Amplitude analysis of $p_\uparrow + p \to \Lambda + X$ and tests of the Mueller theorem

M. Svec
Physics Department, Dawson College, Montréal, Québec, Canada H3Z 1A4
and McGill University, Montréal, Québec, Canada H3A 2T8
(Received 9 January 1990)

Using the Goldstein-Owens angular momentum constraints on forward amplitudes in elastic three-body scattering and the Mueller theorem, we present an explicit form of constraints on the behavior of spin observables in inclusive measurements of $p_\uparrow + p \to \Lambda + X$ with a polarized proton beam. We examine how these constraints can be used to test the validity of the Mueller theorem at small p_T. In addition, we find that the condition $|D_{zz}| > 1$ on the element D_{zz} of the polarization transfer tensor is an independent signature of violation of the Mueller theorem at any value of p_T. The proposed tests of the Mueller theorem will be possible at the Fermilab spin facility and in high-precision experiments at high-intensity hadron facilities.

I. INTRODUCTION

Experiments with unpolarized beams and targets measure spin-independent observables which are always expressed in terms of certain sums and averages of explicitly spin-dependent amplitudes, each of which carries its own specific information on dynamics. Unpolarized experiments provide useful information, e.g., about large structures in the amplitudes and their general behavior. However, the averaging and summing over the spin dependence of the amplitudes results in a loss of important information. Many structures in the amplitudes are not seen while some of the apparent structures in the unpolarized observables are open to misinterpretation. In the case of inclusive measurements, the unpolarized experiments do not even measure the "spin average" of all inclusive amplitudes. In contrast with exclusive reactions, not all inclusive amplitudes contribute to the unpolarized inclusive cross sections. Information about a certain set of inclusive amplitudes is accessible only through polarization experiments.[1-14]

Mueller's generalized optical theorem relates inclusive amplitudes to forward amplitudes in three-body elastic scattering.[2] Unlike two-body forward amplitudes, the three-body scattering amplitudes are not measurable. The Mueller theorem thus cannot be subjected to a direct experimental test. However, the fundamental features of the three-body amplitudes must be shared by the observables in inclusive measurements with spin. This consequence of the Mueller theorem affords useful tests of its validity.

Goldstein and Owens showed[11] that parity and angular momentum conservation and rotational invariance impose kinematic constraints on three-body amplitudes. Via the Mueller theorem these constraints propagate also to spin observables in single-particle inclusive measurements $a + b \to c + X$. As a result, certain spin observables are strongly suppressed over a sizable kinematic region when the particle c is produced either forward or backward.

Kinematic suppression of spin observables is obviously important for planning and interpretation of inclusive measurements with polarized beams and targets. Usually the detector system's trigger logic would be designed to skip events in the suppressed region to increase the data collection rate and to reduce the running time. However, the suppressed spin observables may carry potentially very important information: Any deviations from the expected suppressions would indicate violations of the Mueller theorem and possibly signal a nonconservation of parity P or even nonconservation of kinematic angular momentum in production processes.

There has been considerable experimental interest in the polarization of inclusively produced hyperons.[15,16] In this work we will focus on measurements of $p_\uparrow + p \to \Lambda + X$ with a polarized beam. Such studies are free from problems with an unpolarized background in polarized targets and are now accessible at the polarized-beam facility at Fermilab.[17,18] Experiments dedicated to the study of suppressed spin observables were proposed previously.[19]

This paper is organized as follows. The basic notation, the Mueller theorem, and Goldstein-Owens angular constraints are summarized in Sec. II. The spin observables in the reaction $p_\uparrow + p \to \Lambda + X$ and their expression in terms of inclusive amplitudes are given in Sec. III. The proposed tests of the Mueller theorem are described in Sec. IV. In Sec. V we briefly comment on the possible outcome of such tests.

