Exact computation of the small-fluctuation determinant around a sphaleron

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We compute exactly, in the high-temperature limit, the determinant κ of small fluctuations around the sphaleron configuration of electroweak theory, exploiting the symmetry of the sphaleron under spatial rotations combined with isospin and custodial SU(2) transformations. For the ratio λ/g^2 of scalar four-point coupling λ to gauge coupling g^2 near unity, we find that κ is 0.03. For λ/g^2 large corresponding to a strongly coupled Higgs phase, or for λ/g^2 very small tending to the Coleman-Weinberg limit, we find that the determinant strongly suppresses the rate of baryon-number-changing processes.

I. INTRODUCTION AND PRELIMINARIES

Since the pioneering work of 't Hooft,[1] it has been widely appreciated that, due to the axial $U_A(1)$ anomaly, baryon and lepton number are not conserved in the standard model, but are violated by calculable quantum effects. If N_f is the number of fermion left-isodoublet generations, then the conservation equations are altered to read

$$\partial_\mu j_B^\mu = \partial_\mu j_L^\mu = \frac{N_f}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}(g^2 F_{\mu\nu}^a F_{\rho\sigma}^a + g'^2 f_{\mu\nu} f_{\rho\sigma}) ,$$

where g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, respectively, and $F_{\mu\nu}^a(A)$ and $f_{\mu\nu}(a)$ are their corresponding field strengths. This implies that over a time interval from t_1 to t_2 baryon and lepton number will be violated by an amount given by

$$\Delta B = \Delta L = N_f[N_{CS}(t_2) - N_{CS}(t_1)] ,$$

where $N_{CS}(t)$ is the Chern-Simons number,

$$N_{CS}(t) = \int d^3x\, K_0(t,x) ,$$

$$K_\mu(x) = \frac{\epsilon_{\mu\nu\rho\sigma}}{16\pi^2}(g^2 A_\nu^a \partial_\rho A_\sigma^a - \tfrac{2}{3} g^3 \epsilon^{abc} A_\nu^a A_\rho^b A_\sigma^c + g'^2 a_\nu \partial_\rho a_\sigma) .$$

For vacuum states described by

$$A_\mu = \frac{i}{g} U^\dagger \partial_\mu U, \quad \lim_{|\mathbf{x}|\to\infty} U = 1 ,$$

and

$$a_\mu = \frac{1}{g'}\partial_\mu \theta ,$$

the Chern-Simons number is an integer given by the winding of the gauge function U. Thus in vacuum-to-vacuum transitions, baryon number is violated by an amount proportional to the change in the gauge winding number of the vacuum. In 't Hooft's original analysis,[1,2] the simplest transitions with $\Delta N_{CS} = 1$ were mediated by single instantons tunneling through the energy barrier separating topologically distinct vacua. These occur with a rate proportional to $\exp(-4\pi/\alpha_W)$ (where $\alpha_W \equiv g^2/4\pi$), corresponding to an energy barrier of height $\sim M_W/\alpha_W$. This rate is so small that it appeared at the time that such anomaly induced $\Delta B \neq 0$ processes are completely negligible.

Recent developments suggest that this initial assessment was overly pessimistic. One observation[3-7] is that, at finite temperature, the rate for transitions between states separated by an energy barrier of height ΔV should be governed by the Boltzmann factor $\exp(-\Delta V/T)$. Thus, for electroweak theory at temperatures on the order of $T \sim M_W/\alpha_W \sim 1$ TeV, thermal fluctuations alone will give particles sufficient energy to classically overcome the energy barrier. Preliminary estimates[7,8] have shown that the rate per unit volume Γ/V of classical thermal activation could be quite large, as much as 12 orders of magnitude larger than the expansion rate of the Universe. The implications of such a large rate have been discussed elsewhere.[9-12] One consequence is that any $B+L$ excess will be greatly modified. However, it is still a matter of debate whether these processes could be incorporated in a natural scenario correctly predicting the observed baryon asymmetry of the Universe.[11,12]

This paper is concerned with refining the calculation of the rate,

$$\Gamma/V = \text{prefactor} \times \exp(-2AM_W/\alpha_W T) ,$$

of baryon-violating processes at high temperature according to the mechanism described above. Here A is a function of λ/g^2 of order unity. While naive arguments based simply on the Boltzmann factor suggest that Γ is unsuppressed at temperatures $T \geq 1$ TeV, there is still the uncertainty represented by the prefactor. Fortunately, given a crucial assumption discussed below, it is possible[7] to calculate this prefactor in a reliable way if we work in the temperature range

$$M_W(T) \ll T \ll M_W(T)/\alpha_W , \tag{1}$$

where

$$M_W(T)=M_W(0)\sqrt{1-T^2/T_c^2} \tag{2}$$

is the temperature-dependent W-boson mass, $M_W(0)=gv/2$ is the zero-temperature mass given in terms of the vacuum expectation value v of the Higgs field and the SU(2) weak coupling g, and

$$T_c=v\Bigg/\left[1+\frac{3g^2}{32\lambda}\right]^{1/2}$$

is the critical temperature for symmetry restoration (λ is the scalar four-point coupling). The lower limit of Eq. (1) ensures that high-temperature transition processes proceed at a rate faster than the low-temperature instanton-induced process.[1,13] The upper limit is equivalent to the criteria that the magnetic screening mass gap is large enough that infrared effects will not invalidate the use of weak coupling expansions.[13,14]

The limits of Eq. (1) also permit great simplification in our task of calculating Γ. The condition $T \gg M_W(T)$ implies that we may work in the high-temperature limit where all fermions effectively decouple from the theory. The remaining boson fields are described by the finite-temperature action

$$S=\int_0^{1/T}dt\int d^3x[\tfrac{1}{4}F_{\mu\nu}^aF_a^{\mu\nu}+(D\Phi)^\dagger(D\Phi)+\lambda(\Phi^\dagger\Phi-v^2/2)^2] \,. \tag{3}$$

[Here and for the remainder of the paper we study only the pure SU(2) gauge theory, corresponding to electroweak theory in the limit that the Weinberg angle vanishes.] The application of the high-temperature limit to (3) is more subtle, but results in the following simplification:[15-17] we can take all fields to be static in the Euclidean time t provided that we replace the Higgs vacuum expectation value by its temperature-dependent expression, $v\to v(T)=2M_W(T)/g$, as given by Eq. (2). Then, after rescaling coordinates and fields according to the prescription

$$\mathbf{r}\to\boldsymbol{\xi}/gv,\quad A\to vA,\quad \Phi\to v\Phi\,, \tag{4}$$

the effective high-temperature action becomes

$$S\to S_3=\frac{1}{g_3^2(T)}\int d^3\xi\left[\tfrac{1}{4}F_{\mu\nu}^aF_a^{\mu\nu}+(D\Phi)^\dagger(D\Phi)+\frac{\lambda}{g^2}(\Phi^\dagger\Phi-\tfrac{1}{2})^2\right], \tag{5}$$

where $g_3(T)$ is the effective coupling constant

$$g_3^2=\frac{g^2T}{2M_W(T)}\,.$$

The upper limit $T \ll M_W(T)/\alpha_W$ also simplifies the task of calculating the rate per unit volume of baryon-number-changing processes. As $M_W(T)/\alpha_W$ gives the approximate height of the energy barrier separating vacua of distinct Chern-Simons number, it is clear that in temperature regime (1) the rate will be dominated by thermal fluctuations which pass through or close to the saddle points of the energy barrier. Since, up to an overall normalization, S_3 is the energy functional of electroweak theory, these saddle points will be given as its extrema. By extending the ideas of Morse theory to the infinite-dimensional configuration space of electroweak theory, Manton has shown how to construct the saddle point of interest.[4,5] In terms of the fields rescaled according to (4), it is the sphaleron configuration given by

$$A_0=0\,,$$
$$\mathbf{A}=2\frac{f(\xi)}{\xi}\hat{\xi}\times\tau\,, \tag{6}$$
$$\Phi=\sqrt{2}\,h(\xi)\hat{\xi}\cdot\tau u_{-1/2}\,,$$

where $u_{-1/2}=(0,1)$ and $\xi=|\boldsymbol{\xi}|=gvr$. One may verify that (6) obeys the equations of motion derived from S_3 provided the profile functions $f(\xi)$ and $h(\xi)$ satisfy the coupled equations

$$-\frac{d^2f}{d\xi^2}+\frac{2}{\xi^2}f(1-f)(1-2f)-\tfrac{1}{4}h^2(1-f)=0\,,$$
$$-\frac{d^2h}{d\xi^2}-\frac{2}{\xi}\frac{dh}{d\xi}+\frac{2}{\xi^2}h(1-f)^2+\frac{\lambda}{g^2}(h^2-1)h=0\,,$$

subject to the boundary conditions

$$\lim_{\xi\to 0}\frac{f(\xi)}{\xi}=\lim_{\xi\to 0}h(\xi)=0\,,$$
$$\lim_{\xi\to\infty}f(\xi)=\lim_{\xi\to\infty}h(\xi)=1\,.$$

The energy of the sphaleron is

$$E_{\text{sp}}=A(\lambda/g^2)2M_W(T)/\alpha_W\,,$$

where, for all values of λ/g^2, A is a number of order 1. One also finds the Chern-Simons number of the sphaleron to be $\frac{1}{2}$, in accordance with the notion that the sphaleron sits midway on a one-parameter set of configurations interpolating between vacuum states with N_{CS} equal to 0 and 1.

We are now in a position to state the crucial assumption alluded to above: namely, that the sphaleron configuration is the saddle point of *minimal energy* and hence dominates the transition rate in the temperature range (1). This must be honestly stated, since Manton's application of Morse theory to S_3 only proves the existence of *at least one* saddle point. Indeed, for $\lambda/g^2 \geq 18$, the existence of deformed sphaleron solutions of lower energy has recently been established.[18] As we work in the range of couplings $0.1 \leq \lambda/g^2 \leq 10$, this last result will not be an issue for us. However, even for range considered, it not known whether other saddle points exist or whether they have lower energy.

Having clearly given the setting of our computation we may now construct the expression for the rate per unit volume Γ/V, and isolate the quantity κ which will be the focus of our calculations in the following sections. With the assumption of sphaleron dominance of the transition rate we may now use the techniques of nonequilibrium statistical mechanics developed by Langer, Affleck and Linde[14,19-21] to compute the rate. Let ω_- denote the rate of decay in small fluctuations around the sphaleron.

Then the formula for the rate of decay is

$$\Gamma = \frac{\omega_-}{\pi T} \mathrm{Im} F ,$$

where F is the free energy of the system evaluated in the saddle-point approximation around the sphaleron configuration. To quadratic order in thermal fluctuations the imaginary part of the free energy is found to be[7]

$$\Gamma / V = \frac{\omega_-}{2\pi} \mathcal{N}_{tr} (\mathcal{N} V)_{rot} \left[\frac{\alpha_W T}{4\pi} \right]^3 \alpha_3^{-6} e^{-E_{sp}/T} \kappa ,$$

where $\alpha_W = g^2/4\pi \simeq \frac{1}{29}$ is the weak fine-structure constant. With the exception of the Boltzmann factor $e^{-E_{sp}/T}$ and the quantity κ, the additional factors that appear in this formula are due to contributions from the sphaleron zero modes. The quantities $\mathcal{N}_{tr}$ and $\mathcal{N}_{rot}$ are certain normalization integrals relating the "natural" coordinates describing the translations and rotations of the sphaleron to the "canonical" coordinates of these zero modes as they appear in the Euclidean path integral. These dimensionless functionals of the profile functions $f(\xi)$ and $h(\xi)$ are constructed in full detail in a previous paper,[22] and are displayed in Fig. 1. In Fig. 2 we also present a plot of the negative-mode frequency ω_-^2 as a function of λ/g^2. The volume of the rotation group V_{rot} is $8\pi^2$. Finally, up to an overall scale factor of $(gv)^{-3}$ absorbed by the translation volume V, there is a factor of $1/g_3$ for each of the six zero modes:

$$(g_3)^{-6} = (gv)^{-3} \left[\frac{\alpha_W T}{4\pi} \right]^3 \alpha_3^{-6} .$$

Here $\alpha_3 = g_3^2/4\pi$.

