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Photo- and electroproduction of N^* in a quark model

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We review and update the theory on photo- and electroproduction in the nonrelativistic constituent quark model (NRCQM). The electromagnetic interaction Hamiltonian for a three-quark system is discussed, particularly the nonadditive term and the role of the binding potential which have been neglected in previous calculations. We find that the successes of the NRCQM survive these relativistic corrections and make predictions for baryon-resonance excitation that can be tested in forthcoming experiments; however, we suggest that some QCD improvements to the NRCQM are incomplete. Finally we discuss the limits of application of the long-wavelength approximation in resonance photo- and electroproduction.

I. INTRODUCTION

The nonrelativistic constituent quark model[1-3] (NRCQM) has existed for nearly 25 years. It is highly successful phenomenologically, has very little theoretical justification, and as a "true" description of nature must be wrong. Progress in understanding could follow if first we make the model break down in experiment and then learn from the nature of its failure.

Photon interactions, such as provided at the Continuous Electron Beam Accelerator Facility (CEBAF), could provide important information in this regard. Surprisingly, the literature on electromagnetic interactions in the NRCQM is incomplete and even inconsistent. It is our purpose here to clarify the state of the literature, identify the areas of inconsistency, and bring it up to date, with a view to setting targets for the forthcoming program at CEBAF. This is not simply a rerun of old ground. Given the interest in gluonic degrees of freedom, and the possible existence of hybrid baryons in the mass range accessible to the new machines, it is important to have the best understanding of the "conventional" baryons. Photoexcitation of the low-lying hybrid states contain some rather interesting selection rules[4] and so electromagnetic interactions may enable these states to be isolated from QQQ resonance states.

The photon is a clean probe in that it couples to the spin and flavor of the constituent quarks and reveals the correlation among the flavors and spin inside the target. The photon has spin 1, so helicity amplitudes where the photon and target spin parallel (net helicity $\frac{3}{2}$) or antiparallel (net helicity $\frac{1}{2}$) give rich structure. Furthermore, since the photon has a mixed isospin zero or one, both proton and neutron targets give independent information.

By exciting nucleon resonances in various helicity states from proton and neutron targets, one can form ratios or linear combinations of amplitudes where specific features of confinement, common to all the amplitudes, factor out. The information remaining thereby probes more directly the electromagnetic interaction of the quarks, revealing their spin-flavor correlations. Knowledge of these correlations can give insight into configuration mixing, such as predicted by QCD.

Such a program had its genesis with the early work of Copley, Karl, and Obryk[2] and Feynman, Kislinger, and Ravndal,[3] which gave the first clear evidence for an underlying SU(6)⊗O(3) structure to the hadron spectrum. Subsequent to QCD, the importance of gluon exchange in generating spin-dependent energy shifts has become established.[5] These spin-spin, spin-orbital, and Thomas-precession modifications to the nonrelativistic binding Hamiltonian ($H_{\rm QCD}$) break the SU(6) symmetry of the spectroscopy in significant, though imperfect, agreement with data. They also induce mixing between the SU(6) basis states causing the physical baryon states to be linear combinations of them.

The standard work on the electromagnetic transitions among baryons, incorporating these QCD mixing effects, is due to Koniuk and Isgur.[5] This is our point of departure. The electromagnetic interaction used there, as in Refs. 2, 3, and 6, is incomplete on phenomenological and theoretical grounds. We shall show that it is inconsistent to restrict oneself to the electric (convention) and magnetic moment interaction operator familiar in atomic and nuclear physics:

$$H_{\rm em}=\sum_{i=1}^{3} H_i \ ,$$
$$H_i \equiv -\frac{e_i}{2m_i}[\mathbf{p}_i\cdot\mathbf{A}(r_i)+\mathbf{A}(r_i)\cdot\mathbf{p}_i]-\mu_i\sigma_i\cdot\mathbf{B}(r_i) \ , \tag{1.1}$$

where quark i at position r_i has mass, charge, and magnetic moment m_i, e_i, and μ_i.

There are spin-orbit and Wigner-rotation contributions to $H_{\rm em}$ which arise at the same order as their analogues in $H_{\rm QCD}$, and are required by gauge invariance (Sec. II); we shall refer to these as $H_{\rm rel}$.

$H_{\rm em}$ in Eq. (1.1) was the sum of single-quark interactions. The electric term flips the L_z of the quark in the simple harmonic-oscillator (SHO) potential; on absorbing a positive-helicity photon, it transforms as L_+. Similarly, the magnetic term flips the spin and transforms as S_+. Thus Eq. (1.1) transforms as

$$H \sim L_+ + \tilde{\mu} S_+ \ . \tag{1.2}$$

It has been found to be in remarkable agreement with experiment. Based on this model, Close and Gilman[7] successfully predicted a rapid change with q^2 of the helicity structure of the resonances $F_{15}(1688)$ and $D_{13}(1520)$ in electroproduction.

Koniuk and Isgur[5] considered QCD effects on the SU(6)⊗O(3) configuration mixing, and investigated the consequences for real photoproduction. This gave some interesting improvement in the model predictions. However we believe the $H_{\rm em}$ at Eq. (1.1) is overly restricted. Even if the $H_{\rm QCD}$ were absent, the $H_{\rm em}$ at Eq. (1.1) has long been known to be inconsistent as manifested by its failure to satisfy general low-energy theorems for Compton scattering.[8–10] Koniuk and Isgur[5] discuss the phenomenological consequences only of the hyperfine piece $H_{\rm hyp}$ of $H_{\rm QCD}$ and so do not directly confront the need for $H_{\rm rel}$ on the grounds of *gauge invariance,* but the satisfaction of the low-energy theorems still require the presence of $H_{\rm rel}$ in general at this order.

Given the fact that the mass of three valence quarks is small, the nonrelativistic approximation is not *a priori* justified and the relativistic correction should be studied in order to give a consistent theoretical prediction. The importance of the spin-orbit interaction

$$H_{\rm SO} = \frac{1}{2m}\left[2\mu - \frac{e}{2m}\right]\boldsymbol{\sigma}\cdot\mathbf{p}\times\mathbf{E} \tag{1.3}$$

has been discussed in many places,[2,8–10] and this completes the most general algebraic structure of $H_{\rm em}$ at the single-quark level[11,12] (apart from recoil effects, see later), the spin-orbit interaction transforming as

$$H_{\rm SO} \sim S_z L_+ + S_+ \ . \tag{1.4}$$

There have been some limited investigations of the phenomenological consequences of including this term in the $H_{\rm em}$. Kubota and Ohta[6] added this relativistic correction to the transition operator and, based on Kubota's calculation, Foster and Hughes[13] calculated the transition amplitudes of electroproduction. However, as discussed in Refs. 8 and 9, and applied to the quark model in Ref. 10, this interaction cannot be treated in absence of recoil effects and the em interaction is incomplete. One can no longer write $H = \sum H_i$ at this order; to separate the internal excitation from the motion of the center of mass, there should be a "nonadditive" term associated with the Wigner rotation of the quark spins transformed from the frame of the recoiling quark to the frame of the recoiling baryon. According to Close and Copley[8] (see also Ref. 9), this introduces "nonadditive" terms into $H_{\rm em}$:

$$H_{\rm NA} = \sum_{j>i=1}^{3} \frac{1}{4M_T}\left[\frac{\boldsymbol{\sigma}_i}{m_i} - \frac{\boldsymbol{\sigma}_j}{m_j}\right]\cdot(e_j\mathbf{E}_j\times\mathbf{p}_i - e_i\mathbf{E}_i\times\mathbf{p}_j) \ , \tag{1.5}$$

where M_T is the total mass of the quark system, which in the absence of binding is

$$M_T = \sum_{i=1}^{3} m_i \ . \tag{1.6}$$

Recently Le Yaouanc *et al.*[10] have shown that the binding potential also plays an explicit role in $H_{\rm em}$ at this order. Ohta[14] included this nonadditive term and the binding potential in his calculation for the Roper resonance $P_{11}(1410)$, and showed much better agreement with experiment. But the calculation for other resonances has not been done, nor has any study been made for electroproduction. Furthermore, it is still an open question how the amplitudes for other resonances are modified by the nonadditive term and the binding potential. We will discuss the role of the binding potential in the next section.

In this paper we study the consequences of $H_{\rm SO}$ and $H_{\rm NA}$ (we refer to these collectively as $H_{\rm rel}$) on their own in order to compare the relative magnitudes of $H_{\rm rel}$ and $H_{\rm QCD}$ as in Ref. 5. We find that they are comparable in magnitude, and hence that the conclusions of Ref. 5 are incomplete empirically.