II. INCLUSIVE AMPLITUDES, UNITARITY, AND ANGULAR CONSTRAINTS

Consider the one-particle inclusive measurements $a + b \to c + X$. The exclusive subprocess $a + b \to c + X_k$ is described by helicity amplitudes

$$f^{(k)}_{\Delta c, ab}(s,t,M^2) , \tag{1}$$

where a, b, and c are helicities in the s-channel helicity frames of particles a, b, and c, respectively. The symbol Δ denotes the set of individual helicities of particles in X_k. In general the multiparticle system X_k does not have a definite value of total spin and total helicity. In Eq. (1), $s=(p_a+p_b)^2$, $t=(p_a-p_c)^2$, and $M^2=(p_a+p_b-p_c)^2$ and we omit the additional kinematic variables. Following Goldstein and Owens,[11] we define the inclusive amplitudes

$$Q_{a'b'c',abc}(s,t,M^2)=\sum_{k=1}^{N}\sum_{\Delta}\int d\phi_k f^{(k)}_{\Delta c,ab} f^{(k)*}_{\Delta c',a'b'} , \qquad (2)$$

where N is the number of open channels at energy s and the integration $d\phi_k$ is over the phase space of X_k. In our normalization,[11]

$$\Sigma \equiv s\frac{d^2\sigma}{dt\,dM^2} = \frac{1}{(2s_a+1)(2s_b+1)}\frac{1}{32\pi^2 s}\sum_{a,b,c} Q_{abc,abc} . \qquad (3)$$

Unitarity connects the inclusive amplitudes (2) with forward-scattering amplitudes in the three-body process $a+b+\bar{c}\to a+b+\bar{c}$:

$$A_{a'b'\bar{c}',ab\bar{c}}(s,t,M^2) , \qquad (4)$$

where s is the invariant mass squared of the (ab) pair, t is the spacelike invariant mass of the $(a\bar{c})$ pair, and M^2 is the square of the total three-body energy in the center-of-mass system (c.m.s.). The Mueller theorem is a generalization of the two-body optical theorem[1] to three-body forward elastic scattering.[2] For particles with spin it reads[3-5]

$$Q_{a'b'c',abc}(s,t,M^2)=2\,\mathrm{Disc}_{M^2}A_{a'b'\bar{c}',ab\bar{c}}(s,t,M^2) , \qquad (5)$$

where the discontinuity is over M^2. In Eq. (5) the helicities $\bar{c}=c$ and $\bar{c}'=c'$.

Goldstein and Owens derived[11] important angular momentum constraints on the three-body amplitudes:

$$A_{a'b'c',abc}=(\sin\tfrac{1}{2}\theta)^m(\cos\tfrac{1}{2}\theta)^n B_{a'b'c',abc} , \qquad (6)$$

where

$$m=|(a-b-\bar{c})-(a'-b'-\bar{c}')| ,$$
$$n=|(a-b+\bar{c})-(a'-b'+\bar{c}')| . \qquad (7)$$

The angle θ in Eq. (6) is the c.m.s. scattering angle. The amplitudes $B_{a'b'c',abc}$ are regular at $\theta=0$ and $\theta=\pi$. The unitary relation (5) implies angular constraints on inclusive amplitudes (2) similar to (6).

In the following p_T and p_L are the transverse and longitudinal components of momentum of particle c in the c.m.s., and $x=p_L/p_{L_{\max}}$ is the Feynman variable. The variables p_T and x are related to t and M^2 (e.g., Ref. 14). The scattering angle θ is given by $\sin\theta=p_T/|\mathbf{p}_c|$ where $\mathbf{p}_c$ is the c.m.s. momentum of c. For large s we have

$$\sin\tfrac{1}{2}\theta=+\frac{p_T}{x\sqrt{s}} \quad \text{for } x>0 ,$$
$$\cos\tfrac{1}{2}\theta=-\frac{p_T}{x\sqrt{s}} \quad \text{for } x<0 . \qquad (8)$$

At fixed p_T, the region of x in which either $\sin\frac{1}{2}\theta$ or $\cos\frac{1}{2}\theta$ is small will be increasing with s. Consequently, spin observables which involve amplitudes with $m>0$ and/or $n>0$ will be suppressed over a considerable region of x increasing with s in the beam fragmentation region (small $\sin\frac{1}{2}\theta$) or target-fragmentation region (small $\cos\frac{1}{2}\theta$), or both.

