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QCD mixing effects in a gauge-invariant quark model for photo- and electroproduction of baryon resonances

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The photo- and electroproduction of baryon resonances are calculated in the constituent quark model with chromodynamics of Isgur and Karl consistently to $O(v^2/c^2)$ for the quarks. We find that the successes of the nonrelativistic quark model are preserved, some problems are removed, and QCD mixing effects may become important with increasing q^2 in electroproduction. For the first time both spectroscopy and transitions receive a unified treatment within the framework of a potential quark model.

I. INTRODUCTION

For any composite system bound by a known potential, gauge invariance dictates the form of $H_{\rm em}$ for radiative transitions between eigenstates. This has not always been handled consistently in the literature. A particular example is hadrons treated as systems of constituent quarks bound in a potential with QCD-inspired single-gluon-exchange spin-dependent effects. There are $O(v^2/c^2)$ terms in the $H_{\rm QCD}$ that are important in fitting the spin-dependent splittings in hardon spectroscopy[1,2] and which induce configuration mixing among the SU(6)⊗O(3) symmetry states. Electromagnetic gauge invariance and consistency to $O(v^2/c^2)$ then dictate that the $H_{\rm em}$ includes spin-orbit and Wigner rotation contributions and may be written

$$H_{\rm em(rel)}=\sum_{i=1}^{2}\left[e_i\mathbf{r}_i\cdot\mathbf{E}_i+i\frac{e_i}{2m_i}(\mathbf{p}_i\cdot\mathbf{k}\,\mathbf{r}_i\cdot\mathbf{A}_i+\mathbf{r}_i\cdot\mathbf{A}_i\mathbf{p}_i\cdot\mathbf{k})-\mu_i\boldsymbol{\sigma}_i\cdot\mathbf{B}_i\right.$$
$$\left.-\frac{1}{2}\left[2\mu_i-\frac{e_i}{2m_i^*}\right]\boldsymbol{\sigma}_i\cdot\left[\mathbf{E}_i\times\frac{\mathbf{p}_i}{2m_i^*}-\frac{p_i}{2m_i^*}\times\mathbf{E}_i\right]+e_i\phi_i\right]$$
$$+\sum_{i<j}\frac{1}{4M_T}\left[\frac{\boldsymbol{\sigma}_i}{m_i^*}-\frac{\boldsymbol{\sigma}_j}{m_j^*}\right]\cdot(e_j\mathbf{E}_j\times\mathbf{p}_i-e_i\mathbf{E}_i\times\mathbf{P}_j)\ . \tag{1.1}$$

The first line is essentially $H_{\rm em(NR)}$ used in most work in the nonrelativistic constituent quark model (NRCQM) but where the first two terms replace the more usual $\mathbf{p}\cdot\mathbf{A}/m$ contribution. The second and third lines are $O(v^2/c^2)$ contributions which have long been known to be necessary *even for systems of free particles* if low-energy theorems and the Drell-Hearn-Gerasimov (DHG) sum rule are to be satisfied.[3,4] For particles bound by a scalar potential, the effect of the potentials is absorbed into a redefinition of the mass parameter m^*; one can avoid the explicit appearance of any vector potential at the price of replacing the $\mathbf{p}\cdot\mathbf{A}/m$ form of the usual $H_{\rm em(NR)}$ by the first two terms in the first line of Eq. (1.1) (this is discussed in detail in Ref. 5).

In Ref. 5 we studied how these $O(v^2/c^2)$ corrections to $H_{\rm em(NR)}$ (giving $H_{\rm em(rel)}$) affect the well-known successes of the NRCQM description of electromagnetic transitions. We found that they give perturbations comparable in magnitude to those found from QCD wave-function mixing in Ref. 6. In this paper we confront data by combining these two $O(v^2/c^2)$ effects, namely, $H_{\rm em(rel)}$ at Eq. (1.1) with the QCD mixings of Refs. 1 and 2.

The QCD-inspired quark model of Isgur and Karl gives a good description of baryon spectroscopy and predicts that there is significant wave-function mixing in some baryon states. Electromagnetic interactions are rather clean probes of the internal structure of baryons and, as transition probabilities are sensitive to wave-function mixings, the photo- and electroproduction of baryon resonances promise to provide sharp tests of any model. This is where the QCD-inspired relativistic corrections become important; a question to be investigated is whether the model can simultaneously describe the energy spectrum and the (electromagnetic) transitions. We find the phenomenological successes of $H_{\rm QCD}$ and $H_{\rm rel}$ separately combine "constructively," improving the overall fit to data. Perhaps most important, this is the first attempt to study photo- and electroproduction consistently in a quark model which gives a good description of the baryon spectroscopy and other properties of baryons. Therefore, both spectroscopy and electromagnetic transition can be and should be described within one model.