The main purpose of this paper is to define $H_{\rm rel}$ theoretically, investigate its consequences, and to compare our results with those in the literature that have (incompletely) dealt with this problem.[6,13,14] Note that $H_{\rm rel}$ is necessary on general grounds even if $H_{\rm QCD}$ is ignored; one can consider $H_{\rm rel}$ without $H_{\rm QCD}$ but the converse is invalid, namely, if one discusses electromagnetic transitions including $H_{\rm QCD}$ then one should also include $H_{\rm rel}$ on the grounds of gauge invariance.

The applications will be primarily to real photoproduction. However, there are new features of the hadron's internal dynamics that can be accessed by varying the q^2 of the photon probe as in electroproduction. There has been relatively little study of this q^2 dependence theoretically. However, one of the early successes of the quark model was the prediction[7] that helicity amplitudes, which may be suppressed when $q^2=0$, become dominant very rapidly with q^2, even by $q^2=0.5$ GeV^2. We shall discuss the limit of validity of the model with q^2, and the q^2 dependence of various amplitude ratios can be abstracted from the tables in Sec. IV.

In the conclusions we consider, in particular, the impact of this work on the phenomenological discussion of Ref. 5. It suggests that a realistic confrontation with data requires *both* $H_{\rm QCD}$, as in Ref. 5, and $H_{\rm rel}$ as here. This is developed in detail in Ref. 15.

II. THE ROLE OF THE BINDING POTENTIAL

Consider the following Hamiltonian for the three-quark system:

$$H=\sum_{i=1}^{3}[\boldsymbol{\alpha}_i\cdot(\mathbf{p}_i-e_i\mathbf{A}_i)+\beta_i m_i+e_i\phi_i]+\sum_{i<j}[V_v(r_i-r_j)+\beta_i\beta_j V_s(r_i-r_j)]\ , \tag{2.1}$$

where $\mathbf{A}_i$ and ϕ_i are the electromagnetic fields, $V_v(r_i-r_j)$ and $V_s(r_i-r_j)$ denote the vector and scalar binding potentials for the quark system (later we will write V_v and V_s for simplicity where, as particular examples, V_s could be a long-range scalar simple harmonic potential and V_v be $H_{\rm QCD}$, single-gluon exchange.)

The corresponding relativistic Dirac equation is

$$H\Psi=i\frac{\partial\Psi}{\partial t} \tag{2.2}$$

with the wave function[9]

$$\Psi=N(p_1,p_2,p_3)\prod_{i=1}^{3}\begin{bmatrix}1+\dfrac{\sigma_i\cdot\mathbf{P}}{2M_T+T}\dfrac{\sigma_i\cdot\mathbf{p}_i'}{2m_i+t_i}\\ \sigma\cdot\left[\dfrac{\mathbf{P}}{2M_T+T}+\dfrac{\mathbf{p}_i'}{2m_i+t_i}\right]\end{bmatrix}\Phi\chi_s\ , \tag{2.3}$$

where $\mathbf{P}$ is the total momentum of the system, $\mathbf{p}_i'=\mathbf{p}_i-(m_i/M_T)\mathbf{P}$, T (t_i) is the kinetic energy of the center-of-mass system (the particle i), which can be neglected to $O(1/m^2)$, $N(p_1,p_2,p_3)$ is the normalization factor with respect to the spinor wave function, so Φ is corresponding to the normalized nonrelativistic wave function, and χ_s is a two-component spin wave function. In the case of electromagnetic fields, the momentum $\mathbf{p}_i$ should be replaced by $\mathbf{p}_i-e_i\mathbf{A}_i$. The transformation from four-component relativistic Hamiltonian to the two-component Hamiltonian follows from

$$H_{\rm NR}=N(p_1,p_2,p_3)\prod_{i=1}^{3}\begin{bmatrix}1+\dfrac{\sigma_i\cdot\mathbf{P}}{2M_T+T}\dfrac{\sigma_i\cdot\mathbf{p}_i'}{2m_i+t_i}\\ \sigma_i\cdot\left[\dfrac{\mathbf{P}}{2M_T+T}+\dfrac{\mathbf{p}_i'}{2m_i+t_i}\right]\end{bmatrix}^{\dagger}\left[H-\frac{\partial}{i\partial t}\right]\prod_{i=1}^{3}\begin{bmatrix}1+\dfrac{\sigma_i\cdot\mathbf{P}}{2M_T+T}\dfrac{\sigma_i\cdot\mathbf{p}_i'}{2m_i+t_i}\\ \sigma_i\cdot\left[\dfrac{\mathbf{P}}{2M_T+T}+\dfrac{\mathbf{p}_i'}{2m_i+t_i}\right]\end{bmatrix}N(p_1,p_2,p_3)\ . \tag{2.4}$$

Thus up to $O(1/m^2)$, we get the nonrelativistic Hamiltonian corresponding to Eq. (2.1):

$$H_{\rm NR}=H_b+H_{\rm em}\ , \tag{2.5}$$

where

$$\begin{aligned}H_b=\sum_{i=1}^{3}\left[\frac{p_i^2}{2m_i}-\frac{p_i^4}{8m_i^3}\right]+\sum_{i<j}\Bigg[&V_v+V_s+\frac{1}{8m_i^2}\nabla^2V_v+\frac{1}{4m_i^2}\sigma_i\cdot[\nabla(V_v-V_s)\times\mathbf{p}_i]\\&-\frac{1}{4m_j^2}\sigma_j\cdot[\nabla(V_v-V_s)\times\mathbf{p}_j]-\left\{\left[\frac{1}{8m_i^2}p_i^2+\frac{1}{8m_j^2}p_j^2\right],V_s\right\}\\&-\frac{1}{4m_i^2}\mathbf{p}_iV_s\mathbf{p}_i-\frac{1}{4m_j^2}\mathbf{p}_jV_s\mathbf{p}_j+\frac{1}{4M_T}\left[\frac{\sigma_i}{m_i}-\frac{\sigma_j}{m_j}\right]\cdot[\nabla(V_v+V_s)\times\mathbf{P}]\Bigg]\end{aligned} \tag{2.5a}$$

and

$$\begin{aligned}H_{\rm em}=\sum_{i=1}^{3}\Bigg[&-\frac{e_i}{2m_i}(\mathbf{A}_i\cdot\mathbf{p}_i+\mathbf{p}_i\cdot\mathbf{A}_i)-\frac{e_i}{2m_i}\sigma_i\cdot\mathbf{B}_i+e_i\phi_i\\&+\frac{e_i}{4m_i}\sigma_i\cdot\left[\mathbf{E}_i\times\frac{\mathbf{p}_i}{2m_i}-\frac{\mathbf{p}_i}{2m_i}\times\mathbf{E}_i\right]+\frac{e_i}{8m_i^3}\{p_i^2,\mathbf{A}_i\cdot\mathbf{p}_i+\mathbf{p}_i\cdot\mathbf{A}_i+\sigma_i\cdot\mathbf{B}_i\}\Bigg]\\+\sum_{i<j}\Bigg[&-\frac{e_i}{4m_i^2}\sigma_i\cdot[\nabla(V_v-V_s)\times\mathbf{A}_i]+\frac{e_j}{4m_j^2}\sigma_j\cdot[\nabla(V_v-V_s)\times\mathbf{A}_j]\\&+\left\{V_s,\frac{1}{8m_i^2}(\mathbf{p}_i\cdot\mathbf{A}_i+\mathbf{A}_i\cdot\mathbf{p}_i)+\frac{1}{8m_j^2}(\mathbf{p}_j\cdot\mathbf{A}_j+\mathbf{A}_j\cdot\mathbf{p}_j)\right\}\end{aligned}$$

$$+\left[\frac{e_i}{2m_i^2}\boldsymbol{\sigma}_i\cdot\mathbf{B}_i+\frac{e_j}{2m_j^2}\boldsymbol{\sigma}_j\cdot\mathbf{B}_j\right]V_s+\frac{e_i}{4m_i^2}(\mathbf{p}_i\cdot\mathbf{A}_iV_s+V_s\mathbf{A}_i\cdot\mathbf{p}_i)$$

$$+\frac{e_j}{4m_j^2}(\mathbf{p}_j\cdot\mathbf{A}_jV_s+V_s\mathbf{A}_j\cdot\mathbf{p}_j)$$

$$+\frac{1}{4M_T}\left[\frac{\boldsymbol{\sigma}_i}{m_i}-\frac{\boldsymbol{\sigma}_j}{m_j}\right]\cdot[e_j\mathbf{E}_j\times\mathbf{p}_i-e_i\mathbf{E}_i\times\mathbf{p}_j-\nabla(V_v+V_s)\times(e_1\mathbf{A}_1+e_2\mathbf{A}_2+e_3\mathbf{A}_3)]\Bigg]\ , \tag{2.5b}$$

where $\{A,B\}$ represents the anticommutator. The effects of the anomalous magnetic moment have been excluded in this calculation and will be included later; terms with higher powers of $\mathbf{A}_i$ also are excluded. From this calculation, we can see that the nonrelativistic electromagnetic interaction can be divided into two parts: the first part is associated with the gauge invariance of the nonrelativistic H_b in which $\mathbf{p}_i$ is replaced by $\mathbf{p}_i-e_i\mathbf{A}_i$, and the second part is the interaction between the magnetic moments of the quark system and the external magnetic fields.