The relations between spin observables in $a+b\to c+X$ measurements and the inclusive amplitudes (2) arise as follows. Consider an exclusive subprocess $a+b\to c+X_k$. In this reaction the spin-density matrix (SDM) of the particle c is given by

$$\rho^{f(k)}_{cc'}\sigma^{(k)}=\frac{1}{32\pi^2 s}\sum_{a'b'}\sum_{ab}\sum_{\Delta}\int d\phi_k f^{(k)}_{\Delta c,ab}\rho^i_{ab,a'b'}f^{(k)*}_{\Delta c',a'b'} , \qquad (9)$$

where ρ^i is the initial-state SDM of particles a and b, and

$$\sigma^{(k)}=\frac{1}{(2s_a+1)(2s_b+1)}\frac{1}{32\pi^2 s}\sum_{a,b,c}\sum_{\Delta}\int d\phi_k f^{(k)}_{\Delta c,ab}f^{(k)*}_{\Delta c,ab} \qquad (10)$$

is the subprocess reaction cross section. Evidently $\Sigma=\sum_{k=1}^{N}\sigma^{(k)}$. The SDM of particle c in the inclusive process $a+b\to c+X$ is the weighted average

$$\rho^f_{cc'}=\sum_{k=1}^{N}\rho^{f(k)}_{cc'}\sigma^{(k)}/\Sigma . \qquad (11)$$

From Eq. (9) we get finally

$$\rho^f_{cc'}\Sigma=\frac{1}{32\pi^2 s}\sum_{a'b'}\sum_{ab}\rho^i_{ab,a'b'}Q_{a'b'c',abc} . \qquad (12)$$

The polarization of particle c along an axis k in the rest frame of c is given by $P_k=\mathrm{Tr}(\rho^f S_k)$ where S_k is the appropriate spin operator.[13] We note that c can be polarized even when both a and b are unpolarized.

III. INCLUSIVE MEASUREMENTS OF $p_\uparrow+p\to\Lambda+X$ USING POLARIZED PROTON BEAMS

We first consider a correlated decay production of Λ in an exclusive process $a+b\to\Lambda+X_k$ where a and b can be polarized or unpolarized. The decay $\Lambda\to p+\pi^-$ is described by polar and azimuthal angles θ and ϕ of the decay proton momentum in the rest frame of Λ with the z axis along the direction of Λ momentum. The decay angular distribution $w^{(k)}(\theta,\phi)$ is then given by[20,13]

$$w^{(k)}(\theta,\phi)\sigma^{(k)}=\frac{1}{d}\mathrm{Tr}(U\rho^{(k)}U^+) , \qquad (13)$$

where $\rho^{(k)}$ is the final-state SDM of Λ in the reaction $a+b\to\Lambda+X_k$, U is the Λ decay matrix, and d is a Λ decay normalization factor.[20,13] We define the angular distribution of Λ in the inclusive measurement as the process average

$$w(\theta,\phi)=\sum_{k=1}^{N}w^{(k)}(\theta,\varphi)\sigma^{(k)}/\Sigma ,$$

where N is the number of open channels. As the result of linearity of the trace operator and the definition of ρ^f in Eq. (12) we get

$$w(\theta,\phi)\Sigma=\frac{1}{d}\,\mathrm{Tr}(U\rho^f U^\dagger)\ . \tag{14}$$

We can thus express the measured distribution $w(\theta,\phi)$ in terms of spin observables for the inclusive process using a spin formalism analogous to exclusive processes.

Consider now the inclusive reaction $p_{\uparrow}+p\rightarrow\Lambda+X$. Let $\mathbf{P}_a=(P_{ax},P_{ay},P_{az})$ be the vector of the incident-proton-beam polarization. The measured decay proton angular distribution is

$$\frac{d^4\sigma}{dx\,dp_T^2 d\Omega}=\frac{d^2\sigma}{dx\,dp_T^2}W(x,p_T^2,s,\mathbf{P}_a,\theta,\phi)\ , \tag{15}$$

where the normalized distribution is given[12] by

$$\begin{aligned}W=\frac{1}{16\pi^2}[1&+\alpha P_\Lambda\sin\theta\sin\phi\\ &+P_{ax}(\alpha D_{xx}\sin\theta\cos\phi+\alpha D_{xz}\cos\theta)\\ &+P_{ay}(\alpha D_{yy}\sin\theta\sin\phi+A)\\ &+P_{az}(\alpha D_{zx}\sin\theta\cos\phi+D_{zz}\cos\theta)]\ .\end{aligned} \tag{16}$$

