Top-quark mass dependence of the decay $B_s \to \gamma\gamma$ in the standard electroweak model

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An explicit calculation for the decay $b \to s\gamma\gamma$ is carried out in the standard electroweak model. We find that terms with the lowest powers of external momenta are only proportional to the dipole moment form factor responsible for the decay $b \to s\gamma$. It is shown that this result can be regarded as a non-soft-bremsstrahlung extension of Low's low-energy theorem. Without QCD corrections, the branching ratio $B(B_s \to \gamma\gamma)$ is estimated to be of the order of 10^{-7}. In addition, the photon spin orientations are found to be fairly sensitive to the top-quark mass. Its implications are also discussed.

I. INTRODUCTION

There is mounting experimental evidence that the top quark is much heavier than previously anticipated. Short of direct production, one may have to resort to careful studies of its footprints through radiative corrections. The structure of the Kobayashi-Maskawa (KM) matrix[1] seems to suggest that processes involving $b \to s$ transition might offer appealing possibilities. Thus, it has been an ongoing effort for one to study, for instance, "penguin"-type processes such as $b \to s\gamma$ (Ref. 2) and $b \to sg$ (Ref. 3). Recently we have initiated a study of $B_s \to \gamma\gamma$ with the quarks fused into a single hadron B_s. This is a more detailed report, expanding an earlier Letter by us.[4]

As far as short-distance contributions are concerned, the calculation of $B_s \to \gamma\gamma$ is very similar to that of $K_0 \to \gamma\gamma$ which has been studied by Galliard and Lee[5] and later by Ma and Pramudita.[6] However, in contrast with $K_0 \to \gamma\gamma$ where long-distance effects seem to play a major role,[7] short-distance contributions are likely to be the dominant source for $B_s \to \gamma\gamma$. It is therefore of importance to have a more careful study on the decay $b \to s\gamma\gamma$ and to explore its consequences.

As will be elaborated below, an interesting feature of this decay is that the photon spin orientations are fairly sensitive to the top-quark mass m_t. For a light top, i.e., $m_t < M_W$ (W-boson mass), the situation is very similar to that in the discussion of short-distance contributions to $K_0 \to \gamma\gamma$ (Refs. 5 and 6). There, all the relevant internal quarks are light in mass compared with M_W. As it turns out, the diagrams are the one-particle-irreducible (1PI) ones with the two photons emitted from internal quark lines. As a result, the two photons are in a state given by $F^{\mu\nu}\tilde{F}_{\mu\nu}$ which is odd in CP; here, $F_{\mu\nu}$ is the electromagnetic field tensor with its dual $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$. In momentum space this corresponds to a state with a photon spin orientation given by $\mathbf{k}_1 \cdot [\boldsymbol{\epsilon}_1(k_1) \times \boldsymbol{\epsilon}_2(k_2)]$, where $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$ are the two transverse photon polarizations.

The situation changes drastically for a heavy top ($m_t \gtrsim M_W$). In this case, contributions to $B_s \to \gamma\gamma$ from the one-particle-reducible diagrams become sizable and even dominant. These terms have the feature that they contain equal amounts of $F^{\mu\nu}\tilde{F}_{\mu\nu}$ and $F^{\mu\nu}F_{\mu\nu}$. The state FF has a spin orientation given by $\boldsymbol{\epsilon}_1(k_1) \cdot \boldsymbol{\epsilon}_2(k_2)$ in momentum space. Thus, a measurement on the photon spin structures should provide a useful limit on m_t. Also, the states FF and $F\tilde{F}$ have opposite CP transformations, a result which may arouse interest in the study of CP violation. Details of this part of discussions are presented in Sec. IV, which will be followed by a conclusion in Sec. V.

As will become clear, to handle such a lengthy computation, it is helpful to classify terms by their dimensionality with respect to external momenta and fields (b,s,γ). With such a classification we find that the lowest-dimensional terms, which by the way are the most complicated ones to compute, are only proportional to the one-photon transition $b \to s\gamma$ dipole moment. More importantly, these terms can actually be obtained from the one-particle-reducible external-bremsstrahlung diagrams with the $b \to s\gamma$ vertex given by its dipole moment form factor. This result is a consequence of a non-soft-bremsstrahlung extension of Low's low-energy theorem.[8] In the next section we will formulate this extension. It then follows from this theorem that in the limit to be discussed below, for any flavor-changing two-photon decays (not just for $b \to s\gamma\gamma$), the lowest-dimensional terms can always be represented in a specific form [see Eq. (11)] with only the dipole-moment form factor to be determined by the specific decay process and the theoretical model. Since it is very easy to calculate the higher-dimensional terms (Refs. 5 and 6), with our theorem computations of a flavor-changing two-photon decay now become fairly simple.

The rest of the paper, which covers Sec. III and two Appendixes, is given to an explicit calculation of the decay $b \to s\gamma\gamma$ in the standard electroweak model (without QCD corrections). The result of the calculation agrees with our theorem.

II. A NON-SOFT-BREMSSTRAHLUNG EXTENSION OF THE LOW-ENERGY THEOREM

It is useful to begin with some considerations of the constraints imposed by gauge invariance on the decay amplitude. To do so we follow the idea of Low (Ref. 8).

According to Low's theorem (Ref. 8), leading terms of a soft bremsstrahlung-scattering cross section can be calculated exactly in terms of the cross section of the hard scattering (without the photon emission) and an external photon bremsstrahlung (without the scattering). Two important conditions are required for the theorem to hold. First the energy emitted by the photon must be much smaller than the energy scale for producing the scattering, and second, the photon energy must be much smaller than the external momenta of the participating particles. The first condition permits the separation of radiation from the scattering, whereas the second one amounts to allowing for a momentum expansion to single out the leading terms. The usefulness of Low's theorem is that, thanks to gauge invariance, potentially complicated calculations of internal bremsstrahlungs are now no longer necessary.

It is straightforward to extend Low's theorem to places where a scattering is replaced by a decay process provided the two necessary conditions are satisfied. For the case in point, one can think of the decay $b \to s\gamma\gamma$ as a decay $b \to s\gamma$ followed by a bremsstrahlung (both internal and external). The first condition required for Low's theorem to hold is clearly satisfied, because the energy scale responsible for the flavor-changing decay $b \to s\gamma$ is of the order M_W, which is much higher than the photon energy. However, the second condition, which requires the photon energy to be much smaller than m_b and m_s, cannot be satisfied. Still, we will show below that in the lowest-dimensional terms, the photons can be regarded as if they were soft and thus an extension to Low's theorem follows. The price for relaxing the second condition is that these terms do not necessarily dominate the process.