In the next section we discuss electromagnetic transitions between baryon resonances including QCD mixing effects, and compare with the results obtained in the absence of QCD mixing. In Sec. III we will extend our calculation to electroproduction, where we expect the QCD configuration mixing to give characteristic q^2 dependences to amplitudes. In the final section the paper is summarized and the role of QCD mixing emphasized by comparison with our earlier calculation.[5]

II. THE PHOTOPRODUCTION OF BARYON RESONANCES

The quark potential model introduced by Isgur and Karl[1,2] shows that the QCD-inspired relativistic corrections to the quark binding potential are very important in describing the baryon spectroscopy. Generally, the quark-quark interaction can be divided into long- and short-range potentials; the long-range potential corresponds to the confinement of QCD which is treated as linear or harmonic, while De Rújula, Georgi, and Glashow[6] attribute the short-range potential to the one-gluon-exchange interaction, of which the nonrelativistic limit corresponds to the Coulomb potential. The important spin-dependent mass splittings arise from the hyperfine interaction, i.e., the $O(v^2/c^2)$ Breit-Fermi generalization of one-gluon exchange.

Two main features have been shown in the model of Isgur and Karl. First, the perturbations of the spin-independent interaction including the Coulomb potential are anharmonic; the largest energy shift is for the SU(6)⊗O(3) state $(56',0^+)$, and vanishes for the SU(6)⊗O(3) state $(20,1^+)$. This is why the resonance $P_{11}(1470)$ in which the SU(6)⊗O(3) state $(56',0^+)$ is dominant has a relatively low mass while the SU(6)⊗O(3) $(20,1^+)$ has not been observed experimentally, in part because its mass scale is predicted to be relatively high. Second, because of the tensor structure of the hyperfine interaction, the total spin or total orbital angular momentum are no longer conserved separately, which induces the SU(6)⊗O(3) configuration mixing. The corresponding baryon states are therefore superposition of the SU(6)⊗O(3) basis states,

$$|\Phi_{\rm baryon}\rangle=\sum_i C_i|\Phi^i_{{\rm SU}(6)\otimes {\rm O}(3)}\rangle \ , \tag{2.1}$$

and the coefficients C_i can be found in Refs. 1 and 2. Following the notation of Isgur and Karl, the ground-state wave function in terms of the SU(6)⊗O(3)[7] basis is

$$|N\rangle \simeq 0.90|N\,^2S_S\rangle-0.34|N\,^2S_{S'}\rangle -0.27|N\,^2S_M\rangle-0.06|N\,^2D_M\rangle \ . \tag{2.2}$$

Therefore, since the configuration mixing determined mostly by the Breit-Fermi interaction is at $O(v^2/c^2)$, we should for consistency employ $H_{\rm em}$ to the same order, i.e., that given in Eq. (1.1).

Based on this model, the transition matrix elements can be calculated in the SU(6)⊗O(3) basis. Following the same procedure as in our previous calculation,[5] we write the electromagnetic interaction in Eq. (1.1) as

$$H=H_{\rm AD}+H_{\rm NA} \ , \tag{2.3}$$

where $H_{\rm AD}$ is the additive part of Eq. (1.1) (namely, the first two lines) and $H_{\rm NA}$ is the so called "nonadditive" interaction due to the Wigner rotation of the quark spins transferred from the frame of the recoiling quark to the baryon center frame of the recoiling baryon.[4] To emphasize the group structure we have

$$H_{\rm AD}=q^{(3)}(AL_+ +BS_+ +CS_zL_+) \tag{2.4a}$$

and

$$H_{\rm NA}=q^{(2)}(B_{\rm NA}S_+^{(1-2)}-C_{\rm NA}S_z^{(1-2)}L_+) \ . \tag{2.4b}$$

According Eq. (1.1), the coefficients in Eq. (2.4) can be written as

$$A=6\left[\frac{\pi}{k_0}\right]^{1/2}\mu\frac{im_q}{g}\left\langle\Psi_f\left|\exp(ikz_{(3)})r_+^{(3)}\left[k_0-\frac{k}{m_q}\left[P_z^{(3)}+\frac{k}{2}\right]\right]\right|\Psi_i\right\rangle \ , \tag{2.5a}$$

$$B=6\left[\frac{\pi}{k_0}\right]^{1/2}k\mu\left\langle\Psi_f\left|\exp(ikz_{(3)})\left[1-\frac{k_0}{2m_qk}\left[2-\frac{1}{g}\right]\left[P_z^{(3)}+\frac{k}{2}\right]\right]\right|\Psi_i\right\rangle, \tag{2.5b}$$

$$C=6\sqrt{\pi k_0}\frac{\mu}{2m_q}\left[2-\frac{1}{g}\right]\langle\Psi_f|\exp(ikz_{(3)})P_+^{(3)}|\Psi_i\rangle \ , \tag{2.5c}$$

TABLE I. Photon-decay amplitudes of the P-wave baryon resonances (theory versus experiment). The experimental data are given by the most recent Particle Data Group compilation (Ref. 23). $A^{\rm NR}$ is the nonrelativistic result, $A^{\rm RE}$ are the nonrelativistic results plus the relativistic correction. Both $A^{\rm NR}$ and $A^{\rm RE}$ are calculated in c.m. frame with the harmonic-oscillator strength $\alpha^2=0.175\ \mathrm{GeV}^2$, see Ref. 8. The A_1^M is the result with QCD mixing effects and the relativistic corrections, in which the harmonic-oscillator strength is $\alpha^2=0.175\ \mathrm{GeV}^2$. A_2^M is the same as A_1^M except the α^2 is 0.09 GeV^2. All amplitudes have 100 GeV$^{1/2}$ unit.