In Eq. (16) the constant α is the weak decay parameter, P_Λ is the polarization of Λ, A is the polarized proton beam asymmetry, and $D_{xx},D_{xz},\ldots,D_{zz}$ are the components of the depolarization tensor. These observables are related to the inclusive amplitudes (2) as follows:[11]

$$\begin{aligned}\Sigma&=Q_{+++'+++}+Q_{+-+'+-+}\\ &\quad+Q_{++-'++-}+Q_{+--'+--}\ ,\\ D_{zz}\Sigma&=Q_{+++'+++}+Q_{+-+'+-+}\\ &\quad-Q_{++-'++-}-Q_{+--'+--}\ ,\end{aligned} \tag{17a}$$

$$\begin{aligned}D_{xx}\Sigma&=Q_{+++'-+-}+Q_{+-+'---}\\ &\quad+Q_{++-'-++}+Q_{+--'--+}\ ,\\ D_{yy}\Sigma&=Q_{+++'-+-}+Q_{+-+'---}\\ &\quad-Q_{++-'-++}-Q_{+--'--+}\ ,\end{aligned} \tag{17b}$$

$$\begin{aligned}P_\Lambda\Sigma&=+2\,\mathrm{Im}(Q_{+++'++-}+Q_{+-+'+--})\ ,\\ D_{zx}\Sigma&=+2\,\mathrm{Re}(Q_{+++'++-}+Q_{+-+'+--})\ ,\end{aligned} \tag{17c}$$

$$\begin{aligned}A\Sigma&=-2\,\mathrm{Im}(Q_{+++'-++}+Q_{+-+'--+})\ ,\\ D_{xz}\Sigma&=+2\,\mathrm{Re}(Q_{+++'-++}+Q_{+-+'--+})\ .\end{aligned} \tag{17d}$$

First we notice that the cross section Σ and the spin observables are expressed linearly in terms of inclusive amplitudes rather than as bilinear terms. Considering that the amplitudes Q are discontinuities of forward three-body scattering amplitudes, this linearity is similar to the linearity in the usual two-body optical theorem.

We now note that the four real and positive helicity-nonflip amplitudes which enter into the unpolarized inclusive cross section Σ enter only into one spin variable $D_{zz}\Sigma$.

The four-complex double-flip helicity amplitudes enter only into the other two diagonal components of the polarization tensor: $D_{xx}\Sigma$ and $D_{yy}\Sigma$. The amplitudes must conspire to produce the real-valued spin observables D_{xx} and D_{yy}.

There are two pairs of complex single-flip helicity amplitudes, with one pair contributing only to spin observables $P_\Lambda\Sigma$ and $D_{zx}\Sigma$, and the other pair appearing only in spin observables $A\Sigma$ and $D_{xz}\Sigma$.

In exclusive reactions the same set of complex helicity amplitudes determines the cross section and spin observables. We see that this is not the case in inclusive measurements where experiments with spin reach specific inclusive amplitudes which contribute only to a few spin observables.

The experiment with the polarized proton beam determines eight observables in terms of four real and eight complex inclusive amplitudes. More inclusive amplitudes are measured in experiments with the beam and target both polarized. Such experiments are required for a complete measurement and determination of the 32 real-valued amplitudes describing[21] the inclusive measurements with spin in $p+p\rightarrow\Lambda+X$.

In practice, the spin observables will be determined from the experimental angular distribution $W(\theta,\phi)$ in small bins of (x,p_T) using the methods of maximum likelihood. The optimization process has to take into account the positivity[22] of the spin-density matrix of Λ, and certain nonlinear constraints on the inclusive amplitudes of La France–Winternitz type.[23] Imposing inequality and nonlinear constraints during optimization involves methods of nonlinear programming[24] which requires special optimization programs, such as MINOS 5.0 developed at Stanford,[25] or others,[26] as well as a special treatment of the error matrix.[27] The last point reflects the fact that constraints carry and provide independent information. As well, the sensitivity of the solution point on the uncertainties in the acceptance of the apparatus should be examined.

IV. TESTS OF THE MUELLER THEOREM

We propose a systematic experimental determination of spin observables in $p_{\uparrow}+p\rightarrow\Lambda^0+X$ in the kinematic regions where the suppression of spin observables is expected to occur. To achieve high statistics in small bins of (x,p_T) as $p_T\rightarrow 0$, such experiments may require a kinematic trigger bias.