Now, we consider the decay $b \to s\gamma\gamma$ without QCD loops. Following Low (Ref. 8) we write the transition amplitude $T_{\mu_1\mu_2}$ as

$$T_{\mu_1\mu_2}(b(P) \to s(P')\gamma(k_1)\gamma(k_2)) = (Q_d e)^2 (T_{1,\mu_1\mu_2} + T_{2,\mu_1\mu_2}) \quad (1)$$

with $Q_d = -\frac{1}{3}$ and

$$T_{1,\mu_1\mu_2} = \gamma_{\mu_2} \frac{1}{\not{P}' + \not{k}_2 - m_s} \Gamma_{\mu_1}(P,k_1) + \Gamma_{\mu_1}(P-k_2,k_1) \frac{1}{\not{P} - \not{k}_2 - m_b} \gamma_{\mu_2} + (1 \leftrightarrow 2) \quad (2)$$

corresponding to contributions from external bremsstrahlung diagrams. In Eq. (2), $\Gamma_\mu(P,k)$ represents the $b \to s\gamma$ vertex. Contributions from the rest of the diagrams are represented by $T_{2,\mu_1\mu_2}$, which are to be determined by gauge invariance:

$$k_1^{\mu_1} T_{\mu_1\mu_2} = 0 \ , \quad k_2^{\mu_2} T_{\mu_1\mu_2} = 0 \ . \quad (3)$$

The most general form of $\Gamma_\mu(P,k)$ can be written as follows. Consider the Ward-Takahashi identity for the renormalized quantities

$$k^\mu \Gamma_\mu(P,k) = \Sigma(P-k) - \Sigma(P) \ . \quad (4)$$

Here Σ is the on-shell renormalized $b \to s$ self-energy. It can always be written as $\Sigma(P) = (\not{P} - m_s)\tilde{\Sigma}(P)(\not{P} - m_b)$. The explicit expression of $\tilde{\Sigma}$ is model dependent. However, for our purpose, one only needs to know that $\tilde{\Sigma}$ is finite when operating on the external fields b and s. In other words, $\tilde{\Sigma}$ does not have Dirac projections in its denominator. This result is a consequence of the on-shell conditions

$$\bar{s}\Sigma(P^2 = m_s^2) = 0, \quad \Sigma(P^2 = m_b^2) b = 0 \ , \quad (5)$$

required by our renormalization procedure. It then follows from Eq. (4) that

$$\Gamma_\mu(P,k) = i\sigma_{\mu\nu} k^\nu \frac{mF}{M^2} + (\not{k} k_\mu - k^2 \gamma_\mu) L \frac{H}{M^2} + [(\not{P} - \not{k} - m_s) O_\mu(P,k) + Q_\mu(P,k)(\not{P} - m_b)] \frac{I}{M^2} \ . \quad (6)$$

Here M, which is typically of the order of M_W or higher, represents the energy scale responsible for the flavor-changing transition. m is a mass that flips helicities. O_μ and Q_μ are two combinations of operators. They have the dimension of mass and satisfy

$$k^\mu O_\mu(P,k) \neq 0 \ , \quad (7)$$

$$k^\mu Q_\mu(P,k) \neq 0 \ . \quad (8)$$

That Eqs. (7) and (8) must be true is a consequence that the relation $k^\mu \Gamma_\mu(P,k) = 0$ holds only if the external quark fields are on shell. Finally, F, H, and I are dimensionless functions of $P^2/M_W^2, (P-k)^2/M_W^2$ and other internal participating particle masses. In Eq. (6) the first term is the usual dipole moment term and the second one is the charge radius. The last term gives rise to off-shell effects. It vanishes in $b \to s\gamma$ when both b and s are on shell. In the case of $b \to s\gamma\gamma$, the charge-radius term vanishes also because the photon emitted from Γ is real.

It is useful to classify the relevant operators by their dimensionality with respect to external fields (b,s,γ) and momenta. Such a classification is possible provided $m_b \ll M$ (the first condition of Low's theorem). Now, we can write F and I as series expansions of external momenta with respect to M. Their first terms in these series, denoted by F_0 and I_0 hereafter, are external momenta independent, and hence have the lowest dimensionality. From Eqs. (2) and (6) we have explicitly

$$T_{1,\mu_1\mu_2} = \left[\gamma_{\mu_2} \frac{1}{\not{P}' + \not{k}_2 - m_s} i\sigma_{\mu_1\nu} k_1^\nu + i\sigma_{\mu_1\nu} k_1^\nu \frac{1}{\not{P} - \not{k}_2 - m_b} \gamma_{\mu_2} \right] \frac{mF_0}{M^2} + [\gamma_{\mu_2} O_{\mu_1}(P,k_1) + Q_{\mu_1}(P-k_2,k_1)\gamma_{\mu_2}] \frac{I_0}{M^2} + (1 \leftrightarrow 2) + (\text{higher-dimensional terms}) \ . \quad (9)$$

Here all the external fields in the decay $b \to s\gamma\gamma$ are on their mass shell, and higher-dimensional terms represent the rest in the series expansion. Now, the calculation of $k_1^{\mu_2} T_{1,\mu_1\mu_2}$ and $k_2^{\mu_2} T_{1,\mu_1\mu_2}$ are straightforward. We find that the most general solution to Eq. (3) that satisfies the constraints given by Eqs. (7) and (8) is

$$T_{2,\mu_1\mu_2} = -[\gamma_{\mu_2} O_{\mu_1}(P,k_1) + Q_{\mu_1}(P-k_2,k_1)\gamma_{\mu_2}] + (1 \leftrightarrow 2) + (\text{higher-dimensional terms}) \; . \qquad ((10)$$