McClary and Byers[16] in their calculation of the transition in heavy quarkonium have shown how to avoid the explicit appearance of the binding potential in the first part. Notice that

$$\frac{\mathbf{p}_i}{m_i}=i\left[\frac{p_i^2}{2m_i},\mathbf{r}_i\right] \tag{2.6}$$

and let

$$H_b=\sum_{i=1}^{3}\frac{p_i^2}{2m_i}+\sum_{i<j}[V_v(r_i-r_j)+V_s(r_i-r_j)]+H'\ , \tag{2.7}$$

where H' is the higher-order terms in Eq. (2.5a). Then we can write Eq. (2.6) as

$$i[H_b,\mathbf{r}_i]=\frac{\mathbf{p}_i}{m_i}+i[H',\mathbf{r}_i] \tag{2.8}$$

and hence Eq. (2.5b) becomes

$$H_{\rm em}=\sum_{i=1}^{3}e_i\Bigg[-i\tfrac{1}{2}([H_b,\mathbf{r}_i]\cdot A_i+\mathbf{A}_i\cdot[H_b,\mathbf{r}_i])-\frac{e_i}{2m_i}\boldsymbol{\sigma}_i\cdot\mathbf{B}_i+e_i\phi_i$$
$$+\frac{e_i}{4m_i}\boldsymbol{\sigma}_i\cdot\left[\mathbf{E}_i\times\frac{\mathbf{p}_i}{2m_i}-\frac{\mathbf{p}_i}{2m_i}\times\mathbf{E}_i\right]+\frac{e_i}{8m_i^3}\{p_i^2,\boldsymbol{\sigma}_i\cdot\mathbf{B}_i\}\Bigg]$$
$$+\sum_{i<j}\left[\left[\frac{e_i}{2m_i^2}\boldsymbol{\sigma}_i\cdot\mathbf{B}_i+\frac{e_j}{2m_j^2}\boldsymbol{\sigma}_j\cdot\mathbf{B}_j\right]V_s+\frac{1}{4M_T}\left[\frac{\boldsymbol{\sigma}_i}{m_i}-\frac{\boldsymbol{\sigma}_j}{m_j}\right]\cdot(e_j\mathbf{E}_j\times\mathbf{p}_i-e_i\mathbf{E}_i\times\mathbf{p}_j)\right]\ . \tag{2.9}$$

Furthermore the $\boldsymbol{\sigma}\cdot\mathbf{B}$ terms can be made more transparent

$$\sum_{i=1}^{3}\left[-\frac{e_i}{2m_i}\boldsymbol{\sigma}_i\cdot\mathbf{B}_i+\frac{e_i}{8m_i^3}\{p_i^2,\boldsymbol{\sigma}_i\cdot\mathbf{B}_i\}\right]+\sum_{i<j}\left[\frac{e_i}{2m_i^2}\boldsymbol{\sigma}_i\cdot\mathbf{B}_i+\frac{e_j}{2m_j^2}\boldsymbol{\sigma}_j\cdot B_j\right]V_s=\sum_{i=1}^{3}\frac{e_i}{2m_i^*}\boldsymbol{\sigma}_i\cdot\mathbf{B}_i+O(1/m^2)\ , \tag{2.10}$$

where the effective mass m_i^* is

$$m_i^*=m_i+\frac{p_i^2}{2m_i}+\sum_{j\neq i}V_s(r_i-r_j)\ . \tag{2.11}$$

After including the contribution of the anomalous magnetic moment, the electromagnetic interaction of the quark system can be written as

$$H_{\rm em}=\sum_{i=1}^{3}\left[-ie_i\tfrac{1}{2}([H_b,\mathbf{r}_i]\cdot\mathbf{A}_i+\mathbf{A}_i\cdot[H_b,\mathbf{r}_i])-\mu_i\boldsymbol{\sigma}_i\cdot\mathbf{B}_i-\frac{1}{2}\left[2\mu_i-\frac{e_i}{2m_i^*}\right]\boldsymbol{\sigma}_i\cdot\left[\mathbf{E}_i\times\frac{\mathbf{p}_i}{2m_i^*}-\frac{\mathbf{p}_i}{2m_i^*}\times\mathbf{E}_i\right]+e_i\phi_i\right]$$
$$+\sum_{i<j}\frac{1}{4M_T}\left[\frac{\boldsymbol{\sigma}_i}{m_i^*}-\frac{\boldsymbol{\sigma}_j}{m_j^*}\right]\cdot(e_j\mathbf{E}_j\times\mathbf{p}_i-e_i\mathbf{E}_i\times\mathbf{p}_j) \tag{2.12}$$

which in the long-wavelength approximation may be written as

$$H_{\rm em}=\sum_{i=1}^{3}\left[e_i\mathbf{r}_i\cdot\mathbf{E}_i+i\frac{e_i}{2m_i}(\mathbf{p}_i\cdot\mathbf{k}\mathbf{r}_i\cdot\mathbf{A}_i+\mathbf{r}_i\cdot\mathbf{A}_i\mathbf{p}_i\cdot\mathbf{k})-\mu_i\boldsymbol{\sigma}_i\cdot\mathbf{B}_i-\frac{1}{2}\left[2\mu_i-\frac{e_i}{2m_i^*}\right]\boldsymbol{\sigma}_i\cdot\left[\mathbf{E}_i\times\frac{\mathbf{p}_i}{2m_i^*}-\frac{\mathbf{p}_i}{2m_i^*}\times\mathbf{E}_i\right]+e_i\phi_i\right]$$
$$+\sum_{i<j}\frac{1}{4M_T}\left[\frac{\boldsymbol{\sigma}_i}{m_i^*}-\frac{\boldsymbol{\sigma}_j}{m_j^*}\right]\cdot(e_j\mathbf{E}_j\times\mathbf{p}_i-e_i\mathbf{E}_i\times\mathbf{p}_j) \; . \tag{2.13}$$

This form for $H_{\rm em}$ satisfies the low-energy theorem.[8-10] Note that the spin-orbit term in the first line and the nonadditive term in the second line in Eq. (2.13), omitted in much of the literature[2,3,5,8,17,11,18] are crucially necessary in this regard.

Here, μ_i and m_i^* can be treated as free parameters. The physics behind this is very clear, as McClary and Byers[16] pointed out, since the first part of the electromagnetic interaction containing the binding potential is generated from the corresponding term in H_b by the gauge-covariant substitution $\mathbf{p}\rightarrow\mathbf{p}-e\,\mathbf{A}$. Thus the tensorial transformation properties of $H_b(\mathbf{p})$ are manifested in $H_{\rm em}(\mathbf{A})$. If there is no configuration mixing in the quark model, the matrix elements of Eq. (2.12) between eigenstates of the binding potential are identical to those of the electromagnetic interaction at Eq. (2.5b) neglecting the binding potential $V_{s,v}$ terms. This point has not been consistently addressed in the literature. For example, Ohta[14] generates a spin-orbit term in $H_{\rm em}$ by substituting $\mathbf{p}\rightarrow\mathbf{p}-e\,\mathbf{A}$ into the spin-orbit term of H_b; however, in the Isgur-Karl study of the baryon spectrum, it is necessary to ignore the $\mathbf{L}\cdot\mathbf{S}$ terms in H_b in order to fit the data.[19] This can generate an inconsistency of electromagnetic transition operators when the binding potential is included. The binding potential may induce configuration mixing among the SU(6)⊗O(3) basis states but need not play an explicit role in the $H_{\rm em}$ if Eq. (2.12) is used.