The measurement of $p_{\uparrow}+p\rightarrow\Lambda+X$ does not determine the individual amplitudes. But it does determine spin observables each of which is expressed in terms of amplitudes that satisfy the same angular constraints of Goldstein and Owens. We will use this fact to establish the signatures of violation of the Mueller theorem. There is also an additional signature independent of the angular constraints themselves.

We start by factoring out the angular constraints in the observables (17) which we rewrite as

$$\Sigma = K_1 A_1 + K'_1 B_1 \ , \quad (18a)$$
$$D_{zz}\Sigma = K_1 A_1 - K'_1 B_1 \ ,$$
$$D_{xx}\Sigma = K_2 A_2 + K'_2 B_2 \ , \quad (18b)$$
$$D_{yy}\Sigma = K_2 A_2 - K'_2 B_2 \ ,$$
$$P_\Lambda \Sigma = K_3 A_3, \quad D_{zx}\Sigma = K'_3 B_3 \ , \quad (18c)$$
$$A\Sigma = K_4 A_4, \quad D_{xz}\Sigma = K'_4 B_4 \ . \quad (18d)$$

The angular factors $K_i, K'_i, i=1,\ldots,4$ have a generic form:

$$K_i = (\sin\tfrac{1}{2}\theta)^{2m_i}(\cos\tfrac{1}{2}\theta)^{2n_i} \ ,$$
$$K'_i = (\sin\tfrac{1}{2}\theta)^{2m'_i}(\cos\tfrac{1}{2}\theta)^{2n'_i} \ . \quad (19)$$

Unitarity and angular momentum conservation require that all conditions (20) and (21) become satisfied, namely, that

$$m_1 = m'_1 = n_1 = n'_1 = 0 \ , \quad (20a)$$
$$m_2 = 0, \quad m'_2 = 2, \quad n_2 = 2, \quad m'_2 = 0 \ , \quad (20b)$$
$$m_3 = m'_3 = n_3 = n'_3 = 1 \ , \quad (20c)$$
$$m_4 = m'_4 = n_4 = n'_4 = 1 \ , \quad (20d)$$

and that the limits of reduced spin observables

$$\lim_{\theta\to 0} A_i = \overline{A}_i, \quad \lim_{\theta\to 0} B_i = \overline{B}_i \ , \quad (21a)$$
$$\lim_{\theta\to \pi} A_i = \overline{\overline{A}}_i, \quad \lim_{\theta\to \pi} B_i = \overline{\overline{B}}_i \ , \quad (21b)$$

exist and are finite or zero for $i=1,\ldots,4$ and at all values of (x,s).

In practice we can assume the constraints (20), compute A_i, B_i, $i=1,\ldots,4$ and extrapolate this data to $p_T \to 0$ for fixed (x,s). There are two possible signatures of violation of the Mueller theorem in these reduced spin observables. We may find in some region or regions of (x,s) (a) either a divergence of one or more of the limits (21), or (b) a nonexistence of one or more of the limits (21) due to a chaotic behavior of the observable as a function of p_T.

Notice that the cases $i=1,\ldots,4$ are separate because they involve different amplitudes. The measured spin observables can approach zero as $\theta\to 0$ or $\theta\to\pi$ faster than the unitarity limit (20). It is when some of the spin observables approach zero at a slower rate than is prescribed by the unitarity limit (20) that the corresponding reduced spin observable will diverge as $p_T\to 0$. The divergence can be removed only by reducing some of the powers in (19) to positive values below the unitarity limit (20). Since we do not expect the measured spin observables to diverge as $p_T\to 0$, the violation of the Mueller theorem in A_1 and B_1 which involve nonflip amplitudes can be manifested only through their chaotic behavior.

In an actual fitting the expressions (18) and (19) to measure data in any (x,s) bin, the powers in Eq. (19) can be left as free parameters. In general, the fits then may not provide the powers m_i, m'_i, n_i, n'_i expected from Eq. (20) but rather powers $\mu_i(x,s)$, $\mu'_i(x,s)$, $\nu_i(x,s)$, and $\nu'_i(x,s)$, $i=1,\ldots,4$, some or all of which may be different from the values given in Eq. (20). Assuming the limits (21) exist, the differences (for $i=1,\ldots,4$)

$$\epsilon_i(x,s) = \mu_i(x,s) - m_i \ ,$$
$$\epsilon'_i(x,s) = \mu'_i(x,s) - m'_i \ , \quad (22a)$$
$$\delta_i(x,s) = \nu_i(x,s) - n_i \ ,$$
$$\delta'_i(x,s) = \nu'_i(x,s) - n'_i \quad (22b)$$

become important observables. For negative values, the deviations (22) from the unitary powers (20) measure the violation of the Mueller theorem. Note that in general, the deviations (22) need not be all negative in the same (x,s) bin.