Adding $T^{\alpha\beta}_{2,\mu_1\mu_2}$ to $T^{\alpha\beta}_{1,\mu_1\mu_2}$, we find that the off-shell terms are canceled out, resulting in a theorem

$$T_{\mu_1\mu_2} = i(Q_d e)^2 \frac{mF_0}{M^2} \left\{ \left[\frac{P'_{\mu_2}}{P'\cdot k_2} - \frac{P_{\mu_2}}{P\cdot k_2} \right] \sigma_{\mu_1\nu} k_1^\nu + \left[\frac{P'_{\mu_1}}{P'\cdot k_1} - \frac{P_{\mu_1}}{P\cdot k_1} \right] \sigma_{\mu_2\nu} k_2^\nu \right.$$
$$\left. - \frac{i}{2} \left[\left[\frac{1}{P'\cdot k_2} - \frac{1}{P\cdot k_1} \right] \sigma_{\mu_2\mu}\sigma_{\mu_1\nu} k_2^\mu k_1^\nu + \left[\frac{1}{P'\cdot k_1} - \frac{1}{P\cdot k_2} \right] \sigma_{\mu_1\mu}\sigma_{\mu_2\nu} k_1^\mu k_2^\nu \right] \right\}$$
$$+ (\text{higher-dimensional terms}) \; . \qquad (11)$$

Equation (11) is manifestly gauge invariant. Clearly, it holds for any flavor-changing charged-fermion two-photon decays in any theoretical models, provided the photon energy is much smaller than the energy scale responsible for the flavor-changing transition. One can see by inspection that Eq. (11) can simply be obtained by calculating the effective tree diagrams with the $b \to s\gamma$ vertex given by its dipole-moment form factor. That $T_{\mu_1\mu_2}$ ought to take the form shown in Eq. (11) is of course not surprising. We know that in momentum space, a gauge-invariant two-photon state FF or $\tilde{F}F$ must be bilinear in (external) momenta. To construct a gauge-invariant term that has the lowest dimension with respect to external momenta and fields, one has to introduce terms with external momenta in the denominator. Thus, the lowest-dimensional terms can always be represented in forms given by Eq. (11) with only the dipole moment to be determined by the specific (1) decay process and (2) theoretical model. As far as the lowest-dimensional terms are concerned, potentially complicated calculations can now be simplified to a much simpler calculation of the corresponding dipole-moment form factor.

It should be pointed out that F_0 may depend on the internal participating quark masses in a very complicated way, which we have so far ignored. Thus, there is no reason in general to expect that the lowest-dimensional terms given by Eq. (11) are the dominant terms. To assess their importance, complete model calculations are called for.

III. $b \to s\gamma\gamma$ IN THE STANDARD ELECTROWEAK MODEL

We now consider the decay $b \to s\gamma\gamma$ in the standard electroweak model. The complete answer to the amplitude of $b \to s\gamma\gamma$ can be written as

$$T_{\mu_1\mu_2} = \sum_{j=u,c,t} V_{bj} V^*_{sj} T_{j,\mu_1\mu_2} \; , \qquad (12)$$

where V is the KM matrix and T_j $(j=u,c,t)$ represents contributions from an internal u- or c- or t-quark exchange.

For $T_{t,\mu_1\mu_2}$, at one-loop level, all the external momenta are much smaller than the internal particle masses. Consequently, the usual zero external momentum approximation can be applied. At this point, it is however important to realize that for $T_{c,\mu_1\mu_2}$ and particularly for $T_{u,\mu_1\mu_2}$ such a convenient approximation is no longer applicable, because now the internal quark masses are much smaller than the external momenta, which are typically of the order of m_b. Thus, these graphs have to be treated separately. Details of the calculations for the light-quark contributions can be found in Appendix B.

By a unitarity condition, $\sum_j V_{bj} V^*_{sj} = 0$, Eq. (12) can now be conveniently rewritten as

$$T_{\mu_1\mu_2} = V_{bu} V^*_{su} (T_{u,\mu_1\mu_2} - T_{c,\mu_1\mu_2}) + V_{bt} V^*_{st} (T_{t,\mu_1\mu_2} - T_{c,\mu_1\mu_2}) \; . \qquad (13)$$

To a good approximation, the first term in Eq. (13) can be ignored because of the small value of $V_{bu} V^*_{su}$. As we already mentioned, contributions to $T_{\mu_1\mu_2}$ can be classified in terms of their dimensionality with respect to the external momenta and fields. To the order of $1/M_W^2$, we find that[9] (details can be found in Appendix B) that the lowest-dimensional contributions are indeed given by Eq. (11). The dipole moment form factor $D \equiv mF_0/M^2$ is

$$D(m_c^2) = -\frac{\sqrt{2}\, G_F}{\pi^2} (m_b R + m_s L) \tfrac{23}{48} \qquad (14)$$

for $T_{c,\mu_1\mu_2}$ and

$$D(m_t^2) = \frac{\sqrt{2}\, G_F}{\pi^2} (m_b R + m_s L) \frac{C}{16} \qquad (15)$$

for $T_{t,\mu_1\mu_2}$, where the charge factors have been factorized according to Eq. (1) and C, corresponding to $4C_2 - C_6$ in Ref. 4, is a function of $x = m_t^2/M_W^2$ given by

$$C = \frac{22x^3 - 153x^2 + 159x - 46}{6(1-x)^3} + \frac{3(2-3x)x^2}{(1-x)^4} \ln x \; . \qquad (15a)$$

Higher-dimensional terms $T'_{\mu_1\mu_2}$ which are generated from the one-particle-irreducible graphs, will also contribute to the decay. For a heavy top, i.e., $m_t \gtrsim M_W$, the only term of the order of $1/M_W^2$ comes from the diagram with an internal c quark and W. The result of the calculation is

$$T'_{\mu_1\mu_2} = \frac{i16\sqrt{2}}{9\pi} G_F \alpha R_{\mu_1\mu_2\sigma} \gamma^\sigma L I \ , \tag{16}$$

where

$$R_{\mu_1\mu_2\sigma} = k_{1,\mu_2} \epsilon_{\mu_1\nu\rho\sigma} k_1^\nu k_2^\rho - k_{2,\mu_1} \epsilon_{\mu_2\nu\rho\sigma} k_1^\nu k_2^\rho + k_1 \cdot k_2 \epsilon_{\mu_1\mu_2\nu\sigma} (k_1 - k_2)^\nu \tag{17}$$

is the third rank tensor, which is symmetric under the interchange $1 \leftrightarrow 2$, constructed originally by Rosenberg[10] and Adler.[11] The function I is given by

$$I = \int_0^1 dx \int_0^{1-x} dy \frac{xy}{m_c^2 - 2xyk_1 \cdot k_2 - i\epsilon} \ . \tag{18}$$

This part of the calculation is very similar to those reported in Refs. 5 and 6. The final result is a sum of Eqs. (11) and (16) according to Eq. (13).

