Gauge theories in the light-cone gauge

Kurt Haller
Department of Physics, University of Connecticut, Storrs, Connecticut 06269
(Received 19 September 1988; revised manuscript received 7 May 1990)

A canonical formulation, using equal-time commutation rules for canonically conjugate operator-valued fields, is given for quantum electrodynamics and Yang-Mills theory in the light-cone gauge. A gauge-fixing term is used that avoids all operator constraints by providing a canonically conjugate momentum for every field component. The theory is embedded in a space in which the light-cone gauge condition and all of Maxwell's equations hold. Interaction picture fields and the photon and gluon propagators in the light-cone gauge are evaluated for two alternate representations of the longitudinal and timelike components of gauge fields. One representation makes use of the entire momentum space to represent these gauge field components as superpositions of ghost annihilation and creation operators. The other uses only ghost excitations with $k_3>0$ for the longitudinal modes of A_i, but restricts the gauge-fixing field to ghost excitations with $k_3<0$. It is shown that the former mode leads to a formulation that corresponds to the principal-value (PV) prescription for the extra pole in this gauge, the latter to the Mandelstam-Leibbrandt (ML) prescription. Nevertheless the underlying theory for these two cases is identical. In particular, for QED both modes give identical time evolution, within a physical subspace in which constraints are implemented, as does QED in the Coulomb gauge. It is therefore concluded that canonical formulations of the light-cone gauge cannot be a basis for preferring the ML to the PV prescription for the extra pole at $k_0=k_3$ in the light-cone propagator.

I. INTRODUCTION

Work on light-cone formulations of gauge theories was used as early as twenty years ago to study scaling behavior in large momentum transfer processes.[1-6] More recently there has been interest in the light-cone gauge as an example of an axial gauge that represents the gauge theory in arbitrary reference frames, and without replacing equal-time commutation rules by light-cone commutation rules.[7-19] In this work the propagator for the gauge field is given by

$$D_{\mu\nu}(k)=\frac{\delta_{ab}}{k^2-i\epsilon}\left[\delta_{\mu\nu}-\frac{k_\mu n_\nu+k_\nu n_\mu}{k\cdot n}+\frac{n^2k_\mu k_\nu}{(k\cdot n)^2}\right], \tag{1.1}$$

with $n_1=n_2=0$, and $n_3=n_0=1$ in the light-cone gauge. The ambiguity in the spurious pole at $k_0=k_3$ was originally resolved by choosing the principal-value (PV) prescription, but detailed analysis of gluon loops has shown that the PV prescription for resolving this ambiguity leads to uncontrollable infinities and that use of the Mandelstam-Leibbrandt (ML) prescription avoids this dilemma.[14-20] An argument has furthermore been made that consistent canonical quantization supports, indeed requires, the use of the ML prescription.[21]

The canonical quantization of the light-cone gauge given here applies procedures used in earlier work on the temporal gauge[22,23] to show that the same canonical formulation can lead to either the PV or the ML propagator prescriptions, depending only on the representation of the gauge fields in terms of ghost excitations. This demonstrates that a consistent canonical formulation cannot distinguish between the PV and the ML prescriptions. Moreover, when implementing the constraints that fix the gauge and impose Gauss's law in a time-independent fashion, this work includes all interactions in the time-evolution operator.

II. CANONICAL FORMULATION

In this work we will use the Lagrangian

$$\mathcal{L}=-\tfrac{1}{4}F_{ij}F_{ij}+\tfrac{1}{2}F_{i0}F_{i0}+j_1A_i-j_0A_0+[(\partial_0+\partial_3)(A_3-A_0)]G+\mathcal{L}_{\text{matter}}, \tag{2.1}$$

