Multipole expansion in quantum chromodynamics and the radiative decays $J/\psi \to \gamma + \eta$ and $J/\psi \to \gamma + \pi^0$

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A generalized formalism of QCD multipole expansion including constituent-quark-field quantization is proposed. It can be applied to study soft-gluon emission processes of heavy-quark–antiquark systems with quark-flavor changing or quark pair annihilation. As an application, the radiative decays $J/\psi \to \gamma + \eta$ and $J/\psi \to \gamma + \pi^0$ are studied in this framework. It is shown that $J/\psi \to \gamma + \eta$ is dominated by a vector-meson-dominance mechanism in which the vector mesons are $n\,^3S_1$ states of $c\bar{c}$ and the calculated rate is in agreement with the experiment. The decay $J/\psi \to \gamma + \pi^0$ is dominated by a different vector-meson-dominance mechanism in which the vector meson is ρ^0.

I. INTRODUCTION

Multipole expansion in quantum chromodynamics (QCD) has been studied by many authors.[1,2] A gauge-invariant formalism in terms of a constituent quark field is given by Yan.[2] This has proved to be a powerful tool for studying soft-gluon emission in hadronic transitions in the heavy-quark (Q) and antiquark ($\bar{Q}$) systems.[3,4] Let a be the size of the heavy-$Q\bar{Q}$ system and k the typical momentum of the emitted gluons. The expansion parameter in QCD multipole expansion is ka, so that the expansion is good when ka is small, even though the QCD coupling constant g_s is large. It is shown in nuclear physics that this expansion works even when $ka \sim 1$. In hadronic transitions, the quarks in the initial- and final-quarkonium states are the same. Therefore the transition rates can be calculated in the standard way of quantum mechanics without quantizing the quark field.[2,3] If we want to generalize the idea of QCD multipole expansion to study soft-gluon emission in processes with quark-flavor changing or quark pair annihilation, quantization of the quark field will be necessary. In this paper we will generalize Yan's formalism[2] to include electroweak interactions and quark-field quantization, and will derive a general formula for the S-matrix element in the multipole-expansion approach which can be applied to various decay processes. These will be given in Sec. II of this paper.

As an application of our general formula we calculate the rates $\Gamma(J/\psi \to \gamma\eta)$ and $\Gamma(J/\psi \to \gamma\pi^0)$ in Sec. III. We shall see that the dominant mechanisms in the two decays are different. The decay $J/\psi \to \gamma + \eta$ is dominated by a vector-meson-dominance mechanism in which the vector mesons are $n\,^3S_1$ states of $c\bar{c}$ and the calculated $\Gamma(J/\psi \to \gamma\eta)$ is in agreement with the experiment. On the contrary, the decay $J/\psi \to \gamma + \pi^0$ is dominated by a different vector-meson-dominance mechanism with the ρ meson as the vector meson, as was first calculated by Fritzsch and Jackson.[5]

A brief concluding remark will be given in Sec. IV.

II. GENERALIZED FORMALISM OF QCD MULTIPOLE EXPANSION WITH QUANTIZED QUARK FIELD

Let us consider a system consisting of a heavy quark Q and its antiquark $\bar{Q}$. (Generalization to a system with different quarks is trivial.) The fundamental Lagrangian of QCD is

$$\mathcal{L}_{\rm QCD} = -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{\psi}\left[\gamma^\mu\left[i\partial_\mu - g_s\frac{\lambda_a}{2}A^a_\mu\right] - m\right]\psi + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}\,, \qquad (1)$$

where ψ is the field of Q, A^a_μ ($a = 1, \ldots, 8$) is the gluon field, λ_a is the SU(3) Gell-Mann matrix, $\mathcal{L}_{\rm GF}$ and $\mathcal{L}_{\rm FP}$ are the gauge-fixing term and the gauge-compensation (Faddeev-Popov) term, respectively. Starting from (1), Yan obtained the gauge-invariant effective Lagrangian for the heavy-quark system suitable for studying soft-gluon emissions[2]

$$L_Q = \int d^3x\, \bar{\Psi}\left[\gamma^\mu\left[i\partial_\mu - g_s\frac{\lambda_a}{2}A^{a\prime}_\mu\right] - m\right]\Psi$$
$$-\tfrac{1}{2}\int d^3x\, d^3y \sum_{a=0}^{8} \bar{\Psi}(\mathbf{x},t)\gamma^0\tfrac{1}{2}\lambda_a\Psi(\mathbf{x},t)[\delta_{a0}V_1(|\mathbf{x}-\mathbf{y}|) + (1-\delta_{a0})V_2(|\mathbf{x}-\mathbf{y}|)]\bar{\Psi}(\mathbf{x},t)\gamma^0\tfrac{1}{2}\lambda_a\Psi(\mathbf{x},t) \qquad (2)$$

in which $\frac{1}{2}\lambda_0 \equiv 1$, V_1 and V_2 are static potentials related to the interactions between Q and $\bar{Q}$ in color-singlet and color-octet states, $\Psi(\mathbf{x},t)$ and $A_\mu^{a\prime}(\mathbf{x},t)$ are related to the original $\psi(x)$ and $A_\mu^a(x)$ by

$$\Psi(\mathbf{x},t)=U^{-1}(\mathbf{x},t)\psi(x) , \tag{3a}$$

$$\frac{1}{2}\lambda_a A_\mu^{a\prime}(\mathbf{x},t)= U^{-1}(\mathbf{x},t)\frac{1}{2}\lambda_a A_\mu^a(x)U(\mathbf{x},t) - \frac{i}{g_s}U^{-1}(\mathbf{x},t)\partial_\mu U(\mathbf{x},t) , \tag{3b}$$

with $U(\mathbf{x},t)$ defined by

$$U(\mathbf{x},t)=P\exp\left[ig_s\int_0^{\mathbf{x}} d\mathbf{x}'\cdot\frac{\lambda_a}{2}\mathbf{A}^a(\mathbf{x}',t)\right] , \tag{3c}$$

where P is the path-ordering operation and the line integral is along the straight-line segment connecting the two ends. In (3c) $\mathbf{x}=0$ is identified with the center of mass (cm) of the $Q\bar{Q}$ system. We see from (2) that it is the field Ψ, not ψ, that represents Q and $\bar{Q}$ interacting via the static potential. Therefore Ψ is regarded as the constituent quark field in the potential model of heavy quarkonia.