The remaining contribution to the prefactor is the quantity κ, defined as

$$\kappa = \mathrm{Im} \left[\frac{\det(\delta^2 S_{gf}/\delta\phi^2)|_{\phi=\phi_{vac}} \Delta_{FP}|_{\phi=\phi_{sp}}}{\det'(\delta^2 S_{gf}/\delta\phi^2)|_{\phi=\phi_{sp}} \Delta_{FP}|_{\phi=\phi_{vac}}} \right]^{1/2} , \tag{7}$$

In this equation, S_{gf} is the gauge-fixed version of S_3, Δ_{FP} is the associated Faddeev-Popov determinant, and ϕ generically represents the scalar and vector fields of electroweak theory. ϕ_{sp} and ϕ_{vac} denote the fields of the sphaleron, given by (6), and the perturbative vacuum, respectively. The prime on the determinant of $\delta^2 S_{gf}/\delta\phi^2|_{\phi=\phi_{sp}}$ denotes that the zero-frequency modes of the sphaleron are to be deleted from its evaluation. It is this quantity κ which is the object of the computations presented in this paper.

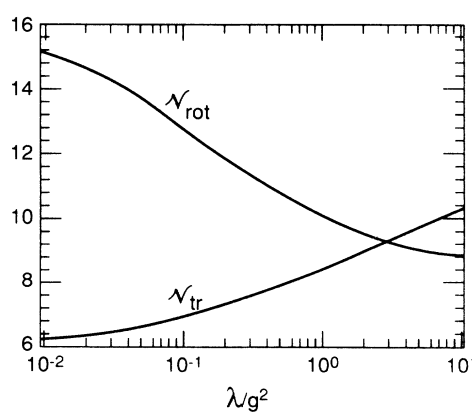

FIG. 1. The zero-mode normalization factors $\mathcal{N}_{tr}$ and $\mathcal{N}_{rot}$ as a function of λ/g^2.

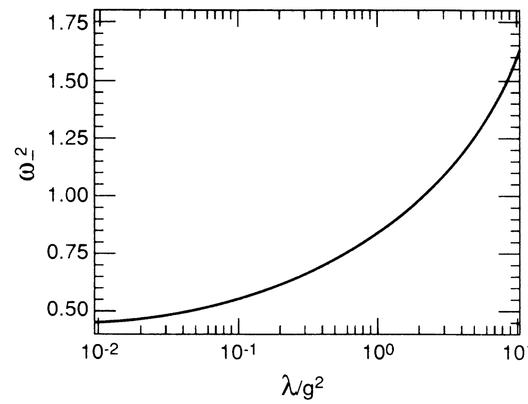

FIG. 2. The negative-mode frequency ω_-^2 [in units of $(gv)^2$] as a function of λ/g^2

The quantity κ has a physical interpretation that makes it evident why it is of importance to evaluate. From the general expression $F = E - T\sigma$ relating the free energy F to the internal energy E and entropy σ, we see that $\kappa \sim e^{\sigma}$ measures the number of non-zero-frequency modes available in the vicinity of the sphaleron relative to that of the perturbative vacuum. It is reasonable that the rate should be enhanced or suppressed depending on whether this quantity is large or small.

In the work of Arnold and McLerran,[7] the quantity κ, since it is a dimensionless function of λ/g^2 was taken to be unity. While this is a resonable estimate for $\lambda/g^2 \sim 1$, κ may be a singular function of λ/g^2 and thus could diverge strongly for extreme values of this ratio. This expectation was borne out in the approximate calculation of Carson and McLerran,[22] which suggested that κ, while of order unity for $\lambda/g^2 \sim 1$, is strongly suppressed in the large λ/g^2 limit, corresponding to a strongly coupled Higgs sector, and in the Coleman-Weinberg limit of λ/g^2 very small. However, in the range $10^{-2} \leq \lambda/g^2 \leq 10$ this suppression resulted in a total rate per unit volume that is still some 9–10 orders of magnitude larger than expansion rate of the Universe.

While suggestive, the calculation of κ by Carson and McLerran may be criticized on at least two counts, both due to the method of approximation used. The method, developed by Diakonov, Petrov, and Yung [23](DPY), utilizes the Schwinger proper-time representation of the ratio:

$$\frac{\det \mathcal{M}}{\det \mathcal{M}^0} = \exp \left[-\mathrm{Tr} \int_0^\infty \frac{dt}{t} (e^{-t\mathcal{M}} - e^{-t\mathcal{M}^0}) \right] , \tag{8}$$

where Tr denotes a functional trace. Here $\mathcal{M}$ may represent an operator such as the fluctuation operator

$\delta^2 S_{gf}/\delta\phi^2|_{\phi=\phi_{sp}}$ introduced in the definition of κ above, while $\mathcal{M}^0$ is the same operator evaluated in the perturbative vacuum. The DPY approximation, as applied to (8), consists of expanding the integrand to finite order in the proper time t, evaluating the integral with an upper (infrared) cutoff δ, and then minimizing with respect to this cutoff. This method has the advantage of producing an approximation for $\ln(\det\mathcal{M}/\det\mathcal{M}^0)$ expressed in terms of gauge-invariant functionals of the background fields, and the minimization with respect to δ is relatively easy to implement. However, the small-t expansion, since it neglects infrared effects, is destined to fail in the Coleman-Weinberg limit ($\lambda/g^2 \ll 1$) when the Compton wavelength of the Higgs field becomes large. Also, since representation (8) is well defined only when both $\mathcal{M}$ and $\mathcal{M}^0$ have the same number of eigenvalues, the presubtraction of zero-frequency modes from $\mathcal{M}=\delta^2 S_{gf}/\delta\phi^2|_{\phi=\phi_{sp}}$, as required by the definition of κ, necessitated the subtraction of an equal number of modes from $\mathcal{M}^0$. If the eigenvalue of the modes subtracted from $\mathcal{M}^0$ is ϵ, then this subtraction introduced an (almost) arbitrary parameter in the evaluation of the determinant.

Therefore, in view of these shortcomings of the DPY method, it is desirable to have a definitive and exact evaluation of κ. At first glance, such a calculation appears prohibitively difficult, since the operators involved in (7) are (ultimately) defined in terms of the profile functions $f(\xi)$ and $h(\xi)$, which are numerically determined. Hence the evaluation of κ is equivalent to the solution of the Schrödinger equation in a potential which is known only in numerical form. However, as the next sections will demonstrate, such a calculation is indeed feasible due to the symmetry of the sphaleron configuration (6) under spatial rotations combined with isospin and custodial SU(2) transformations. If $\mathbf{L}+\mathbf{S}$, $\mathbf{I}$, and $\mathbf{K}$ denote the corresponding generators ($\mathbf{L}$ and $\mathbf{S}$ are the orbital and spin pieces of the spatial angular momentum operator), then the sphaleron is invariant under transformations generated by

$$\mathbf{J}=\mathbf{L}+\mathbf{S}+\mathbf{I}+\mathbf{K}\ .$$

Moreover, the sphaleron transforms simply under spatial inversion. Consequently, the determinants appearing in κ may be decomposed into products of determinants of one-dimensional operators labeled by partial-wave j and parity π:

$$\det\mathcal{M}(\xi)=\prod_{j=0}^{\infty}\prod_{\pi=\pm}\det\mathcal{M}_{j\pi}(\xi)\ . \tag{9}$$

A method of computation then suggests itself. Truncating (9) at some $j_{\max}$ and numerically evaluating the simpler partial-wave determinants for $j=0,\ldots,j_{\max}$, the behavior of the full determinant on $j_{\max}$ may be evaluated and the limit $j_{\max}\to\infty$ numerically extracted. Actually, this procedure is too simple minded, since it overlooks a linear divergence present in $\ln\kappa$. However, this problem is surmountable, as will be shown in more detail below.

In the next section we perform the decomposition of the operators defined in (7) with respect to partial wave j and parity π. In this endeavor we shall utilize the SO(4) representation of scalar fields in which the isospin and custodial transformations take a particularly convenient form. Expanding the fluctuation gauge potentials a_i^a in tensor spherical harmonics, and the complex scalar field η in scalar and vector spherical harmonics, the partial-wave operators $\mathcal{M}_{j\pi}$ are constructed. In Sec. III we discuss how to isolate the linear divergence in $\ln\kappa$ and subtract it out. In Sec. IV we conclude with a presentation of our final, exact results for $\ln\kappa$. An appendix gives details of our operator construction of vector and tensor spherical harmonics.

II. SPHERICAL HARMONIC DECOMPOSITION

In this section we first construct the gauge-fixed fluctuation action δS_{gf} for the fields of electroweak theory in the background of the sphaleron configuration (6). Then we explicitly display the invariance of the sphaleron under SU(2) transformations generated by the operator $\mathbf{J}$ representing spatial rotations combined with isospin and custodial SU(2) transformations. Exploiting this symmetry we decompose the fluctuation fields in spherical harmonics defined with respect to $\mathbf{J}$ and parity π. For a fixed partial wave j, this reduces the fluctuation operator defining δS_{gf} to a 13-channel operator, whose form we present. We make comments regarding the isolation of negative mode from the $j=0$, $\pi=+$operator and the zero modes from the $j=1$, $\pi=\pm$ operators. Finally we also perform the partial-wave decomposition of the Faddeev-Popov operator.

A. The action of the small fluctuations

We begin with the SU(2) Yang-Mills-Higgs action S_3 in three dimensions defined by Eq. (5), corresponding to the high-temperature limit of electroweak theory with vanishing Weinberg angle ($\Theta_W=0$). Explicitly separating the terms involving the field A_0, we obtain

$$S_3=\frac{1}{g_3^2(T)}\int d^3\xi\tfrac{1}{2}A_0[-D_i(A)D_i(A)+\tfrac{1}{2}\Phi^\dagger\Phi]A_0 +\frac{1}{g_3^2(T)}\int d^3\xi\left[\tfrac{1}{4}F_{ij}^aF_{ij}^a+(D_i\Phi)^\dagger(D_i\Phi)+\frac{\lambda}{g^2}(\Phi^\dagger\Phi-\tfrac{1}{2})^2\right].$$

Here $D_i(A)=\partial_i-iA_i^a\mathcal{T}^a$, $i=1,2,3$, is the covariant derivative in the appropriate representation of the generator $\mathcal{T}^a$. For example, when acting on the isovector A_0^b, $\mathcal{T}^a$ assumes the form $\mathcal{T}^aA_0^b=(\Delta^a)^{bc}A_0^c$ where

$$(\Delta^a)^{bc}\equiv-i\epsilon^{abc}\ . \tag{10}$$

In the case of the isospinor Φ, $\mathcal{T}^a$ is given by τ^a which are the Pauli matrices divided by 2:

$$\tau^a=\tfrac{1}{2}\sigma^a\ .$$

We expand this action in the small fluctuations around a general background configuration by replacing

$$A_0 \to A_0 + g_3(T)a_0 ,$$
$$\mathbf{A} \to \mathbf{A} + g_3(T)\mathbf{a} ,$$
$$\Phi \to \Phi + g_3(T)\eta ,$$

where a_0, **a**, and η denote the fluctuation fields. Gauge degrees of freedom are eliminated by imposing the background $R_{\xi=1}$ gauge condition,

$$D_k(A)a_k + i(\Phi^\dagger \tau\eta - \eta^\dagger \tau\Phi) = 0 .$$

The Faddeev-Popov determinant corresponding to this gauge is

$$\Delta_{\rm FP} = \det[-D(A)^2 + \tfrac{1}{2}\Phi^\dagger\Phi] . \tag{11}$$

Since we shall be working with background fields with $A_0=0$, we immediately infer from the formula for S_3 above that its dependence on a_0 is quadratic. Furthermore, functional integration over the a_0 fields yields a factor $(\Delta_{\rm FP})^{-1/2}$ which partially cancels with the Faddeev-Popov determinant to produce the factor $(\Delta_{\rm FP})^{+1/2}$ in the expression (7) for κ.