IV. IMPLICATIONS FOR THE DECAY $B_s \to \gamma\gamma$

Having established the amplitude of $b \to s\gamma\gamma$ we now consider the decay $B_s \to \gamma\gamma$. As part of our efforts of estimating short-distance contributions, in what follows we will model the B_s meson by assuming, for simplicity, that both b and $\bar{s}$ are static.[12] The radiation takes place by the annihilation of b and $\bar{s}$. In terms of this picture, the quark masses should take values given by their constituent (rather than current) masses, i.e., $m_b \approx m_B$ and $m_s \approx m_K$. We expect that such a simplified meson picture should at least provide a correct order-of-magnitude estimate. In any case we have, from Eqs. (11), (13), and (16),

$$T(B_s \to \gamma\gamma) \approx iAF_{\mu\nu}\tilde{F}^{\mu\nu} + BF_{\mu\nu}F^{\mu\nu} \tag{19}$$

with

$$A = \frac{\sqrt{2}}{\pi^2} G_F f_B m_B^2 (eQ_d)^2 V_{bt} V_{st}^* \times \left[I(m_c^2) - \frac{1}{32 m_B m_K} (C + \tfrac{23}{3}) \right] \tag{20}$$

and

$$B = -\frac{\sqrt{2}\, m_B}{32\pi^2 m_K} G_F f_B (eQ_d)^2 V_{bt} V_{st}^* (C + \tfrac{23}{3}) \ . \tag{21}$$

Here we have written the hadronic matrix element as

$$\langle 0 | \bar{s} \gamma_\mu \gamma_5 b | B_s(P) \rangle = -i f_B P_\mu \ . \tag{22}$$

The form factor f_B is probably of the order of 200 MeV in view of lattice calculations. Also, following Eq. (22) we estimated

$$\langle 0 | \bar{s} \gamma_5 b | B_s \rangle = i \frac{f_B m_B^2}{m_b + m_s} \approx i f_B m_B \ . \tag{23}$$

Notice, Eq. (21) and the last term of Eq. (20) arise from $D(m_c^2) - D(m_t^2)$ [see Eqs. (14) and (15)]. According to our classifications, these terms represent the lowest-dimensional contributions. They contain, as shown explicitly by Eqs. (19)–(21), equal amounts of FF and $F\tilde{F}$. Terms proportional to $I(m_c^2)$ come from the higher-dimensional operators. The integral $I(m_c^2)$, which is defined originally in Eq. (18), can be carried out analytically. In the case of $B_s \to \gamma\gamma$ (implying $2k_1 \cdot k_2 = m_B^2$), the result of the calculation is

$$I(m_c^2) = -\frac{1}{2m_B^2} - \frac{m_c^2}{2m_B^4} \left[\ln^2 \left[\frac{1+\beta}{1-\beta} \right] - \pi^2 - 2i\pi \ln \left[\frac{1+\beta}{1-\beta} \right] \right] , \tag{24a}$$

where $\beta = \sqrt{1 - 4m_c^2/m_B^2}$.

The decay width of $B_s \to \gamma\gamma$ is then given by

$$\Gamma(B_s \to \gamma\gamma) = \frac{m_B^2}{16\pi} (A^*A + B^*B) \ . \tag{24b}$$

As the higher dimensional terms [see Eq. (16)] are m_t independent, the dependence of $\Gamma(B_s \to \gamma\gamma)$ arises primarily from the lowest-dimensional terms via the flavor-changing dipole-moment form factor. Taking $V_{bt} V_{st}^* = 5 \times 10^{-2}$, the influence of m_t on the decay width is plotted in Fig. 1. One can see that it varies from 2×10^{-11} eV for $m_t = 40$ GeV to 8.5×10^{-11} eV for $m_t = 200$ GeV. Formally it reaches a limit $\sim 1.8 \times 10^{-10}$ eV as $m_t \to \infty$ and asymptotically approaches to $\sim 1.8 \times 10^{-11}$ eV as $m_t \ll M_W$. The upper limit arises because for large m_t, the flavor-changing dipole-moment form factor and hence $\Gamma(B_s \to \gamma\gamma)$ become independent[13] of m_t. The lower limit follows because for $m_t \ll M_W$, contributions from the higher-dimensional terms (which are m_t independent) take over those from the lowest-dimensional terms, which are now suppressed due to a

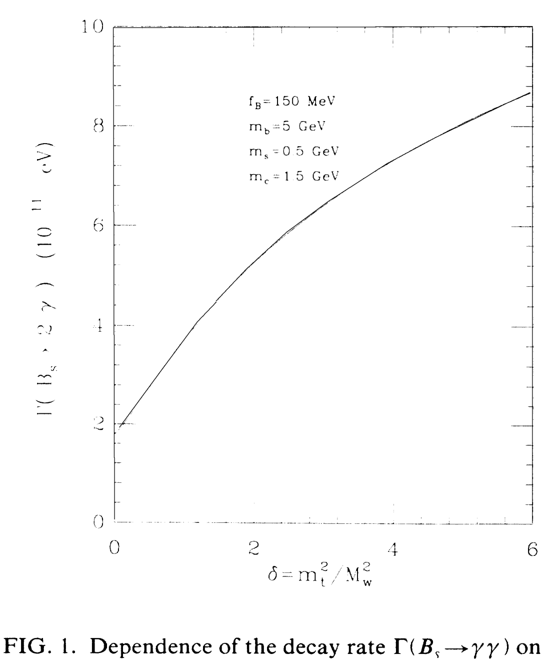

FIG. 1. Dependence of the decay rate $\Gamma(B_s \to \gamma\gamma)$ on m_t.

strong Glashow-Iliopoulos-Maiani cancellation. The situation here resembles that in the studies of $\mu \to e\gamma\gamma$ (Ref. 14) and $\nu' \to \nu\gamma\gamma$ (Ref. 15), where it was pointed out that in some theoretical models it is possible to have a two-photon decay exceeding a one-photon decay, even though a two-photon decay appears to be suppressed additionally by a small factor α/π.

Now, assuming the decay width of B_s is the same as the total decay width of B (including $B_d, B_s, \ldots$) Γ_B, it then follows from the experimental value of $\Gamma_B = 5 \times 10^{-4}$ eV we have roughly

$$B(B_s \to \gamma\gamma) \sim 10^{-7} . \tag{25}$$

Should future experiments find that the width of B_s is much smaller than Γ_B, the result given by Eq. (25) should, of course, be scaled up accordingly.

