrm{MeV},\quad m_dB_d=39\ \mathrm{MeV}\,,\\ &m_sB_s=376\ \mathrm{MeV},\quad \left\langle n\left|\frac{\alpha_s}{4\pi}GG\right|n\right\rangle=-113\ \mathrm{MeV}\,, \end{aligned}\tag{37}$$

and hence

$$f_\sigma=-(\sqrt{2}G_F)^{1/2}0.58m_N\,. \tag{38}$$

The result of $-(\sqrt{2}G_F)^{1/2}0.27m_N$ was obtained by Anselm *et al.*[6] as the effect of strange quarks was not considered by them. As to the pseudoscalar coupling, we have outlined in Ref. 36 a scheme for computing the nucleon matrix elements of light-quark pseudoscalar densities and of $G\tilde{G}$ based on the $1/N_c$ argument. Using Eqs. (3.8) and (3.9) of Ref. 36 for the neutron and noticing that the singlet axial-vector coupling $g_A^{(0)}\approx 0$ inferred from the recent European Muon Collaboration measurement on the proton polarized structure functions,[38] we find[39,40]

$$\begin{aligned} &m_uC_u=419\ \mathrm{MeV},\quad m_dC_d=-772\ \mathrm{MeV}\,,\\ &m_sC_s=-165\ \mathrm{MeV},\quad \left\langle n\left|\frac{\alpha_s}{4\pi}G\tilde{G}\right|n\right\rangle=-236\ \mathrm{MeV}\,. \end{aligned}\tag{39}$$

This leads to[41]

$$f_H=(\sqrt{2}G_F)^{1/2}0.36m_N\,, \tag{40}$$

which is to be compared with the result $(\sqrt{2}G_F)^{1/2}2.5m_N$ obtained in Ref. 6 in which $g_A^{(0)}$ is taken to be $\frac{3}{5}g_A^{(6)}=0.75$. As for the form factors F_σ and F_H, we shall follow Ref. 7 to take

$$\begin{aligned} F_\sigma(k^2)&=\left[1+\frac{k^2}{M^2}\right]^{-1},\\ F_H(k^2)&=\left[1+\frac{k^2}{m_\eta^2}\right]^{-1}, \end{aligned}\tag{41}$$

with $M\sim 1.5$ GeV. Since the Higgs-boson mass is much larger than the characteristic scales of the form factors, the Higgs-boson propagator can be set at $k^2=0$.

Substituting Eqs. (38), (40), and (41) into (33) yields

$$d_n=1.6\times 10^{-22}\langle\sigma H\rangle_0\ e\ \mathrm{cm\,GeV^2}\,, \tag{42}$$

where $\langle\sigma H\rangle_0$ is the average mixed propagator of the neutral Higgs boson at $k^2=0$. It is customary to assume that *CP* violation in the charged or neutral Higgs propagator is approximately the same, i.e., $\mathrm{Im}A_{11}\sim\mathrm{Im}A_{22}\sim\mathrm{Im}A_{12}$. As a result

$$\mathrm{Im}\langle T\{\phi^0,\phi^0\}\rangle_0=2\langle\sigma H\rangle_0\sim\frac{\mathrm{Im}Z_{11}}{m_{H^0}^2}\sim\frac{\mathrm{Im}(\alpha_1\beta_1^*)}{m_{H^+}^2}\,. \tag{43}$$

It follows from Eqs. (18), (42), and (43) that $d_n\sim 1\times 10^{-24}$ e cm ($m_{H^+}=20$ GeV is being used), which is still too large by an order of magnitude. However, we should notice that the *CP*-odd quantity $\mathrm{Im}A_{12}$ or $\langle T\{\phi_1^{+*},\phi_2^+\}\rangle$ is constrained by the observed $K_L\to\pi\pi$ data, whereas $\mathrm{Im}A_{11}$ or $\langle\sigma H\rangle$ is not. As the charged-Higgs-boson mass increases, the lower limit on $\mathrm{Im}(\alpha_1\beta_1^*)$ must also increase (e.g., by adjusting the VEV's so that $v_3 \gg v_1, v_2$) in order to accommodate ϵ [see Eq. (17)]. Indeed, $\mathrm{Im}(\alpha\beta^*)/m_{H^+}^2$ is fairly insensitive to the change of $m_{H^+}^2$. Since neutral-Higgs-boson exchange does not contribute to $K_L \to \pi\pi$, there is no reason for the neutral Higgs boson to be light and for $\mathrm{Im}Z_{11}$ increasing with the Higgs-boson mass. Hence, it is more natural and plausible to let $\mathrm{Im}Z_{11}$ be of order unity and take a more realistic value of m_{H^0}. We then find