The gauge-fixed action of small fluctuations to quadratic order in the remaining fields **a** and η is

$$\delta S_{\rm gf} = \int d^3\xi \left[\tfrac{1}{2}(D_i a_j)^a (D_i a_j)^a + \epsilon^{abc} F^a_{ij} a^b_i a^c_j + \tfrac{1}{4}\Phi^\dagger\Phi a^a_i a^a_i + 2i[\eta^\dagger a_i\cdot\tau D_i\Phi - (D_i\Phi)^\dagger a_i\cdot\tau\eta] + (D_i\eta)^\dagger(D_i\eta) + \frac{2\lambda}{g^2}(\Phi^\dagger\Phi - \tfrac{1}{2})\eta^\dagger\eta + \tfrac{1}{2}\eta^\dagger\eta\Phi^\dagger\Phi + \left[\frac{\lambda}{g^2} - \frac{1}{8}\right](\Phi^\dagger\eta + \eta^\dagger\Phi)^2 \right] . \tag{12}$$

For notational convenience we shall denote the quadratic operator defined here by $\mathcal{M}$, where

$$\mathcal{M} = \frac{1}{2}\frac{\delta^2 S_{\rm gf}}{\delta\phi^2}$$

and ϕ is a generic symbol for the fluctuation fields **a** and η.

B. Symmetries of the fluctuation operator

Before giving decomposition of the fluctuation fields a^a_i, η, and η^* in spherical harmonic components, we first give the form of all symmetries present in S_3. Here we introduce an SO(4) notation that will prove useful later. By studying the effect of these symmetries on the sphaleron background fields, it is then easy to identify the invariance group of this configuration, according to which we may classify the fluctuation fields.

One symmetry of the full action S_3 is an SO(3) group of rotational symmetries generated by $\mathbf{L}+\mathbf{S}$, where **L** and **S** denote the orbital and spin pieces of the angular momentum operator. Under this group, the fields transform as

$$\Phi(\xi) \to e^{i\epsilon\cdot(\mathbf{L}+\mathbf{S})}\Phi(\xi) = \Phi(R(\epsilon)\xi) ,$$
$$A^a_i(\xi) \to e^{i\epsilon\cdot(\mathbf{L}+\mathbf{S})} A^a_i(\xi) = (e^{i\epsilon\cdot\Delta})_{ij} A^a_j(R(\epsilon)\xi) .$$

Here Δ is the spin-1 matrix operator defined by (10) while

$$R(\epsilon) = e^{\epsilon\cdot(\xi\times\nabla)} .$$

Note that the formal operator **S** annihilates Φ while the gauge potential $\mathbf{A}^a$ transforms as a spin-1 object. Since it will not cause confusion below, we shall not distinguish between the formal operator **L** and its representation as $-i\xi\times\nabla_\xi$.

The action S_3 also possesses an SO(4) invariance. To display it we introduce the four-component notation

$$\Phi = \begin{bmatrix} -\mathrm{Re}\phi_1 \\ \mathrm{Im}\phi_1 \\ \mathrm{Re}\phi_2 \\ \mathrm{Im}\phi_2 \end{bmatrix} , \tag{13}$$

where ϕ_1 and ϕ_2 are the complex components of Φ in the ordinary isospinor representation of this field:

$$\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} .$$

Then under the indicated SO(4) symmetry, the field Φ transforms as a four-vector according to representation (13). Before we give its explicit form, let us also state how the gauge potential transforms under this same symmetry group. Here we recall the well-known isomorphism

$$\mathrm{SO}(4) \sim \mathrm{SU}(2)_{\rm isospin} \otimes \mathrm{SU}(2)_{\rm custodial} ,$$

which is generated by abstract operators **I** and **K**, respectively. Then the isospin transformations are

$$\Phi_i \to e^{i\epsilon\cdot\mathbf{I}}\Phi_i = (e^{i\epsilon\cdot\mathbf{T}})_{ij}\Phi_j ,$$
$$\mathbf{A}^a \to e^{i\epsilon\cdot\mathbf{I}}\mathbf{A}^a = (e^{i\epsilon\cdot\Delta})^{ab}\mathbf{A}^b ,$$

where again Δ is the spin-1 matrix given by (10) while the 4×4 matrices T_i are given by

$$T_1 = \tfrac{1}{2}\sigma_2\otimes\sigma_1 ,$$
$$T_2 = -\tfrac{1}{2}\sigma_2\otimes\sigma_3 ,$$
$$T_3 = \tfrac{1}{2}\sigma_0\otimes\sigma_2 .$$

Here σ_0 is the unit 2×2 matrix. These latter matrices are simply derived by recasting the transformation law of the field Φ in isospinor form

$$\begin{bmatrix}\delta\phi_1\\ \delta\phi_2\end{bmatrix}=i\boldsymbol{\epsilon}\cdot\boldsymbol{\tau}\begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix},$$

to the SO(4) vector representation (13):

$$\delta\Phi=i\boldsymbol{\epsilon}\cdot\mathbf{T}\Phi .$$

Under the custodial SU(2)-symmetry group, the field Φ in isospinor form transforms as

$$\begin{bmatrix}\delta\phi_1\\ \delta\phi_2^*\end{bmatrix}=i\boldsymbol{\epsilon}\cdot\boldsymbol{\tau}\begin{bmatrix}\phi_1\\ \phi_2^*\end{bmatrix},$$

while the gauge potential $\mathbf{A}^a$ remains unchanged. In the SO(4) form, these transformation laws read

$$\Phi_i\to e^{i\boldsymbol{\epsilon}\cdot\mathbf{K}}\Phi_i=(e^{i\boldsymbol{\epsilon}\cdot\mathbf{T}'})_{ij}\Phi_j ,$$
$$\mathbf{A}^a\to e^{i\boldsymbol{\epsilon}\cdot\mathbf{K}}\mathbf{A}^a=\mathbf{A}^a ,$$

where the matrices T_i' are given by

$$T_1'=-\tfrac{1}{2}\sigma_1\otimes\sigma_2 ,$$
$$T_2'=-\tfrac{1}{2}\sigma_2\otimes\sigma_0 ,$$
$$T_3'=\tfrac{1}{2}\sigma_3\otimes\sigma_2 .$$

One may verify that our representation for T_i and T_j' satisfy the SO(4) commutation relations. In particular,

$$[T_i,T_j']=0 .$$

We have thus identified the full symmetry group of S_3 to be

$$\mathcal{G}=\mathrm{SO}(3)_{\mathbf{L}+\mathbf{S}}\otimes\mathrm{SO}(4)_{\mathbf{I},\mathbf{K}} .$$

However, the sphaleron configuration (6) is not invariant under each of these symmetries separately but breaks the full symmetry group $\mathcal{G}$ down to a residual symmetry group $\mathcal{H}$. To identify $\mathcal{H}$, note that, for the sphaleron, the field Φ is proportional $\boldsymbol{\tau}\cdot\hat{\boldsymbol{\xi}}u_{-1/2}$, which may be expressed in SO(4) vector form as

$$\boldsymbol{\tau}\cdot\hat{\boldsymbol{\xi}}\begin{bmatrix}0\\1\end{bmatrix}\to\frac{1}{2}\begin{bmatrix}-\hat{\xi}_1\\-\hat{\xi}_2\\-\hat{\xi}_3\\0\end{bmatrix}.$$

Note that the three components of $\hat{\boldsymbol{\xi}}$ are arranged in natural order as the first three components of this four-vector. On the other hand, the set of matrices $\mathbf{T}+\mathbf{T}'$ has the simple form

$$\mathbf{T}+\mathbf{T}'=\begin{bmatrix}\boldsymbol{\Delta}&0\\0&0\end{bmatrix},$$

where the spin-1 matrix $\boldsymbol{\Delta}$ appears in the upper left 3×3 corner. This implies that, at least for the Φ field, a spatial rotation of the sphaleron background can be compensated by SO(4) transformation. In particular, it is the transformation generated by $\mathbf{I}+\mathbf{K}$. The same trick works for the gauge potential, although in this case a spatial rotation (generated by $\mathbf{L}+\mathbf{S}$) is compensated by a pure isospin transformation. We can state both of these invariances in compact form by noting that the spin angular momentum operator $\mathbf{S}$ annihilates Φ while $\mathbf{K}$ annihilates the gauge potential $\mathbf{A}^a$. We therefore have

$$\begin{aligned}\Phi(\boldsymbol{\xi})&=e^{i\boldsymbol{\epsilon}\cdot(\mathbf{L}+\mathbf{S}+\mathbf{I}+\mathbf{K})}\Phi(\boldsymbol{\xi})=(e^{i\boldsymbol{\epsilon}\cdot(\mathbf{T}+\mathbf{T}')})_{ij}\Phi(R(\boldsymbol{\epsilon})\boldsymbol{\xi})_j ,\\ A_i^a(\boldsymbol{\xi})&=e^{i\boldsymbol{\epsilon}\cdot(\mathbf{L}+\mathbf{S}+\mathbf{I}+\mathbf{K})}A_i^a(\mathbf{x}) \qquad (14)\\ &=(e^{i\boldsymbol{\epsilon}\cdot\boldsymbol{\Delta}})_{ij}(e^{i\boldsymbol{\epsilon}\cdot\boldsymbol{\Delta}})^{ab}A_j^b(R(\boldsymbol{\epsilon})\boldsymbol{\xi}) .\end{aligned}$$

Consequently the residual symmetry group $\mathcal{H}$ is an SU(2) symmetry generated by

$$\mathbf{J}=\mathbf{L}+\mathbf{S}+\mathbf{I}+\mathbf{K} .$$

Furthermore, for background fields obeying (14) we have the following condition for the fluctuation operator $\mathcal{M}$ defined above:

$$[\mathbf{J},\mathcal{M}]=0 .$$

We also conclude that the classical sphaleron lies in the lowest possible representation of the group generator $\mathbf{J}$, namely, $j=0$.

C. The spherical harmonic decomposition of a^a and η

Inserting the sphaleron background field (6) into the expression for the gauge-fixed fluctuation action (12) we obtain (rescaling $\eta\to\eta/\sqrt{2}$)

$$\begin{aligned}\delta S_{\mathrm{gf}}=\int d\Omega_{\hat{\xi}}\int\xi^2d\xi\Bigg[&-\tfrac{1}{2}a_{ka}(D_V^2)_{ab}a_{bk}-\tfrac{1}{2}\eta_a(D_S^2)_{ab}\eta_b-\tfrac{1}{2}\eta_4(D_S^2)_{44}\eta_4-\eta_a(D_S^2)_{a4}\eta_4\\ &+\left[(\hat{\xi}_k\hat{\xi}_a-\delta_{ka})\frac{2f'}{\xi}-\hat{\xi}_k\hat{\xi}_a\frac{4f(1-f)}{\xi^2}\right]\epsilon^{klm}\epsilon^{abc}a_{lb}a_{mc}\\ &+[\eta_4\hat{\xi}_k\hat{\xi}_pa^{kp}-\hat{\xi}_k(\boldsymbol{\eta}\times\hat{\boldsymbol{\xi}})_pa^{kp}]\left[h'-\frac{h}{\xi}+\frac{hf}{\xi}\right]+(\eta_4a^{kp}\delta_{kp}-\eta_a\epsilon^{akp}a_{kp})\frac{(1-f)h}{\xi}\\ &+\frac{\lambda}{2g^2}(h^2-1)\eta^2+\left[\frac{\lambda}{g^2}-\frac{1}{8}\right]h^2(\hat{\boldsymbol{\xi}}\cdot\boldsymbol{\eta})^2+\frac{1}{8}h^2(a_k^aa_k^a+\boldsymbol{\eta}\cdot\boldsymbol{\eta}+\eta_4^2)\Bigg],\end{aligned} \qquad (15)$$

where f and h are the profile functions of ξ defining the sphaleron configuration (6), and

$$-(D_V^2)_{ab}=-\left[\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\xi^2\frac{\partial}{\partial\xi}-\frac{\mathbf{L}^2}{\xi^2}-\frac{4f^2}{\xi^2}\right]\delta_{ab}+\frac{4f}{\xi^2}\mathbf{\Delta}_{ab}\cdot\mathbf{L}+\frac{4f^2}{\xi^2}\hat{\xi}_a\hat{\xi}_b\ ,$$

$$-(D_S^2)_{ab}=-\left[\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\xi^2\frac{\partial}{\partial\xi}-\frac{\mathbf{L}^2}{\xi^2}-\frac{2f^2}{\xi^2}\right]\delta_{ab}+\frac{4f}{\xi^2}\mathbf{T}_{ab}\cdot\mathbf{L}$$

are the covariant derivatives acting on the $\mathbf{a}^a$ and η_a fields, respectively. Note that we have separated the four-vector η into two pieces, namely $\eta_a, a=1, 2, 3$, and η_4. The reason we do this is that, as was pointed out above, the action of the operator $\mathbf{I}+\mathbf{K}$ on η decomposes this field into a triplet and singlet, where the triplet lies precisely in the first three components of η.