where $F_{ij}=\partial_jA_i-\partial_iA_j-2eA_i\times A_j$ for QCD and $F_{ij}=\partial_jA_i-\partial_iA_j$ for QED (we will use Yang-Mills theory as a prototype non-Abelian theory and, at times, refer to it as QCD). Similarly $F_{i0}=\partial_0A_i+\partial_iA_0+2eA_i\times A_0$ for QCD and $F_{i0}=\partial_0A_i+\partial_iA_0$ for QED, and $\mathcal{L}_{\text{matter}}=\bar\psi(m+\gamma\cdot\partial)\psi$; and finally $j_0=e\bar\psi\gamma_4\tau\psi$ and $j_i=ie\bar\psi\gamma_i\tau\psi$ for QCD, while $j_0=e\bar\psi\gamma_4\psi$ and $j_i=ie\bar\psi\gamma_i\psi$ for QED. Superscripts denoting Lie group indices will be suppressed unless necessary to avoid confusion. The Euler-Lagrange equations derived from $\mathcal{L}$ are

$$D_0F_{i0}-D_jF_{ij}-j_i=-\delta_{i,3}(\partial_0+\partial_3)G, \tag{2.2}$$

$$D_iF_{i0}+j_0=(\partial_0+\partial_3)G, \tag{2.3}$$

$$(\partial_0+\partial_3)(D_0+D_3)G=0, \tag{2.4}$$

and

$$(\partial_0+\partial_3)(A_0-A_3)=0, \tag{2.5}$$

where D_0 and D_i represent $D_0=\partial_0-2eA_0\times$ and

$D_i=\partial_i+2eA_i\times$, respectively, in QCD, and ∂_0 and ∂_i, respectively, in QED. Alternate forms of Gauss's law and Ampere's law, that are very useful for our purposes, are based upon the observation that the current J_μ, given by

$$J_0=j_0+2eA_i\times F_{i0}-2e(A_0-A_3)\times G\ , \tag{2.6a}$$

and

$$J_i=j_i+2eA_0\times F_{i0}+2eA_j\times F_{ij}-\delta_{i,3}2e(A_0-A_3)\times G \tag{2.6b}$$

obeys the conservation law $\partial_i J_i+\partial_0 J_0=0$. Equations (2.2) and (2.3) can be rewritten in the form

$$\partial_i F_{i0}+J_0=(D_0+D_3)G \tag{2.7}$$

and

$$\partial_0 F_{i0}-\partial_j F_{ij}-J_i=-\delta_{i,3}(D_0+D_3)G\ , \tag{2.8}$$

and, with the substitution of the conserved current j_μ for J_μ in QED, Eqs. (2.7) and (2.8) apply in that case too. These equations indicate that, to maintain the validity of Gauss's and Ampere's laws, $(D_0+D_3)G=0$ must be imposed weakly in some appropriately chosen subspace, and state vectors describing physical systems must remain in that subspace under time translation. The Lagrangian $\mathcal{L}$ gives rise to canonical momenta

$$\frac{\partial\mathcal{L}}{\partial(\partial_0 A_0)}=-G=\Pi_0 \tag{2.9}$$

and

$$\frac{\partial\mathcal{L}}{\partial(\partial_0 A_i)}=F_{i,0}+\delta_{i,3}G=\Pi_i \tag{2.10}$$

so that $E_i^a=\delta_{i,3}G^a-\Pi_i^a$ where $E_i^a=-F_{i,0}^a$; when the appropriate expressions for $F_{i,0}$ are used, the same equations apply in QED and QCD. As in earlier works on the temporal gauge, the gauge-fixing term we use avoids primary constraints that arise in formulations in which Π_0 vanishes identically.[22,23] The canonical commutation rules are $[A_i(\mathbf{x}),\Pi_j(\mathbf{y})]=i\delta_{i,j}\delta(\mathbf{x}-\mathbf{y})$, $[G(\mathbf{y}),A_0(\mathbf{x})]=i\delta(\mathbf{x}-\mathbf{y})$, and, consequently, that $[A_i(\mathbf{x}),E_j(\mathbf{y})]=-i\delta_{i,j}\delta(\mathbf{x}-\mathbf{y})$, $[A_0(\mathbf{x}),E_3(\mathbf{y})]=-i\delta(\mathbf{x}-\mathbf{y})$, as well as $[\Phi(\mathbf{x}),E_3(\mathbf{y})]=0$ and $[\Phi(\mathbf{x}),G(\mathbf{y})]=-i\delta(\mathbf{x}-\mathbf{y})$ for $\Phi=A_0-A_3$. In constructing Fock-space representations of these gauge fields that implement these commutation rules we represent the transverse components of the gauge fields in terms of the excitation operators for positive- and negative-helicity photons (and gluons) in a form identical to the one used for transverse gauge fields in the covariant, the temporal, as well as all other gauges; and, as in the temporal gauge, for the representation of longitudinal and timelike components of gauge fields, we use the gluon (or photon) ghost annihilation operators $a_Q(\mathbf{k})$, $a_R(\mathbf{k})$ and the corresponding creation operators which are their adjoints in an indefinite metric space, $a_Q^*(\mathbf{k})$ and $a_R^*(\mathbf{k})$, respectively. These operators obey the commutation rules $[a_Q(\mathbf{k}),a_R^*(\mathbf{k}')]=[a_R(\mathbf{k}),a_Q^*(\mathbf{k}')]=\delta_{\mathbf{k},\mathbf{k}'}$, and combine to form the unit operator as illustrated for the one-particle ghost sector by