III. THE TRANSITION MATRIX ELEMENTS AND CURRENT CONSERVATION

For the three-quark system, the SU(6)⊗O(3) wave function can be written as

$$|\mathrm{SU}(6)\otimes\mathrm{O}(3)\rangle=\Psi(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{R})\phi\chi \; , \tag{3.1}$$

where ϕ is a SU(3)-flavor wave function and χ is a spin wave function, the spatial wave function $\Psi(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{R})$ can be written as

$$\Psi(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{R})=\psi(\boldsymbol{\rho},\boldsymbol{\lambda})\exp(i\mathbf{P}\cdot\mathbf{R}) \tag{3.2}$$

and the relations among $\boldsymbol{\rho}$, $\boldsymbol{\lambda}$, $\mathbf{R}$, and $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$ are

$$\boldsymbol{\rho}=\frac{1}{\sqrt{2}}(\mathbf{r}_1-\mathbf{r}_2) \; , \tag{3.3a}$$

$$\boldsymbol{\lambda}=\frac{1}{\sqrt{6}}(\mathbf{r}_1+\mathbf{r}_2-2\mathbf{r}_3) \; , \tag{3.3b}$$

$$\mathbf{R}=\tfrac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3) \; . \tag{3.3c}$$

The current conservation in the nonrelativistic limit can be written as[17]

$$k_\lambda V^\lambda=k_0V_0-\mathbf{k}\cdot\mathbf{V}=0 \tag{3.4}$$

and in the SU(6)⊗O(3)-symmetry limit, this leads to

$$k_0\langle\Psi_f(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{R})|\exp(i\mathbf{k}\cdot\mathbf{r}_3)|\Psi_i(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{R})\rangle$$
$$=\frac{1}{2m_q}\mathbf{k}\cdot\langle\Psi_f(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{R})|\mathbf{p}_3\exp(i\mathbf{k}\cdot\mathbf{r}_3)+\exp(i\mathbf{k}\cdot\mathbf{r}_3)\mathbf{p}_3|\Psi_i(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{R})\rangle \; , \tag{3.5}$$

where k_λ is the four-momentum transfer to the quark system, and we assumed equal mass quarks.

If we take $\mathbf{k}$ to be in the z direction, then it can be proved that, for a three-quark system with the Hamiltonian

$$H=\sum_{i=1}^{3}\frac{p_i^2}{2m_i}+\sum_{i<j}V(\mathbf{r}_i-\mathbf{r}_j) \; , \tag{3.6}$$

we have

$$\langle\Psi_f(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{R})|p_{3z}\exp(ik_zz_3)+\exp(ik_zz_3)p_{3z}|\Psi_i(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{R})\rangle$$
$$=\frac{1}{k}[\tfrac{1}{3}(P_f^2-P_i^2)+2m_q(E_f-E_i)]$$
$$\times\langle\Psi_f(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{R})|\exp(ik_zz_3)|\Psi_i(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{R})\rangle \; , \tag{3.7}$$

where P_f (P_i) is the final (initial) momentum of the center of mass, and E_f (E_i) is final (initial) internal energy level of the system. Thus if current conservation is true, we should have

$$k_0=\frac{1}{6m_q}(P_f^2-P_i^2)+E_f-E_i \tag{3.8}$$

which is the energy conservation in first-order nonrelativistic approximation. The left-hand side of Eq. (3.7) is used in calculating matrix elements of the spin-orbit term in Ref. 6 which used the center-of-mass frame. This integral is frame dependent and in the c.m. frame the first term on the right-hand side of Eq. (3.7) is not zero; therefore, it should be taken into account, but was ignored in Ref. 6. The recoil term can contribute to the transition matrix elements, and sometimes it is important.

From Eq. (3.7), it is possible to make the recoil contribution vanish by working in the Breit frame where $|P_f|=|P_i|$ and the amplitudes then correspond to total internal transition. The result of the calculation will be shown later.

For the harmonic-oscillator basis, the above constraints are satisfied explicitly since

$$E_f - E_i = \frac{N\alpha^2}{m_q} , \tag{3.9}$$

where α is oscillator strength, and N is the quantum number of the energy level, and where E_i is the energy of the ground state.

IV. PHOTOPRODUCTION OF BARYON RESONANCES

Following Sec. II, and in the absence of configuration mixing, the electromagnetic interaction may be written as

$$H_{em} = H_{NR} + H_{SO} + H_{NA} , \tag{4.1}$$

where

$$H_{NR} = \sum_{i=1}^{3} - \left[\frac{e_i}{2m_i} (\mathbf{p}_i \cdot \mathbf{A}_i + \mathbf{A}_i \cdot \mathbf{p}_i) + \mu_i \boldsymbol{\sigma}_i \cdot \mathbf{B}_i - e_i \phi_i \right] , \tag{4.1a}$$

$$H_{SO} = \sum_{i=1}^{3} -\frac{1}{2} \left[2\mu_i - \frac{e_i}{2m_i} \right] \frac{\boldsymbol{\sigma}_i}{2m_i} \cdot (\mathbf{E}_i \times \mathbf{p}_i - \mathbf{p}_i \times \mathbf{E}_i) , \tag{4.1b}$$

and H_{NA} is the same as Eq. (1.5). Following the procedure in Refs. 1 and 2, simplified expressions can be written as

$$H_{NR} = \exp(ikz_{(3)}) 6 \left[\frac{\pi}{k} \right]^{1/2} \mu q^{(3)} \left[k S_{+}^{(3)} + \frac{1}{g} P_{+}^{(3)} \right] , \tag{4.2a}$$

TABLE I. Photon-decay amplitudes between $[56,0^+]_0$ and excited basis states for spin-orbit terms in naive SU(6) model. These are coefficients multiplying B, C, the expressions for which are in Table IV.

Multiplet	States	$A^p_{\frac{1}{2}}$		$A^p_{\frac{3}{2}}$		$A^n_{\frac{1}{2}}$		$A^n_{\frac{3}{2}}$	
		B	C	B	C	B	C	B	C
$[70,1^-]_1$	$N(^2P_M)\frac{1}{2}^-$	$\frac{-1}{3\sqrt{6}}$	$\frac{-1}{6\sqrt{3}}$			$\frac{1}{9\sqrt{6}}$	$\frac{1}{18\sqrt{3}}$		
	$N(^2P_M)\frac{3}{2}^-$	$\frac{1}{3\sqrt{3}}$	$\frac{-1}{6\sqrt{6}}$	0	$\frac{1}{6\sqrt{2}}$	$\frac{-1}{9\sqrt{3}}$	$\frac{1}{18\sqrt{6}}$	0	$\frac{-1}{18\sqrt{2}}$
	$N(^4P_M)\frac{1}{2}^-$	0	0			$\frac{-\sqrt{2}}{18\sqrt{3}}$	$\frac{-1}{18\sqrt{3}}$		
	$N(^4P_M)\frac{3}{2}^-$	0	0	0	0	$\frac{-1}{9\sqrt{30}}$	$\frac{-2}{9\sqrt{15}}$	$\frac{-1}{3\sqrt{10}}$	$\frac{-1}{9\sqrt{5}}$
	$N(^4P_M)\frac{5}{2}^-$	0	0	0	0	$\frac{1}{3\sqrt{30}}$	$\frac{-1}{6\sqrt{15}}$	$\frac{1}{3\sqrt{15}}$	$\frac{-1}{3\sqrt{30}}$
	$\Delta(^2P_M)\frac{1}{2}^-$	$\frac{-1}{9\sqrt{6}}$	$\frac{-1}{18\sqrt{3}}$						
	$\Delta(^2P_M)\frac{3}{2}^-$	$\frac{1}{9\sqrt{3}}$	$\frac{-1}{18\sqrt{6}}$	0	$\frac{1}{18\sqrt{2}}$				
$[56,2^+]_2$	$N(^2D_S)\frac{3}{2}^+$	$\frac{-\sqrt{2}}{3\sqrt{5}}$	$\frac{-\sqrt{3}}{6\sqrt{5}}$	0	$\frac{-1}{6\sqrt{5}}$	$\frac{2\sqrt{2}}{9\sqrt{5}}$	$\frac{\sqrt{3}}{9\sqrt{5}}$	0	$\frac{1}{9\sqrt{5}}$
	$N(^2D_S)\frac{5}{2}^+$	$\frac{\sqrt{3}}{3\sqrt{5}}$	$\frac{-\sqrt{2}}{6\sqrt{5}}$	0	$\frac{1}{3\sqrt{5}}$	$\frac{-2\sqrt{3}}{9\sqrt{5}}$	$\frac{\sqrt{2}}{9\sqrt{5}}$	0	$\frac{-2}{9\sqrt{5}}$
	$\Delta(^4D_S)\frac{1}{2}^+$	$\frac{-\sqrt{2}}{9\sqrt{5}}$	$\frac{-\sqrt{3}}{9\sqrt{5}}$						
	$\Delta(^4D_S)\frac{3}{2}^+$	$\frac{\sqrt{2}}{9\sqrt{5}}$	0	$\frac{-\sqrt{6}}{9\sqrt{5}}$	$\frac{-2}{9\sqrt{5}}$				
	$\Delta(^4D_S)\frac{5}{2}^+$	$\frac{\sqrt{6}}{9\sqrt{35}}$	$\frac{\sqrt{5}}{9\sqrt{7}}$	$\frac{2}{\sqrt{105}}$	$\frac{\sqrt{2}}{9\sqrt{35}}$				
	$\Delta(^4D_S)\frac{7}{2}^+$	$\frac{-2}{3\sqrt{35}}$	$\frac{2\sqrt{2}}{3\sqrt{105}}$	$\frac{-2\sqrt{3}}{9\sqrt{7}}$	$\frac{2\sqrt{2}}{9\sqrt{7}}$				
$[56,0^+]_2$	$N(^2S'_S)\frac{1}{2}^+$	$\frac{1}{3}$	0			$\frac{-2}{9}$	0		
	$\Delta(^4S'_S)\frac{3}{2}^+$	$\frac{-\sqrt{2}}{9}$	0	$\frac{-\sqrt{6}}{9}$	0				
$[56,0^+]_0$	$\Delta(^4S_S)\frac{3}{2}^+$	$\frac{-\sqrt{2}}{9}$	0	$\frac{-\sqrt{6}}{9}$	0				
$[70,0^+]_2$	$N(^2S_M)\frac{1}{2}^+$	$\frac{1}{3\sqrt{2}}$	0			$\frac{-1}{9\sqrt{2}}$	0		
	$N(^4S_M)\frac{3}{2}^+$	0	0	0	0	$\frac{\sqrt{2}}{18}$	0	$\frac{\sqrt{6}}{18}$	0