We are now ready to perform the spherical harmonic decomposition. First we note that the $\mathbf{J}$ acting on the field η_4 reduces simply to the orbital angular momentum operator $\mathbf{L}$. Consequently we will expand this field as

$$\eta_4=\sum_{j=0}^{\infty}\sum_{m=-j}^{j}\eta_{j,m}(\xi)Y_{j,m}(\Omega)\ ,$$

where $Y_{j,m}$ are the usual spherical harmonic functions, defined as eigenfunctions of the angular momentum operator $\mathbf{L}^2$, viz.,

$$\mathbf{L}^2Y_{j,m}=j(j+1)Y_{j,m},\quad L_3Y_{j,m}=mY_{j,m}\ .$$

Next we recall that $\mathbf{J}$ acting on the three-component η_a field has the form $\mathbf{L}\delta_{ab}+\mathbf{\Delta}_{ab}$, since it is a triplet under the action of $\mathbf{I}+\mathbf{K}$. Thus, in this case, it is appropriate to expand η_a in vector spherical harmonics $Y^a_{j,s,m}(s=0,\pm1)$ as

$$\eta^a=\sum_{j=0}^{\infty}\sum_{m=-j}^{j}\sum_{s=-1}^{1}\eta_{j,s,m}(\xi)Y^a_{j,s,m}(\Omega)\ .$$

Here the vector spherical harmonics are defined as eigenfunctions of $(\mathbf{L}+\mathbf{\Delta})^2$, $L_3+\Delta_3$, and $\mathbf{L}^2$:

$$\begin{aligned}(\mathbf{L}+\mathbf{\Delta})^2_{ab}Y^b_{j,s,m}&=j(j+1)Y^a_{j,s,m}\ ,\\ (L_3+\Delta_3)_{ab}Y^b_{j,s,m}&=mY^a_{j,s,m}\ ,\\ \mathbf{L}^2Y^a_{j,s,m}&=(j+s)(j+s+1)Y^a_{j,s,m}\ .\end{aligned}\tag{16}$$

Note that due to the condition that the orbital angular momentum be non-negative, e.g., $l=j+s\geq0$, only the modes with s equal to 0 and 1 are defined when $j=0$.

Finally, the operator $\mathbf{J}$ acts on the gauge fluctuation field a^{ak} as $\mathbf{L}\delta_{ab}\delta_{kl}+\delta_{ab}\mathbf{\Delta}_{kl}+\mathbf{\Delta}'_{ab}\delta_{kl}$, which we see acts in the direct product space of a spin-1 vector with a spin-1 isovector. (For clarity we have placed a prime on the isospin contribution although the reader should realize that $\mathbf{\Delta}$ and $\mathbf{\Delta}'$ are numerically identical.) Therefore, we must expand $\mathbf{a}^a$ in tensor spherical harmonics as

$$a^{ak}=\sum_{j=0}^{\infty}\sum_{m=-j}^{j}\sum_{s_2=0}^{2}\sum_{s_1=-s_2}^{s_2}a_{j,s_1,s_2,m}(\xi)Y^{ak}_{j,s_1,s_2,m}(\Omega)\ ,$$

where the $Y^{ak}_{j,s_1,s_2,m}(s_2=2,1,0;\ s_1=-s_2,\dots,s_2)$ are eigenfunctions of $(\mathbf{L}+\mathbf{\Delta}+\mathbf{\Delta}')^2$, $L_3+\Delta_3+\Delta'_3$, $\mathbf{L}^2$, and $(\mathbf{\Delta}+\mathbf{\Delta}')^2$:

$$\begin{aligned}(\mathbf{L}+\mathbf{\Delta}+\mathbf{\Delta}')^2_{ab,kl}Y^{bl}_{j,s_1,s_2,m}&=j(j+1)Y^{ak}_{j,s_1,s_2,m}\ ,\\ (L_3+\Delta_3+\Delta'_3)_{ab,kl}Y^{bl}_{j,s_1,s_2,m}&=mY^{ak}_{j,s_1,s_2,m}\ ,\\ L^2Y^{ak}_{j,s_1,s_2,m}&=(j+s_1)(j+s_1+1)Y^{ak}_{j,s_1,s_2,m}\ ,\\ (\mathbf{\Delta}+\mathbf{\Delta}')^2_{ab,kl}Y^{bl}_{j,s_1,s_2,m}&=s_2(s_2+1)Y^{ak}_{j,s_1,s_2,m}\ .\end{aligned}$$

Once again the condition that $l=j+s_1\geq0$ means that only the modes with $s_1\geq0$ are defined for $j=0$ and $s_1\geq-1$ for $j=1$. We should also comment that the tensor harmonic functions that we have defined here incorporate all components $s_2=0$, 1, and 2 in a single set of functions. It is more conventional to define them with $s_2=2$ only, but since the fluctuation operator defined in (15) couples all these components of a^{ak} together, we have found our particular definitions to be more convenient.

The spherical harmonic functions Y_{jm}, $Y^a_{j,s,m}$, and $Y^{ak}_{j,s_1,s_2,m}$ may be constructed using the standard Clebsch-Gordan methodology. An alternative method, defining the vector and tensor harmonics in terms of operators acting on Y_{jm}, is given in an appendix.

Because $\mathbf{J}$ commutes with the fluctuation operator $\mathcal{M}$, the nonzero matrix elements of $\mathcal{M}$ are m independent and are block diagonal in the quantum number j. Therefore, for a given j, the fluctuation operator is reduced to a 13-channel matrix of one-dimensional operators with degeneracy $(2j+1)$. This matrix is further split into two decoupled blocks of size 7×7 and 6×6 due to parity conservation. Because of this decoupling due to parity we shall arrange the expansion coefficients of the fluctuations fields into the following seven- and six-dimensional column matrices, $\Psi^{(\pi)}_{j,m}$, $\pi=\pm$, corresponding to sectors with parity $(-1)^j$ and $(-1)^{j+1}$, respectively:

$$\Psi^{(+)}_{j,m}=\begin{bmatrix}a_{j,2,2,m}(\xi)\\ a_{j,0,2,m}(\xi)\\ a_{j,-2,2,m}(\xi)\\ a_{j,0,1,m}(\xi)\\ a_{j,0,0m}(\xi)\\ \eta_{j,0,m}(\xi)\\ \eta_{j,m}(\xi)\end{bmatrix},\quad \Psi^{(-)}_{j,m}=\begin{bmatrix}a_{j,1,2,m}(\xi)\\ a_{j,-1,2,m}(\xi)\\ a_{j,1,1,m}(\xi)\\ a_{j,-1,1,m}(\xi)\\ \eta_{j,1,m}(\xi)\\ \eta_{j,-1,m}(\xi)\end{bmatrix}\ .$$

Note that the symbol π does not represent the parity quantum directly, but it is related to it by the assignment

$$\pi=+\quad\text{for parity}=(-1)^j\ ,$$

$$\pi=-\quad\text{for parity}=(-1)^{j+1}\ .$$

The corresponding matrix elements of the fluctuation operator are

$$(\mathcal{M}_{j\pi=\pm})_{\alpha\beta}=\left[-\frac{1}{2\xi}\frac{d^2}{d\xi^2}\xi+\frac{1}{2\xi^2}(j+j_\alpha)(j+j_\alpha+1)+\frac{1}{8}h^2+\frac{\lambda}{2g^2}(h^2-1)P_\eta\right]\delta_{\alpha\beta}+\frac{2f}{\xi^2}C^{(\pm)(1)}_{\alpha\beta}$$
$$+\frac{2f^2}{\xi^2}C^{(\pm)(2)}_{\alpha\beta}+\frac{2f'}{\xi}C^{(\pm)(3)}_{\alpha\beta}-\frac{4f(1-f)}{\xi^2}C^{(\pm)(4)}_{\alpha\beta}+\left[\frac{\lambda}{g^2}-\frac{1}{8}\right]h^2C^{(\pm)(5)}_{\alpha\beta}+\frac{h(1-f)}{\xi}C^{(\pm)(6)}_{\alpha\beta}$$
$$+\left[h'-\frac{h(1-f)}{\xi}\right]C^{(\pm)(7)}_{\alpha\beta}\ , \tag{17}$$

where P_η is the projection operator to the η-field subspace, j_α is an integer relating the total angular momentum j with orbital angular momentum l of a given channel α via $l=j+j_\alpha$, while $C^{(\pm)(1)-(7)}_{\alpha\beta}$ are certain symmetric coefficient matrices resulting from the integration over angular coordinates $\hat{\xi}$. The indices α and β label channels and run over the sets of the quantum numbers labeling the modes of fluctuation, e.g., (j,m), (j,s,m), and (j,s_1,s_2,m). However, in our discussion below, the subspace (j,m) we work in shall always be clear from context and it suffices to associate each channel α with the quantum numbers $-$, s or (s_1,s_2), where $-$ denotes the mode $\eta_{j,m}$.

In Tables I–VI we present our results for all coefficient matrices defined by (17). First, we list all integers j_α and the diagonal elements of $C^{(\pm)(1)-(4)}_{\alpha\beta}$ in Tables I(a) and I(b). Next we give the nonzero, off-diagonal matrix elements of $C^{(\pm)(1)}_{\alpha\beta}$ and $C^{(\pm)(2)-(4)}_{\alpha\beta}$ in Tables II, III(a), and III(b), respectively. Here, to save space, we note that all coefficient matrices are symmetric, and so we need only list elements from the upper-right triangle. The matrix $C^{(-)(5)}_{\alpha\beta}$ has nonzero elements only in the $\pi=-$sector; these are given in Table IV. Finally, $C^{(\pm)(6)}_{\alpha\beta}$ and $C^{(\pm)(7)}_{\alpha\beta}$ have nonzero elements only off the diagonal; they are given in Tables V and VI, respectively.

D. The negative and zero modes of the fluctuation operator

Because of the constraints $j+s\geq 0$ and $j+s_1\geq 0$ occurring in the spherical harmonic decomposition of η^a

TABLE I. (a) The integers j_α and the diagonal elements of $C^{(+)(i)}_{\alpha\beta}$, $i=1,2,3$, and 4. (b) The integers j_α and the diagonal elements of $C^{(-)(i)}_{\alpha\beta}$, $i=1,2,3$, and 4.

(a)

α	j_α	$C^{(+)(1)}_{\alpha\alpha}$	$C^{(+)(2)}_{\alpha\alpha}$	$C^{(+)(3)}_{\alpha\alpha}$	$C^{(+)(4)}_{\alpha\alpha}$
2,2	2	$-(j+3)$	$1+\frac{j+2}{2j+3}$	$\frac{2(j+2)}{2j+3}$	$\frac{1}{2j+3}$
0,2	0	$-\frac{3}{2}$	$1+\frac{9+4j(j+1)}{6(2j-1)(2j+3)}$	$\frac{9+4j(j+1)}{3(2j-1)(2j+3)}$	$\frac{2(9-4j(j+1))}{3(2j-1)(2j+3)}$
-2,2	-2	$j-2$	$1+\frac{j-1}{2j-1}$	$\frac{2(j-1)}{2j-1}$	$-\frac{1}{2j-1}$
0,1	0	$-\frac{1}{2}$	$\frac{3}{2}$	-1	0
0,0	0	0	$\frac{4}{3}$	$-\frac{4}{3}$	$\frac{2}{3}$
0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0
$-$	0	0	$\frac{1}{2}$	0	0

(b)

α	j_α	$C^{(-)(1)}_{\alpha\alpha}$	$C^{(-)(2)}_{\alpha\alpha}$	$C^{(-)(3)}_{\alpha\alpha}$	$C^{(-)(4)}_{\alpha\alpha}$
1,2	1	$-\frac{1}{2}(j+4)$	$1+\frac{j+2}{2(2j+1)}$	$\frac{j+2}{2j+1}$	$-\frac{j-1}{2j+1}$
-1,2	-1	$\frac{1}{2}(j-3)$	$1+\frac{j-1}{2(2j+1)}$	$\frac{j-1}{2j+1}$	$-\frac{j+2}{2j+1}$
1,1	1	$-\frac{1}{2}(j+2)$	$1+\frac{j}{2(2j+1)}$	$-\frac{j}{2j+1}$	$\frac{j+1}{2j+1}$
-1,1	-1	$\frac{1}{2}(j-1)$	$1+\frac{j+1}{2(2j+1)}$	$-\frac{j+1}{2j+1}$	$\frac{j}{2j+1}$
1	1	$-\frac{1}{2}(j+2)$	$\frac{1}{2}$	0	0
-1	-1	$\frac{1}{2}(j-1)$	$\frac{1}{2}$	0	0

and a^{ak}, respectively, the number of channels for $j=0$ and $j=1$ are correspondingly reduced. For $j=0$, the $\pi=\pm$ sectors are described by the three- and two-channel vectors

$$\Psi^{(+)}_{j=0,0}=\begin{bmatrix} a_{0,2,2,0}(\xi) \\ a_{0,0,0,0}(\xi) \\ \eta_{0,0}(\xi) \end{bmatrix}, \quad \Psi^{(-)}_{j=0,0}=\begin{bmatrix} a_{0,1,1,0}(\xi) \\ \eta_{0,1,0}(\xi) \end{bmatrix},$$

while for $j=1$, $\pi=\pm$ we have the six- and five-channel vectors

TABLE II. Nonzero, off-diagonal elements of $C^{(\pm)(1)}_{\alpha\beta}$.