Multiplet states	A_J^N	$A^{\rm NR}$	$A^{\rm RE}$	A_1^M	A_2^M	$A^{\rm expt}$
$(70,1^-)_1S_{11}(1535)$	$A^p_{1/2}$	174	163	142	162	73 ± 14
	$A^n_{1/2}$	-128	-106	-77	-90	-76 ± 32
$D_{13}(1520)$	$A^p_{1/2}$	-20	-30	-47	-51	-22 ± 10
	$A^n_{1/2}$	-43	-49	-75	-79	-65 ± 13
	$A^p_{3/2}$	131	146	117	133	167 ± 10
	$A^n_{3/2}$	-131	-146	-127	-153	-144 ± 14
$S_{11}(1700)$	$A^p_{1/2}$	0	25	78	97	48 ± 16
	$A^n_{1/2}$	28	10	-47	-60	-17 ± 37
$D_{13}(1700)$	$A^p_{1/2}$	0	-27	-16	-1	-22 ± 12
	$A^n_{1/2}$	-13	12	35	11	0 ± 56
	$A^p_{3/2}$	0	-47	-42	-18	0 ± 19
	$A^n_{3/2}$	-66	-36	10	-15	-2 ± 44
$D_{15}(1675)$	$A^p_{1/2}$	0	0	8	8	19 ± 12
	$A^n_{1/2}$	-37	-47	-30	-31	-47 ± 23
	$A^p_{3/2}$	0	0	11	11	19 ± 12
	$A^n_{3/2}$	-52	-66	-42	-44	-69 ± 19
$S_{31}(1620)$	$A^{p,n}_{1/2}$	68	109	72	40	19 ± 16
$D_{33}(1675)$	$A^{p,n}_{1/2}$	100	68	81	74	116 ± 17
	$A^{p,n}_{3/2}$	104	78	58	60	77 ± 28

while

$$B_{\rm NA}=\sqrt{\pi k_0}\frac{2\mu}{m_q g}\langle\Psi_f|\exp(ikz_{(2)})P_z^{(1)}|\Psi_i\rangle\ , \qquad (2.5\mathrm{d})$$

$$C_{\rm NA}=\sqrt{\pi k_0}\frac{2\mu}{m_q g}\langle\Psi_f|\exp(ikz_{(2)})P_+^{(1)}|\Psi_i\rangle\ , \qquad (2.5\mathrm{e})$$

where k_0 is the photon's energy and k is the photon's momentum. These are equal in magnitude for the case of real photons but differ if we extend our calculation to virtual photon, i.e., for electroproduction. $|\Psi_i\rangle$ and $|\Psi_f\rangle$ are the harmonic-oscillator wave functions of the three-quark system.

In Tables I and II we show the calculation of the photoproduction amplitudes with and without QCD mixing

TABLE II. Photon-decay amplitudes for the positive-parity baryon resonances. The experimental data are given by the most recent Particle Data Group compilation (Ref. 23). For the significance of $A^{\rm NR}$, $A^{\rm RE}$, A_1^M, and A_2^M, see caption to Table I.

Multiplet states	A_J^N	$A^{\rm NR}$	$A^{\rm RE}$	A_1^M	A_2^M	$A^{\rm expt}$
$[56,2^+]_2P_{13}(1720)$	$A^p_{1/2}$	-113	-75	-68	-112	52 ± 39
	$A^n_{1/2}$	32	16	-4	24	-2 ± 26
	$A^p_{3/2}$	38	49	53	55	-35 ± 24
	$A^n_{3/2}$	0	-5	-33	-17	-43 ± 94
$F_{15}(1680)$	$A^p_{1/2}$	0	6	-8	-15	-17 ± 10
	$A^n_{1/2}$	36	37	11	15	31 ± 13
	$A^p_{3/2}$	76	98	105	107	127 ± 12
	$A^n_{3/2}$	0	-11	-43	-36	-30 ± 14
$P_{31}(1910)$	$A^{p,n}_{1/2}$	-21	-19	-28	-27	-12 ± 30
$P_{33}(1920)$	$A^{p,n}_{1/2}$	-21	-20	-14	-26	$43\pm?$
	$A^{p,n}_{3/2}$	36	21	-7	18	$23\pm?$
$F_{35}(1905)$	$A^{p,n}_{1/2}$	-14	-1	24	13	27 ± 13
	$A^{p,n}_{3/2}$	-60	-53	25	-9	-47 ± 19
$F_{37}(1950)$	$A^{p,n}_{1/2}$	-36	-47	-28	-35	-73 ± 14
	$A^{p,n}_{3/2}$	-48	-65	-36	-45	-90 ± 13
$[56,0^+]_2P_{11}(1470)$	$A^p_{1/2}$	26	10	-93	-80	-69 ± 7
	$A^n_{1/2}$	-18	-11	67	60	37 ± 19
$P_{33}(1600)$	$A^{p,n}_{1/2}$	-22	-15	-38	-33	-22 ± 29
	$A^{p,n}_{3/2}$	-38	-25	-70	-64	1 ± 22
$[56,0^+]_0P_{33}(1232)$	$A^{p,n}_{1/2}$	-101	-113	-94	-93	-141 ± 5
	$A^{p,n}_{3/2}$	-173	-195	-162	-160	-258 ± 19
$[70,0^+]_2P_{11}(1705)$	$A^p_{1/2}$	-40	-18	-18	-16	5 ± 16
	$A^n_{1/2}$	13	4	-22	-23	-5 ± 23