$$d_n \sim 2\times 10^{-26}\left[\frac{100\ \mathrm{GeV}}{m_{H^0}}\right]^2, \tag{44}$$

which we believe is more sensible than previous estimates. Therefore, we conclude that the neutron EDM generated by neutral-Higgs-boson exchange is consistent with experiment for any reasonable Higgs-boson mass.

Several remarks are in order.

(i) Anselm *et al.*[6] obtained an estimate $d_n \approx 7.4\times 10^{-22}\langle \sigma H\rangle_0$ e cm GeV^2, which is different from our Eq. (42) by a factor of 5. The difference comes mainly from the evaluation of Higgs-scalar and -pseudoscalar couplings, as we explained before. Following the assumption $\langle \sigma H\rangle \sim \mathrm{Im}(\alpha\beta^*)/m_{H^+}^2 \approx 0.18\ \mathrm{GeV}^{-2}$ made by Anselm *et al.*, one recovers their original result $d_n \sim 1.3\times 10^{-22}$ e cm. However, we notice that the value of $\mathrm{Im}(\alpha\beta^*)/m_{H^+}^2$ employed in Ref. 6 is too large by a factor of 25. Also, $\langle \sigma H\rangle \sim 0.18\ \mathrm{GeV}^{-2}$ corresponds to *CP* violation mediated by a neutral Higgs boson of mass ~ 3 GeV, which is unrealistically too light. A more plausible estimate yields $d_n \sim 7\times 10^{-26}$ $(100\ \mathrm{GeV}/m_{H^0})^2$ if one utilizes the Higgs-boson couplings given in Ref. 6.

(ii) As pointed out by Weinberg,[8] the coefficient of the *P*- and *T*-odd four-gluon operator,

$$\mathcal{O}_4 = \tfrac{1}{24}\zeta(GG)(G\tilde{G}), \tag{45}$$

produced in integrating out neutral Higgs boson and heavy quarks (Fig. 1), is not suppressed by light-quark masses or small mixing angles. Using the heavy-quark expansion given by Eq. (35), it is straightforward to show that

$$\zeta = \frac{3\sqrt{2}}{64\pi^2}G_F\alpha_s^2\langle \sigma H\rangle_0 . \tag{46}$$

The neutron EDM may be estimated by naive dimensional analysis[8,26] to be of order

$$d_n = \frac{3\sqrt{2}}{64\pi^2}G_F\left[\frac{g_s(\mu)}{4\pi}\right]^4\Lambda_\chi^3\langle \sigma H\rangle_0 , \tag{47}$$

where $\Lambda_\chi = 2\pi f_\pi$ (Ref. 32) is the chiral-symmetry-breaking scale, and $g_s(\mu)$ is the strong coupling at the running scale μ relevant for the evaluation of d_n. Following Ref. 8 to choose $g_s(\mu)/4\pi \simeq 1/\sqrt{6}$, we obtain

$$d_n \simeq 2.5\times 10^{-27}\left[\frac{100\ \mathrm{GeV}}{m_{H^0}}\right]^2 e\ \mathrm{cm}\ . \tag{48}$$

Of course, a concrete estimate of the contribution of the four-gluon operator $\mathcal{O}_4$ to d_n is ready by using Eq. (33) with f_σ and f_H receiving contributions from gluon fields only. Repeating the same calculation yields

$$d_n \simeq 2\times 10^{-27}\left[\frac{100\ \mathrm{GeV}}{m_{H^0}}\right]^2 e\ \mathrm{cm}\ , \tag{49}$$

which demonstrates that the naive dimensional analysis is indeed valid for an order-of-magnitude estimate. The contribution of $\mathcal{O}_4$ to the neutron EDM is smaller than Eq. (44) by an order of magnitude since Higgs-boson couplings f_σ and f_H are not dominated by the heavy-quark contributions.[36,7]

(iii) Meson-loop contributions to d_n have been estimated in Ref. 42 to be of order -1×10^{-24} e cm, but it is difficult to nail down the large uncertainties associated with the calculation of various hadronic matrix elements.