α	β	$C^{(+)(1)}_{\alpha\beta}$
0,1	0,0	$-[\frac{2}{3}j(j+1)]^{1/2}$
0	—	$-\frac{1}{2}\sqrt{j(j+1)}$
0,2	0,1	$\frac{1}{2}[\frac{1}{3}(2j-1)(2j+3)]^{1/2}$
α	β	$C^{(-)(1)}_{\alpha\beta}$
1,2	1,1	$\frac{1}{2}\sqrt{j(j+2)}$
−1,2	−1,1	$\frac{1}{2}\sqrt{(j-1)(j+1)}$

TABLE III. (a) Nonzero, off-diagonal elements of $C^{(+)(2)}_{\alpha\beta}$, $C^{(+)(3)}_{\alpha\beta}$, and $C^{(+)(4)}_{\alpha\beta}$. (b) Nonzero, off-diagonal elements of $C^{(-)(2)}_{\alpha\beta}$, $C^{(-)(3)}_{\alpha\beta}$, and $C^{(-)(4)}_{\alpha\beta}$.

(a)

α	β	$C^{(+)(2)}_{\alpha\beta}$	$C^{(+)(3)}_{\alpha\beta}=C^{(+)(4)}_{\alpha\beta}$
2,2	0,2	$-\left[\frac{j(2j-1)(j+2)}{6(2j+1)(2j+3)^2}\right]^{1/2}$	$-2\left[\frac{j(2j-1)(j+2)}{6(2j+1)(2j+3)^2}\right]^{1/2}$
2,2	0,1	$-\left[\frac{j(j+2)}{2(2j+1)(2j+3)}\right]^{1/2}$	0
2,2	0,0	$\left[\frac{(j+1)(j+2)}{3(2j+1)(2j+3)}\right]^{1/2}$	$-\left[\frac{(j+1)(j+2)}{3(2j+1)(2j+3)}\right]^{1/2}$
0,2	−2,2	$-\left[\frac{(j-1)(j+1)(2j+3)}{6(2j+1)(2j-1)^2}\right]^{1/2}$	$-2\left[\frac{(j-1)(j+1)(2j+3)}{6(2j+1)(2j-1)^2}\right]^{1/2}$
0,2	0,1	$-\frac{1}{2}\left[\frac{3}{(2j-1)(2j+3)}\right]^{1/2}$	0
0,2	0,0	$-\frac{1}{3}\left[\frac{2j(j+1)}{(2j-1)(2j+3)}\right]^{1/2}$	$\frac{1}{3}\left[\frac{2j(j+1)}{(2j-1)(2j+3)}\right]^{1/2}$
−2,2	0,1	$\left[\frac{(j-1)(j+1)}{2(2j+1)(2j-1)}\right]^{1/2}$	0
−2,2	0,0	$\left[\frac{j(j-1)}{3(2j+1)(2j+3)}\right]^{1/2}$	$-\left[\frac{j(j-1)}{3(2j+1)(2j+3)}\right]^{1/2}$

(b)

α	β	$C^{(-)(2)}_{\alpha\beta}$	$C^{(-)(3)}_{\alpha\beta}=C^{(-)(4)}_{\alpha\beta}$
1,2	−1,2	$\frac{1}{2}\left[\frac{(j-1)(j+2)}{(2j+1)^2}\right]^{1/2}$	$\left[\frac{(j-1)(j+2)}{(2j+1)^2}\right]^{1/2}$
1,2	1,1	$-\frac{1}{2}\left[\frac{j(j+2)}{(2j+1)^2}\right]^{1/2}$	0
1,2	−1,1	$\frac{1}{2}\left[\frac{(j+1)(j+2)}{(2j+1)^2}\right]^{1/2}$	0
−1,2	1,1	$-\frac{1}{2}\left[\frac{j(j-1)}{(2j+1)^2}\right]^{1/2}$	0
−1,2	−1,1	$\frac{1}{2}\left[\frac{(j-1)(j+1)}{(2j+1)^2}\right]^{1/2}$	0
1,1	−1,1	$-\frac{1}{2}\left[\frac{j(j+1)}{(2j+1)^2}\right]^{1/2}$	$\left[\frac{j(j+1)}{(2j+1)^2}\right]^{1/2}$

TABLE IV. Nonzero elements of $C^{(-)(5)}_{\alpha\beta}$.

α	β	$C^{(-)(5)}_{\alpha\beta}$
1	1	$\frac{j+1}{2j+1}$
—	—	$\frac{j}{2j+1}$
1	-1	$\left[\frac{j(j+1)}{2j+1}\right]^{1/2}$

$$\Psi^{(+)}_{j=1,m}=\begin{bmatrix} a_{1,2,2,m}(\xi) \\ a_{1,0,2,m}(\xi) \\ a_{1,0,1,m}(\xi) \\ a_{1,0,0,m}(\xi) \\ \eta_{1,0,m}(\eta) \\ \eta_{1,m}(\xi) \end{bmatrix}, \quad \Psi^{(-)}_{j=1,m}=\begin{bmatrix} a_{1,1,2,m}(\xi) \\ a_{1,1,1,m}(\xi) \\ a_{1,-1,1m}(\xi) \\ \eta_{1,1,m}(\xi) \\ \eta_{1,-1,m}(\xi) \end{bmatrix},$$

where $m=0,\pm 1$. After performing the channel reductions indicated above, the corresponding fluctuation operators $\mathcal{M}_{j=0,\pm}$ and $\mathcal{M}_{j=1,\pm}$ are obtained from the general expression (17) above for $\mathcal{M}_{j,\pm}$ by taking $j\to 0$ and $j\to 1$, respectively.

This explicit display of the $j=0$ and $j=1$ channels is opportune, as it facilitates our current task of isolating the negative and zero modes of the fluctuation operator $\mathcal{M}$. This is required since, as we recall from our discussion in Sec. I, κ is proportional to $\text{Im}(\det'\mathcal{M})^{-1/2}$, where the prime denotes deletion of the zero modes of the sphaleron. With the partial-wave decomposition in hand, we may now perform this subtraction rather easily and furthermore evaluate the imaginary part of the determinant by explicitly removing the negative mode eigenvalue, $-\omega_-^2$ [in units of $(gv)^2$].

The channel in which the negative mode occurs has been identified by Akiba, Kikuchi, and Yanagida.[24] It is given by the ansatz

$$a_i^a=(\delta_{ia}-\hat{\xi}_i\hat{\xi}_a)B(\xi)+\hat{\xi}_i\hat{\xi}_a C(\xi),$$
$$\eta_4=H(\xi),$$

TABLE V. Nonzero elements of $C^{(\pm)(6)}_{\alpha\beta}$.

α	β	$C^{(+)(6)}_{\alpha\beta}$
0,1	0	$\frac{1}{\sqrt{2}}$
0,0	—	$\frac{\sqrt{3}}{2}$
α	β	$C^{(-)(6)}_{\alpha\beta}$
1,1	1	$-\frac{1}{\sqrt{2}}$
$-1,1$	-1	$-\frac{1}{\sqrt{2}}$

where the functions $B(\xi)$, $C(\xi)$ and $H(\xi)$ satisfy an eigenvalue equation of the form (for details see Ref. 22)

$$\left[-\frac{d^2}{d\xi^2}-\frac{2}{\xi}\frac{d}{d\xi}+\mathcal{V}[f(\xi),h(\xi)]\right]\begin{bmatrix}B\\C\\H\end{bmatrix}=-\omega_-^2\begin{bmatrix}B\\C\\H\end{bmatrix}.$$

It has been shown that in the sphaleron background this is the unique negative mode for $\lambda/g^2\le 18$ (Ref. 18). In the partial-wave formulation, this negative mode occurs in the three-channel sector with $j=0$ and $\pi=+$. The correspondence is given by

$$a_{0,2,2,0}=\left[\frac{8\pi}{3}\right]^{1/2}(C-B),$$
$$a_{0,0,0,0}=\left[\frac{4\pi}{3}\right]^{1/2}(C+2B),$$
$$\eta_{0,0}=\sqrt{4\pi}H.$$

One nontrivial check on our calculations was to verify that the operator $\mathcal{M}_{0,+}$ acting on this three-vector gave the same eigenvalue equations as produced by Carson and McLerran.[22]

The zero modes of the sphaleron come in two varieties, translational and rotational, and occur in the partial waves with $j=1$ and positive and negative parity, respectively. The translational zero mode fluctuations, which are induced by translations $\xi\to\xi+\epsilon$ of the background field, take the form

$$a_k^a=\epsilon_i F_{ik}^a,\quad \frac{\eta}{\sqrt{2}}=\epsilon_i D_i\Phi,$$

where the canonical transformation has been supplemented by a gauge transformation in order to preserve the $R_{\xi=1}$ gauge condition. Under the spherical wave decomposition, this expression is recast as

$$a_k^a=\frac{\sqrt{4\pi}}{3}\epsilon_i U_{im}\bar{a}_{1,s_1,s_2}(\xi)Y^{ak}_{1,s_1,s_2,m}(\Omega),$$
$$\eta^a=\frac{\sqrt{8\pi}}{3}\epsilon_i U_{im}\eta_{1,s}(\xi)Y^a_{1,s,m}(\Omega), \quad (18)$$

where U_{im} is the unitary matrix relating the Cartesian indices i $(i=1,2,3)$ to the spherical indices m $(m=-1,0,1)$,

$$U=\begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix},$$

while the functions $\bar{a}_{1,s_1,s_2}(\xi)$ and $\bar{\eta}_{1,s}(\xi)$ are related to the sphaleron profile functions $f(\xi)$ and $h(\xi)$ as

TABLE VI. Nonzero elements of $C^{(\pm)(7)}_{\alpha\beta}$.