$$H_{\rm SO}=\exp(ikz_{(3)})3\sqrt{\pi k}\,\frac{\mu}{m_q g}\left[2-\frac{1}{g}\right]q^3 \times\left[S_z^{(3)}P_+^{(3)}-S_+^{(3)}\left[P_z^{(3)}+\frac{k}{2}\right]\right], \tag{4.2b}$$

$$H_{\rm NA}=\exp(ikz_{(2)})2\sqrt{\pi k}\,\frac{\mu}{m_q g}q^{(2)} \times(S_+^{(1-2)}P_z^{(1)}-S_z^{(1-2)}P_+^{(1)}), \tag{4.2c}$$

where $S^{(1-2)}=S^{(1)}-S^{(2)}$, the direction of the photon momentum $\mathbf{k}$ is chosen as z direction. Here the phase convention by Koniuk and Isgur[5] is used. It is convenient to emphasis the group structure of Eq. (4.2): namely,

$$H_{\rm NR}=q^{(3)}(A_nL_+ + B_nS_+) . \tag{4.3a}$$

$$H_{\rm SO}=q^{(3)}(B_sS_+ + C_sS_zL_+) , \tag{4.3b}$$

$$H_{\rm NA}=q^{(2)}(B_{n0}S_+^{(1-2)}-C_{n0}S_z^{(1-2)}L_+) , \tag{4.3c}$$

where

$$A_n=6\left[\frac{\pi}{k}\right]^{1/2}\mu\frac{1}{g}\langle\Psi_f|\exp(ikz_{(3)})P_+^{(3)}|\Psi_i\rangle , \tag{4.4a}$$

$$B_n=6\sqrt{\pi k}\,\mu\langle\Psi_f|\exp(ikz_{(3)})|\Psi_i\rangle , \tag{4.4b}$$

$$B_s=-6\sqrt{\pi k}\,\frac{\mu}{2m_q g}\left[2-\frac{1}{g}\right] \times\left\langle\Psi_f\left|\exp(ikz_{(3)})\left[P_z^{(3)}+\frac{k}{2}\right]\right|\Psi_i\right\rangle , \tag{4.4c}$$

$$C_s=6\sqrt{\pi k}\,\frac{\mu}{2m_q g}\left[2-\frac{1}{g}\right]\langle\Psi_f|\exp(ikz_{(3)})P_+^{(3)}|\Psi_i\rangle , \tag{4.4d}$$

$$B_{na}=\sqrt{\pi k}\,\frac{2\mu}{m_q g}\langle\Psi_f|\exp(ikz_{(2)})P_z^{(1)}|\Psi_i\rangle , \tag{4.4e}$$

$$C_{na}=\sqrt{\pi k}\,\frac{2\mu}{m_q g}\langle\Psi_f|\exp(ikz_{(2)})P_+^{(1)}|\Psi_i\rangle . \tag{4.4f}$$

In Table I, we show the contribution of the spin-orbit

TABLE II. Photon-decay amplitudes between $[56,0^+]_0$ and excited 70 basis states for nonadditive terms in naive SU(6) model. ρ and λ denote the permutation symmetry of the wave function in final states. Explicit forms for the various B, C are in Table IV.

States	N	$A_{\frac{1}{2}}$				$A_{\frac{3}{2}}$			
		B^ρ	B^λ	C^ρ	C^λ	B^ρ	B^λ	C^ρ	C^λ
$N(^2P_M)\frac{1}{2}^-$	p	$\frac{\sqrt{2}}{9}$	0	$\frac{-1}{9}$	0				
	n	0	0	0	0				
$N(^2P_M)\frac{3}{2}^-$	p	$\frac{-2}{9}$	0	$\frac{-1}{9\sqrt{2}}$	0	0	0	$\frac{1}{3\sqrt{6}}$	0
	n	0	0	0	0	0	0	0	0
$N(^4P_M)\frac{1}{2}^-$	p	$\frac{1}{18\sqrt{2}}$	$\frac{1}{6\sqrt{6}}$	$\frac{-1}{36}$	$\frac{-1}{12\sqrt{3}}$				
	n	$\frac{1}{18\sqrt{2}}$	$\frac{-1}{6\sqrt{6}}$	$\frac{-1}{36}$	$\frac{1}{12\sqrt{3}}$				
$N(^4P_M)\frac{3}{2}^-$	p	$\frac{1}{18\sqrt{10}}$	$\frac{1}{6\sqrt{30}}$	$\frac{-1}{9\sqrt{5}}$	$\frac{-1}{3\sqrt{15}}$	$\frac{1}{2\sqrt{30}}$	$\frac{1}{2\sqrt{10}}$	$\frac{-1}{6\sqrt{15}}$	$\frac{-1}{6\sqrt{5}}$
	n	$\frac{1}{18\sqrt{10}}$	$\frac{-1}{6\sqrt{30}}$	$\frac{-1}{9\sqrt{5}}$	$\frac{1}{3\sqrt{15}}$	$\frac{1}{2\sqrt{30}}$	$\frac{-1}{2\sqrt{10}}$	$\frac{-1}{6\sqrt{15}}$	$\frac{1}{6\sqrt{5}}$
$N(^4P_M)\frac{5}{2}^-$	p	$\frac{-1}{6\sqrt{10}}$	$\frac{-1}{2\sqrt{30}}$	$\frac{-1}{6\sqrt{20}}$	$\frac{-1}{2\sqrt{60}}$	$\frac{-1}{6\sqrt{5}}$	$\frac{-1}{2\sqrt{15}}$	$\frac{-1}{6\sqrt{10}}$	$\frac{-1}{2\sqrt{30}}$
	n	$\frac{-1}{6\sqrt{10}}$	$\frac{1}{2\sqrt{30}}$	$\frac{-1}{6\sqrt{20}}$	$\frac{1}{2\sqrt{60}}$	$\frac{-1}{6\sqrt{5}}$	$\frac{1}{2\sqrt{15}}$	$\frac{-1}{6\sqrt{10}}$	$\frac{1}{2\sqrt{30}}$
$\Delta(^2P_M)\frac{1}{2}^-$	p,n	$\frac{1}{3}$	$\frac{-1}{3\sqrt{3}}$	$\frac{-1}{3\sqrt{2}}$	$\frac{1}{3\sqrt{6}}$				
$\Delta(^2P_M)\frac{3}{2}^-$	p,n	$\frac{-\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3\sqrt{3}}$	$\frac{-1}{6}$	$\frac{1}{6\sqrt{3}}$	0	0	$\frac{1}{2\sqrt{3}}$	$\frac{-1}{6}$
$N(^2S_M)\frac{1}{2}^+$	p	$\frac{-2}{3\sqrt{6}}$	0	0	0				
	n	0	0	0	0				
$N(^4S_M)\frac{3}{2}^+$	p	$\frac{-1}{6\sqrt{6}}$	$\frac{-1}{6\sqrt{2}}$	0	0	$\frac{-1}{6\sqrt{2}}$	$\frac{-1}{2\sqrt{6}}$	0	0
	n	$\frac{-1}{6\sqrt{6}}$	$\frac{1}{6\sqrt{2}}$	0	0	$\frac{-1}{6\sqrt{2}}$	$\frac{1}{2\sqrt{6}}$		

term to photoproduction amplitudes in terms of B_s and C_s. The phase convention of Koniuk and Isgur[5] is used here, so the contribution of the nonrelativistic term in Eq. (4.3a) can be found in Ref. 5. In Tables II and III, we show the contribution of the nonadditive term for 70 and 56 multiplets. Notice that for the different SU(6) multiplets, the expressions for the quantities in Eq. (4.4) are different (see Table IV).