Expanding $A_\mu^{a\prime}(\mathbf{x},t)$ in powers of $\mathbf{x}$ we get the multipole expansion.[2]

Now we generalize (2) to include electroweak interactions and Ψ-field quantization. We add the electromagnetic (e.m.) interaction

$$\mathcal{L}_{\rm em}=-e\bar{\psi}\gamma^\mu Q\mathcal{A}_\mu\psi \tag{4}$$

and weak interactions

$$\mathcal{L}_W=-\frac{g}{\sqrt{2}}\bar{\psi}\gamma^\mu\frac{1-\gamma_5}{2}(t_+W_\mu^+ + t_-W_\mu^-)\psi - \frac{g}{\cos\theta_W}\bar{\psi}\gamma^\mu\left[\frac{1-\gamma_5}{2}t_3-\sin^2\theta_W Q\right]Z_\mu\psi \tag{5}$$

to (1). Here e is the electromagnetic coupling constant, Q is the electric charge operator of the quark ($Q=\frac{2}{3}$ for c quark, $Q=-\frac{1}{3}$ for b quark), $\mathcal{A}_\mu$ is the photon field, g is the SU(2) coupling constant, θ_W is the Weinberg angle, $W_\mu^\pm, Z_\mu$ are the intermediate-vector-boson fields, and $t_\pm, t_3$ are SU(2) generators. The total Lagrangian is then

$$\mathcal{L}=\mathcal{L}_{\rm QCD}+\mathcal{L}_{\rm em}+\mathcal{L}_W . \tag{6}$$

Let $\mathbf{x}_1$ and $\mathbf{x}_2$ be the space coordinates of Q and $\bar{Q}$, respectively. The coordinate of the c.m. is

$$\mathbf{X}=\frac{1}{2}(\mathbf{x}_1+\mathbf{x}_2) .$$

Translational invariant of the system with respect to $\mathbf{X}$ leads to the conservation of momentum. We will not take $\mathbf{X}=0$ in the following discussions.

In most soft-gluon emission processes, the emitted gluon energy is much smaller than the quarkonium masses. We shall neglect the small recoil of the quarkonium due to the emission and regard $\mathbf{X}$ as a constant. The transformation (3c) is now written as

$$U(\mathbf{x},t)=P\exp\left[ig_s\int_{\mathbf{X}}^{\mathbf{x}} d\mathbf{x}'\cdot\frac{\lambda_a}{2}\mathbf{A}^a(\mathbf{x}',t)\right] \tag{7}$$

and $\Psi(\mathbf{x},t)$ and $A_\mu^{a\prime}(\mathbf{x},t)$ are given in (3a) and (3b). Since the electroweak sector is independent of the QCD sector, the ψ's in (4) and (5) can be transformed directly into Ψ's by (3a), and thus the total effective Lagrangian is

$$L_Q=L_{\rm QCD}^{(0)}+L_{\rm QCD}^{(1)}+L_{\rm em}+L_W , \tag{8a}$$

where

$$L_{\rm QCD}^{(0)}=\int d^3x\,\bar{\Psi}(\mathbf{x},t)(i\partial\!\!\!/-m)\Psi(\mathbf{x},t)-\frac{1}{2}\int d^3x_1 d^3x_2\sum_{a=0}^{8}\bar{\Psi}(\mathbf{x}_1,t)\gamma^a\frac{\lambda_a}{2}\Psi(\mathbf{x}_1,t) \times[\delta_{a0}V_1(|\mathbf{x}_1-\mathbf{x}_2|)+(1-\delta_{a0})V_2(|\mathbf{x}_1-\mathbf{x}_2|)]\bar{\Psi}(\mathbf{x}_2,t)\gamma^0\frac{\lambda_a}{2}\Psi(\mathbf{x}_2,t) , \tag{8b}$$

$$L_{\rm QCD}^{(1)}=-g_s\int d^3x\,\bar{\Psi}(\mathbf{x},t)\gamma^\mu\frac{\lambda_a}{2}A_\mu^{a\prime}(\mathbf{x},t)\Psi(\mathbf{x},t) , \tag{8c}$$

$$L_{\rm em}=-e\int d^3x\,\bar{\Psi}(\mathbf{x},t)\gamma^\mu Q\mathcal{A}_\mu(x)\Psi(\mathbf{x},t) , \tag{8d}$$

$$L_W=-\int d^3x\left[\frac{g}{\sqrt{2}}\bar{\Psi}(\mathbf{x},t)\gamma^\mu\frac{1-\gamma_5}{2}[t_+W_\mu^+(x)+t_-W_\mu^-(x)]\Psi(\mathbf{x},t) + \frac{g}{\cos\theta_W}\bar{\Psi}(\mathbf{x},t)\gamma^\mu\left[\frac{1-\gamma_5}{2}t_3-\sin^2\theta_W Q\right]Z_\mu(x)\Psi(\mathbf{x},t)\right] . \tag{8e}$$

Expanding $A_\mu^{a\prime}(\mathbf{x},t)$ in powers of $(\mathbf{x}-\mathbf{X})$ via the procedure given in Ref. 2 we get the multipole expansion

$$A_0^{a\prime}(\mathbf{x},t)=A_0^a(\mathbf{X},t)-(\mathbf{x}-\mathbf{X})\cdot\mathbf{E}^a(\mathbf{X},t)+\cdots ,$$
$$\mathbf{A}^{a\prime}(\mathbf{x},t)=-\frac{1}{2}(\mathbf{x}-\mathbf{X})\times\mathbf{B}^a(\mathbf{X},t)+\cdots , \tag{9}$$

where $\mathbf{E}^a$ and $\mathbf{B}^a$ are the color-electric and -magnetic fields. In (8) we have separated $L_{\rm QCD}$ into two parts. The part $L_{\rm QCD}^{(0)}$ gives the bound-state properties of the $Q\bar{Q}$ system, while $L_{\rm QCD}^{(1)}$ gives multipole-gluon emission.

In the nonrelativistic approximation, the Hamiltonian of the system can be written as

$$H=H_{\rm QCD}^{(0)}+H_{\rm QCD}^{(1)}+H_{\rm em}+H_W\ , \tag{10a}$$

where

$$H_{\rm QCD}^{(0)}=\int d^3x_1d^3x_2\Psi^\dagger(\mathbf{x}_1,t)\Psi(\mathbf{x}_1,t)\hat{H}\Psi^\dagger(\mathbf{x}_2,t)\Psi(\mathbf{x}_2,t)\ ,$$
$$H_{\rm QCD}^{(1)}=-L_{\rm QCD}^{(1)},\quad H_{\rm em}=-L_{\rm em},\quad H_W=-L_W\ , \tag{10b}$$

and

$$\hat{H}\equiv-\frac{1}{2m}(\partial_1^2+\partial_2^2)+V_1(|\mathbf{x}_1-\mathbf{x}_2|)+\sum_{a=1}^{8}\frac{\lambda_a}{2}\frac{\bar{\lambda}_a}{2}V_2(|\mathbf{x}_1-\mathbf{x}_2|)+2m \tag{10c}$$

is just the quantum-mechanical Hamiltonian of the $Q\bar{Q}$ system.