Besides measuring the decay width, another "cleaner" way of exploring the influence of m_t on the decay $B_s \to \gamma\gamma$ would be to measure the photon spin polarizations. Indeed, we can rewrite Eq. (19) in the rest frame of B_s as

$$T(B_s \to \gamma\gamma) = 2m_B A \mathbf{k}_1 \cdot [\boldsymbol{\epsilon}_1(k_1) \times \boldsymbol{\epsilon}_2(k_2)] + m_B^2 B[\boldsymbol{\epsilon}_1(k_1) \cdot \boldsymbol{\epsilon}_2(k_2)] . \tag{26}$$

Here $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$ are two transverse photon polarizations. As we already mentioned that in the limit $m_t \ll M_W$, one has $B \ll A$. As a consequence, the decay final state would be in a configuration with the photon spin polarizations given by $\mathbf{k}_1 \cdot [\boldsymbol{\epsilon}_1(k_1) \times \boldsymbol{\epsilon}_2(k_2)]$. In contrast, for $m_t \gtrsim M_W$, one would have a mixture of $\mathbf{k}_1 \cdot [\boldsymbol{\epsilon}(k_1) \times \boldsymbol{\epsilon}(k_2)]$ and $\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_2$ because now B and A become equally important. The influence of m_t on the mixing is plotted in Fig. 2. One can see that for large m_t the mixture can reach to 50%. Potentially this could lead to interesting discussions in the study of CP violation.[16,17]

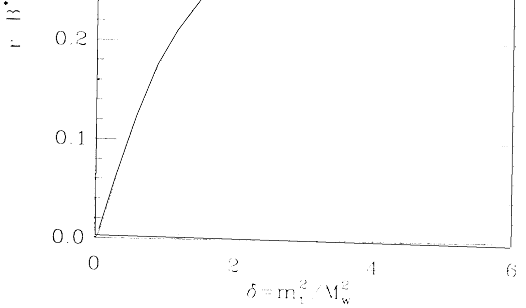

FIG. 2. Dependence of the ratio B^*B/A^*A on m_t.

We should point out that, unlike in $b \to s\gamma$, no attempt has yet been made to include QCD corrections to the decay $b \to s\gamma\gamma$. First, all operators appearing here are highly nonlocal. Second, the complete set of the operators are rather large. Although the lowest-dimensional terms are related to $b \to s\gamma$, it is not clear whether the results with QCD corrections for $b \to s\gamma$ can be used for $b \to s\gamma\gamma$. This is because, for the two-photon decay, one of the quark lines connecting to the $b \to s\gamma$ vertex is necessarily off shell, whereas the result in existing literature[18] for $b \to s\gamma$ is obtained by assuming both lines on shell. Even so, it is fairly reasonable to believe that our estimate gives the correct order of magnitude.

Finally, what are the prospects for observing rare decays such as $B_s \to \gamma\gamma$ at a branching ratio of the order of 10^{-7}? The prospect of searching for rare B decays at the Superconducting Super Collider (SSC) has been a subject of discussion in the past few years. It has been estimated[19] that about 5×10^8 usable B_s could be produced at SSC annually. In addition, a program[20] proposed to upgrade the PEP storage ring at SLAC could, if materialized, reach the production rate of 3×10^8 pairs of $B_s \bar{B}_s$ a year. Spectrometers especially suited for rare B_s decays have also been designed (Refs. 19 and 20). It is therefore feasible to search for $B_s \to \gamma\gamma$ in these machines. Besides, a much larger branching ratio for $B_s \to \gamma\gamma$ is expected for some nonstandard theoretical models (i.e., minimal supersymmetric and two-Higgs-doublet models). Thus, any observation of such a decay at an unusual rate would be an indication of new physics.

V. CONCLUSION

We have calculated the decay $b \to s\gamma\gamma$ in the standard electroweak model explicitly. Its implications, particularly the influence of m_t, on the decay $B_s \to \gamma\gamma$ are investigated. We find that the branching ratio for a heavy top (~ 200 GeV), which is approximately of the order of 10^{-7}, is about four times larger than that for a light top ($\lesssim 40$ GeV). In particular, the photon spin polarizations in these two limits become distinctively different. Specifically, for a light top, the photons would be in a state with spin polarizations given by $\mathbf{k}_1 \cdot [\boldsymbol{\epsilon}_1(k_1) \times \boldsymbol{\epsilon}_2(k_2)]$, whereas for a heavy top they would be in a mixture of $\mathbf{k}_1 \cdot [\boldsymbol{\epsilon}_1(k_1) \times \boldsymbol{\epsilon}_2(k_2)]$ and $\boldsymbol{\epsilon}_1(k_1) \cdot \boldsymbol{\epsilon}_2(k_2)$. The mixing could reach to 50% depending on the choice of m_t. Thus, a precise measurement on the photon spin polarizations of the decay should provide useful information on the value of m_t.

Another interesting result obtained in our study of $b \to s\gamma\gamma$ is to establish[21] a non-soft-bremsstrahlung extension of the low-energy theorem. Thus, contributions to a flavor-changing two-photon decay with the lowest powers of external momenta can be simply calculated in terms of a flavor-changing one-photon decay and external bremsstrahlungs. The only condition which needs to be satisfied is that all the external momenta of the decay are much smaller than the energy scale responsible for the flavor-changing transition, which in our case is of the order of M_W. As long as this condition is satisfied, the lowest-dimensional terms of any flavor-changing two-photon decays (for example, $\mu \rightarrow e\gamma\gamma$, $e \rightarrow \mu\gamma\gamma, \ldots$) are simply related [see Eq. (11)] to the flavor-changing dipole-moment form factor, which in turn is determined by the specific (1) decay process and (2) theoretical model. Our new theorem makes the computations of a flavor-changing two-photon decay fairly simple.

ACKNOWLEDGMENTS

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APPENDIX A: FEYNMAN RULES AND WARD IDENTITIES

For convenience we perform our calculation in the background-field gauge.[22] In this gauge, the gauge-fixing terms are given by

$$L_{gf} = -(D_\mu W_\mu^+ - iM_W \phi^+)(D_\mu^\dagger W_\mu^- + iM_W \phi^-) - (\partial_\mu A_\mu)^2/2 - (\partial_\mu Z_\mu + M_Z \phi^0)^2/2 \ .$$

Here $D_\mu = \partial_\mu - ieA_\mu$ and $\phi^{0,\pm}$ are Goldstone bosons in the Higgs sector. Except for the covariant derivatives, this gauge-fixing function is identical to that in the 't Hooft–Feynman gauge. An interesting feature of the background-field gauge is that there is no $A\phi W$-type vertices. The $AAWW$ and AWW couplings now differ from those in the more familiar 't Hooft–Feynman gauge. For completeness we list part of the Feynman rules in Fig. 3.