C. d_n from three-gluon operator

Recently, Weinberg has ascertained that the following dimension-6, *P*- and *T*-violating three-gluon operator,[6]

$$\mathcal{O}_3 = -\tfrac{1}{6}Cf_{abc}G_a^{\mu\rho}G_{b\rho}^{\nu}\tilde{G}_{c\mu\nu}\ , \tag{50}$$

obtained from a heavy-quark loop with a Higgs-boson exchange can threaten to produce an excessive d_n since it involves neither light-quark masses nor small mixing angles.

Considering a two-loop diagram with a charged-Higgs-boson exchange as depicted in Fig. 2. As pointed out in Ref. 43, Weinberg's three-gluon operator begins to appear only at the scale when the b quark is further eliminated from the effective theory obtained in integrating out t quarks. It is thus necessary to take into account the difference in the renormalization-group equations be-

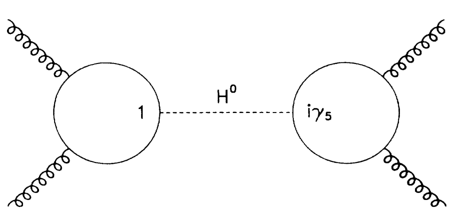

FIG. 1. The Feynman diagram contributing to the operator $\mathcal{O}_4$ given in Eq. (45).

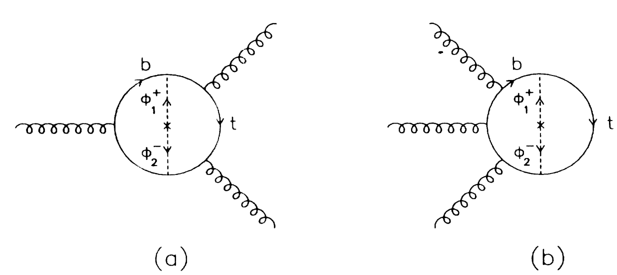

FIG. 2. Feynman diagrams contributing to the operator $\mathcal{O}_3$ given in Eq. (50). Figure 2(b) actually does not contribute to $\mathcal{O}_3$.

tween the t and b-quark masses and below the b-quark mass. The QCD renormalization factor is given by[43]

$$\zeta=\left[\frac{g_s(m_b)}{g_s(m_t)}\right]^{\gamma_b/\beta_5}\left[\frac{g_s(m_c)}{g_s(m_b)}\right]^{\gamma_g/\beta_4}\left[\frac{g_s(\mu)}{g_s(m_c)}\right]^{\gamma_g/\beta_3}, \tag{51}$$

where $\gamma_g=-18$ (Ref. 9), $\gamma_b=-14/3$, $\beta_n=(33-2n)/6$ for n flavors of quarks. This is to be compared with the evolution factor

$$\left[\frac{g_s(\mu)}{g_s(m_t)}\right]^{-108/23} \tag{52}$$

found in the neutral-Higgs-boson-exchange mode. Numerically, the QCD renormalization effect amounts to increasing the contribution of charged-Higgs-boson exchange to d_n by a factor of 5 compared to neutral-Higgs-boson exchange.