The canonical momentum $\pi(\mathbf{x},t)$ conjugate to $\Psi(\mathbf{x},t)$ is now

$$\pi(\mathbf{x},t)=\frac{\delta\mathcal{L}}{\delta\dot{\Psi}(\mathbf{x},t)}=\frac{\delta\mathcal{L}_{\rm QCD}^{(0)}}{\delta\dot{\Psi}(\mathbf{x},t)}=i\Psi^\dagger(\mathbf{x},t)\ . \tag{11}$$

Therefore the canonical quantization of the Ψ field is given by

$$\{\Psi(\mathbf{x},t),\Psi^\dagger(\mathbf{x}',t)\}=\delta^3(\mathbf{x}-\mathbf{x}')\ ; \tag{12}$$

others anticommute. With this equal-time commutation relation, we can evaluate the S-matrix elements for soft-gluon emission processes.

The gauge-invariant expression for the $Q\bar{Q}$ bound state has been introduced in Ref. 2 with $\mathbf{X}$ taken to be zero. Here we give a more general expression. A gauge-invariant color-singlet $Q\bar{Q}$ bound state with momentum $\mathbf{k}$ (momentum of the c.m.), a set of spatial quantum number λ and spin quantum number σ, can be expressed as

$$|\lambda,\sigma,\mathbf{k}\rangle=\frac{1}{\sqrt{3}}\int d^3x_1d^3x_2f_\lambda(\mathbf{x}_1-\mathbf{x}_2)\bar{\Psi}_\alpha(\mathbf{x}_1,t)\Gamma_\sigma\Psi_\alpha(\mathbf{x}_2,t)e^{i\mathbf{k}\cdot\mathbf{X}}|0\rangle\ , \tag{13}$$

where α ($\alpha=1,2,3$) is the color index of the quark, $f_\lambda(\mathbf{x}_1-\mathbf{x}_2)$ is the spatial wave function of the bound state, and Γ_σ is the matrix determining the addition of spins of Q and $\bar{Q}$. This state is normalized as

$$\int\frac{d^3k}{(2\pi)^3}\langle\lambda,\sigma,\mathbf{k}|\lambda',\sigma',\mathbf{k}'\rangle=\delta_{\lambda\lambda'}\delta_{\sigma\sigma'}\ ,$$
$$\sum_{\lambda,\sigma}\int\frac{d^3k}{(2\pi)^3}|\lambda,\sigma\mathbf{k}\rangle\langle\lambda,\sigma,\mathbf{k}|=1\ . \tag{14}$$

With the same normalization, a color-octet $Q\bar{Q}$ bound state with color index a ($a=1,2,\ldots,8$) can be expressed as

$$|a,\lambda,\sigma,\mathbf{k}\rangle=\int d^3x_1d^3x_2f_\lambda(\mathbf{x}_1-\mathbf{x}_2)\bar{\Psi}_\alpha(\mathbf{x}_1,t)\left[\frac{\lambda_a}{\sqrt{2}}\right]_{\alpha\beta}\times\Gamma_\sigma\Psi_\beta(\mathbf{x}_2,t)e^{i\mathbf{k}\cdot\mathbf{X}}|0\rangle\ . \tag{15}$$

In the following calculations we shall take $H_{\rm QCD}^{(0)}$ as the zeroth-order Hamiltonian and regard $H_{\rm QCD}^{(1)}+H_{\rm em}+H_W$ as a perturbation. This is different from the conventional perturbation in quantum field theory since $H_{\rm QCD}^{(0)}$ is not the free quark Hamiltonian. We shall use the following shortened notation:

$$H_0\equiv H_{\rm QCD}^{(0)},\quad H_{\rm int}\equiv H_{\rm QCD}^{(1)}+H_{\rm em}+H_W\ , \tag{16}$$

and

$$H_{\rm QCD}^{(1)}=H_1+H_2\ ,$$
$$H_1\equiv Q_aA_0^a(\mathbf{X},t)\ , \tag{17a}$$
$$H_2\equiv-\mathbf{d}_a\cdot\mathbf{E}^a(\mathbf{X},t)-\mathbf{m}_a\cdot\mathbf{B}^a(\mathbf{X},t)\cdots\ ,$$

where

$$Q_a\equiv g_s\int d^3x\,\Psi^\dagger(\mathbf{x},t)\frac{\lambda_a}{2}\Psi(\mathbf{x},t)\ ,$$
$$\mathbf{d}_a\equiv g_s\int d^3x\,(\mathbf{x}-\mathbf{X})\Psi^\dagger(\mathbf{x},t)\frac{\lambda_a}{2}\Psi(\mathbf{x},t)\ , \tag{17b}$$
$$\mathbf{m}_a\equiv\tfrac{1}{2}g_s\int d^3x\,(\mathbf{x}-\mathbf{X})\times\Psi^\dagger(\mathbf{x},t)\gamma\frac{\lambda_a}{2}\Psi(\mathbf{x},t)$$

are the color charge, color-electric dipole moment, and color-magnetic dipole moment of the $Q\bar{Q}$ system, respectively. Here we have separated $H_{\rm QCD}^{(1)}$ into H_1+H_2. In soft-gluon emission processes, H_1 is not small, so that we should treat H_1 to all orders.

The time evolution of the gauge fields can be written as

$$F(\mathbf{x},t)=e^{t\partial_0}F(\mathbf{x},0)\ , \tag{18}$$

where $F(\mathbf{x},t)$ represents $A_0^a(\mathbf{x},t)$, $\mathbf{E}^a(\mathbf{x},t)$, $\mathbf{B}^a(\mathbf{x},t)$, $\mathcal{A}_\mu(\mathbf{x},t)$, etc. In the present picture of perturbation theory, the time evolution of operators composed of heavy-quark fields is determined by H_0. Therefore the time evolution of Q_a, $\mathbf{d}_a$, $\mathbf{m}_a$, etc., can be further expressed as

$$G(\mathbf{x},t)=e^{t\partial_0}G(\mathbf{x},0)=e^{iH_0t}G(\mathbf{x},0)e^{-iH_0t}\ , \tag{19}$$

where $G(\mathbf{x},t)$ represents $Q_a(\mathbf{x},t)$, $\mathbf{d}_a(\mathbf{x},t)$, $\mathbf{m}_a(\mathbf{x},t)$, etc. From (18) and (19) we have

$$H_{\rm int}(t)=e^{it(H_0-i\partial_0)}H_{\rm int}(0)e^{-iH_0t}\ , \tag{20}$$

in which ∂_0 operates only on the gauge fields.