α	β	$C^{(+)(7)}_{\alpha\beta}$	α	β	$C^{(-)(7)}_{\alpha\beta}$
2,2	0	$\frac{1}{2}\left[\frac{j(j+2)}{(2j+1)(2j+3)}\right]^{1/2}$	1,2	1	$-\frac{1}{2}\left[\frac{j(j+2)}{2(2j+1)^2}\right]^{1/2}$
2,2	—	$\frac{1}{2}\left[\frac{(j+1)(j+2)}{(2j+1)(2j+3)}\right]^{1/2}$	1,2	-1	$\frac{1}{2}\left[\frac{(j+1)(j+2)}{2(2j+1)^2}\right]^{1/2}$
0,2	0	$\frac{1}{2}\left[\frac{3}{2(2j-1)(2j+3)}\right]^{1/2}$	$-1,2$	1	$-\frac{1}{2}\left[\frac{j(j-1)}{2(2j+1)^2}\right]^{1/2}$
0,2	—	$-\frac{1}{2}\left[\frac{2j(j+1)}{3(2j-1)(2j+3)}\right]^{1/2}$	$-1,2$	-1	$\frac{1}{2}\left[\frac{(j+1)(j-1)}{2(2j+1)^2}\right]^{1/2}$
$-2,2$	0	$-\frac{1}{2}\left[\frac{(j-1)(j+1)}{(2j-1)(2j+1)}\right]^{1/2}$	1,1	1	$-\frac{1}{2}\left[\frac{j^2}{2(2j+1)^2}\right]^{1/2}$
$-2,2$	—	$\frac{1}{2}\left[\frac{j(j-1)}{(2j-1)(2j+1)}\right]^{1/2}$	1,1	-1	$\frac{1}{2}\left[\frac{j(j+1)}{2(2j+1)^2}\right]^{1/2}$
0,1	0	$\frac{1}{2\sqrt{2}}$	$-1,1$	1	$\frac{1}{2}\left[\frac{j(j+1)}{2(2j+1)^2}\right]^{1/2}$
0,0	—	$\frac{1}{2\sqrt{3}}$	$-1,1$	-1	$-\frac{1}{2}\left[\frac{(j+1)^2}{2(2j+1)^2}\right]^{1/2}$

$$\bar{a}_{1,1,2}=2\sqrt{3}\left[-\frac{1}{\xi}f'+\frac{2}{\xi^2}f(1-f)\right],$$
$$\bar{a}_{1,1,1}=2\left[\frac{1}{\xi}f'-\frac{2}{\xi^2}f(1-f)\right],$$
$$\bar{a}_{1,-1,1}=4\sqrt{2}\left[\frac{1}{\xi}f'+\frac{1}{\xi^2}f(1-f)\right],$$
$$\bar{\eta}_{1,1}=\sqrt{2}\left[h'-\frac{1}{\xi}(1-f)h\right],$$
$$\bar{\eta}_{1,-1}=h'+\frac{2}{\xi}(1-f)h\ .$$

Again, it proved a valuable check on our formulas to verify that the operator $\mathcal{M}_{1,-}$ annihilated this set of modes.

The same checks may be performed on the operator $\mathcal{M}_{1,+}$ with the rotational zero modes, given by

$$a_k^a=\epsilon_i\epsilon_{imn}\xi_m F^a_{nk}+\epsilon_i(D_k\tilde{\Lambda}_i)^a\ ,\qquad \frac{\eta}{\sqrt{2}}=\epsilon_i\epsilon_{imn}\xi_m D_n\Phi+i\epsilon_i\tilde{\Lambda}_i\Phi\ . \tag{19}$$

Here one has the additional complication that, in order to restore the $R_{\xi=1}$ gauge condition, the transformation law of the background fields A^a_k and Φ under $\boldsymbol{\xi}\to\boldsymbol{\epsilon}\times\boldsymbol{\xi}$ must be supplemented by a residual gauge transformation $\tilde{\Lambda}_i$ solving the inhomogeneous equation

$$[-D(A)^2+\tfrac{1}{2}\Phi^\dagger\Phi]\tilde{\Lambda}_i=-\epsilon_{ijk}F_{jk}\ .$$

In the sphaleron background (6) the function $\tilde{\Lambda}_i$ is parametrized by the ansatz

$$\tilde{\Lambda}_i^a=4(\delta_{ia}-\hat{\xi}_i\hat{\xi}_a)P(\xi)+4\hat{\xi}_i\hat{\xi}_a Q(\xi)\ ,$$

where $P(\xi)$ and $Q(\xi)$ satisfy a certain set of linear, coupled, inhomogeneous equations which have been given elsewhere.[22] We may transcribe (19) to a decomposition with respect to $j=1$, $\pi=+$ spherical harmonics using (18), together with a similar expansion for η_4:

$$\eta_4=\frac{\sqrt{8\pi}}{3}\epsilon_i U_{im}\bar{\eta}_1(\xi)Y_{1,m}(\Omega)\ .$$

After pages of algebra, one finds

$$\bar{a}_{1,2,2}=\frac{2\sqrt{30}}{5}\left[f'+2P'-2Q'-\frac{2}{\xi}f(1-f)-\frac{4}{\xi}(1-f)P+\frac{4}{\xi}(1-f)Q\right],$$
$$\bar{a}_{1,0,2}=\frac{2\sqrt{5}}{5}\left[3f'+6P'+4Q'-\frac{6}{\xi}f(1-f)-\frac{2}{\xi}(1-6f)P+\frac{2}{\xi}(1+4f)Q\right],$$
$$\bar{a}_{1,0,1}=2\sqrt{3}\left[f'+2P'+\frac{2}{\xi}f(1-f)+\frac{2}{\xi}(1-2f)P-\frac{2}{\xi}Q\right],$$
$$\bar{a}_{1,0,0}=4\left[-Q'+\frac{2}{\xi}P-\frac{2}{\xi}(1-2f)Q\right],$$
$$\bar{\eta}_{1,0}=\sqrt{6}[2Ph-(1-f)h]\ ,$$
$$\bar{\eta}_1=-2\sqrt{3}Qh\ .$$

E. The Faddeev-Popov determinant

Finally, we must perform the partial-wave decomposition of the Faddeev-Popov operator defined by (11). In this case, the invariance group is given by **L**+**I**, where **I** is the isospin operator, realized in the isovector representation. Thus the Faddeev-Popov operator

$$-D(A)^2+\Phi^\dagger\Phi/2$$

acts on isovector functions $f_a(\xi)$ which we expand in vector spherical harmonics:

$$f_a=\sum_{j=0}^{\infty}\sum_{m=-j}^{j}\sum_{\alpha=-1}^{1} f_{j,\alpha,m}(\xi)Y^a_{j,\alpha,m}(\Omega)\ .$$

Evaluating matrix elements of the Faddeev-Popov operator with respect to $Y^a_{j,\alpha,m}$, we reduce it, for fixed j, to a 3×3 matrix of one-dimensional operators with degeneracy $(2j+1)$:

$$\begin{aligned}(\mathcal{F}_j)_{\alpha\beta}&=\frac{1}{2}\int d\Omega\, Y^{a\dagger}_{j,\alpha,m}[-D(A)^2+\tfrac{1}{2}\Phi^\dagger\Phi]_{ab}Y^b_{j,\beta,m}\\ &=\left[-\frac{1}{2\xi}\frac{d^2}{d\xi^2}\xi+\frac{(j+\alpha)(j+\alpha+1)}{2\xi^2}\right.\\ &\quad\left.+\frac{h^2}{8}+\frac{2f}{\xi^2}D^{(1)}_\alpha\right]\delta_{\alpha\beta}+\frac{2f^2}{\xi^2}D^{(2)}_{\alpha\beta}\ . \end{aligned} \tag{20}$$

The nonzero elements of $D^{(1)}_\alpha$ and $D^{(2)}_{\alpha\beta}$ are given in Table VII.

TABLE VII. Matrix elements $D^{(1)}_\alpha$ and $D^{(2)}_{\alpha\beta}$ defining the Faddeev-Popov operator $(\mathcal{F}_j)_{\alpha\beta}$ [cf. Eq. (20)].

α	β	$D^{(1)}_\alpha$	$D^{(2)}_{\alpha\beta}$
1	1	$-j-2$	$1+\frac{j+1}{2j+1}$
0	0	-1	1
-1	-1	$j-1$	$1+\frac{j}{2j+1}$
1	0	—	0
1	-1	—	$\frac{\sqrt{j(j+1)}}{2j+1}$
0	-1	—	0

III. RENORMALIZATION

Using the machinery devised in the last section we have reduced the task of evaluating $\ln\kappa$ to a sum of reduced determinants each labeled by the partial-wave quantum numbers j and π. If we let

$$\mathcal{M}^0_{j\pi}\equiv\mathcal{M}_{j\pi}(f\to1,h\to1)$$

denote the partial-wave fluctuation operator evaluated in the perturbative vacuum, then we have

$$\begin{aligned}\ln\kappa=&-\ln\omega_- -\tfrac{1}{2}\ln\det{}'\mathcal{M}_{0,+}-\tfrac{1}{2}\ln\det\mathcal{M}_{0,-}-\tfrac{3}{2}\ln\det{}'\mathcal{M}_{1,+}-\tfrac{3}{2}\ln\det{}'\mathcal{M}_{1,-}\\ &-\tfrac{1}{2}\sum_{j=2}^{\infty}\sum_{\pi=\pm}(2j+1)\ln\det\mathcal{M}_{j\pi}+\tfrac{1}{2}\sum_{j=0}^{\infty}\sum_{\pi=\pm}(2j+1)\ln\det\mathcal{M}^0_{j\pi}+\tfrac{1}{2}\sum_{j=0}^{\infty}(2j+1)(\ln\det\mathcal{F}_j-\ln\det\mathcal{F}^0_j)\ .\end{aligned} \tag{21}$$

The primes on the determinants of $\mathcal{M}_{0,+}$, $\mathcal{M}_{1,+}$, and $\mathcal{M}_{1,-}$ indicate the removal of the lowest eigenvalue from the determinant of each of these operators, since the corresponding eigenvectors are the negative, rotation-zero, and translation-zero modes of the sphaleron, respectively.

In practical calculation, the sums over j appearing in (21) are truncated to some maximum partial-wave $j_{\max}$ and the operators defining the partial-wave determinants are formulated on a discrete grid with lattice spacing $\Delta\xi$, producing the intermediate quantity $\ln\kappa(j_{\max},\Delta\xi)$. The final result is then obtained by extrapolating this function to the limit

$$j_{\max}\to\infty,\quad \Delta\xi\to0\ , \tag{22}$$

such that $j_{\max}\Delta\xi$ is helo constant. In our case, however, this procedure produces a sequence of values for $\ln\kappa$ that diverges linearly with $j_{\max}$ (or $1/\Delta\xi$).

The presence of this linear divergence could have been anticipated from the form of the effective three-dimensional action S_3 [Eq. (5)]. By power counting, one finds that the only primitive divergence in the theory is the tadpole diagram in the scalar field Φ, and its divergence is linear.

Presumably, if one returns to the original four-dimensional finite-temperature theory and renormalizes fields, couplings, and masses according to standard prescriptions in the zero-temperature limit, then a finite theory would result which would be valid at all temperatures. In particular, one should be able to take the high-temperature limit of any gauge-invariant quantities in the renormalized theory and obtain finite results. This expectation is only partially correct, for it does not allow for the fact that quantities may also diverge as $T\to\infty$. The tadpole diagram, contributing to the self-mass of the scalar field, is such a quantity, resulting in a term in the logarithm of the partition function that diverges linearly with T: $\ln Z=\beta F\sim\beta T^2+\cdots=T+\cdots$. The resolution of this difficulty is well known.[15–17] The bare vacuum expectation value v of the scalar field is readjusted to absorb the zero-temperature (quadratic) divergence while the T dependence is incorporated by replacing v by $v(T)$ as described in the Introduction. One implication of these adjustments for the effective, high-temperature, three-dimensional field theory is that in the loop expansion of $\ln Z$ all tadpole (sub)diagrams may be dropped provided the replacement $v \to v(T)$ is made *holding the temperature dependence of $v(T)$ fixed.*

Therefore our task of renormalizing $\ln\kappa$ in one-loop order is equivalent to isolating and subtracting the tadpole contribution to the quantity

$$\ln\frac{\det\mathcal{M}(A,\Phi)}{\det\mathcal{M}^0}\ , \tag{23}$$

where $\mathcal{M}$ is an operator of the form

$$\mathcal{M}(A,\Phi)=-D(A)^2+V(A,\Phi) \tag{24}$$

and $\mathcal{M}^0$ is the same operator evaluated in the vacuum:

$$\mathcal{M}^0=\mathcal{M}\left[A\to 0,\Phi\to\frac{1}{\sqrt{2}}u_{1/2}\right]=-\partial^2+V^0\ . \tag{25}$$