To derive Ward identifies in this gauge,[23] one may proceed by using the conservation of electromagnetic current, which is of course maintained by this particular choice of gauge. For simplicity we skip the details and present our results in terms of diagrams as depicted in Fig. 4. If one replaces W bosons in the internal loops by Goldstone bosons, all the identities remain valid. These identities are very useful for checking our calculations.

APPENDIX B: EXPLICIT CALCULATIONS

With the necessary machineries developed in Appendix A, we can now carry out the calculations in a straightforward way.[24] Our results can easily be checked by Ward identities derived in Appendix A. In this appendix we will only present some details of the light-quark (u,c) contributions $T_{u,c,\mu_1\mu_2}$ [for definitions see Eq. (13)], heavy-quark contributions $T_{t,\mu_1\mu_2}$ can be computed in a similar manner or by applying the operator expansion method introduced in our earlier paper (Ref. 4). In the gauge we use, Feynman graphs contributing to the decay $b \rightarrow s\gamma\gamma$ are listed in Fig. 5.

1. *One-particle-irreducible diagrams*

We begin by considering the one-particle-irreducible graphs. For light-quark contributions, graphs with an unphysical-Higgs-boson exchange can be ignored. To the order of $1/M_W^2$ our results of the calculation for the different graphs are (for simplicity, the KM matrix elements are not shown explicitly)

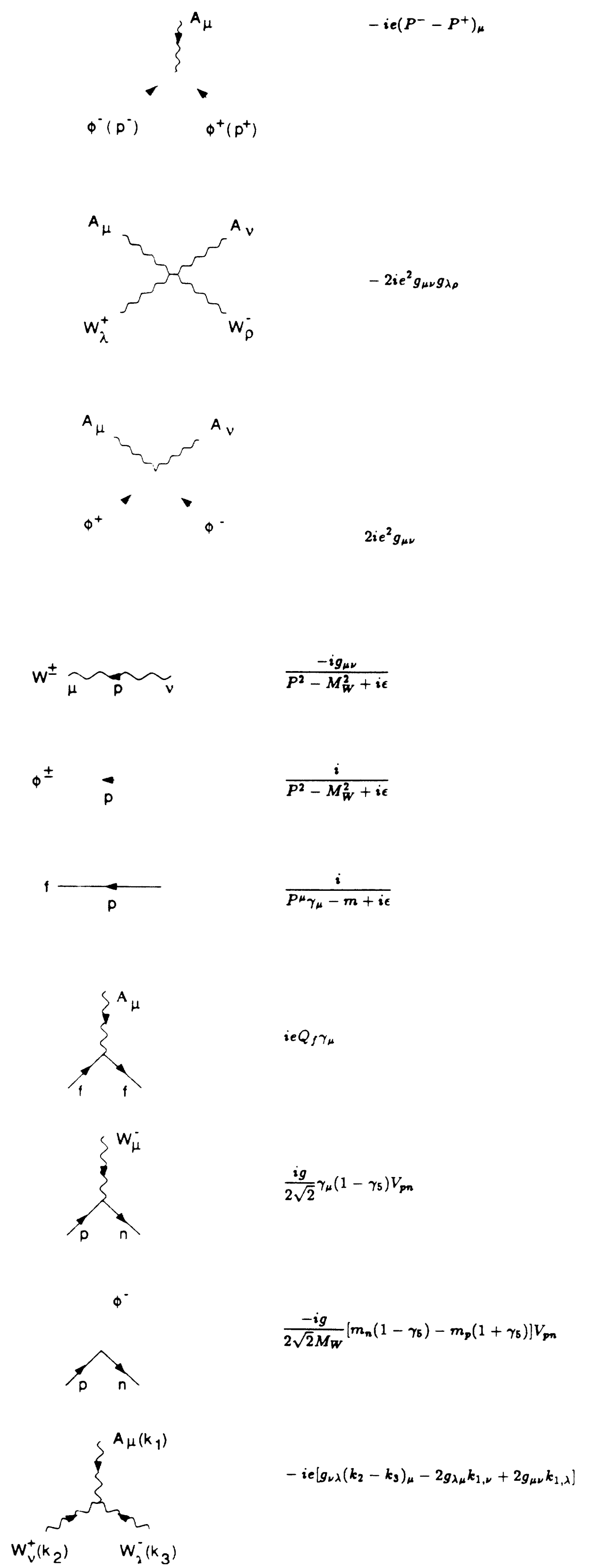

FIG. 3. Feynman rules in the background-field gauge, where $p = (u,c,t)$ and $n = (d,s,b)$. V_{pn} is the corresponding KM matrix element.