Table I shows that there are interesting constraints implied among some of the amplitudes which are rather general and independent of the binding potential or choice of parameters. First we note that for nonrelativistic, spin-orbit and nonadditive terms with arbitrary strengths one has

$$D_{15}(1680): A^n_{3/2} = \sqrt{2} A^n_{1/2} \ . \qquad (4.5)$$

Data are consistent with this at $q^2=0$; if SU(6) symmetry applies for large $q^2 \neq 0$ then this relationship should hold for all q^2. Second, in the harmonic-oscillator basis, if the g factor is chosen to be unity, the relation

$$D_{13}(1520)(^2 8): A^p_{3/2} = -A^n_{3/2} \qquad (4.6)$$

familiar in the nonrelativistic models (pp. 146 and 147 of Ref. 1), but violated by the *additive* spin-orbit terms, is recovered when the nonadditive terms of Eq. (4.2) are included.

These results are frame independent, so these relations determined by SU(6) symmetry can directly test the model. In Table IV we see that the terms B depend explicitly upon recoil effects and hence their strengths relative to the C (and A) terms are thus frame dependent. The most pleasing symmetry among these expressions occurs in the Breit frame where the recoil effects vanish resulting in simple relations. In particular, in the Breit frame the contributions of the nonadditive terms to the neutron in $^4 8$ of the $[70,1^-]_1$ resonant states and to the Δ resonances of $[70,1^-]_1$ are all zero. Thus in other frames the nonadditive terms contribute to these particular excitations only through recoil effects.

V. COMPARISON WITH EXPERIMENT

The calculation of the photoproduction amplitudes is straightforward. In Tables V and VI, the calculation is made in the c.m. frame with the parameters of Koniuk and Isgur.[5] In order to show the frame dependence of the numerical results, we made similar calculations in the Breit frame, the results being shown in Table VII, where the parameters given by Foster and Hughes[13] are used. Comparing results in Table VII with those in Tables V and VI reveals that there is no significant change, giving a measure of "theoretical systematic uncertainty" of the model.

We have neglected QCD mixing effects in this paper. Modulo this we confirm the nonrelativistic results given by Koniuk and Isgur;[5] the results of Kubota and Ohta[6] incorporating H_{SO} are also confirmed but for their failure to include the recoil term. Given our confirmation of the existing nonrelativistic and incomplete relativistic matrix elements in the literature, our Table V shows the full impact of the relativistic treatment for electromagnetic transitions in the constituent quark model.

When we began this investigation we expected that the successes of the NRCQM would break down and its significance thereby be questioned. Somewhat to our surprise we find that not only do its successes survive, but there are even some improvements. One of these may be particularly significant, namely, the small amplitudes for excitations of $P_{11}(1705)$. Koniuk and Isgur[5] found bad agreement for this state; Kubota and Ohta[6] appear to

TABLE III. Photon-decay amplitudes between $[56,0^+]_0$ and excited 56 basis states for nonadditive terms in naive SU(6) model. S denotes the permutation symmetry of the wave function in final states. The various B, C are in Table IV.

States	$A^p_{\frac{1}{2}}$		$A^p_{\frac{3}{2}}$		$A^n_{\frac{1}{2}}$		$A^n_{\frac{3}{2}}$	
	B^S	C^S	B^S	C^S	B^S	C^S	B^S	C^S
$N(^2D_S)\frac{3}{2}^+$	$\frac{\sqrt{2}}{3\sqrt{5}}$	$\frac{-1}{2\sqrt{15}}$	0	$\frac{-1}{6\sqrt{5}}$	$\frac{-\sqrt{2}}{3\sqrt{5}}$	$\frac{1}{2\sqrt{15}}$	0	$\frac{1}{6\sqrt{5}}$
$N(^2D_S)\frac{5}{2}^+$	$\frac{-1}{2\sqrt{15}}$	$\frac{-1}{3\sqrt{10}}$	0	$\frac{1}{3\sqrt{5}}$	$\frac{1}{2\sqrt{15}}$	$\frac{1}{2\sqrt{10}}$	0	$\frac{-1}{3\sqrt{5}}$
$\Delta(^4D_S)\frac{1}{2}^+$	$\frac{1}{3\sqrt{10}}$	$\frac{-1}{2\sqrt{15}}$						
$\Delta(^4D_S)\frac{3}{2}^+$	$\frac{-1}{3\sqrt{10}}$	0	$\frac{1}{\sqrt{30}}$	$\frac{-1}{3\sqrt{5}}$				
$\Delta(^4D_S)\frac{5}{2}^+$	$\frac{-1}{\sqrt{210}}$	$\frac{\sqrt{5}}{6\sqrt{7}}$	$\frac{-\sqrt{3}}{\sqrt{35}}$	$\frac{1}{3\sqrt{70}}$				
$\Delta(^4D_S)\frac{7}{2}^+$	$\frac{1}{\sqrt{35}}$	$\frac{\sqrt{2}}{\sqrt{105}}$	$\frac{1}{\sqrt{21}}$	$\frac{\sqrt{2}}{3\sqrt{7}}$				
$N(^2S'_S)\frac{1}{2}^+$	$\frac{-1}{3}$	0			$\frac{1}{3}$	0		
$\Delta(^4S'_S)\frac{3}{2}^+$	$\frac{1}{3\sqrt{2}}$	0	$\frac{1}{\sqrt{6}}$	0				
$\Delta(^4S_S)\frac{3}{2}^+$	$\frac{1}{3\sqrt{2}}$	0	$\frac{1}{\sqrt{6}}$	0				

have a good agreement but this is spurious due to their neglect of the recoil contribution. Barnes and Close[4] suggested that the small (proton) amplitudes would be natural if this were a hybrid state, citing this as a significant failure of the NRCQM. However, we see (Table V) that inclusion of the relativistic contributions improves the situation (but note that we have ignored any $[70,2^+]_2$ mixture, in contrast with Ref. 5).

Photoproduction data are therefore inconclusive as to the hybrid or QQQ constitutions of this state. If we believe SU(6) symmetry, the calculation by Barnes and Close[4] implies that the electroproduction amplitudes for proton hybrid resonance P_{11} will always be zero, while it is not the case for a QQQ baryon resonance. Phenomenological consequences of this have not yet been studied in detail.

VI. LONG-WAVELENGTH APPROXIMATION AND ELECTROPRODUCTION

Our calculation has concentrated on photoproduction. For (transverse) electroproduction the expressions in the tables can be formally taken over, with **k** suitably modified in magnitude, but some technical problems arise in their application. These problems appear, in particular, in the calculation by Foster and Hughes[13] who use a form factor which is outside of the model and fitted to the experimental data. The question is how far can we extend our result of photoproduction to electroproduction and maintain theoretical consistency?

The nonrelativistic quark model has used for $H_{\rm em}$ Eq. (4.2a), and the relativistic corrections to it have to date been limited to Eq. (4.2b). In our work we have adopted

TABLE IV. The expressions of the coefficients in Tables I–III in harmonic-oscillator basis.