effects for the P-wave baryon resonances and the positive-parity baryon resonances. We show how these magnitudes are built up: the nonrelativistic results and the relativistic contribution (spin-orbit and nonadditive terms combined) were calculated in the center-of-mass frame with harmonic-oscillator strength $\alpha^2=0.175$ GeV^2, the value used in earlier electromagnetic calculations.[8–10] The calculation including the QCD mixing effect is performed in the Breit frame, in which recoil effects vanish and so electromagnetic transitions correspond directly to internal excitations. (We have also made similar calculations in the center-of-mass frame, and some fluctuations exist, but generally do not affect the qualitative behavior of the photoproduction amplitudes.) We show the photoproduction amplitudes with the harmonic-oscillator strength $\alpha^2=0.175$ GeV^2 (A_1^M in the tables), the standard value used in nonrelativistic calculation, and $\alpha^2=0.09$ GeV^2 (A_2^M in the tables), the value suggested by Isgur and Karl in their fitting baryon spectroscopy. Qualitatively, our results show that the photoproduction amplitudes are quite stable from $\alpha^2=0.175$ GeV^2 to $\alpha^2=0.09$ GeV^2, if the QCD mixing effects are included, and the success of the nonrelativistic calculation survives the relativistic corrections in the transition operator as well as in the binding potential. To the extent that the A^M [the calculated amplitudes including mixing and (v^2/c^2) effects] agree with $A^{\rm expt}$ we thus have, for the first time, a unified consistent description of spectroscopy and electromagnetic transitions within the framework of the potential quark model by Isgur and Karl. [The small component $|N\,^2D_M\rangle$ in Eq. (2.2) is neglected in our calculation.]

The highlight of the original nonrelativistic calculation by Copley, Karl, and Obryk[9] is that, in addition to the selection rules dictated by SU(6)⊗O(3) symmetry, there are also dynamical selection rules which arise because the photoproduction amplitudes for the resonances $D_{13}(1520)$ and $F_{15}(1688)$ have the form

$$A_{1/2}^p \sim \begin{cases} 1-\dfrac{k^2}{\alpha^2} & \text{for } D_{13}(1520)\,, \\[2ex] 1-\dfrac{k^2}{2\alpha^2} & \text{for } F_{15}(1688)\,. \end{cases} \tag{2.6}$$

Empirically these amplitudes are small and Copley *et al.* forced this in the center-of-mass frame by choosing a value for the oscillator strength $\alpha^2=0.175$ GeV. The excited energies of baryon resonances can qualitatively be described with this value and so the calculation is consistent within the nonrelativistic framework. These dynamical selection rules become nontrivial with the QCD mixing effects; the formalism becomes more complicated because of large configuration mixing and relativistic corrections, and the harmonic-oscillator strength required to fit the resonance mass spectrum is changed due to the anharmonic perturbation of the spin-independent interactions (both in the short range $1/r$ and, possibly, long-range linear).

In some cases the QCD mixings make significant contributions. The most noticeable examples are probably the amplitudes for the Roper resonance, $P_{11}(1470)$. According to the results of Isgur and Karl; the SU(6)⊗O(3) state $|N\,^2S_{S'}\rangle$ dominates the wave function of the resonance $P_{11}(1470)$, the matrix element between SU(6)⊗O(3) states $|N\,^2S_{S'}\rangle$ in Eq. (2.2) and in the resonance $P_{11}(1470)$ contributes significantly to the photoproduction amplitudes. Indeed, the QCD mixing effects play a crucial role in determining the mass and photoproduction amplitude of this state. But it is premature to say the nature of the Roper resonance has been fully understood; the calculation for the pionic decay of the resonance with the relativistic correction and the mixed wave function has not yet been done. Presumably one could expect that the QCD mixing effect might also play an important role in the pion decay of the Roper resonance. Furthermore, without the relativistic correction $H_{\rm em(rel)}$ the ratio between the Roper excitation amplitudes $A_{1/2}^p$ and $A_{1/2}^n$ should be $-\frac{2}{3}$, which is the ratio of the magnetic moments of proton and neutron, but this is destroyed by the relativistic correction. More accurate experimental information on the Roper excitation amplitudes could therefore help to establish the importance of the relativistic effects.