$$1=\sum_{\mathbf{k}}[a_Q^*(\mathbf{k})|0\rangle\langle 0|a_R(\mathbf{k})+a_R^*(\mathbf{k})|0\rangle\langle 0|a_Q(\mathbf{k})]\ . \tag{2.11}$$

As in other axial gauges, there is no need for scalar fermion (Faddeev-Popov) ghosts in this gauge. $A_i(\mathbf{x})$ will be given as $A_i(\mathbf{x})=A_i^T(\mathbf{x})+A_i^G(\mathbf{x})$, where $A_i^T(\mathbf{x})$, the transverse part, has the form

$$A_i^T(\mathbf{x})=\sum_{\mathbf{k}}\left[\frac{\epsilon_i^n(\mathbf{k})}{\sqrt{(2k)}}[a_n(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}+a_n^\dagger(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}]\right] \tag{2.12}$$

and the index n is summed over the two transverse helicity modes. $A_i^G(\mathbf{x})$ is represented in terms of ghosts operators, as well as the functions $\eta(\mathbf{k})$, $\xi(\mathbf{k})$, $\overline{\eta}(\mathbf{k})$, and $\overline{\xi}(\mathbf{k})$, which will be given two alternate sets of values, which we refer to as the PV and the ML values, respectively. All of these functions are set equal to 1 in the PV case, but in an adaptation of a procedure used by Bassetto *et al.*,[21] by Lazzizzera,[24] and by Landshoff and Taylor,[25] they are given by $\eta(\mathbf{k})=2\Theta(k_3)$, $\xi(\mathbf{k})=\Theta(k_3)$, $\overline{\eta}(\mathbf{k})=2\Theta(-k_3)$, and $\overline{\xi}(\mathbf{k})=\Theta(-k_3)$ in the ML case. One consequence of these definitions is that $\eta(\mathbf{k})\overline{\eta}(\mathbf{k})=\xi(\mathbf{k})\overline{\xi}(\mathbf{k})=1$ for the PV case, but $\eta(\mathbf{k})\overline{\eta}(\mathbf{k})=\xi(\mathbf{k})\overline{\xi}(\mathbf{k})=0$ for the ML case. $A_i^G(\mathbf{x})$ is given by

$$A_i^G(\mathbf{x})=\sum_{\mathbf{k}}\frac{\eta(\mathbf{k})k_i}{2k^{3/2}}\{[a_R(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}+a_R^*(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}]+\lambda\overline{\eta}(\mathbf{k})[a_Q(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}+a_Q^*(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}]\}\ , \tag{2.13}$$

and representations of other gauge fields needed in this theory are

$$E_i(\mathbf{x})=i\sum_{\mathbf{k}}\left[\epsilon_i^n(\mathbf{k})\left[\frac{k}{2}\right]^{1/2}[a_n(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}-a_n^\dagger(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}]+\frac{\xi(\mathbf{k})k_i}{\sqrt{k}}[a_Q(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}-a_Q^*(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}]\right]\ , \tag{2.14}$$

$$G(\mathbf{x})=i\alpha\sum_{k}\frac{\overline{\xi}(\mathbf{k})k_3}{\sqrt{k}}\left[\frac{1}{\lambda}[a_R(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}-a_R^*(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}]-\xi(\mathbf{k})[a_Q(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}-a_Q^*(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}]\right]\ , \tag{2.15}$$

and

$$\Phi(\mathbf{x})=\frac{\lambda}{2\alpha}\sum_{\mathbf{k}}\frac{\overline{\eta}(\mathbf{k})\sqrt{k}}{k_3}[a_Q(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}+a_Q^*(\mathbf{k})e^{-\mathbf{k}\cdot\mathbf{x}}]\ . \tag{2.16}$$