Multiplet	Expression
$[70,1^-]_1$	$B=\sqrt{3}\mu\sqrt{\pi k}\frac{\alpha}{m_q}(2-\frac{1}{g})[1+\frac{1}{6\alpha^2}(P_f^2-P_i^2)]e^{\frac{-k^2}{6\alpha^2}}$
	$C=\sqrt{6}\mu\sqrt{\pi k}\frac{\alpha}{m_q}(2-\frac{1}{g})e^{\frac{-k^2}{6\alpha^2}}$
	$B^\rho=2\mu\sqrt{\pi k}\frac{\alpha}{m_q}[1-\frac{1}{6\alpha^2}(P_f^2-P_i^2)]e^{\frac{-k^2}{6\alpha^2}}$
	$B^\lambda=\frac{1}{3}B\frac{g}{2-\frac{1}{g}}$
	$C^\rho=-\frac{1}{\sqrt{3}}C\frac{g}{2-\frac{1}{g}}$
	$C^\lambda=-\frac{1}{3}C\frac{g}{2-\frac{1}{g}}$
$[56,2^+]_2$	$B=\sqrt{\frac{2}{3}}\mu\sqrt{\pi k}\frac{k}{m_q}(2-\frac{1}{g})[1+\frac{1}{12\alpha^2}(P_f^2-P_i^2)]e^{\frac{-k^2}{6\alpha^2}}$
	$C=\mu\sqrt{\pi k}\frac{k}{m_q}(2-\frac{1}{g})e^{\frac{-k^2}{6\alpha^2}}$
	$B^S=\frac{1}{3}\sqrt{\frac{2}{3}}\mu\sqrt{\pi k}\frac{k}{m_q}[1-\frac{1}{6\alpha^2}(P_f^2-P_i^2)]e^{\frac{-k^2}{6\alpha^2}}$
	$C^S=-\frac{1}{3}C\frac{g}{2-\frac{1}{g}}$
$[56,0^+]_2$	$B=-\frac{1}{\sqrt{3}}\mu\sqrt{\pi k}\frac{k}{m_q}(2-\frac{1}{g})[1+\frac{1}{12\alpha^2}(P_f^2-P_i^2)]e^{\frac{-k^2}{6\alpha^2}}$
	$B^S=\frac{1}{3\sqrt{3}}\mu\sqrt{\pi k}\frac{k}{m_q g}[1-\frac{1}{6\alpha^2}(P_f^2-P_i^2)]e^{\frac{-k^2}{6\alpha^2}}$
$[56,0^+]_0$	$B=-\mu\sqrt{\frac{\pi}{k}}\frac{1}{m_q}(2-\frac{1}{g})(P_f^2-P_i^2)e^{\frac{-k^2}{6\alpha^2}}$
	$B^S=-\frac{1}{3}\mu\sqrt{\frac{\pi}{k}}\frac{1}{m_q g}(P_f^2-P_i^2)e^{\frac{-k^2}{6\alpha^2}}$
$[70,0^+]_2$	$B=-\frac{1}{\sqrt{3}}\mu\sqrt{\pi k}(2-\frac{1}{g})[1+\frac{1}{12\alpha^2}(P_f^2-P_i^2)]e^{\frac{-k^2}{6\alpha^2}}$
	$B^\rho=-\frac{1}{36}\mu\sqrt{\pi k}\frac{k}{m_q g\alpha^2}(P_f^2-P_i^2)e^{\frac{-k^2}{6\alpha^2}}$
	$B^\lambda=-\frac{1}{6\sqrt{3}}\mu\sqrt{\pi k}\frac{k}{m_q g}[1+\frac{1}{6\alpha^2}(P_f^2-P_i^2)]e^{\frac{-k^2}{6\alpha^2}}$

TABLE V. Photon-decay amplitudes between $[56,0^+]_0$ and excited 70 basis states (theory vs experiment) in c.m. frame. The experimental data are given by the most recent Particle Data Group (Ref. 20). $A^{\rm NR}$ is the nonrelativistic results from Eqs. (4.2a) and (4.3a). This column essentially agrees with those of Ref. 5 except insofar as QCD mixing effects have not been included in this work. $A^{\rm SO}$ shows the contribution from additive spin-orbit contributions [Eqs. (4.2b) and (4.3b)]. These agree with Ref. 6 except for their neglect of recoil (see text). $A^{\rm NA}$ show the nonadditive spin orbit. The sum of $A^{\rm NA}$ and $A^{\rm SO}$ is the true measure of the relativistic correction and hence to be considered together; $A^{\rm T}$ is the total contribution. Comparing $A^{\rm T}$ and $A^{\rm NR}$ shows the relative importance of relativistic effects.

Multiplet States	A_J^N	$A^{\rm NR}$	$A^{\rm SO}$	$A^{\rm NA}$	$A^{\rm T}$	$A^{\rm expt}$
$[70,1^-]_1$ $S_{11}(1535)$	$A_{1/2}^p$	174	−66	55	163	73±14
	$A_{1/2}^n$	−128	22	0	−106	−76±32
$D_{13}(1520)$	$A_{1/2}^p$	−20	15	−25	−30	−22±10
	$A_{1/2}^n$	−43	−6	0	−49	−65±13
	$A_{3/2}^p$	131	46	−31	146	167±10
	$A_{3/2}^n$	−131	−15	0	−146	−144±14
$S_{11}(1700)$	$A_{1/2}^p$	0	0	25	25	48±16
	$A_{1/2}^n$	28	−21	3	10	−17±37
$D_{13}(1700)$	$A_{1/2}^p$	0	0	−27	−27	−22±12
	$A_{1/2}^n$	−13	26	−1	12	0±56
	$A_{3/2}^p$	0	0	−47	−47	0±19
	$A_{3/2}^n$	−66	38	−8	−36	−2±44
$D_{15}(1675)$	$A_{1/2}^p$	0	0	0	0	19±12
	$A_{1/2}^n$	−37	−5	−5	−47	−47±23
	$A_{3/2}^p$	0	0	0	0	19±12
	$A_{3/2}^n$	−52	−7	−7	−66	−69±19
$S_{31}(1620)$	$A_{1/2}^{p,n}$	68	22	19	109	19±16
$D_{33}(1675)$	$A_{1/2}^{p,n}$	100	−3	−29	68	116±17
	$A_{3/2}^{p,n}$	104	−26	0	78	77±28
$[70,0^+]_2$ $P_{11}(1705)$	$A_{1/2}^p$	−40	26	−4	−18	5±16
	$A_{1/2}^n$	13	−9	0	4	−5±23

TABLE VI. Photon-decay amplitudes between $[56,0^+]_0$ and excited 56 basis states (theory vs experiment) in c.m. frame. The experimental data are given by the most recent Particle Data Group (Ref. 20). For significance of $A^{\rm NR,SO,NA,T}$, see Table V.

Multiplet states	A_J^N	$A^{\rm NR}$	$A^{\rm SO}$	$A^{\rm NA}$	$A^{\rm T}$	$A^{\rm expt}$
$[56,2^+]_2$ $P_{13}(1720)$	$A_{1/2}^p$	−113	65	−27	−76	52±39
	$A_{1/2}^n$	32	−43	27	16	−2±26
	$A_{3/2}^p$	38	17	−6	49	−35±24
	$A_{3/2}^n$	0	−11	6	−5	−43±94
$F_{15}(1680)$	$A_{1/2}^p$	0	18	−12	6	−17±10
	$A_{1/2}^n$	36	−11	12	37	31±13
	$A_{3/2}^p$	76	33	−11	98	127±12
	$A_{3/2}^n$	0	−22	11	−11	−30±14
$P_{31}(1910)$	$A_{1/2}^{p,n}$	−21	24	−22	−19	−12±30
$P_{33}(1920)$	$A_{1/2}^{p,n}$	−21	12	−11	−20	43±?
	$A_{3/2}^{p,n}$	36	−47	32	21	23±?
$F_{35}(1905)$	$A_{1/2}^{p,n}$	−14	32	−19	−1	27±13
	$A_{3/2}^{p,n}$	−60	41	−34	−53	−47±19
$F_{37}(1950)$	$A_{1/2}^{p,n}$	−36	−5	−6	−47	−73±14
	$A_{3/2}^{p,n}$	−48	−6	−11	−65	−90±13
$[56,0^+]_2$ $P_{11}(1470)$	$A_{1/2}^p$	26	−28	12	10	−69±39
	$A_{1/2}^n$	−18	19	−12	−11	37±19
$P_{33}(1600)$	$A_{1/2}^{p,n}$	−22	17	−10	−15	−22±29
	$A_{3/2}^{p,n}$	−38	30	−17	−25	1±22
$[56,0^+]_0$ $P_{33}(1232)$	$A_{1/2}^{p,n}$	−101	−6	−6	−113	−141±5
	$A_{3/2}^{p,n}$	−173	−11	−11	−195	−258±19

TABLE VII. Photon-decay amplitudes between $[56,0^+]_0$ and excited basis states in the Breit frame.

Multiplet states	$A^p_{1/2}$	$A^p_{3/2}$	$A^n_{1/2}$	$A^n_{3/2}$
$[70,1^-]_1$ $S_{11}(1535)$	151		−98	
$D_{13}(1520)$	−28	143	−46	−143
$S_{11}(1700)$	27		5	
$D_{13}(1700)$	−31	−53	16	−21
$D_{15}(1675)$	0	0	−42	−60
$S_{31}(1620)$	85			
$D_{33}(1700)$	90	76		
$[56,2^+]_2$ $P_{13}(1720)$	−95	52	13	−7
$F_{15}(1680)$	4	105	37	−14
$P_{31}(1910)$	−5			
$P_{33}(1920)$	−18	17		
$F_{35}(1905)$	−2	−33		
$F_{37}(1950)$	−46	−59		
$[56,0^+]_2$ $P_{11}(1470)$	8		−10	
$[56,0^+]_0$ $P_{33}(1232)$	−110	−190		
$[70,0^+]_2$ $P_{11}(1705)$	−19		6	

this formalism and completed it with the necessary nonadditive contributions Eq. (4.2c). However, our derivation of $H_{\rm em}$ in Sec. II shows that in general it has a complicated form with an explicit appearance of the binding potentials Eq. (2.5). McClary and Byers[16] have shown that, to the order $1/m^2$ employed here, this $H_{\rm em}$ reduces to Eq. (2.13), and equivalently to our Eq. (4.2), only to the extent that the long-wavelength approximation is valid. If this approximation fails then the $H_{\rm em}$ at Eq. (4.2) and all results following from it, will be inconsistent.