For the $L=1$ negative-parity baryon resonances we find a large contribution from the matrix element between the state $|N\,^2S_M\rangle$ in Eq. (2.2) and $|N\,^2P_M\rangle$. There is a major unresolved problem with the most prominent of these resonances, namely, the photoproduction amplitudes for the resonance $S_{11}(1535)$. These were found to be too large in the nonrelativistic model[8] and remained large even after QCD mixing was incorporated.[6] Our inclusion of $H_{\rm em(rel)}$ does not essentially change this. The transition from ground state to $|N\,^2P_M\rangle$ dominates the photoproduction amplitudes of the resonance $S_{11}(1700)$, and so QCD mixing affects these amplitudes significantly; the effect of $H_{\rm em(rel)}$ is small here in comparison.

It is interesting to notice that the relation

$$D_{15}(1675), \quad A_{3/2}=\sqrt{2}\,A_{1/2} \tag{2.7}$$

for both the neutron and proton survives the QCD mixing effects, and is independent of the choice of the frame. This has been shown by Isgur *et al.*[7]; the result presented here shows that this relation survives the use of $H_{\rm em(rel)}$ as well though the absolute magnitudes have changed because of our relativistic corrections and the change of the harmonic-oscillator strength. This is very important; in the Isgur-Karl model, the resonance $D_{15}(1675)$ is a pure SU(6)⊗O(3) state; therefore, relation (2.7) is determined by the symmetry and the experimental data are consistent with this relation.

The QCD mixing and use of $H_{\rm em(rel)}$ play an important role for the resonances $S_{31}(1620)$ and $D_{33}(1675)$ as can be seen by comparing Table I with the results in Table VII of Ref. 8.

Another important resonance is the $P_{33}(1600)$; it is the partner of the Roper resonance in the 56-plet of SU(6) in the unmixed spectrum, but as the QCD mixing plays an essential role in making these states so light in mass one may expect that photoproduction amplitudes may also be significantly modified. If the state $|\Delta\,^4S_{S'}\rangle$ dominates the

$P_{33}(1600)$ resonance, as implied by the Isgur-Karl calculation, then the photoproduction amplitude should satisfy the relation

$$A_{3/2}=\sqrt{3}\,A_{1/2} \tag{2.8}$$

because it is pure magnetic dipole transition. This result obtains in calculations either with or without QCD mixing; however, the data look as if they may violate this, though the error bars are too great to draw this conclusion definitively. Clearly this is an example where better data may be crucial. The status of the resonance $P_{33}(1600)$ is far from clear, and more accurate information is needed; good data on the electromagnetic transitions could help establish both the existence and the internal-symmetry structure of this state.

Notice that for the $P_{33}(1232)$ the calculated magnitudes after the inclusion of both relativistic corrections and the QCD mixing effects are essentially unchanged from the nonrelativistic magnitudes (see Table II) and as such remain unsatisfactory.

There has been a suggestion that the resonance $P_{11}(1705)$ could be the lightest hybrid baryon in view of a selection rule[11] that says this hybrid state is not photoproduced from proton targets. The data were consistent with this zero and inconsistent with the predictions of the nonrelativistic quark model (column 1 of Table II). However, we see that the relativistic effect and QCD mixing change the predicted magnitudes significantly such that they are now in better agreement with data. Improved data and extension to electroproduction may help to elucidate the nature of this resonance.

Overall the results show that the success of the nonrelativistic calculation is preserved by the QCD mixing and corresponding relativistic effects, and there are improvements for some cases where the QCD mixing effects become important. In the next section we discuss electroproduction, where we expect that the QCD mixing effects may become more noticeable as q^2 increases.

III. ELECTROPRODUCTION

Foster and Hughes[12] have attempted to extend the quark-model calculations to electroproduction using the interaction at Eq. (2.4a). In order to make a most direct comparison we shall follow the procedure of Foster and Hughes whose calculations were for an equal-velocity frame (EVF) (which is very close to the Breit frame). When the calculation is extended to electroproduction, it becomes highly relativistic due to the large momentum transfer, and the nonrelativistic wave function of a three-quark system is no longer adequate to describe such dynamic process, so these authors introduced a Lorentz-boost factor in the spatial integrals of Eq. (2.5); thus,

$$R(k)\to\frac{1}{\gamma^2}R\left[\frac{k}{\gamma}\right], \tag{3.1}$$

which helps to remove the exponential decay with q^2 (same motivation is in Ref. 13). In the EVF frame, the Lonretz-boost factor is

$$\gamma=\left[1+\frac{k^2}{(M_r+M_p)^2}\right]^{1/2}, \tag{3.2}$$

where M_r and M_p are the masses of the resonance and nucleon. The relation between the momentum k and the mass of the virtual photon in the EVF is

$$k^2(\mathrm{EVF})=\frac{(M_r^2-M_p^2)^2}{4M_rM_p}+\frac{q^2(M_r+M_p)^2}{4M_rM_p}. \tag{3.3}$$

Substitute Eqs. (3.1), (3.2), and (3.3) into the photodecay amplitudes $A_{1/2}$ and $A_{3/2}$, so we have the dependence of helicity amplitude on virtual-photon mass q^2.

