Brief Reports

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Gravitational shielding

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Results of the lunar laser ranging experiment show that the Majorana gravitational shielding constant is $h \leq 1.0\times10^{-21}\ \mathrm{g^{-1}cm^2}$. This is 6 orders of magnitude smaller than geophysical constraints.

Majorana[1] proposed that Newton's inverse-square law of gravitation for the attractive force F between point masses M_1 and M_2 separated by distance r be modified to

$$F = GM_1M_2r^{-2}\exp\left[-\int h\rho\, dr\right], \tag{1}$$

where G is the Newtonian gravitational constant, ρ is the density of matter along the line joining the two points, and h is a universal constant. Russell[2] noted that with Majorana's theory the ratio η of gravitational to inertial mass of the Earth is different from that of the Moon and, consequently, lunar theory and observations impose a constraint on the maximum admissible value for h. Let a_0 be the radius of a spherical planet or satellite, and let ρ_0 be its mean density. For $p = ha_0\rho_0 \ll 1$, Russell showed that $\eta = 1-\frac{3}{4}p$ if the density is uniform, and $\eta = 1-\frac{25}{24}p$ if the density at the distance a from the center varies as $a_0^2-a^2$, this being a much higher central condensation than that of the Earth or Moon. The uniform density model provides a suitable approximation for both the Earth and Moon so, relative to the Earth, the Moon receives an excess acceleration $\kappa hGM_{\odot}R^{-2} = \kappa hn^2R$ in the direction of the Sun, where

$$\kappa = \tfrac{3}{4}(\rho_{\oplus}a_{\oplus} - \rho_{\rm moon}a_{\rm moon}) = 2.20\times10^9\ \mathrm{g\,cm^{-2}}, \tag{2}$$

$R = 1.50\times10^{13}$ cm is the Sun-Earth distance, and n is the Earth's mean angular rate about the Sun. Nordtvedt[3] analyzed the effect of such a term on the lunar orbit, and Williams *et al.*[4] and Shapiro *et al.*[5] searched for a Nordtvedt term by analyzing lunar laser ranging data. The results were negative, but they were not interpreted in terms of Majorana shielding.

Let L and L' be the mean longitudes of the Moon and Sun, and let $D = L - L'$ be the mean elongation of the Moon from the Sun (Fig. 1). Assume that the Moon, Earth, and Sun lie in the same plane and that the only perturbation in the elliptical orbit of the Moon around the Earth is due to Majorana shielding. Let r be the Earth-Moon distance and let ω be the Moon's angular rate about the Earth. Set

$$T = \tfrac{1}{2}[(dr/dt)^2 + \omega^2r^2] \tag{3}$$

and

$$V = -GM_{\oplus}/r - \kappa hn^2Rr\cos D\ . \tag{4}$$

Then $T - V$ is the Lagrangian and $T + V = $const is the energy integral for the perturbed orbit. The following equation holds:

$$r\left[\frac{d}{dt}\left[\frac{\partial(T-V)}{\partial(dr/dt)}\right] - \frac{\partial(T-V)}{\partial r}\right] + 2T + 2V = \text{const}\ , \tag{5}$$

which, on substituting (3) and (4), is

$$\tfrac{1}{2}d^2(r^2)/dt^2 - GM_{\oplus}/r = 3\kappa hn^2Rr\cos D + \text{const}\ . \tag{6}$$

Setting $r = r_0 + \delta r$ and $GM_{\oplus} = \omega_0^2r_0^3$, where r_0 and ω_0 are the values for an undisturbed circular reference orbit, and expanding (6) to the first degree in δr (neglecting the constant part of δr) gives

$$d^2(\delta r)/dt^2 + \omega_0^2\delta r = 3\kappa hn^2R\cos D\ , \tag{7}$$

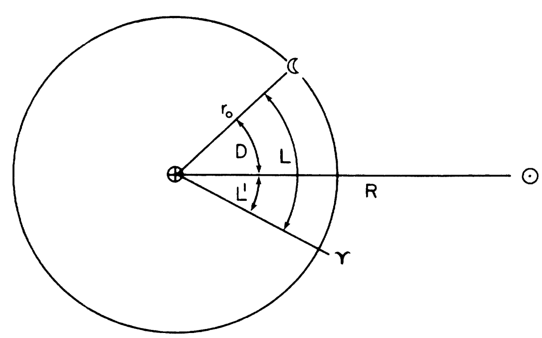

FIG. 1. Geometrical relationships between the mean longitudes of the Moon L and the Sun L', and the elongation of the Moon from the Sun (argument of the synodic month) $D = L - L'$. Their rates of change are $dL'/dt = n$, $dL/dt = \omega_0 = n(1+\chi)$, and $dD/dt = n\chi$.

which, with $\chi=(dD/dt)/n=12.4$, has the solution

$$\delta r = 3(2\chi+1)^{-1}\kappa h R \cos D$$
$$= 3.8\times 10^{21} h \cos D \text{ cm} . \quad (8)$$

According to Williams *et al.*[4] the amplitude of this term is 0±4 cm; therefore $h=0.0\pm1.0\times10^{-21}\text{g}^{-1}\text{ cm}^2$, a null result with an uncertainty 6 orders of magnitude smaller than those of the geophysical estimates of Harrison and of Slichter *et al.*[6]

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