Comparison of the results with the two $H_{\rm em}$ [Eqs. (2.13) and (4.1)] shows that the criterion for the long-wavelength approximation is that we can expand the common factor $\exp(-k^2/6\alpha^2)$ to the order of k^2: that is,

$$\exp\left[-\frac{k^2}{6\alpha^2}\right]\approx 1-\frac{k^2}{6\alpha^2} . \tag{6.1}$$

Let η be a factor defined as

$$\eta=\frac{1-\dfrac{k^2}{6\alpha^2}}{\exp\left[-\dfrac{k^2}{6\alpha^2}\right]} \tag{6.2}$$

then if the long-wavelength approximation is a good approximation, η should be close to unity.

We stress that the literature, Refs. 2-7, 13, and 14 and this paper, are using a $H_{\rm em}$ [Eq. (4.1)] which is an approximation valid only if $\eta\approx 1$. Essentially, if $H_{\rm em}$ is used to order $(v/c)^2$ then it is only consistent to expand the exponential to $(k/\alpha)^2$. We show the relation between η^2 and virtual-photon mass in Fig. 1 for three different final states in the Breit frame. From this calculation, we can see that for small q^2, the value of η is indeed very close to unity, but deviates very quickly with the increasing of q^2. This implies that the long-wavelength approximation breaks down in electroproduction and the model's validity becomes dubious. To confront large q^2 for individual

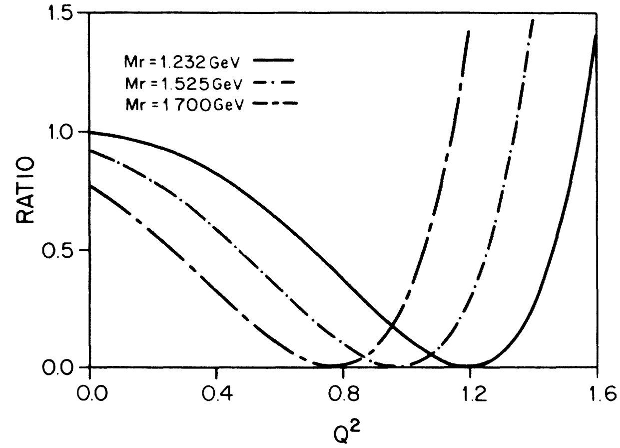

FIG. 1. The relation η^2 (ratio) with virtual-photon mass Q^2 for three different final states in the Breit frame. A similar result is obtained in the c.m. frame.

amplitudes requires more sophisticated approaches, higher-order terms in Eq. (6.1) as in the nonrelativistic Hamiltonian and also detailed evaluation of the role of gluon dynamics. These go far beyond this paper and highlight the limited application of the model, something that seems not to have been addressed much in the literature.

However one may still be able to make predictions for the ratios of helicity amplitudes with q^2 where many of the uncertainties may cancel out. For example, we find that with increasing q^2 the dramatic change in helicity structure for $D_{13}(1520)$ and $F_{15}(1680)$, predicted in the NRCQM (Ref. 7) survives with remarkable accuracy under the relativistic extension as shown in Fig. 2.

VII. CONCLUSION AND DISCUSSION

We confirm the results of Koniuk and Isgur[5] in the c.m. frame with $H_{\rm NR}$ in the limit of ignoring QCD configuration mixing. We also confirm the effect of including $H_{\rm SO}$ as studied by Kubota and Ohta[6] except for their neglect of recoil effects. It is necessary to include these and also $H_{\rm NA}$ in a complete treatment [to $O(v/c)^2$] since these terms have a clear physical origin related to spin rotation in transforming between the quark and baryon rest frame.

We find that in most cases these relativistic effects do not distort the nonrelativistic, unmixed results of Koniuk and Isgur[5] and it is for this reason that nonrelativistic models appear to be so successful. There are some cases where the relativistic corrections, in particular the nonadditive terms, give potentially measurable deviations from the nonrelativistic results. Eventually, one may hope that such deviations may provide a challenge to experiment, namely, to test for their presence or absence by precision data that can distinguish relativistic effects from nonrelativistic results.

However, our calculations are not yet mature enough to warrant such faith. Isgur, Karl and Koniuk,[18,19] in the nonrelativistic framework, noted there are contributions

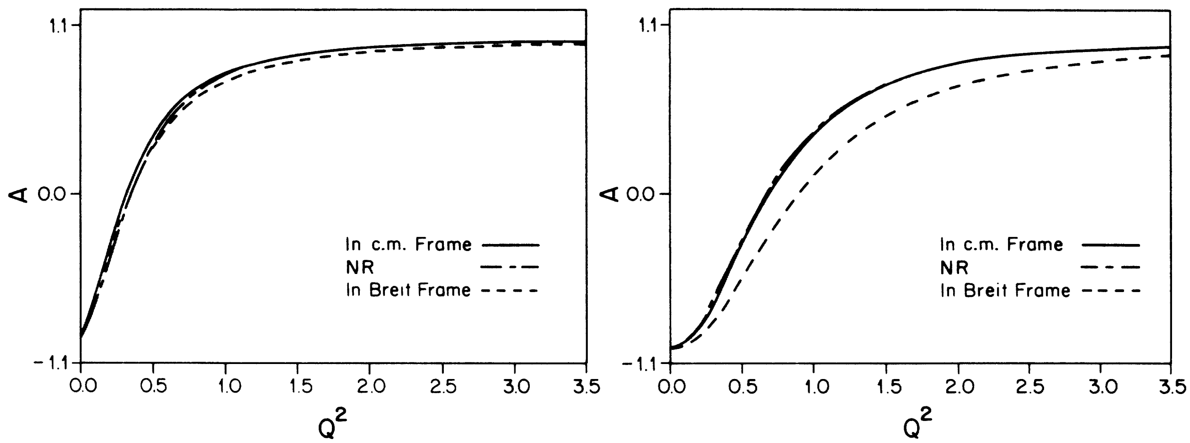

FIG. 2. The helicity structure A defined as $A=(A_{1/2}^2-A_{3/2}^2)/(A_{1/2}^2+A_{3/2}^2)$ for $D_{13}(1520)$ (left) and $F_{15}(1680)$ (right) in c.m. frame (solid line), in Breit frame (dotted line) and the nonrelativistic prediction (Ref. 7) (dotted-dashed line) in c.m. frame.

due to QCD-induced mixing in the wave function. We have not yet included these into our study preferring first to evaluate the stability of the nonrelativistic model to relativistic corrections in $H_{\rm em}$. The result that these effects do not destroy the NRCQM is a mixed blessing. It adds credibility to the model for $q^2=0$ but raises doubts about its extension to $q^2\neq 0$ (see also Ref. 12 which has investigated relativistic effects and extended to $q^2\neq 0$. They plot cross sections for direct confrontation with data rather than break them down into helicity amplitudes, but insofar as we can tell their results and ours seem to agree for $q^2=0$. The validity of the $q^2\neq 0$ work may be subject to the concerns outlined here, but requires more study). Ratios of amplitudes as a function of q^2 may be within the application of the present approximations. Indeed it is interesting to note that with increasing q^2 the dramatic change in helicity structure for $D_{13}(1520)$ and $F_{15}(1680)$, predicted in the NRCQM survives with remarkable accuracy under the relativistic extension as shown in Fig. 2.

Furthermore the results imply that for light-quark systems, the relativistic effects tend to be comparable in some cases to QCD induced effects and so studies of $H_{\rm QCD}$ mixings which ignore $H_{\rm rel}$ are presently incomplete. The contributions of $H_{\rm rel}$ tend to improve the agreement between model and data but the sign of the Roper resonance excitation amplitudes, a classical problem, is incorrect. This sign is correctly described when QCD mixing is taken into account,[5] there being a sensitive cancellation between two contributions in the mixed wave function, and as such highlights the need for mixings such as provided by QCD.

In a separate paper we combine the $H_{\rm QCD}$ and $H_{\rm rel}$ contributions. We find the phenomenological successes of each separately combine "constructively," improving the overall fit to data. The successful sign of the Roper excitation amplitudes that arises in $H_{\rm QCD}$ is preserved and the magnitude *improved* when $H_{\rm rel}$ is included. Finally, and perhaps most significant, we find that both spectroscopy and electromagnetic transition amplitudes are simultaneously described by a single set of parameters (contrast the present literature where spectroscopy[19] and transitions[5] are separately confronted with different values for the oscillator strength parameter).

The q^2 dependence of QCD mixing effects will in general differ from that of relativistic effects and so, in principle, may help to distinguish them. However, this in turn requires better modeling for extending to $q^2\neq 0$ and eventually comparing with the predictions of perturbative QCD (e.g., Ref. 21).

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