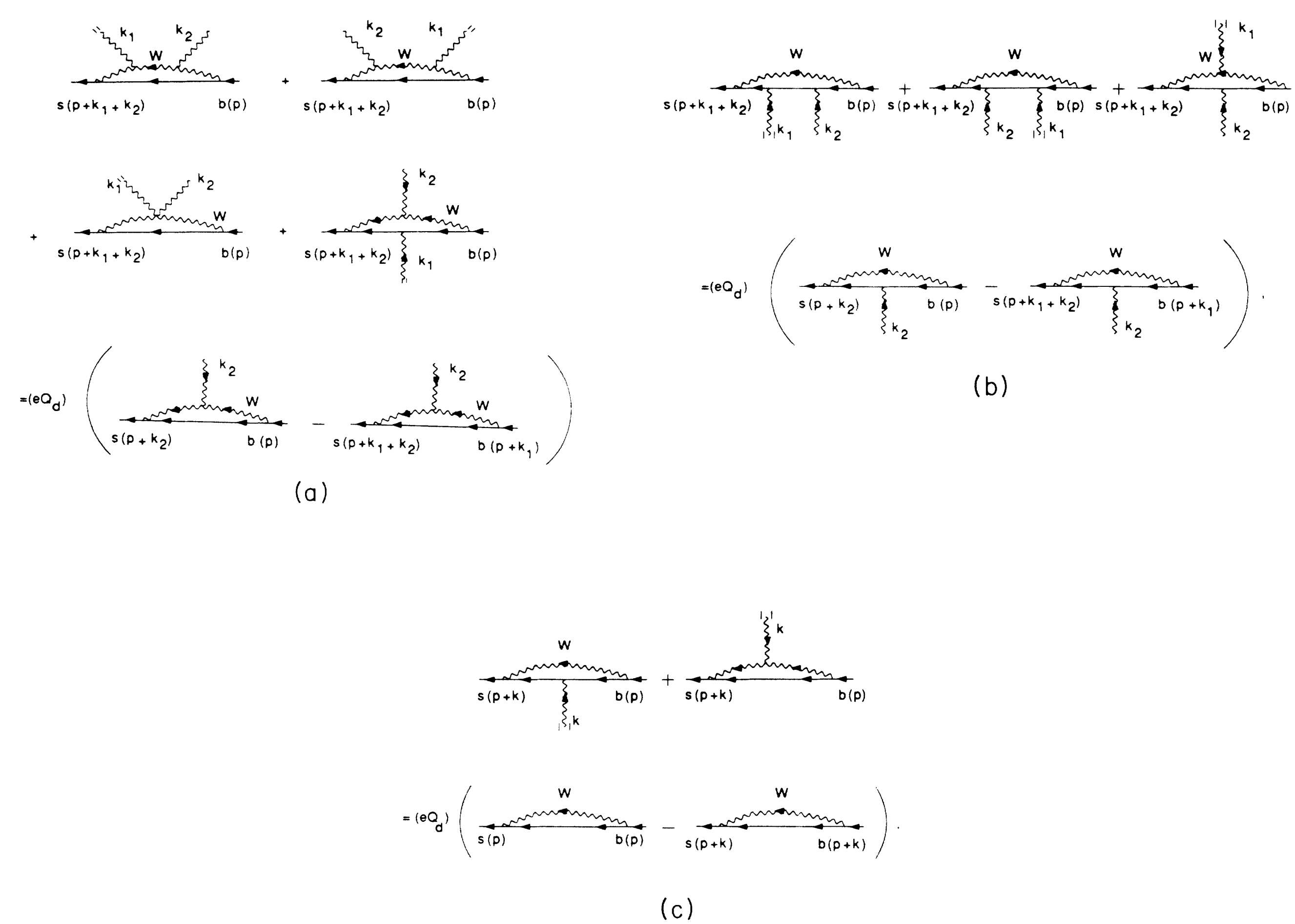

FIG. 4. Diagrammatical expressions of the Ward identities in the background-field gauge. The double lines on the photon line symbolize the contraction of the proper vertex with photon momentum. Also, $Q_d=-\frac{1}{3}$.

$$\begin{aligned}
&[\text{Fig. 5(a)}]\frac{e^2G_F}{27\sqrt{2}\pi^2}[g_{\mu_1\mu_2}(\not{P}+\not{k}_1)+\gamma_{\mu_1}(2P_{\mu_2}+k_{1,\mu_2})-\tfrac{3}{2}i\epsilon_{\mu_1\mu_2\mu\rho}(k_1-k_2)^\rho\gamma^\mu+12iR_{\mu_1\mu_2\mu}\gamma^\mu I(m^2)]L+(1\leftrightarrow 2)\;,\\
&[\text{Fig. 5(b)}]\frac{e^2G_F}{12\sqrt{2}\pi^2}[-g_{\mu_1\mu_2}(2\not{P}+\not{k}_1+\not{k}_2)+\gamma_{\mu_1}(2P_{\mu_2}+k_{1,\mu_2})+3i\epsilon_{\mu_1\mu_2\mu\rho}(k_1-k_2)^\rho\gamma^\mu]L+(1\leftrightarrow 2)\;,\\
&[\text{Fig. 5(c)}]\frac{e^2G_F}{4\sqrt{2}\pi^2}g_{\mu_1\mu_2}(\not{P}+\not{k}_1)L+(1\leftrightarrow 2)\;,\\
&[\text{Fig. 5}(d)]\frac{-e^2G_F}{9\sqrt{2}\pi^2}[g_{\mu_1\mu_2}(\not{P}+\not{k}_1)+\gamma_{\mu_1}(2P_{\mu_2}+k_{1,\mu_2})-\tfrac{1}{2}i\epsilon_{\mu_1\mu_2\mu\rho}(6k_2+3k_1)^\rho\gamma^\mu]L+(1\leftrightarrow 2)\;.
\end{aligned} \tag{B1}$$

These results are symmetric under the interchange $(1\leftrightarrow 2)$. Terms antisymmetric under the interchange cancel after adding together the corresponding exchange diagrams. In (B1), $R_{\mu_1\mu_2\mu}$ and $I(m^2)$ are given by Eqs. (17) and (18), respectively. The sum of the 1PI contributions is

$$T'_{\mu_1\mu_2}+\delta T'_{\mu_1\mu_2}\;, \tag{B2}$$

where $T'_{\mu_1\mu_2}$, defined in Eq. (16), is the higher-dimensional gauge-invariant terms, and $\delta T'_{\mu_1\mu_2}$ is given by

$$\delta T'_{\mu_1\mu_2}=\frac{G_F\alpha}{27\sqrt{2}\pi}[g_{\mu_1\mu_2}(\not{P}+\not{k}_1)+\gamma_{\mu_1}(2P_{\mu_2}+k_{1,\mu_2})+12i\epsilon_{\mu_1\mu_2\mu\rho}(k_1-k_2)^\rho\gamma^\mu]L+(1\leftrightarrow 2)\;. \tag{B3}$$

This term is not gauge invariant. Thus, it must be canceled by terms generated from the one-particle-reducible (IPR) diagrams which we discuss below.

2. One-particle-reducible diagrams

For the 1PR diagrams it is convenient to compute the irreducible subdiagram first, and then calculate the remaining effective tree graphs.