In the SU(6)⊗O(3) basis, the photoproduction amplitudes can be derived analytically, so we could easily extend our calculation to electroproduction with the help of Eqs. (3.1)–(3.3). In Figs. 1(a)–1(d) we show the results of our calculations of the transverse amplitudes for the resonances $S_{11}(1535)$, $D_{13}(1520)$, $F_{15}(1688)$, and the Roper resonance $P_{11}(1470)$.

For real photoproduction the amplitudes are not particularly sensitive to the value of $\alpha^2=0.09$ to $0.175\ \mathrm{GeV}^2$. However for electroproduction α^2 determines the scale of q^2 at which the form factor begins to die off and, in particular, controls the magnitude of the charge radius of the nucleon. If one chooses $\alpha^2=0.175\ \mathrm{GeV}^2$ the corresponding charge radius of the nucleon is about 0.5 fm, which is too small compared to the data; therefore, one cannot expect the NR calculation to give a good description of the q^2 dependence. In order to overcome this, Foster and Hughes introduced an extra *ad hoc* form factor, which is outside of the model, and fitted to the data. If a smaller value of α^2, such as $0.09\ \mathrm{GeV}^2$, is used, there

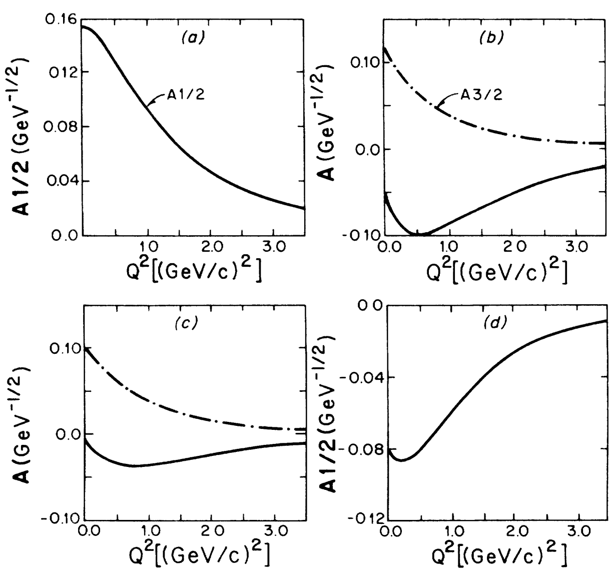

FIG. 1. The q^2 dependence of the helicity amplitudes (a) $A_{1/2}^p$ for the $S_{11}(1535)$, (b) A for the $D_{13}(1520)$, (c) $F_{15}(1688)$, where the solid (dot-dash) line represents $A_{1/2}^p(A_{3/2}^p)$, and (d) $A_{1/2}^p$ for the $P_{11}(1470)$. The QCD mixing effects and relativistic corrections are included. For the experimental situation, see Ref. 14.

is no need for such an *ad hoc* manipulation and the corresponding charge radius is closer to the data as indicated by Isgur and Karl.[1]

The relative importance of different components in the mixed wave functions varies with q^2 for the following reason. The transition form factors to components with large-N quantum numbers has a larger threshold behavior and hence falls less rapidly with q^2 than does the transition to the small-N components. Therefore large-q^2 electroproduction feels increasingly the large-N components in the wave functions which could cause the ratios of helicity amplitudes to vary, perhaps significantly.

Comparing our calculations with the data,[14] we find that although the QCD mixing effects have improved the photoproduction amplitude for the Roper resonance they do not solve the problem of why this amplitude empirically appears to change sign with increasing q^2. A similar result has been shown by Warn *et al.*[15] This might imply that the wave functions given by the calculation of Isgur and Karl do not have the correct internal structure for the Roper resonance or we need another dynamical approach to electroproduction. Gavela *et al.*[16] have shown that the quark-pair-creation model (QPCM) may give a natural explanation of such behavior. It would be a challenge to show that the success of the QPCM survives the QCD mixing effect and also gives a consistent description for other baryon resonances. So, the q^2 dependence of helicity amplitudes may help us to distinguish the different dynamical approaches. Furthermore, because of the large QCD mixing effect for the Roper resonance, the restriction of mixing to a single harmonic-oscillator shell in the Isgur-Karl calculation becomes unrealistic, especially as q^2 increases. A recent study[17] suggests that two resonances may be associated with the Roper resonance which might support a scenario that one of these is dominantly a hybrid $gQQQ$ state, and the other is a conventional QQQ state. Comparison of γN and πN data will be interesting, since on proton targets two states would contribute to πN, but only one to γN (Ref. 12). The reversed process $\pi_- p \rightarrow N^* \rightarrow \gamma n$ could access the neutron coupling of both conventional and hybrid baryon states. In electroproduction, this leads to a zero contribution from the proton targets but a nonzero one for neutron targets. Further study should be made on the resonance $P_{33}(1600)$, which would be the 56-plet partner of the Roper resonance in the QQQ model; but for the hybrid baryons, no such partner exists.