Direct substitution confirms that these representations implement the equal-time commutation rules for any real values of λ and α. λ and α therefore remain unspecified parameters in these representations. The canonical prescription $\mathcal{H}=\Pi_0\partial_0 A_0+\Pi_i\partial_0 A_i-\mathcal{L}$ leads to the

Hamiltonian density which we represent as $\mathcal{H}=\mathcal{H}_0+\mathcal{H}_1$, where $\mathcal{H}_0$ is given by

$$\mathcal{H}_0=\tfrac{1}{2}E_i^aE_i^a+\tfrac{1}{4}(\partial_j A_i^a-\partial_i A_j^a)(\partial_j A_i^a-\partial_i A_j^a) - A_0^a\partial_i E_i^a+G^a\partial_3(A_0^a-A_3^a)+\mathcal{H}_{q\bar{q}} \quad (2.17)$$

for QCD, and by an identical expression for QED, but with the Lie group superscripts omitted. $\mathcal{H}_{q\bar{q}}$ represents the Hamiltonian density for noninteracting quarks (electrons in the case of QED). $\mathcal{H}_1$ is given by $\mathcal{H}_1=j_0A_0-j_iA_i$ for QED, and, for the non-Abelian case, it is given by

$$\mathcal{H}_1=j_0\cdot A_0-j_i\cdot A_i-2eA_0\cdot(A_i\times E_i) +2e\partial_j A_i\cdot(A_j\times A_i)+e^2(A_j\times A_i)\cdot(A_j\times A_i)\ . \quad (2.18)$$

$\mathcal{H}_1$ is free of operator-ordering problems in QCD as well as in QED, because noncommuting gauge field operators never appear in operator products. Operator-ordering problems that arise in $\mathcal{H}_0$ are not serious, and are always resolved in favor of normal-ordered bilinear products. It is useful, for later analysis, to represent the Hamiltonian H_0 in terms of the particle excitation operators used in Eqs. (2.13)–(2.16). In this representation H_0 is given by

$$\begin{aligned}H_0=\sum_{\mathbf{k}}\Bigg\{&k[a_n^\dagger(\mathbf{k})a_n(\mathbf{k})+\xi(\mathbf{k})a_Q^*(\mathbf{k})a_Q(\mathbf{k})]+k_3[a_R^*(\mathbf{k})a_Q(\mathbf{k})+a_Q^*(\mathbf{k})a_R(\mathbf{k})]\\&+\left[[a_Q^*(\mathbf{k})a_Q^*(-\mathbf{k})+a_Q(\mathbf{k})a_Q(-\mathbf{k})]\left[\frac{k}{2}\xi(\mathbf{k})\bar{\xi}(\mathbf{k})-\frac{\lambda k^2}{\alpha k_3}[\eta(\mathbf{k})-\xi(\mathbf{k})]\right]+\frac{\lambda}{\alpha}\xi(\mathbf{k})\bar{\xi}(\mathbf{k})\left[\frac{k^2}{k_3}\right]a_Q^*(\mathbf{k})a_Q(\mathbf{k})\right]\\&+\frac{k}{\sqrt{2}}\epsilon_3^n(\mathbf{k})\{a_n^\dagger(\mathbf{k})[a_Q(\mathbf{k})\xi(\mathbf{k})+a_Q^*(-\mathbf{k})\bar{\xi}(\mathbf{k})]+a_n(\mathbf{k})[a_Q^*(\mathbf{k})\xi(\mathbf{k})+a_Q(-\mathbf{k})\bar{\xi}(\mathbf{k})]\}\Bigg\}\\&+\sum_{\mathbf{q}}\omega_q(e^\dagger_{\mathbf{q},s}e_{\mathbf{q},s}+\bar{e}^\dagger_{\mathbf{q},s}\bar{e}_{\mathbf{q},s})\ ,\end{aligned} \quad (2.19)$$

where $\omega_q=(q^2+m^2)^{1/2}$ and m is the mass of the fermion (electron in QED, quark in QCD). H_0 applies equally to QED and QCD with the sole exception that in the case of QCD all a, $a^\dagger$, or a^* operators carry an additional Lie group index, which is contracted over the two elements of each bilinear product of such operators.