In (24), A_i^a is the background gauge field, $D_i(A)=\partial_i-iT^aA_i^a$ is the covariant derivative in some representation R of SU(2), and V is a potential function of the background fields transforming under SU(2) according to some linear combination of the singlet and $R\otimes\bar{R}$ representations. One can show[22] that $\ln\kappa$ is precisely composed of terms of the form (23)–(25). For example, in the case of the Faddeev-Popov determinant (11), R is the adjoint representation with $(T^a)_{bc}=-i\epsilon_{abc}$, while V is the pure singlet field, $\Phi^\dagger\Phi/2$. One property shared by all the potentials V entering in the expression for κ is

$$\mathrm{tr}V(A,\Phi)=\mathrm{tr}V(0,\Phi)\ . \tag{26}$$

We can isolate the tadpole contribution to (23) quite easily if we retain Cartesian components of all tensor quantities (as opposed to the spherical harmonic expansions employed in the previous section) and utilize the Schwinger proper time representation

$$\ln\frac{\det\mathcal{M}}{\det\mathcal{M}^0}=\mathrm{Tr}\int_0^\infty\frac{dt}{t}(e^{-t\mathcal{M}}-e^{-t\mathcal{M}^0})\ ,$$

where Tr denotes a functional trace. The functional trace can be resolved over a complete set of plane-wave states plus a residual trace (tr) over group and spinor indices. We obtain

$$\ln\left[\frac{\det\mathcal{M}}{\det\mathcal{M}^0}\right]=-\int_0^\infty\frac{dt}{t}\int d^3x\int\frac{d^3p}{(2\pi)^3}e^{-tp^2}\times\mathrm{tr}(e^{-t\delta\mathcal{M}}-e^{-t\delta\mathcal{M}^0})\times 1\ , \tag{27}$$

where

$$\delta\mathcal{M}(p,x)=-2ip\cdot D(A)-D(A)^2+V(A,\Phi)\ ,$$

$$\delta\mathcal{M}^0(p,x)=-2ip\cdot\partial-\partial^2+V^0\ .$$

Keeping the factor of e^{-tp^2} intact, the factors of $e^{-t\delta\mathcal{M}}$ and $e^{-t\delta\mathcal{M}^0}$ are expanded in powers of t and the resulting expression is integrated over momenta p. To lowest order in t, one finds

$$\ln\left[\frac{\det\mathcal{M}}{\det\mathcal{M}^0}\right]=\int_0^\infty dt\frac{1}{(4\pi t)^{3/2}}\left[\int d^3x\ \mathrm{tr}[V(A,\Phi)-V^0]+O(t)\right]\ . \tag{28}$$

Cutting off the lower (ultraviolet) limit of the t integral at $1/\Lambda^2$ we obtain a contribution to $\ln(\det\mathcal{M}/\det\mathcal{M}^0)$ from the first term of the right-hand side of

$$\frac{\Lambda}{4\pi^{3/2}}\int d^3x\ \mathrm{tr}[V(A,\Phi)-V^0]\ ,$$

which diverges as $\Lambda\to\infty$. It is simple to show that higher-order terms in (28) are finite in the same limit. We have thus isolated the linear divergence.

In performing the expansions of $e^{-t\delta\mathcal{M}}$ and $e^{-t\delta\mathcal{M}^0}$ in the calculation above, it is important to note that due to the exponential factor e^{-tp^2}, the momentum p is implicitly of order $t^{-1/2}$. Thus the operators $tD(A)^2$ and $(t^2/2!)[2ip\cdot D(A)]^2$ encountered in the steps leading from (27) to (28), are of the same order in t. In fact, after performing the p integrations, one finds that these two terms cancel, leaving in lowest order only the term involving $\mathrm{tr}[V(A,\Phi)-V^0]$.

This last observation, combined with the trace property (26), permits the following simplification in isolating the linear divergence: By setting the A dependence of $\mathcal{M}(A,\Phi)$ in (27) to zero, one can obtain the entire contribution to the linear divergence by expanding $e^{-t\delta\mathcal{M}}$ and $e^{-t\delta\mathcal{M}^0}$ to first (naive) order in t.

To formalize this procedure in a manner that will be helpful to us later, let us define the inverse propagator

$$(G^\Lambda)^{-1}=-\partial^2+\Lambda^2\ ,$$

with an arbitrary mass term Λ^2. Then, after setting A to zero in (23), one has

$$\ln\left[\frac{\det\mathcal{M}(0,\Phi)}{\det\mathcal{M}^0}\right]=\ln\left[\frac{\det[(G^0)^{-1}+V(0,\Phi)]}{\det[(G^0)^{-1}+V^0]}\right]=\mathrm{Tr}\ln\left[\frac{1+G^0V(0,\Phi)}{1+G^0V^0}\right]=\mathrm{Tr}\,G^0[V(0,\Phi)-V^0]+\cdots\ .$$

Expanding the functional trace over a complete set of plane waves, this first term is manifestly linearly divergent:

$$\int d^3\xi\ \mathrm{tr}[V(0,\Phi)-V^0]\int\frac{d^3p}{(2\pi)}\frac{1}{p^2}\ .$$

Alternatively we could have obtained this expression from Eq. (27) had we first performed the t integration, taking care to retain terms of equal order in p^{-2} by noting that implicitly $t\sim p^{-2}$.

To renormalize one must regulate. We shall employ Pauli-Villars, which in the full one-loop expression (23) corresponds to the replacement

$$\ln\frac{\det\mathcal{M}}{\det\mathcal{M}^0}\to\frac{\det\mathcal{M}\det(\mathcal{M}^0+\Lambda^2)}{\det\mathcal{M}^0\det(\mathcal{M}+\Lambda^2)} .$$

From our discussion above, it is evident that if we subtract from this regulated expression the quantity

$$\mathrm{Tr}\{(G^0-G^\Lambda)[V(0,\Phi)-V^0]\}$$

containing the (regulated) linear divergence, then the remaining terms will be finite as $\Lambda\to\infty$. The value of this regulated, subtracted result in the limit $\Lambda\to\infty$ is the desired contribution to $\ln\kappa$.

The entire procedure outlined above can be adopted to the partial-wave expansion with only a few modifications. The three-dimensional Laplacian

$$-\partial^2=-\frac{1}{\xi}\frac{d^2}{d\xi^2}\xi+\frac{\mathbf{L}^2}{\xi^2}$$

reduces in the subspace of orbital angular momentum l to give the following expression for the inverse propagator:

$$(G_l^\Lambda)^{-1}=-\frac{1}{\xi}\frac{d^2}{d\xi^2}\xi+\frac{l(l+1)}{\xi^2}+\Lambda^2 .$$

Let us also relabel the orbital angular momentum quantum number l by $j+j_\alpha$ where j_α is the integer relating l of a particular channel α to the total angular momentum j. (These integers are listed in Table I for the operators $\mathcal{M}_{j\pi}$, while for the Faddeev-Popov operators $\mathcal{F}_j$ one simply has $j_\alpha=\alpha$.) Then the Pauli-Villars-regulated, subtracted expression we wish to evaluate is

$$\sum_{j=0}^{\infty}\sum_{\pi=\pm}(2j+1)\ln\left[\frac{\det\mathcal{M}_{j\pi}(A,\Phi)\det[\mathcal{M}_{j\pi}^0+\Lambda^2]}{\det\mathcal{M}_{j\pi}^0\cdot\det[\mathcal{M}_{j\pi}(A,\Phi)+\Lambda^2]}\right] -\sum_{j=0}^{\infty}\sum_{\pi=\pm}(2j+1)\sum_\alpha\int_0^\infty d\xi\,\xi^2[G_{j+j_\alpha}^0(\xi,\xi)-G_{j+j_\alpha}^\Lambda(\xi,\xi)][V_{j\pi}(0,\Phi)-V_{j\pi}^0]_{\alpha\alpha} . \quad (29)$$

As mentioned above, we shall cutoff the sum over j at $j_{\max}$ and discretize the radial coordinate ξ with lattice spacing $\Delta\xi$. However, with expression (29) we may now safely extrapolate to the continuum limit (22). In this way we extract the (finite) contribution $\ln\kappa$ of interest, up to power-law corrections in $1/\Lambda$. These corrections are finally eliminated by taking Λ to infinity.

It is not hard to convince oneself from expressions (17) and (20) for $\mathcal{M}_{j\pi}$ and $\mathcal{F}_j$ that the quantities $[V_{j\pi}(0,\Phi)-V_{j\pi}^0]_{\alpha\alpha}$ entering in Eq. (29) are proportional to $h^2(\xi)-1$, where $h(\xi)$ is the profile function defining the scalar field in the sphaleron background [cf. Eq. (6)]:

$$[V_{j\pi}(0,\Phi)-V_{j\pi}^0]_{\alpha\alpha}\equiv V_{j\pi\alpha}[h^2(\xi)-1] .$$

For the Faddeev-Popov determinant (11), the constant of proportionality $V_{j\pi\alpha}$ is $\frac{1}{4}$, independent of j, π, and channel number α. In the case of fluctuation operators $\mathcal{M}_{j\pi}$, the constants $V_{j\pi\alpha}$ depend on all these quantum numbers. However, for the integrals defined in (29), it is clear we only require $V_{j\pi\alpha}$ summed over parity and channels α with fixed j_α. The relevant sums in this case are given in Table VIII.

IV. RESULTS

The partial-wave expansion (21) of $\ln\kappa$, together with expressions (17) and (20) for $\mathcal{M}_{j\pi}$ and $\mathcal{F}_j$ and the regularization scheme summarized by Eq. (29), constitute the final, analytical results of this paper. To proceed further with the evaluation of $\ln\kappa$ entails a number of tasks, including the computation of the sphaleron profile functions $f(\xi)$ and $h(\xi)$, the negative-mode frequency ω_-, the partial-wave determinants $\det\mathcal{M}_{j\pi}$, $\det\mathcal{M}_{j\pi}^0$, $\det\mathcal{F}_j$, and $\det\mathcal{F}_j^0$, and the diagonal elements of Green's functions $G_{j+j\alpha}^\Lambda(\xi,\xi)$. These are all problems that may be treated by standard methods of numerical analysis; we shall not give any details here save to mention that by representing the interval $\xi\in[0,\infty]$ by a discrete mesh, $\xi_n=n\cdot\Delta\xi, n=0,\dots,N$, one has introduced two additional parameters, the lattice spacing $\Delta\xi$ and size $\Xi=N\Delta\xi$, into the calculation. Thus one has a set of regularized quantities $\ln\kappa_{\rm reg}(\Xi,\Delta\xi,j_{\max},\Lambda)$, which must be extrapolated to the physical limit $\Xi\to\infty$, $\Delta\xi\to 0$, $j_{\max}\to\infty$, and $\Lambda\to\infty$. These limits are not independent, but are constrained by the relation

$$1=j_{\max}\Delta\xi .$$

We have found that the extrapolations may be reliably

TABLE VIII. Sums of coefficients $V_{j\pi\alpha}$ required for the subtraction of the linear divergence from $\det\mathcal{M}/\det\mathcal{M}^0$. The prime on the channel sum denotes summation only over channels α such that j_α is fixed.

	$\sum_{\pi=\pm}\sum_\alpha{}' V_{j\pi\alpha}$		
j_α	$j=0$	$j=1$	$j>1$
2	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
1	$3\frac{\lambda}{g^2}+\frac{1}{4}$	$\frac{7}{3}\frac{\lambda}{g^2}+\frac{7}{12}$	$\frac{\lambda}{g^2}+\frac{3}{4}+\frac{j+1}{2j+1}\left[2\frac{\lambda}{g^2}-\frac{1}{4}\right]$
0	$\frac{\lambda}{g^2}+\frac{1}{2}$	$2\frac{\lambda}{g^2}+\frac{5}{4}$	$2\frac{\lambda}{g^2}+\frac{5}{4}$
−1	—	$\frac{5}{3}\frac{\lambda}{g^2}+\frac{5}{12}$	$\frac{\lambda}{g^2}+\frac{3}{4}+\frac{j}{2j+1}\left[2\frac{\lambda}{g^2}-\frac{1}{4}\right]$
−2	—	—	$\frac{1}{4}$

TABLE IX. Calculated values for $\ln\kappa$.

$\frac{\lambda}{g^2}$	(a) $\ln \mathrm{Im}\left[\frac{\det'\mathcal{M}}{\det\mathcal{M}^0}\right]^{-1/2}$	(b) $\ln\left[\frac{\Delta_{\rm FP}}{\Delta^0_{\rm FP}}\right]^{1/2}$	(a)+(b)	Approx. (Ref. 22)
0.1	−10.5	2.6	−7.9	−0.9
0.3	−3.8	1.6	−2.2	0.6
1.0	−4.0	0.4	−3.6	0.3
10.0	−12.6	0.1	−12.5	−5.7

performed from a three-parameter set of points given by

$$\Xi=4,6,8,$$

$$j_{\max}=25,50,100,200\ ,$$

and

$$\Lambda=1,3,5\ .$$

For $\lambda/g^2=0.1$, the set of Ξ points were replaced by 8, 12, and 16 to accommodate the larger Compton wavelength of the Higgs field.