In order to see the QCD effects on the structure of the wave function, we turn to the helicity asymmetry of baryon resonances, which is defined as

$$A = \frac{A_{1/2}^2 - A_{3/2}^2}{A_{1/2}^2 + A_{3/2}^2} \tag{3.4}$$

which is more sensitive to the structure of the wave function and independent of the radial behavior of the wave function. Furthermore, the Drell-Hearn-Gerasimov[4] (DHG) sum rule and the Bjorken[18] sum rule suggest a rather nontrivial dependence on q^2 of the helicity asymmetry. Indeed, Close and Gilman[19] have shown that there is a dramatic change of the helicity asymmetry for the resonance $D_{13}(1520)$ and $F_{15}(1688)$ with increasing q^2 using the nonrelativistic quark model. Our calculation with the relativistic correction and the QCD mixing effects (shown in Fig. 2) has consistently confirmed the prediction of Close and Gilman, which is also in agreement with experimental data. Such behavior shows that the magnetic-multipole moments become dominant at high q^2. This is a very important result; if the Bloom-Gilman duality assumption is correct,[20] the dramatic change in helicity asymmetry implies that the Bjorken structure polarized function $g_1(x)$ could become negative as $x \rightarrow 0$. Quantitatively, the QCD mixing effects have improved the agreement with data for the resonance $F_{15}(1688)$, and made little change for $D_{13}(1520)$.

Another interesting quantity is the ratio $A_{1/2}^p(D_{13})/A_{1/2}^p(S_{11})$. In the nonrelativistic quark model, both resonances $S_{11}(1535)$ and $D_{13}(1520)$ belong to the same SU(6)⊗O(3) representation $|N\,^2P_M\rangle$. Because the magnetic-dipole transition dominates at high q^2, this ratio should be the ratio of the magnetic-dipole matrix elements in SU(6)⊗O(3) states, which is the Clebsch-Gordan coupling coefficient. Quantitatively, it can be shown that the ratio should be $\sqrt{2}$ with $q^2 \rightarrow \infty$ for the SU(6)⊗O(3) state $|N\,^2P_M\rangle$, which is also true under the

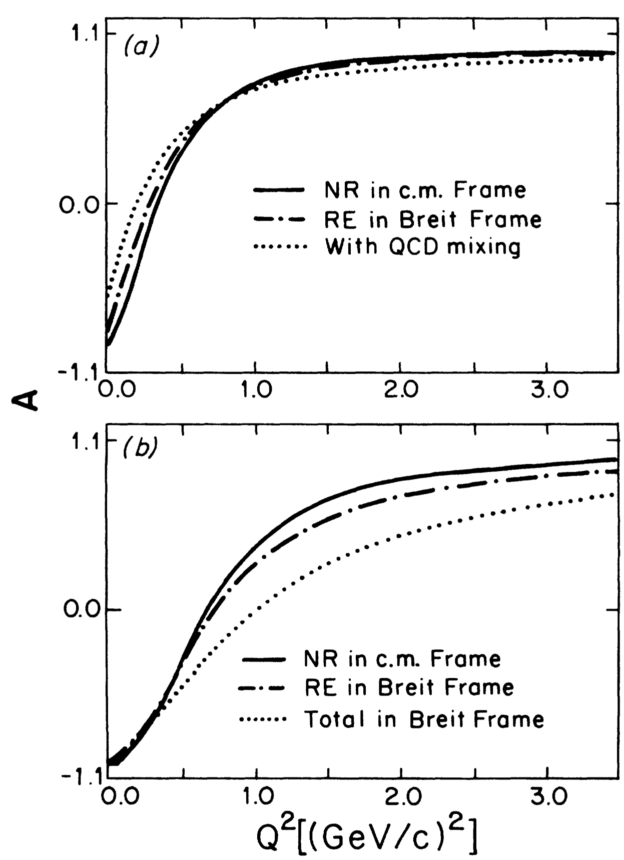

FIG. 2. The helicity asymmetry for (a) $D_{13}(1520)$ and (b) $F_{15}(1688)$. The solid line represents the nonrelativistic results (Ref. 18), dot-dash line is the results with relativistic correction (Ref. 5), and the dotted line is the calculation with the QCD mixing effects and the relativistic correction. See text.

relativistic extension. Therefore it would be interesting to see how QCD mixing affects this quantity. In Fig. 3 we show the results with the QCD mixing effects and the nonrelativistic calculation from the calculation of Copley, Karl, and Obryk. The two calculations are quite close to each other, and neither gives a good description of the experimental data at higher q^2. According to the Isgur-Karl result, there is a large configuration mixing for $S_{11}(1535)$, but rather little mixing for $D_{13}(1520)$. Because the matrix element between the state $|N\,{}^2P_M\rangle$ and the ground state dominates the transition amplitudes, the qualitative behavior would remain the same as the nonrelativistic calculation. Therefore, here the QCD mixing effects fail to improve the agreement with data at high q^2. It may be that higher configurations in the QCD mixed wave function are becoming increasingly important and that we are seeing a transition where some other approximation or model approach needs to be developed. One such suggestion (e.g., Carlson[21]) is that there are effects arising from the perturbative QCD which begin to show up at moderately small q^2.