Let $|i\rangle$ and $|f\rangle$ be the initial and final states. To nth order, the S-matrix element is

$$\langle f|S^{(n)}|i\rangle=\left\langle f\left|\int_{-\infty}^{\infty}dt_1\int_{-\infty}^{t_1}dt_2\cdots\int_{-\infty}^{t_{n-1}}dt_n H_{\rm int}(t_1)H_{\rm int}(t_2)\cdots H_{\rm int}(t_n)\right|i\right\rangle . \tag{21}$$

Let

$$t_1=\tau_1,\quad t_2=\tau_1-\tau_2,\quad t_3=\tau_1-\tau_2-\tau_3\ ,\ldots,\quad t_n=\tau_1-\tau_2-\cdots-\tau_n\ . \tag{22}$$

Equation (21) can be written as

$$\begin{aligned}\langle f|S^{(n)}|i\rangle&=(-i)^n\left\langle f\left|\int_{-\infty}^{\infty}d\tau_1\int_0^{\infty}d\tau_2\cdots\int_0^{\infty}d\tau_n e^{i\tau_1(H_0-i\partial_0)}H_{\rm int}(0)e^{-i\tau_2(H_0-i\partial_0)}\cdots H_{\rm int}(0)e^{-i(\tau_1-\cdots-\tau_n)H_0}\right|i\right\rangle\\&=(-i)^n\left\langle f\left|\int_{-\infty}^{\infty}d\tau_1\int_0^{\infty}d\tau_2\cdots\int_0^{\infty}d\tau_n e^{i\tau_1(E_f+\omega_f-E_i)}H_{\rm int}(0)e^{i\tau_2(E_i-H_0+i\partial_0)}\cdots e^{i\tau_n(E_i-H_0+i\partial_0)}H_{\rm int}(0)\right|i\right\rangle\\&=-i2\pi\delta(E_f+\omega_f-E_i)\left\langle f\left|H_{\rm int}(0)\frac{1}{e_i-H_0+i\partial_0}H_{\rm int}(0)\cdots\frac{1}{E_i-H_0+i\partial_0}H_{\rm int}(0)\right|i\right\rangle ,\end{aligned} \tag{23}$$

where E_i and E_f are energies of the $Q\bar{Q}$ systems in the initial and final states, respectively, and ω_f is the energy of the emitted gauge fields in the final states. In (23), it is understood that the resolvent $(E_i-H_0+i\partial_0)^{-1}$ is the shortened notation of $(E_i-H_0+i\partial_0+i\epsilon)^{-1}$ with $\epsilon\to+0$.

Taking into account H_1 contributions to all orders and $(H_2+H_{\rm em}+H_m)$ to nth order, the formula for the S-matrix element is

$$\begin{aligned}\langle f|S|i\rangle&=-i2\pi\delta(E_f+\omega_f-E_i)\\&\quad\times\sum_{l_1=0}^{\infty}\sum_{l_2=0}^{\infty}\cdots\sum_{l_{n+1}=0}^{\infty}\left\langle f\left|\left[H_1\frac{1}{E_i-H_0+i\partial_0}\right]^{l_1}(H_2+H_{\rm em}+H_W)\right.\right.\\&\qquad\times\left[\frac{1}{E_i-H_0+i\partial_0}H_1\right]^{l_2}\frac{1}{E_i-H_0+i\partial_0}(H_2+H_{\rm em}+H_W)\\&\qquad\left.\left.\times\cdots(H_2+H_{\rm em}+H_W)\left[\frac{1}{E_i-H_0+i\partial_0}H_1\right]^{l_{n+1}}\right|i\right\rangle\\&=-i2\pi\delta(E_f+\omega_f-E_i)\left\langle f\left|\frac{E_i-H_0+i\partial_0}{E_i-H_0+i\partial_0-H_1}(H_2+H_{\rm em}+H_W)\right.\right.\\&\qquad\left.\left.\times\frac{1}{E_i-H_0+i\partial_0-H_1}\cdots(H_2+H_{\rm em}+H_W)\frac{E_i-H_0+i\partial_0}{E_i-H_0+i\partial_0-H_1}\right|i\right\rangle .\end{aligned} \tag{24}$$

When $|i\rangle$ and $|f\rangle$ are color-singlet states of $Q\bar{Q}$, we have $Q_a|i\rangle=Q_a|f\rangle=0$, or $H_1|i\rangle=H_1|f\rangle=0$. In this case,

$$\langle f|S|i\rangle=-i2\pi\delta(E_f+\omega_f-E_i)\left\langle f\left|(H_2+H_{\rm em}+H_w)\frac{1}{E_i-H_0+i\partial_0-H_1}\cdots\frac{1}{E_i-H_0+i\partial_0-H_1}(H_2+H_{\rm em}+H_W)\right|i\right\rangle . \tag{25}$$

This is our general formula for the S-matrix element. It can be evaluated by using the equal-time commutation relation (12).

As an example, let us consider the $E1$-$E1$ hadronic transition process

$$\Phi_i\to\Phi_f+g_1+g_2\ ,$$

where Φ_i and Φ_f are the initial- and final-quarkonium states, respectively, and g_1 and g_2 are the emitted gluons. Φ_i and Φ_f are of the form of (13). The S-matrix element in the process is

$$\langle\Phi_f g_1 g_2|S|\Phi_i\rangle=-i2\pi\delta(E_f+\omega_1+\omega_2-E_i)g_s^2\left\langle\Phi_f g_1 g_2\left|\mathbf{d}_a\cdot\mathbf{E}^a\frac{1}{E_i-H_0+i\partial_0-H_1}\mathbf{d}_b\cdot\mathbf{E}^b\right|\Phi_i\right\rangle . \tag{26}$$