Double-flip amplitudes offer another test of unitarity. Assuming the angular constraints (20b), we can write (18b) as

$$D_{xx}\Sigma = (\cos\tfrac{1}{2}\theta)^4 A_2 + (\sin\tfrac{1}{2}\theta)^4 B_2 \ ,$$
$$D_{yy}\Sigma = (\cos\tfrac{1}{2}\theta)^4 A'_2 - (\sin\tfrac{1}{2}\theta)^4 B'_2 \ . \quad (23)$$

Unitarity requires $A_2 \equiv A'_2$ and $B_2 \equiv B'_2$. This implies

$$D_{xx} = +D_{yy} \quad \text{for } \theta\to 0 \ , \quad (24a)$$
$$D_{xx} = -D_{yy} \quad \text{for } \theta\to \pi \ . \quad (24b)$$

Let us now consider the observables Σ and $D_{zz}\Sigma$ which involve the nonflip amplitudes, and write (18a) as

$$\Sigma = A_1 + B_1 \ ,$$
$$D_{zz}\Sigma = A'_1 - B'_1 = A_1 - B_1 + \Delta \ . \quad (25)$$

Unitarity requires that $A_1 \equiv A'_1 \geq 0$ and $B_1 \equiv B'_1 \geq 0$, i.e., $\Delta = 0$. Assuming $\Delta \neq 0$ we get

$$X = \tfrac{1}{2}(1+D_{zz})\Sigma = A_1 + \tfrac{1}{2}\Delta \ ,$$
$$Y = \tfrac{1}{2}(1-D_{zz})\Sigma = B_1 - \tfrac{1}{2}\Delta \ . \quad (26)$$

For as long as $X \geq 0$ and $Y \geq 0$, the measurement of Σ and $D_{zz}\Sigma$ will not detect violation of unitarity. However, where A_1 or B_1 is sufficiently small and/or Δ is large, we may see $X<0$ or $Y<0$. The equivalent signature is

$$|D_{zz}| > 1 \quad (27)$$

for some values of (p_T, x, s). Notice that this test does not require the limit $p_T\to 0$ and is thus accessible to current measurements of $p_\uparrow + p \to \Lambda + X$ at the Fermilab spin facility.[17,18] The proper use of the maximum-likelihood function, optimization procedures, and statistical hypotheses testing will be crucial for this test.

The parity P conservation in three-body elastic scattering leads to parity relations for amplitudes (4) which read

$$A_{-a'-b'-\bar{c}',-a-b-\bar{c}} = (-)^{\xi} A_{a'b'\bar{c}',abc} \ , \quad (28)$$

where the helicity factor

$$\xi = (a-a') - (b-b) - (\bar{c} - \bar{c}') \ . \quad (29)$$

The Mueller theorem (5) implies similar parity relations for the inclusive amplitudes (2). These relations were used in the derivation of Eqs. (17) and (18). Conditions (24) and (27) subject to a simple test this aspect of the Mueller theorem.

V. DISCUSSION

The first tests of the Goldstein-Owens angular constraints (6) in $p + p \rightarrow \Lambda + X$ were done for Λ polarization P_Λ, and appear to be well satisfied with P_Λ showing a linear dependence[15,16] on p_T for $p_T \lesssim 1$ GeV/c. If the same conclusions can be reached for the remaining spin observables, then the use of the Mueller theorem in theoretical studies of hadron production based, e.g., on Regge-type models,[14,28] will receive new experimental justification. Moreover, the inclusive measurements with spin can then be viewed as an indirect method to measure the forward amplitudes in three-body scattering. Development of relativistic kinetics[29] of high-density hadron matter at high temperatures may require such information. While two-body forward amplitudes are related, e.g., to the refractive index of a medium, higher virial coefficients in the equation of state and transport equations require three-body scattering.[30] Spin experiments could be thus useful for studies of inaccessible states of hadron matter of interest to cosmology.