One still can test the validity of the static symmetry of the quark model in electroproduction in order to understand at what value of q^2 the static symmetry begins to fail. One such example is the helicity asymmetry for the resonance $P_{33}(1232)$ which the model predicts should be $-\frac{1}{2}$, and for $D_{15}(1675)$ which the model predicts $-\frac{1}{3}$ for the neutrons and protons independent of QCD mixing effects.

IV. CONCLUSION AND DISCUSSION

In our earlier paper, Ref. 5, we showed that theoretical consistency implies that calculations of electromagnetic transition amplitudes between baryons whose quark wave functions including QCD configuration mixing at $O(v^2/c^2)$ require $H_{\rm em}$ to be developed consistently to $O(v^2/c^2)$ too. In this paper we have applied this $H_{\rm em(rel)}$ to calculate the electromagnetic transition amplitudes between baryons whose wave functions include QCD mixings inspired by the Isgur-Karl QCD potential model. We find that the successes of the NRCQM are preserved and some, though not all, of its problems are removed. A significant advance over previous work is that both spectroscopy and transitions can be described with the same values of parameters, and with no need of the *ad hoc* form factors, which are essential prerequisites for understanding the underlying dynamics of quarks within a baryon in the nonperturbative region. We find the overall results with QCD mixing effects do not change very much in the photoproduction amplitudes from $\alpha^2=0.175$ GeV^2 to $\alpha^2=0.09$ GeV^2, but the q^2 dependence of helicity amplitudes clearly favors $\alpha^2=0.09$ GeV^2, which is also consistent with fitting of the baryon spectroscopy. This calculation also confirms the dramatic change of the helicity asymmetry predicted in the nonrelativistic quark model. In order to confirm whether the model is indeed consistent, one should reevaluate the pion decays of baryon resonances, including both the QCD mixed wave functions and related relativistic corrections in the transition operator. Only if this is phenomenologically successful will one be able to claim self-consistent treatment of baryon spectroscopy and dynamics.

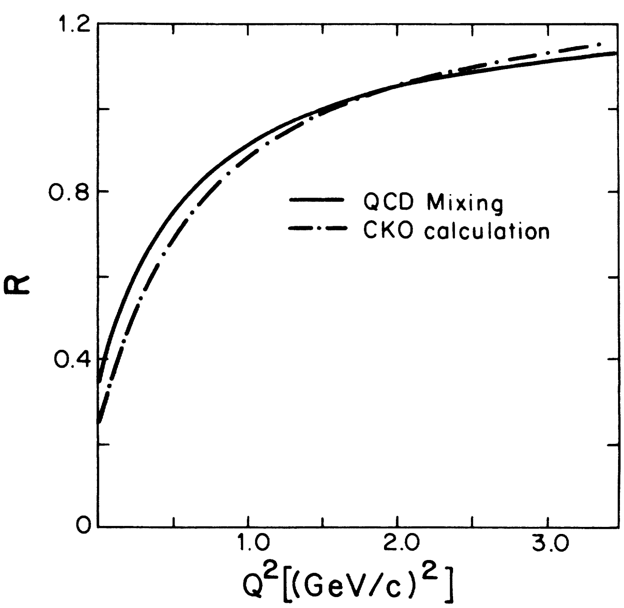

FIG. 3. The ratio $R=|A_{1/2}^p(D_{13})/A_{1/2}^p(S_{11})|$, the solid line represents the results of the QCD mixing and dot-dash line is the results by Copley *et al.* (Ref. 8). For the experimental data, see Ref. 14.

Despite the successes of $H_{FB(\rm QCD)}$ there are still many problems unanswered in this model such as the role of the spin-orbit interaction in spectroscopy. Furthermore, if the energy shift due to the anharmonic perturbation of the spin-independent interaction is quite large, one could question the validity of the perturbation method itself. In the wave-function mixing there is also the question of how results are modified when one relaxes the Isgur-Karl restriction of a single harmonic-oscillator shell. As noted in the text, when

$$\frac{q^2}{2M} > \frac{N\alpha^2}{M_q} \tag{4.1}$$

mixings involving N shells may become increasingly important, especially for the q^2 dependence of helicity amplitudes. Therefore continuing effort should be made in the relativized model of Capstick and Isgur,[22] in which there are many improvements and the calculations are not restricted to a single harmonic-oscillator shell. Furthermore, if hybrid baryons exist below 2 GeV there should be a mixing between the hybrid and conventional baryons, and gluon degrees of freedom will play an explicit role in both the hadron spectrum and transition processes.

In summary, when $H_{\rm em}$ is consistently developed to $O(v^2/c^2)$ the quark model with QCD mixing effects at this same order is successful in describing simultaneously the spectrum and electromagnetic transition processes. There are however many unanswered questions, in particular, concerning the extension of $q^2 \neq 0$ and certainly future experiments at CEBAF will provide more information and challenges to the potential quark model.

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