To impose constraints we employ a procedure we previously applied in the temporal gauge.[22,23] We note that $D_TG=-\partial_iE_i+J_0$ where $D_T=D_0+D_3$. By substituting j_0 for J_0, and making the other changes indicated after Eq. (2.5), we arrive at the corresponding form of this equation for QED. This permits us to write

$$[\partial_iE_i^c(\mathbf{x})-J_0^c(\mathbf{x})]=-\sum_{\mathbf{k}}\xi(\mathbf{k})k^{3/2}[\Omega^c(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}+\Omega^{c*}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}]\ , \quad (2.20)$$

where $\Omega^c(\mathbf{k})=a_Q^c(\mathbf{k})+\eta(\mathbf{k})J_0^c(\mathbf{k})/(2k^{3/2})$. We can also express $\Phi=A_0-A_3$ as

$$\Phi^c(\mathbf{x})=\frac{\lambda}{2\alpha}\sum_{\mathbf{k}}\bar{\eta}(\mathbf{k})\frac{\sqrt{k}}{k_3}[\Omega^c(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}+\Omega^{c*}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}]\ . \quad (2.21)$$

Equation (2.4) together with $\partial_3[\partial_iE_i^c(\mathbf{x})-J_0^c(\mathbf{x})]=-i\sum k_3\xi(\mathbf{k})k^{3/2}[\Omega^c(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}-\Omega^{c*}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}]$ determine the space-time dependence of $\partial_iE_i^c(\mathbf{x})-J_0^c(\mathbf{x})$ to be

$$[\partial_iE_i^c(\mathbf{x},t)-J_0^c(\mathbf{x},t)] =-\sum_{\mathbf{k}}\xi(\mathbf{k})k^{3/2}[\Omega^c(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{x}-k_3t)}+\Omega^{c*}(\mathbf{k})e^{-i(\mathbf{k}\cdot\mathbf{x}-k_3t)}]\ . \quad (2.22)$$

Equation (2.22) is verified by the observation that $\partial_0\Omega^c(\mathbf{k})=i[H,\Omega^c(\mathbf{k})]=-ik_3\Omega^c(\mathbf{k})$ both in QED and QCD. We can therefore use $\Omega^c(\mathbf{k})$ to define a subspace $\{|\nu\rangle\}$ of an indefinite metric space with the time-independent constraint