With some algebra, (26) can be expressed as

$$\langle\Phi_f g_1 g_2|S|\Phi_i\rangle=-i2\pi\delta(E_f+\omega_1+\omega_2-E_i)\frac{g_s^2}{N_c^2-1}\left\langle\Phi_f g_1 g_2\left|\mathbf{d}_a\cdot\mathbf{E}^b\left[\frac{1}{E_i-H_0+iD_0}\right]_{bc}\mathbf{d}_a\cdot\mathbf{E}^c\right|\Phi_i\right\rangle , \tag{27}$$

where N_c is the number of colors and

$$(D_0)_{bc}\equiv\delta_{bc}\partial_0-g_s f_{acb}A_0^a\ . \tag{28}$$

Equation (27) has the same form as the quantum-mechanical formula given in Ref. 2, but now $\mathbf{d}_a, H_0$ are all quantum-field-theoretical operators. The resolvent $(E_i - H_0 + iD_0)^{-1}$ can be evaluated by inserting complete sets of intermediate states $\sum_N |N\rangle\langle N|$ into the matrix element. The intermediate state $|N\rangle$ contains a $Q\bar{Q}$ pair in the color octet and a soft gluon. $H_0 - iD_0$ is the Hamiltonian of this system. Such a three-body bound state is difficult to study from the first principles of QCD. What people used to do is to take a proper model to imitate this state. For example, we can take a vibrational state of the quark-confining-string model[6] or the hybrid state in the bag-model approach[7] as the state $|N\rangle$ (Refs. 3 and 8). Then $H_0 - iD_0$ is taken to be the energy eigenvalue E'_N of that state $|N\rangle$, and the S-matrix element is then easy to evaluate by using (12). Lengthy but elementary calculations give

$$\langle \Phi_f g_1 g_2 | S | \Phi_i \rangle = -i(2\pi)^4 \delta^4(k_f + \omega_1 + \omega_2 - k_i) \frac{g_s^2}{2N_c} \sum_N \frac{\int d^3x' f_f^*(\mathbf{x}') f'_N(\mathbf{x}') x'^j \int d^3x\, f'_N(\mathbf{x}) f_i(\mathbf{x}) x^l}{E_i - E'_N} \times \langle g_1 g_2 | E_j^a(0) E_l^a(0) | 0 \rangle , \tag{29}$$

where $f_i(\mathbf{x})$, $f_f(\mathbf{x})$, and $f'_n(\mathbf{x})$ are the spatial wave functions of the state $|i\rangle$, $|f\rangle$, and $|N\rangle$, respectively. Equation (29) is just the quantum-mechanical formula given in Ref. 3.

III. CALCULATIONS OF $\Gamma(J/\psi \to \gamma\eta)$ AND $\Gamma(J/\psi \to \gamma\pi^0)$

There have been some phenomenological studies of $\Gamma(J/\psi \to \gamma\eta)$ and $\Gamma(J/\psi \to \gamma\pi^0)$ in the literature.[5,9] We will give here a more theoretical calculation in the framework of Sec. II. Of course, in our calculation, we still have to take a potential model for the bound states of $c\bar{c}$, and as we have mentioned in Sec. II we have to take a certain model for the complicated intermediate states. The dependence of the calculation on the model for the intermediate state can be compensated by taking effective electric and magnetic multipole-expansion coupling constants g_E, g_M determined by taking certain measured hadronic transition rates as inputs.[3]

The reason why $\Gamma(J/\psi \to \gamma\eta)$ and $\Gamma(J/\psi \to \gamma\pi^0)$ can be calculated in the framework of our generalized QCD multipole expansion can be seen as follows. From the point of view of QCD, the decay process contains two steps. The first step is the emission (including $Q\bar{Q}$ annihilation) of a photon and at least two gluons to form a color-singlet state. The second step is the conversion of the gluons into η or π^0. Examples of the diagrams are shown in Fig. 1. In the two-body decay processes $J/\psi \to \gamma + \eta$ and $J/\psi \to \gamma + \pi^0$, the momenta of η and π^0 are fixed:

$$q_\eta = \frac{M_\psi^2 - m_\eta^2}{2M_\psi} = 1.50 \text{ GeV} ,$$

$$q_\pi = \frac{M_\psi^2 - m_\pi^2}{2M_\psi} = 1.55 \text{ GeV} .$$

If η or π^0 is produced from the hadronization of two gluons, the typical momentum of the emitted gluon will be

$$k \sim \tfrac{1}{2} q_\eta \sim \tfrac{1}{2} q_\pi \sim 750 \text{ MeV} . \tag{30}$$

For such a low momentum scale, perturbative QCD does not work well, while QCD multipole expansion can be a better approach.[3] Since η and π^0 are pseudoscalars, the leading multipole-gluon emission in Fig. 1(a) is $E1$-$M2$ since there should be no flip of spin. The leading multipole emission calculation may work fairly well for the following reason. Take the first gluon emission as an example. It may be emitted via electric dipole emission. We know that J/ψ is a color-singlet state so that the color charge distribution in J/ψ is antisymmetric with respect to the interchange of c and $\bar{c}$. Since the electric quadrupole moment tensor is symmetric in $\mathbf{x}_1$ and $\mathbf{x}_2$, there cannot be electric quadrupole emission and the next electric multipole emission is the octupole emission. To make an order-of-magnitude estimate of the electric octupole emission correction, we take the formula for electric multipole emissions in classical electrodynamics. The related coefficient is[10]

$$A_E(l,m) = \frac{4\pi k^{l+2}}{i(2l+1)!!} \left[\frac{l+1}{l} \right]^{1/2} Q_{lm} , \tag{31a}$$

where

$$Q_{lm} = \int d^3x\, r^l Y_{lm}^*(\theta,\phi) \rho(\mathbf{x}) \tag{31b}$$

and $\rho(\mathbf{x})$ is the electric charge density. As an order-of-magnitude estimate we take $Q_{lm} \propto a^l$, where $a = \langle r^2 \rangle^{1/2}$ is the size of J/ψ. Then

$$\frac{A_E(\text{octupole})}{A_E(\text{dipole})} \sim \frac{\sqrt{2/3}}{35} (ka)^2 = 0.02(ka)^2 . \tag{32}$$

For k given in (30), $ka \sim 1.6$. Thus the ratio (32) is about 5%.

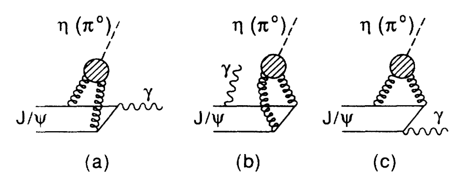

FIG. 1. Examples of three different kinds of diagrams in $J/\psi \to \gamma + \eta(\pi^0)$. The solid line, wavy line, spiral line, and dashed line denote the c quark, photon, gluon, and η or π^0, respectively. Every gluon or photon can be emitted by both c and $\bar{c}$.