The contribution of Fig. 2 to the neutron EDM can be estimated by the naive dimensional analysis to be[44]

$$\begin{aligned} d_n &\simeq 4\times 10^{-21}\zeta\left[\frac{g_s(\mu)}{4\pi}\right]^3 \mathrm{Im}Z_{12}h'(m_t,m_b,m_H) \\ &=1.96\times 10^{-24}\mathrm{Im}Z_{12}h'(m_t,m_b,m_h)\ e\ \mathrm{cm} \end{aligned} \tag{53}$$

with[44]

$$h'\approx\frac{1}{4}\frac{m_t^2}{m_H^2}\left[1-\frac{m_t^2}{m_H^2}\right]^{-3}\left[-\ln\frac{m_t^2}{m_H^2}-\frac{3}{2}+2\frac{m_t^2}{m_H^2}-\frac{1}{2}\frac{m_t^4}{m_H^4}\right]. \tag{54}$$

Since $\mathrm{Im}Z_{12}\simeq 2\ \mathrm{Im}(\alpha_1\beta_1^*)$, it follows from Eqs. (17), (53), and (54) that

$$d_n\simeq\begin{cases}1\times 10^{-24}\ e\ \mathrm{cm} & \text{for } m_{H^+}=20\ \mathrm{GeV}, \\ 4\times 10^{-24}\ e\ \mathrm{cm} & \text{for } m_{H^+}=45\ \mathrm{GeV},\end{cases} \tag{55}$$

where $m_t\sim 100$ GeV is being assumed. It is evident that even if charged Higgs bosons are very light, the predicted neutron EDM is too large by an order of magnitude. As for neutral-Higgs-boson exchange, the quantity $\mathrm{Im}Z_{11}/m_{H^0}^2$ is not subject to the constraint derived from observed CP violation in the $K^0-\bar{K}^0$ system. Moreover, since Weinberg's operator is already present at the top-quark mass scale, QCD renormalization effects are stronger than that in the case of charged-Higgs-boson exchange [see Eqs. (51) and (52)]. Therefore, the neutron EDM induced by the operator $\mathcal{O}_3$ due to neutral-Higgs-boson exchange is safely below the present experimental bound for reasonable neutral-Higgs-boson mass.

VI. DISCUSSION AND CONCLUSIONS

In the present paper we examine the CP-violating effects ϵ'/ϵ and the neutron EDM within the framework of the Weinberg three-Higgs-doublet model of CP violation. In order to get ϵ large enough, one of the charged Higgs bosons in this model should be light enough (of order 20 GeV) if the vacuum expectation values v_1, v_2, and v_3 are of the same order of magnitude. However, if the light Higgs boson is not seen experimentally, in order to implement the observed CP violation in the K^0-$\bar{K}^0$ system it will require that $v_3 \gg v_1, v_2$ or $\mathrm{Im}(\alpha\beta^*)$ be uncomfortably large.

There is an additional tadpole term which plays the role of chiral suppression on the physical CP-odd $K\to 2\pi$ amplitude. Owing to the improved estimate of the $\eta-\eta'$ mixing and of the K^0-η_0 transition, we found that the sign of ϵ'/ϵ is the same as that of the chiral suppression factor D and is most likely to be positive, contrary to previous calculations. The NA31 measurements of ϵ'/ϵ is easily accommodated in the Weinberg CP-violating model.

The neutron EDM due to charged-Higgs-boson exchange is potentially larger than that from neutral-Higgs-boson exchange since CP-nonconserving parameters in the former are subject to the constraint derived from ϵ, while the latter does not, that is, the neutral Higgs particle is not necessarily light. We advocate that since the scalar and pseudoscalar couplings of the Higgs boson are not dominated by heavy quarks, a plausible d_n induced by neutral-Higgs-boson exchange is about an order of magnitude smaller than the present experimental bound. However, the Weinberg's three-gluon operator arising from charged-Higgs-boson exchange will produce an excessive d_n even if the charged Higgs bosons are very light.

We conclude that the Weinberg three-Higgs-doublet model is *not* necessarily inconsistent with experiment of measuring ϵ'/ϵ, but it tends to give too large a value of d_n. The model will be definitely ruled out if the neutron EDM is not seen in forthcoming experiments capable of meausring the d_n to the accuracy of 1 part of 10^{-26} e cm. This will then imply that CP violation mediated by Higgs-boson exchange is not solely responsible for $K_L\to\pi\pi$.

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