Let us now consider the possibility that some violations of the Mueller theorem are found. At small p_T they will show up either as a divergence of some limits in (21), or as a chaotic behavior of some spin observables leading to the nonexistence of corresponding limits in (21), or as a violation of conditions (24). At small and larger p_T, an independent signature of the Mueller theorem violation will be the nonphysical values (27), for the element D_{zz} of the polarization transfer tensor.

Since the Goldstein-Owens angular constraints (6) are based on accepted assumptions of conservation of angular momentum and parity P in three-body elastic scattering, any possible violations of these constraints must be traced to the assumptions involved in the derivation of the Mueller theorem for particles with spin. A detailed discussion of these questions will be presented elsewhere.

A question arises how seriously one should consider the possibility of chaotic behavior in some spin observables. Recently intermittent behavior was observed in nucleus-nucleus, hadron-nucleus, and to a larger degree in $\pi^+ p$ and $K^+ p$ multiparticle production.[31] The fluctuations in factorial moments of multiparticle distributions have been found largest in $e^+ e^-$ hadron production, indicating that the origin of possible chaos is in the dynamics of hadronization and not in the reaction dynamics.[32] The spin dependence of the hadronization processes[33–36] is not yet fully understood. Helicity transfer in quark hadronization was recently studied for processes with polarized heavy-quark production and subsequent jet formation.[37–39] In the context of semiclassical models for spin observables in inclusive production of hyperons based on the precession of quark spins[40,41] in confining color fields, the interaction of slow sea quarks with fast valence quarks via coupled precessions could lead to a chaotic component in the quark recombination dynamics. Observable effects of such chaos could be detected in sensitive observables in high-statistics experiments with spin. Previous models ignored the precession of fast valence quarks entirely.

To conclude, we suggest that the Mueller theorem should be tested in inclusive measurements with spin, such as $p_\uparrow + p \rightarrow \Lambda^0 + X$ discussed in this paper. The experiments will be feasible at the Fermilab spin facility[18] and, at lower energies but with a higher precision, at the proposed high-intensity hadron facilities.[42–48]

ACKNOWLEDGMENTS

I wish to thank L. van Rossum, A. Yokosawa, D. Underwood, R. Stanek, and other members of Fermilab E-704 Collaboration for discussions and interest. This work was supported by Fonds pour la Formation de Chercheurs et l'Aide à la Recherche (FCAR), Ministère de l'Education du Québec, Canada.

[1] R. J. N. Phillips, Nucl. Phys. **43**, 413 (1963).
[2] A. H. Mueller, Phys. Rev. D **2**, 2963 (1970).
[3] H. D. I. Abarbanel and D. J. Gross, Phys. Rev. D **5**, 699 (1972).
[4] R. J. N. Phillips *et al.*, Phys. Lett. **40B**, 239 (1972).
[5] M. G. Doncel and A. Méndez, Phys. Lett. **41B**, 83 (1972).
[6] Ph. Salin and J. Soffer, Nucl. Phys. **B71**, 125 (1974).
[7] J. P. Ader *et al.*, Nucl. Phys. **B82**, 237 (1974).
[8] R. D. Field, in *Proceedings of the Brookhaven National Laboratory Workshop on Physics with Polarized Targets,* Upton, New York, 1974, edited by J. S. Russ (National Technical Information Service, Springfield, VA, 1975), p. 155.
[9] J. Randa and A. Donnachie, Nucl. Phys. **B109**, 495 (1976).
[10] F. E. Paige and D. P. Sidhu, Phys. Rev. D **14**, 2307 (1976).
[11] G. R. Goldstein and J. F. Owens, Nucl. Phys. **B103**, 145 (1976).
[12] J. F. Owens, Phys. Lett. **63B**, 341 (1976).
[13] C. Bourrely, E. Leader, and J. Soffer, Phys. Rep. **59**, 98 (1980).
[14] S. N. Ganguli and D. P. Roy, Phys. Rep. **67**, 201 (1980).
[15] K. Heller, J. Phys. (Paris) Colloq. **46**, C2-121 (1985).
[16] L. G. Pondrom, Phys. Rep. **122**, 57 (1985).
[17] Fermilab Experiment E-704, A. Yokosawa, spokesperson.
[18] A. Yokosawa, in *High-Energy Spin Physics: Eighth International Symposium,* proceedings of the Eighth International Symposium at the University of Minnesota, 1988, edited by Kenneth J. Heller (AIP Conf. Proc. No. 187) (AIP, New York, 1989), Vol. 1, p. 210.
[19] M. Svec, in Workshop on Fermilab Experiment E-704, Fermilab 1983 (unpublished).
[20] J. D. Jackson, in *High Energy Physics, Les Houches 1965,* edited by C. DeWitt and M. Jacob (Gordon and Breach, New York, 1965), p. 367.
[21] G. R. Goldstein *et al.*, Int. J. Mod. Phys. A **3**, 1847 (1988).
[22] J. Soffer, Polarisation dans les réactions inclusives, CPT, C.N.R.S., Marseille, 1973 (unpublished); also in Proc. Colloque de la Division des Hautes Energies, Orsay, France,