In the following calculation we take the Cornell Coulomb plus linear potential model[11] for the $c\bar{c}$ bound states, and take the string vibrational states[6] as the model for the intermediate states. The effective electric dipole emission coupling constant $\alpha_E=g_E^2/4\pi$ can be determined by taking the datum of $\Gamma(\psi'\to J/\psi\pi\pi)$ as input and it is[3]

$$\alpha_E=0.54 \ . \quad (33)$$

There is not yet an ideal datum for determining the effective magnetic dipole emission coupling constant $\alpha_M=g_M^2/4\pi$. A reasonable range for α_M discussed in Ref. 3 is

$$\alpha_E \le \alpha_M \le (2-3)\alpha_E \ . \quad (34)$$

A. Calculation of $\Gamma(J/\psi\to\gamma\eta)$

From our general formula (25), the S-matrix element for $J/\psi\to\gamma+\eta$ is

$$\begin{aligned}\langle\gamma\eta|S|J/\psi\rangle = -i2\pi\delta(\omega_\gamma+E_\eta-M_\psi)\Bigg[&\left\langle\gamma\eta\left|H_{\rm em}\frac{1}{M_\psi-H_0+i\partial_0-H_1}H_2\frac{1}{M_\psi-H_0+i\partial_0-H_1}H_2\right|J/\psi\right\rangle\\ &+\left\langle\gamma\eta\left|H_2\frac{1}{M_\psi-H_0+i\partial_0-H_1}H_2\frac{1}{M_\psi-H_0+i\partial_0-H_1}H_{\rm em}\right|J/\psi\right\rangle\\ &+\left\langle\gamma\eta\left|H_2\frac{1}{M_\psi-H_0+i\partial_0-H_1}H_{\rm em}\frac{1}{M_\psi-H_0+i\partial_0-H_1}H_2\right|J/\psi\right\rangle\Bigg] \ . \quad (35)\end{aligned}$$

The three terms in (35) correspond to the three kinds of diagrams in Fig. 1. Since bound-state contributions to the intermediate states are to be taken into account in the evaluation of (35) in our approach, the physics is different for different kinds of diagrams. In the nonrelativistic approach, Fig. 1(a) describes a process that a virtual hadronic transition $J/\psi\to\psi(n\,^3S_1)+\eta$ first takes place and then the c and $\bar{c}$ in $\psi(n\,^3S_1)$ annihilate into a photon. According to the discussions in Sec. II, this amplitude is proportional to $g_Eg_Mf_{n0}(\mathbf{0})$, where $f_{n0}(\mathbf{0})$ is the wave function at the origin of the $\psi(n\,^3S_1)$ state. The other two diagrams, Figs. 1(b) and 1(c) are different. They describe processes in which J/ψ first emits a photon and a gluon and transits into a virtual vector-meson state with c and $\bar{c}$ in color octet, and then the color octet c and $\bar{c}$ annihilate into a gluon. In the nonrelativistic approach, the color octet c and $\bar{c}$ annihilate at $\mathbf{x}_1=\mathbf{x}_2=\mathbf{X}$. Thus the $U(\mathbf{x},t)$ defined in (7) is now unity and hence $\Psi=\psi$ and $A_\mu^{a\prime}=A_\mu^a$. Therefore $Q\bar{Q}$ annihilation in our formalism is exactly the same as that in perturbative QCD. The coupling constant g_s at the annihilation vertex should then be taken to be the conventional QCD coupling constant at the scale M_ψ instead of being g_E or g_M. According to this, the amplitudes of Figs. 1(b) and 1(c) are proportional to $g_Eg_sf'_{n0}(\mathbf{0})$ or $g_Mg_sf'_{n0}(\mathbf{0})$, where $f'_{n0}(\mathbf{0})$ is the wave function at the origin of the vector-meson state in which c and $\bar{c}$ are in color octet (e.g., the string vibrational state). We know that $g_s<g_E(g_M)$ and $|f'_{n0}(\mathbf{0})|<|f_{n0}(\mathbf{0})|$. Therefore the contributions of Figs. 1(b) and 1(c) are smaller than that of Fig. 1(a). Furthermore, QCD corrections to the annihilation vertices are significant.[12] So that the effective $f_{n0}(\mathbf{0})$ and $f'_{n0}(\mathbf{0})$ in the tree-graph formula are not simply the ones calculated from the potential model. Instead, they should be determined by taking the data of related decay widths [say $\Gamma(J/\psi\to e^+e^-)$ and $\Gamma(J/\psi\to ggg)$] as inputs. The ratio $|f'_{n0}(\mathbf{0})|/|f_{n0}(\mathbf{0})|$ determined in this way is even smaller than that calculated from the potential model. Therefore the decay $J/\psi\to\gamma+\eta$ is dominated by the process described by Fig. 1(a) which shows a $\psi(n\,^3S_1)$ vector-meson-dominance mechanism for the interaction between the photon and the hadrons. As an approximation we neglect the contributions of Figs. 1(b) and 1(c) in the following calculation.

When we take the string vibrational states (or hybrid states in bag model) as the model for the intermediate states, the hadronic transition amplitude factorizes into two factors—the first factor describing the soft-gluon emissions and the second factor describing the hadronization $gg\to\eta$,[3] and so does the first term in (35). The evaluation of the first factor is straightforward, while the second factor needs more consideration. This is an $E1$-$M2$ gluon emission process, the hadronization matrix element is of the form

$$\langle\eta|g_Eg_ME_j^aD_jB_l^a|0\rangle \ , \quad (36)$$

where $D_j\equiv\partial_j-g_s(\lambda_a/2)A_j^a$ is the covariant derivative. The operator in (36) can be written as

$$g_Eg_ME_j^aD_jB_l^a=g_Eg_M\partial_j(E_j^aB_l^a)-g_Eg_M(D_jE_j^a)B_l^a \ . \quad (37)$$

Voloshin and Zakharov[4] argued that the second term in (37) is smaller than the first term and they suggested the approximation

$$\langle\eta|g_Eg_ME_j^aD_jB_l^a|0\rangle\simeq iq_{\eta j}\langle\eta|g_Eg_ME_j^aB_l^a|0\rangle \ . \quad (38)$$

The matrix element $\langle\eta|g_Eg_ME_j^aB_l^a|0\rangle$ is then related to the Gross-Treiman-Wilczek formula[13]

$$\langle\eta|\alpha_sF_{\mu\nu}^a\tilde{F}^{a\mu\nu}|0\rangle=2\pi\sqrt{2/3}\,f_\pi m_\eta^2 \ , \quad (39)$$

i.e.,

$$\langle \eta | g_E g_M E_j^a B_l^a | 0 \rangle = \frac{1}{3}\frac{g_E g_M}{\alpha_s}\langle \eta | \alpha_s \mathbf{E}^a \cdot \mathbf{B}^a | 0 \rangle \delta_{jl} = \frac{1}{12}\frac{g_E g_M}{\alpha_s}\langle \eta | \alpha_s F^a_{\mu\nu}\tilde{F}^{a\mu\nu} | 0 \rangle \delta_{jl} = \frac{g_E g_M}{g_s^2}\frac{4\pi^2}{3\sqrt{6}} f_\pi m_\eta^2 \delta_{jl} . \quad (40)$$

Note that the QCD coupling constant g_s in (40) is the one at the scale m_η, so that it is large.