For the renormalized flavor-changing self-energy $\Sigma_{\rm ren}$ we find

$$\Sigma_{\rm ren}(P)=\frac{G_F}{12\sqrt{2}\pi^2}\Lambda_s(P)(\not{P}R+m_bL+m_sR)\Lambda_b(P)\ , \tag{B4}$$

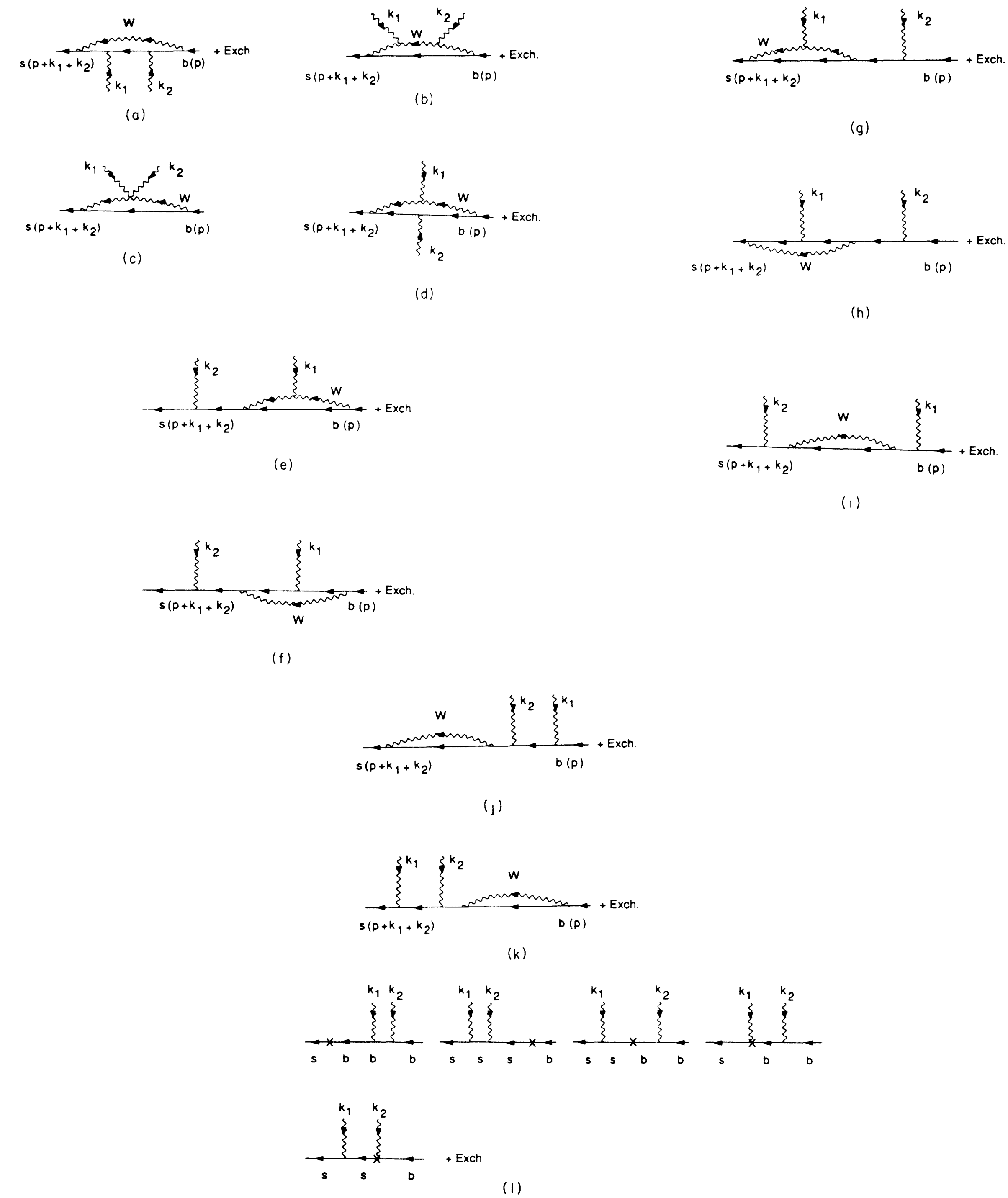

FIG. 5. One-loop diagrams contributing to the decay $b \to s\gamma\gamma$. For simplicity, diagrams with unphysical-Higgs-boson exchanges are not shown here explicitly.

where

$$\Lambda_{b,s}(P)=\not{P}-m_{b,s} \tag{B5}$$

are the Dirac projections which satisfy $\bar{s}\Lambda_s=0$ and $\Lambda_b b=0$. $\Sigma_{\rm ren}$ is generated from the one-loop self-energy diagram and its counterterm diagram. Clearly, it satisfies the on-shell conditions [Eq. (5)] required by our renormalization procedure.[25]

For the renormalized $b\to s\gamma$ transition vertex $\Gamma^{\mu}_{\rm ren}(P,k)$, which is generated from the one-loop and their corresponding counterterm graphs, we have

$$\Gamma^{\mu}_{\rm ren}(P,k)=-\frac{ieG_F}{\sqrt{2}\pi^2}\frac{23}{72}\sigma^{\mu\nu}k_\nu(m_bR+m_sL)+\Gamma'^{\mu}\ , \tag{B6}$$

due to an internal charm quark. The first term is the usual flavor-changing transition dipole moment. Γ'^{μ} represents the "off-shell" terms because it vanishes if the b and s quarks connecting to the vertex are on shell. For a real photon, Γ'^{μ} is given by

$$\Gamma'^{\mu}(P,k)=\frac{-eG_F}{72\sqrt{2}\pi^2}\{\Lambda_s(P+k)[\gamma^{\mu}(m_bR+m_sL)+(\not{P}+\not{k}+m_s)\gamma^{\mu}L+24\not{k}\gamma^{\mu}L+2P^{\mu}L]$$
$$+[(m_bR+m_sL)\gamma^{\mu}+R\gamma^{\mu}(\not{P}+m_b)+24\not{k}\gamma^{\mu}R+2P^{\mu}R]\Lambda_b(P)\}\ . \tag{B7}$$

Here terms proportional to $\Lambda_s(P+k)\Lambda_b(P)$ have been ignored because they will not contribute to the decay $b\to s\gamma\gamma$ if the external quarks are on shell. Evidently, $\Gamma^{\mu}_{\rm ren}$ satisfies the Ward-Takahashi identity [Eq. (4)].

Similar expressions generated from t-quark exchanges can be found in Ref. 4.

Now, it is straightforward to show that contributions from the 1 PR diagrams due to $\Sigma_{\rm ren}$ and Γ'^{μ} cancel $\delta T'_{\mu_1\mu_2}$ (B3). The only terms which survive after this cancellation arise from the effective tree graphs [corresponding to Figs. 5(e)–5(h) and their counterterm diagrams] with the flavor-changing $b\to s\gamma$ vertex given by the dipole-moment term (B6) only. Including the top-quark contributions, the result is shown in Eq. (11) with the dipole-moment form factor given by Eqs. (14) and (15).

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