$$\Omega^c(\mathbf{k})|\nu\rangle=0\ . \quad (2.23)$$

In that subspace $\langle\nu'|\Phi^c|\nu\rangle=0$ and $\langle\nu'|\partial_iE_i-J_0|\nu\rangle=0$ obtain at all times because the subspace $\{|\nu\rangle\}$ remains invariant under time evolution. Moreover, we can verify that gauge-invariant quantities do not project from the "physical" subspace $\{|\nu\rangle\}$. We note that in QCD, quantities that transform according to the adjoint representation of the Lie group, such as the field strengths $F_{i,j}^a(\mathbf{x})$, $E_i^a(\mathbf{x})$, and the fermion charge density $j_0^a(\mathbf{x})$ [here collectively denoted as $\mathcal{V}^a(\mathbf{x})$], obey $[\mathcal{V}^a(\mathbf{x}),\Omega^b(\mathbf{k})]=[2ie/(2k^{3/2})]\eta(\mathbf{k})\epsilon_{abc}\mathcal{V}^c(\mathbf{x})\exp(-i\mathbf{k}\cdot\mathbf{x})$, so that invariants such as $F_{i,j}^a(\mathbf{x})F_{i,j}^a(\mathbf{x})$ or $E_i^a(\mathbf{x})j_i^a(\mathbf{x})$ commute with $\Omega^b(\mathbf{k})$. In QED the field strengths and current densities themselves are gauge invariant and commute with $\Omega(\mathbf{k})$. For any such gauge-invariant quantities $\mathcal{S}(\mathbf{x})$, since $[\mathcal{S}(\mathbf{x}),\Omega^b(\mathbf{k})]=0, \Omega^b(\mathbf{k})\mathcal{S}(\mathbf{x})|\psi\rangle=0$ if $\Omega^b(\mathbf{k})|\psi\rangle=0$. When the unit operator $|\alpha\rangle\langle\bar{\alpha}|$ is inserted [note that, as illustrated in Eq. (2.11), $|\bar{\alpha}\rangle$ and $|\alpha\rangle$ are not always identical], $\Omega^b(\mathbf{k})|\alpha\rangle\langle\bar{\alpha}|\mathcal{S}(\mathbf{x})|\psi\rangle=0$; and for those elements $\{|\lambda\rangle\}$ in $\{|\alpha\rangle\}$ which are not part of the subspace $\{|\nu\rangle\}$, for which $\Omega^b(\mathbf{k})|\lambda\rangle\neq0$, $\langle\bar{\lambda}|\mathcal{S}(\mathbf{x})|\psi\rangle=0$ follows from $\Omega^b(\mathbf{k})\mathcal{S}(\mathbf{x})|\psi\rangle=0$. This demonstrates that only matrix elements $\langle\bar{\nu}|\mathcal{S}(\mathbf{x})|\psi\rangle$, for which both $|\nu\rangle$ and $|\psi\rangle$ are within the physical subspace $\{|\nu\rangle\}$, contribute to products (or commutators) of gauge-invariant quantities. Similarly, application of the equal-time commutation rules demonstrates that for $Q^a=\int J_0^a(\mathbf{x})d\mathbf{x}$, $[Q^a,\Omega^b(\mathbf{k})]=2ie\epsilon_{abc}\Omega^c(\mathbf{k})$, so that for $\Omega^b(\mathbf{k})|\psi\rangle=0, \Omega^b(\mathbf{k})Q^a|\psi\rangle=0$ also.

III. PERTURBATIVE VACUUM STATES AND PROPAGATORS

In a canonical formulation of gauge theory, the propagator is the vacuum expectation value

$$D_{\mu\nu}(x[1],x[2])=\langle 0|T\{A_\mu(x[1]),A_\nu(x[2])\}|0\rangle \; , \tag{3.1}$$

where T indicates time ordering, and where $x[1],x[2]$ refer to two space-time points. The interaction picture field $A_\mu(x)=\exp(-iH_0t)A_\mu(\mathbf{x})\exp(iH_0t)$ can be explicitly evaluated and, for the PV option for the functions $\eta(\mathbf{k})$, $\xi(\mathbf{k})$, $\bar\eta(\mathbf{k})$, and $\bar\xi(\mathbf{k})$, and for $\lambda=0$, $A_i(x)$ is given by

$$A_i(\mathbf{x},t)=\sum_{\mathbf{k}}\left[\frac{a_n(\mathbf{k})}{\sqrt{2k}}e^{i\mathbf{k}\cdot\mathbf{x}}\left[\epsilon_i^n(\mathbf{k})e^{-ikt}-\frac{k_i}{k_3-k}\epsilon_3^n(\mathbf{k})(e^{-ikt}-e^{-ik_3t})\right]+a_R(\mathbf{k})\frac{k_i}{2k^{3/2}}e^{i(\mathbf{k}\cdot\mathbf{x}-k_3t)}\right.$$
$$\left.+a_Q(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}\frac{(k_3k_i+\delta_{i,3}k^2)[\cos(kt)-e^{-ik_3t}]-i(kk_i+\delta_{i,3}k_3k)\sin(kt)}{\sqrt{k}\,(k^2-k_3^2)}\right]+\text{Hermitian adjoint}\; . \tag{3.2}$$