With all these we can evaluate the first term in (35) and calculate the rate $\Gamma(J/\psi \to \gamma\eta)$. Our calculation is in the Coulomb gauge. The calculation is straightforward but lengthy. The result is

$$\Gamma(J/\psi \to \gamma\eta) = \frac{1}{12\pi}\left[\frac{\alpha_M}{\alpha_E}\right]\left[\frac{\alpha_E}{\alpha_s}\right]^2 \frac{M_\psi^2 - m_\eta^2}{M_\psi^2} |\mathbf{q}_\eta(J/\psi \to \gamma\eta)|^2 \left[\frac{2eQ}{3\sqrt{6}\, m_c}\right]^2 \left[\frac{4\pi^2}{3\sqrt{6}} f_\pi m_\eta^2 \sum_n h^{111}_{10n0}\right]^2 \quad (41)$$

in which the amplitude $h^{P_i PL}_{n_i l_i nl}$ is defined by

$$h^{P_i PL}_{n_i l_i nl} \equiv \sum_K \frac{\langle R_{nl} | r^P | R'_{KL}\rangle \langle R'_{KL} | r^{P_i} | R_{n_i l_i}\rangle}{(M_\psi - E_{nl} - \omega_\eta)(M_\psi - E'_{KL})} f_{nl}(\mathbf{0}) , \quad (42)$$

where n_i, n, K are principal quantum numbers; l_i, l, L are orbital angular momentum quantum numbers; and $R_{n_i l_i}, R_{nl}, R'_{KL}$ are the radial wave functions of the initial-, final-, and intermediate-quarkonium states in the hadronic transition process, respectively, $f_{nl}(\mathbf{0})$ is the wave function at the origin of the final-quarkonium state in the hadronic transition process. As has been mentioned, $f_{n0}(\mathbf{0})$ should be determined by the datum of the related leptonic width $\Gamma(\psi(n\,^3S_1) \to e^+e^-)$. For $n=1,2$, the determined $f_{10}(\mathbf{0})$ and $f_{20}(\mathbf{0})$ are smaller than the ones calculated from the Cornell potential model by almost the same factor 0.57. For $n \geq 3$, the states are above the charm threshold and there are significant state mixings, so that the data $\Gamma(\psi(n\,^3S_1) \to e^+e^-)$ for $n \geq 3$ are not useful. We expect that the QCD corrections will not vary seriously with n as is inspired by the case of $n=1,2$. Therefore we use the same factor 0.57 for all n. In our calculation we keep five terms in the summation $\sum_n h^{111}_{10n0}$. The values of h^{111}_{10n0} ($n=1,\ldots,5$) are listed in Table I.

Actually, the values of α_s at the scale m_η is not precisely known. The currently accepted value of α_s is close to our α_E given in (33). So we take $\alpha_s \simeq \alpha_E$ in (41) and this gives

$$\Gamma(J/\psi \to \gamma\eta) = 0.020\left[\frac{\alpha_M}{\alpha_E}\right]\text{keV} = \begin{cases} 0.020\ \text{keV}, & \alpha_M = \alpha_E , \\ 0.059\ \text{keV}, & \alpha_M = 3\alpha_E , \end{cases} \quad (43a)$$

and the corresponding branching ratio is

$$B(J/\psi \to \gamma\eta) = \begin{cases} 2.9\times 10^{-4}, & \alpha_M = \alpha_E , \\ 8.6\times 10^{-4}, & \alpha_m = 3\alpha_E . \end{cases} \quad (43b)$$

This is to be compared with the experimental value[14]

$$B(J/\psi \to \gamma\eta) = (8.6 \pm 0.8)\times 10^{-4} . \quad (44)$$

There are some uncertainties in (41): namely, (α_M/α_E) is not well determined, (α_E/α_s) is not precisely known, and we do not know how good the approximation (38) is. To eliminate these uncertainties, let us consider another $E1$-$M2$ transition process $\psi' \to J/\psi + \eta$. It has been calculated in Ref. 3 with the same uncertainties and the result is

$$\Gamma(\psi' \to (J/\psi)\eta) = \frac{2}{243\pi}\left[\frac{\alpha_M}{\alpha_E}\right]\left[\frac{\alpha_E}{\alpha_s}\right]^2 \left[\frac{1}{3m_c}\right]^2 |\mathbf{q}_\eta(\psi' \to (J/\psi)\eta)|^3 \left[\frac{\pi^2}{\sqrt{3}} f_\pi m_\eta^2 f^{111}_{2010}\right]^2 , \quad (45)$$

where

$$f^{P_i PL}_{n_i l_i nl} \equiv \sum_K \frac{\langle R_{nl} | r^P | R'_{KL}\rangle \langle R'_{KL} | r^{P_i} | R_{n_i l_i}\rangle}{M_{\psi'} - E'_{KL}} . \quad (46)$$

Now we take the ratio

$$R_\eta \equiv \frac{\Gamma(J/\psi \to \gamma\eta)}{\Gamma(\psi' \to (J/\psi)\eta)} = \frac{\frac{4}{81}(eQ)^2[(M_\psi^2 - m_\eta^2)/M_\eta^2]|\mathbf{q}_\eta(J/\psi \to \gamma\eta)|^2 |\sum_n h^{111}_{10n0}|^2}{\frac{2}{243}|\mathbf{q}_\eta[\psi' \to (J/\psi)\eta]|^3 |f^{111}_{2010}|^2} . \quad (47)$$

TABLE I. The calculated h^{111}_{10n0}.

n	h^{111}_{10n0} ($\text{GeV}^{-5/2}$)
1	0.61
2	-0.42
3	-0.02
4	-7×10^{-3}
5	-6×10^{-3}