1973, p. 1 (unpublished).

[23]P. La France and P. Winternitz, Phys. Rev. D **27**, 112 (1983).

[24]Ph. E. Gill, W. Murray, and M. M. Wright, *Practical Optimization* (Academic, New York, 1981).

[25]B. A. Murtagh and M. A. Saunders, MINOS 5.0 Users Guide, Systems Optimization Laboratory Report No. SOL 83-20, Stanford University, 1983 (unpublished).

[26]S. S. Rao, *Optimization: Theory and Applications* (Wiley, New York, 1984), p. 723.

[27]W. T. Eadie *et al.*, *Statistical Methods in Experimental High Energy Physics* (North-Holland, Amsterdam, 1971), p. 159.

[28]R. C. Browner *et al.*, Phys. Rep. C **14**, 257 (1974).

[29]S. R. De Groot, W. A. Van Leeuwen, and C. G. Van Wert, *Relativistic Kinetic Theory—Principles and Applications* (North-Holland, Amsterdam, 1980).

[30]E. M. Lifshitz and L. P. Pitaevski, *Physical Kinetics* (Pergamon, New York, 1981).

[31]I. V. Ajimenko *et al.*, Phys. Lett. B **222**, 306 (1989).

[32]W. Braunschweig *et al.*, Phys. Lett. B **231**, 578 (1989).

[33]O. Nachtman, Nucl. Phys. **B127**, 314 (1977).

[34]J. F. Owens, Phys. Lett. **76B**, 85 (1978).

[35]T. Uematsu, Phys. Lett. **79B**, 97 (1978).

[36]C. S. Lam *et al.*, Phys. Lett. **106B**, 411 (1981).

[37]R. H. Dalitz, R. Goldstein, and R. Marshall, Phys. Lett. B **215**, 783 (1988).

[38]R. H. Dalitz, R. Goldstein, and R. Marshall, Z. Phys. C **42**, 441 (1989).

[39]M. B. Einhorn, in *High-Energy Spin Physics: Eight International Symposium* (Ref. 18), p. 802.

[40]T. de Grand and M. I. Miettinen, Phys. Rev. D **24**, 2419 (1981).

[41]M. Nessi, in *High-Energy Spin Physics: Eight International Symposium* (Ref. 18), p. 66.

[42]E. Vogt, Nucl. Phys. **A450**, 453c (1986); J. Domigo, *ibid.* **A450**, 473c (1986); G. T. Garvey, *ibid.* **A450**, 539c (1986).

[43]*Canadian Kaon Factory* (TRIUMF, Vancouver, 1985).

[44]*Physics and a Plan for a 45 GeV Facility* (LANL Report No. LA-10720-MS, Los Alamos, 1986).

[45]*Proceedings of the International Conference on a European Hadron Facility,* Mainz, Germany, 1986, edited by T. Walcher [Nucl. Phys. **B279**, 2 (1987)].

[46]*International Workshop on Hadron Facility Technology,* Los Alamos, 1987, edited by H. A. Thiessen (LANL Report No. LA-11130-C, Los Alamos, 1987).

[47]Japanese Hadron Facility, in *International Workshop on Hadron Facility Technology* (Ref. 46), p. 67.

[48]M. Svec, in *High-Energy Spin Physics: Eight International Symposium* (Ref. 18), p. 1181.