The results of these computations for four values of λ/g^2 (0.1, 0.3, 1.0, and 10.0) are presented in Table IX, each value representing over 35 CPU hours of vectorized computation on a Cray 2 supercomputer. In this table we display the contribution to $\ln\kappa$ due to the ratio of determinants of the fluctuation matrices $\mathcal{M}$ and $\mathcal{M}^0$ given by the first seven terms of (21) and the remaining contribution due to the ratio of Faddeev-Popov determinants $\Delta_{\rm FP}/\Delta^0_{\rm FP}$. These are summed in the third column to give the final results for $\ln\kappa$ of this paper.

One immediate conclusion that we can draw from these results is that, in confirmation of the work in Ref. 22, the rate for baryon-number-changing processes is suppressed by the κ factor for large and small values of λ/g^2. To take an extreme example we see that the estimate $\kappa\sim 1$ would have been in error by nearly 6 orders of magnitude for $\lambda/g^2=10$. It is therefore crucial to take entropy suppression into account when estimating the rate of thermally activated $\Delta B\neq 0$ transitions

It is also interesting to compare our results with the approximate calculation of $\ln\kappa$ by Carson and McLerran.[22] Besides listing the actual values of this calculation in Table IX we also present a plot of their calculation in Fig. 3. We see that, while the qualitative dependence on λ/g^2 has been reproduced, our calculations give an additional overall suppression by some 2–3 orders of magnitude. We do not have any deep understanding of this discrepancy, save the remarks we have already made in the Introduction.

One check on our computations is provided by the perturbative limit when $\lambda \ll g^2 \ll 1$. In this regime, the effective potential at finite temperature can be reliably evaluated as a perturbative series in λ and g^2. To next-to-leading order in the temperature T, it is given by[11]

$$V(\phi)=-\lambda v^2\phi^2+\lambda\phi^4+\lambda T^2\left[1+\frac{3}{32}\frac{g^2}{\lambda}\right]\phi^2 -\frac{3}{32\pi}g^3T\phi^3+\cdots\ .$$

We recognize the third term in this series to be $\lambda v^2(T/T_c)^2\phi^2$, where T_c is the critical temperature for symmetry restoration. It was precisely this term that was absorbed in the redefinition $v\rightarrow v(T)$ of vacuum expectation value of the Higgs field ϕ. The fourth term linear in T gives a temperature-independent contribution to the logarithm of the partition function, a contribution that should match our exact calculation in the limit that $\lambda \ll g^2 \ll 1$. Evaluating this contribution to $\ln\kappa$ in the sphaleron background (6) we obtain

$$\ln\kappa|_{\rm perturbative}=\frac{3}{16\sqrt{2}}\int_0^{\infty}\xi^2 d\xi[h^3(\xi)-1]\ .$$

This (implicit) function of λ/g^2 is also plotted in Fig. 3. We see that our results are at least consistent with this calculation in the perturbative regime, $\lambda/g^2 \ll 1$ al-

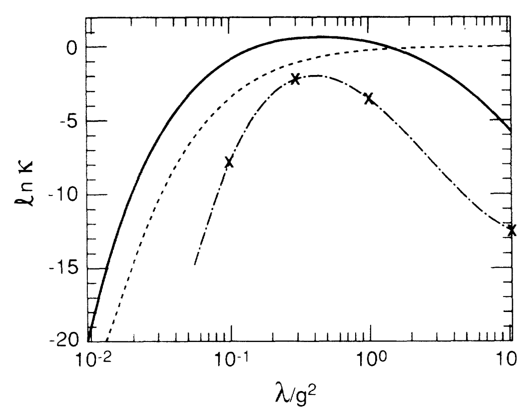

FIG. 3. A plot of $\ln\kappa$, where κ is the determinant of small fluctuations around the sphaleron solution with zero modes removed [Eqs. (7) and (21)], as a function of λ/g^2. The solid curve shows the result of approximate calculations of Ref. 22 using the method of Diakonov, Petrov, and Yung (Ref. 23). The four crosses are the result of the exact computations performed in this paper. These four points are interpolated by a third-order polynomial in $\ln\lambda/g^2$ (dot-dash curve). The dashed line is the result of a perturbative calculation (Ref. 11) to next-to-leading-order in the temperature T, valid for small λ/g^2.

though at $\lambda/g^2=0.1$ the two still differ by 2 orders of magnitude. As expected, the perturbative calculation completely fails for $\lambda/g^2 \geq 1$.

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APPENDIX

In this appendix we show how one may construct vector and tensor spherical harmonic eigenfunctions in terms of vector and tensor operators acting on the ordinary spherical harmonic functions $Y_{j,m}$. The basic vector operators we require are

$$\mathbf{O}_0 = i\mathbf{L} ,$$
$$\mathbf{O}_1 = \hat{\xi}(\bar{L}+1) + i\hat{\xi}\times\mathbf{L} ,$$
$$\mathbf{O}_{-1} = \hat{\xi}\bar{L} - i\hat{\xi}\times\mathbf{L} ,$$

where

$$\mathbf{L} = -i\xi\times\nabla$$

is the orbital angular momentum operator and

$$\bar{L} \equiv (\mathbf{L}^2+\tfrac{1}{4})^{1/2} - \tfrac{1}{2} .$$

This last operator is well defined if it always acts on an eigenstate of $\mathbf{L}^2$, a condition we shall maintain throughout the discussion below. Since the operators $\mathbf{O}_s$ defined above satisfy the commutation relations

$$[\mathbf{L}^2, \mathbf{O}_s] = 2ls + s(s+1) ,$$

we can use $\mathbf{O}_1$ or $\mathbf{O}_{-1}$ as raising or lowering operators to change the orbital angular momentum quantum number of an eigenstate from l to $(l+1)$ or $(l-1)$. Similarly, the action of the operator $\mathbf{O}_0$ will not change l. Our assertion then is that we may define the vector spherical harmonic functions satisfying the defining relations (16) as

$$Y^a_{j,s,m} = N_{j,s}^{-1/2} O_s^a Y_{j,m} , \tag{A1}$$

where the normalization constants $N_{j,s}$ are determined by the orthonormality condition

$$\int d\Omega\, Y^{\dagger a}_{j,s,m} Y^a_{j',s',m'} = \delta_{jj'}\delta_{ss'}\delta_{mm'} .$$

We list O_s^a and the constants $N_{j,s}$ in Table X. The parity of the $Y^a_{j,s,m}$ is $(-1)^{j+s}$.

To prove (A1) we note that by construction the functions $Y^a_{j,s,m}$ are eigenfunctions of $\mathbf{L}^2$ with eigenvalue $l=j+s$. Thus we need only show that they are also eigenfunctions of $\mathbf{J}^2$ and J_3 with eigenvalues j and m, respectively. Here we recall the total angular momentum operator is the vector sum $\mathbf{J}=\mathbf{L}+\mathbf{\Delta}$, where $\Delta^a_{bc} \equiv -i\epsilon_{abc}$. The proof is most easily executed by using the definitions of $\mathbf{L}$ and $\mathbf{O}_s$ above to show that

$$[L^a, O_s^b] = i\epsilon^{abc} O_s^c ,$$
$$\mathbf{L}\times\mathbf{O}_s = i\mathbf{O}_s[sl + \tfrac{1}{2}s(s+1)+1] . \tag{A2}$$

It is then a simple matter to derive the equalities

TABLE X. Table of vector spherical harmonics $Y^a_{j,s,m}$.

s	O_s^a	$N_{j,s}$
1	O_1^a	$(2j+1)(j+1)$
0	O_0^a	$j(j+1)$
-1	O_{-1}^a	$j(2j+1)$

TABLE XI. Table of tensor spherical harmonics $Y^{ak}_{j,s_1,s_2,m}$.

s_1	s_2	$O^{ak}_{s_1 s_2}$	N_{j,s_1,s_2}
2	2	$O_1^{\{a}O_1^{k\}}$	$(2j+1)(j+1)(j+2)(2j+3)$
1	2	$O_0^{\{a}O_1^{k\}} + O_1^{\{a}O_0^{k\}}$	$2j(j+1)(j+2)(2j+1)$
0	2	$-O_0^{\{a}O_0^{k\}} + \frac{1}{3}\delta^{ak}\mathbf{O}_0^2$	$\frac{1}{6}j(j+1)(2j+3)(2j-1)$
-1	2	$O_0^{\{a}O_{-1}^{k\}} + O_{-1}^{\{a}O_0^{k\}}$	$2j(j-1)(2j+1)(j+1)$
-2	2	$O_{-1}^{\{a}O_{-1}^{k\}}$	$(2j+1)j(2j-1)(j-1)$
1	1	$\epsilon^{akd}O_1^d$	$2(2j+1)(j+1)$
0	1	$-\epsilon^{akd}O_0^d$	$2j(j+1)$
-1	1	$\epsilon^{akd}O_{-1}^d$	$2j(2j+1)$
0	0	δ^{ak}	3

$$(J_a)_{bc}O_s^c=(L_a+\Delta_a)_{bc}O_s^c=O_s^bL_a ,$$
$$\mathbf{J}^2_{bc}O_s^c=(\mathbf{L}+\mathbf{\Delta})^2_{bc}O_s^c=O_s^b\mathbf{L}^2 ,$$

which completes the proof.

The same procedure may be used to define the tensor spherical harmonic functions. We take

$$Y^{ak}_{j,s_1,s_2,m}=N^{-1/2}_{j,s_1,s_2}O^{ak}_{s_1s_2}Y_{j,m} , \tag{A3}$$

where the operators $O^{ak}_{s_1s_2}$ are constructed in terms of some combinations of the $\mathbf{O}_s$ while N_{j,s_1,s_2} are normalization constants determined by the condition

$$\int d\Omega\, Y^{\dagger ak}_{j,s_1,s_2,m}Y^{ak}_{j',s_1',s_2',m'}=\delta_{jj'}\delta_{s_1s_1'}\delta_{s_2s_2'}\delta_{mm'} .$$

The operators $O^{ak}_{s_1s_2}$ and constants N_{j,s_1,s_2} are given in Table XI. The parity of the $Y^{ak}_{j,s_1,s_2,m}$ is $(-1)^{j+s_1}$.

Once again, the defining equations for $\mathbf{O}_s$ as well as the relations (A2) can be used to show that the tensor spherical harmonics given by (A3) are eigenfunctions of $\mathbf{L}^2$ (by construction), $\mathbf{J}^2$, J_3, and $(\mathbf{\Delta}+\mathbf{\Delta}')^2$, where the total angular momentum operator is now $\mathbf{J}=\mathbf{L}+\mathbf{\Delta}+\mathbf{\Delta}'$. Here $\mathbf{\Delta}$ and $\mathbf{\Delta}'$ are spin-1 vector matrices which act separately on the indices a and k of $Y^{ak}_{j,s_1,s_2,m}$. From (A2) and the form of the $O^{ak}_{s_1s_2}$, one may show that

$$(J_a)_{bk,cl}O^{cl}_{s_1s_2}=(L_a+\Delta_a+\Delta_a')_{bk,cl}O^{cl}_{s_1s_2}=O^{bk}_{s_1s_2}L^a ,$$
$$\mathbf{J}^2_{bk,cl}O^{cl}_{s_1s_2}=(\mathbf{L}+\mathbf{\Delta}+\mathbf{\Delta}')^2_{bk,cl}O^{cl}_{s_1s_2}=O^{bk}_{s_1s_2}\mathbf{L}^2 ,$$

which immediately proves that the $Y^{ak}_{j,s_1,s_2,m}$ are eigenfunctions of $\mathbf{J}^2$ and J_3 with the appropriate eigenvalues. The remaining operator $(\mathbf{\Delta}+\mathbf{\Delta}')^2$ has eigenvalues $s_2=0,1,2$ which specify the usual index symmetry of the tensor operators O^{ak}: for $s_2=2$, O^{ak} is symmetric under the interchange of a and k and is traceless, for $s_2=1$ it is antisymmetric and for $s_2=0$ it is proportional to δ^{ak}. From Table XI and the properties

$$\mathbf{O}_1^2=\mathbf{O}_{-1}^2=\mathbf{O}_0\cdot\mathbf{O}_{\pm1}=\mathbf{O}_{\pm1}\cdot\mathbf{O}_0=0 ,$$

it is clear that the tensor harmonics $Y^{ak}_{j,s_1,s_2,m}$ have the indicated s_2 eigenvalues.

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