The uncertainties in the two processes cancel each other in R_η, and R_η just test the soft-gluon-emission dynamics in our approach. Our theoretical result is

$$R_\eta|_{\text{theory}}=0.012 \; . \tag{48}$$

It is in agreement with the corresponding experimental value[14]

$$R_\eta|_{\text{expt}}=0.009\pm0.005 \; . \tag{49}$$

B. On the rate $\Gamma(J/\psi\to\gamma\pi^0)$

The decay $J/\psi\to\gamma+\pi^0$ is isospin violating. There are two possible mechanisms that can contribute to this decay process. One possibility is the $\psi(n\,{}^3S_1)$ vector-meson-dominance mechanism shown in Fig. 1(a) and the other is $J/\psi\to\rho^{0*}+\pi^0\to\gamma+\pi^0$ which is a vector-meson-dominance mechanism with ρ^0 being the vector meson. Since the branching ratio of the isospin-conserving decay $J/\psi\to\rho^0+\pi^0$ is large,[14] the second possibility may play an important role in $J/\psi\to\gamma+\pi^0$. [Note that $B(J/\psi\to\rho^0+\eta)$ is about 2 orders of magnitude smaller than $B(J/\psi\to\rho^0+\pi^0)$;[14] therefore, the contribution of $J/\psi\to\rho^{0*}+\eta\to\gamma+\eta$ in $J/\psi\to\gamma+\eta$ is negligible.] Fritszch and Jackson[5] have calculated the contribution of the second mechanism in $J/\psi\to\gamma+\pi^0$ and the result is $B(J/\psi\to\gamma\pi^0)\sim2\times10^{-5}$ which is close to the experimental value $(4\pm1)\times10^{-5}$. What about the first mechanism? To complete the study of $J/\psi\to\gamma+\pi^0$, we must answer this question.[15]

The calculation of $\Gamma(J/\psi\to\gamma\pi^0)$ from Fig. 1(c) is completely similar to the calculation in the preceding subsection. The only difference is that the hadronization matrix element is now[13]

$$\begin{aligned}\langle\pi^0|g_Eg_ME^a_jB^a_l|0\rangle&=\frac{1}{12}\frac{g_Eg_M}{\alpha_s}\langle\pi^0|\alpha_sF^a_{\mu\nu}\tilde{F}^{a\mu\nu}|0\rangle\delta_{jl}\\&=\frac{g_Eg_M}{g_s^2}\frac{4\pi^2}{3\sqrt{2}}\frac{m_d-m_u}{m_d+m_u}f_\pi m_\pi^2\delta_{jl}\;.\end{aligned}\tag{50}$$

Thus the rate contributed by Fig. 1(a) is

$$\begin{aligned}\Gamma_1(J/\psi\to\gamma\pi^0)&=\frac{1}{12\pi}\left[\frac{\alpha_M}{\alpha_E}\right]\left[\frac{\alpha_E}{\alpha_s}\right]^2\frac{M_\psi^2-m_\pi^2}{M_\psi^2}|\mathbf{q}_\pi(J/\psi\to\gamma\pi^0)|^3\\&\quad\times\left[\frac{2eQ}{3\sqrt{6}\,m_c}\right]^2\left[\frac{4\pi^2}{3\sqrt{2}}\frac{m_d-m_u}{m_d+m_u}f_\pi m_\pi^2\sum_n h^{111}_{10n0}\right]^2\;.\end{aligned}\tag{51}$$

The value of $(m_d-m_u)/(m_d+m_u)$ is about 0.3 (Ref. 16). When taking $\alpha_s\simeq\alpha_E$, we obtain

$$\begin{aligned}\Gamma_1(J/\psi\to\gamma\pi^0)&=2.3\left[\frac{\alpha_M}{\alpha_E}\right]\times10^{-5}\ \text{keV}\\&=\begin{cases}2.3\times10^{-5}\ \text{keV}, & \alpha_m=\alpha_E\;,\\6.9\times10^{-5}\ \text{keV}, & \alpha_M=3\alpha_E\;.\end{cases}\end{aligned}\tag{52a}$$

and the corresponding branching ratio is

$$B_1(J/\psi\to\gamma\pi^0)=\begin{cases}3\times10^{-7}, & \alpha_M=\alpha_E\;,\\1\times10^{-6}, & \alpha_M=3\alpha_E\;.\end{cases}\tag{52b}$$

Therefore the contribution of the mechanism of Fig. 1(a) is about an order of magnitude smaller than that of the second mechanism. Thus we conclude that the decay $J/\psi\to\gamma+\pi^0$ is dominated by the mechanism $J/\psi\to\rho^{0*}+\pi^0\to\gamma+\pi^0$.

IV. CONCLUSIONS

We have given in this paper a generalized formalism of QCD multipole expansion in which electroweak interactions and constituent-quark-field quantization are included. A general formula for the S-matrix element [cf. (24) and (25)] is derived in this framework and it can be applied to the study of soft-gluon emissions in processes with quark flavor changing or quark pair annihilation. This is the first attempt to apply QCD multipole expansion to the study of heavy-quark decay processes other than hadronic transitions. In our new formalism, the $Q\bar{Q}$ annihilation calculation is exactly the same as that in perturbative QCD, while for soft-gluon emissions the new approach is essentially different from the perturbation calculation. The general formula is then applied to the calculation of the radiative decay rates $\Gamma(J/\psi\to\gamma\eta)$ and $\Gamma(J/\psi\to\gamma\pi^0)$. We do not expect that our approach can be used to study radiative decays of Υ since the emitted gluons in those processes are not soft and multipole expansion may not work well. Also vector-meson dominance via $\Upsilon(n\,{}^3S_1)$ is more questionable since the b quark is heavy.

In our new approach we have seen that $J/\psi\to\gamma\eta$ is dominated by the vector-meson-dominance mechanism shown in Fig. 1(a) in which the vector mesons are $\psi(n\,{}^3S_1)$ states. The rate $\Gamma(J/\psi\to\gamma\eta)$ obtained in our approach is in agreement with the experiment [cf. (43b) and (44)]; especially, the ratio R_η defined in (47) tests exclusively the soft-gluon-emission dynamics in our approach and our prediction (48) fits the experimental (49) fairly well. The result is encouraging.

We would like to point out that the physical picture of $J/\psi\to\gamma+\eta$ in Ref. 9 is similar to ours. The differences between the two approaches are (1) the mechanism serves as a phenomenological model in Ref. 9, while in our approach it is the consequence of our general formula (25), and (2) in Ref. 9 only the contributions of the $n=1$ and $n=2$ $\psi(n\,{}^3S_1)$ states are taken into account and the relative strengths of these contributions are taken from an assumption, while in our approach the relative strengths of all $\psi(n\,{}^3S_1)$ contributions are calculated from the soft-gluon-emission dynamics in the framework of our generalized QCD multipole expansion. (See, for example, the results listed in Table I.)

Therefore our calculation is more closely related to QCD.

Our calculation also shows that the mechanism shown in Fig. 1(a) is not essential in $J/\psi \rightarrow \gamma + \pi^0$. The decay $J/\psi \rightarrow \gamma + \pi^0$ is dominated by a different mechanism $J/\psi \rightarrow \rho^{0*} + \pi^0 \rightarrow \gamma + \pi^0$.

There are still some effects we have not considered in this paper. For example, coupled-channel effects (state mixing and continuous spectrum contribution to the intermediate states), relativistic corrections, etc. We do not expect that these corrections will affect the main conclusions of this paper. They will be considered elsewhere.

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