Equation (3.2) can be used to evaluate the vacuum expectation value of $T\{A_\mu(x[1]),A_\nu(x[2])\}$ in the perturbative vacuum annihilated by all gauge-field annihilation operators, including those for the ghost excitations. The resulting expression lacks time displacement invariance, in conformance with a previously reported theorem.[26] There is, however, no *a priori* reason for identifying that vacuum state as the proper perturbative vacuum for the propagator. The fundamental requirement that must be satisfied, in order to derive the Dyson-Wick reduction of the S matrix to Feynman rules that include the expression in Eq. (3.1) as the propagator, is that the vacuum and the n-particle Fock state be eigenstates of H_0. However, the vacuum state annihilated by all annihilation operators does not satisfy that requirement because H_0 creates, from the vacuum, gluon (or photon) pairs which consist of a transverse and a Q-type excitation. To construct a satisfactory vacuum state that is an eigenstate of H_0, we make use of a unitary transformation to transform H_0 so that the undesirable term that creates the gluon pair from the vacuum is eliminated. A suitable transformation is given by $\mathcal{U}=e^{\delta}$, where δ is

$$\delta=-\sum_{\mathbf{k}}\frac{k}{\sqrt{2}}\frac{\epsilon_3^n(\mathbf{k})}{k_3-k}\{a_n^\dagger(\mathbf{k})[a_Q(\mathbf{k})\xi(\mathbf{k})+a_Q^*(-\mathbf{k})\bar\xi(\mathbf{k})]-a_n(\mathbf{k})[a_Q^*(\mathbf{k})\xi(\mathbf{k})+a_Q(-\mathbf{k})\bar\xi(\mathbf{k})]\}\; . \tag{3.3}$$

$\mathcal{U}$ transforms H_0 to $(H_0)_\delta$ where $(H_0)_\delta=\mathcal{U}H_0\mathcal{U}^{-1}$ and $(H_0)_\delta$ is given by

$$(H_0)_\delta=\sum_{\mathbf{k}}\left[ka_n^\dagger(\mathbf{k})a_n(\mathbf{k})+k_3[a_R^*(\mathbf{k})a_Q(\mathbf{k})+a_Q^*(\mathbf{k})a_R(\mathbf{k})]\right.$$
$$\left.+\frac{\lambda k^2}{\alpha k_3}\{a_Q^*(\mathbf{k})a_Q(\mathbf{k})\xi(\mathbf{k})\bar\xi(\mathbf{k})-[a_Q^*(\mathbf{k})a_Q^*(-\mathbf{k})+a_Q(\mathbf{k})a_Q(-\mathbf{k})][\eta(\mathbf{k})-\xi(\mathbf{k})]\}\right]$$
$$+\sum_{\mathbf{q}}\omega_q(e_{\mathbf{q},s}^\dagger e_{\mathbf{q},s}+\bar e_{\mathbf{q},s}^\dagger\bar e_{\mathbf{q},s})\; . \tag{3.4}$$

The transformed perturbative vacuum state $\mathcal{U}|0\rangle$, and the Fock space constructed on it, *are* eigenstates of H_0, and the vacuum expectation value

$$D_{\mu\nu}(x[1],x[2])_\delta=\langle 0|\mathcal{U}^{-1}T\{A_\mu(x[1]),A_\nu(x[2])\}\mathcal{U}|0\rangle \tag{3.5}$$

can be used consistently in a perturbative S-matrix expansion. $D_{\mu\nu}(x[1],x[2])_\delta$ can also be represented as $D_{\mu\nu}(x[1],x[2])_\delta=\langle 0|T\{A_\mu(x[1])_\delta,A_\nu(x[2])_\delta\}|0\rangle$, where $A_\mu(\mathbf{x},t)_\delta=\mathcal{U}A_\mu(\mathbf{x},t)\mathcal{U}^{-1}$ and $A_i(\mathbf{x},t)_\delta$ is given by

$$A_i(\mathbf{x},t)_\delta=\sum_{\mathbf{k}}\left[\frac{1}{\sqrt{2k}}\left[\epsilon_i^n(\mathbf{k})-\frac{k_i}{k_3-k}\epsilon_3^n(\mathbf{k})\right][a_n(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{x}-kt)}+a_n^\dagger(\mathbf{k})e^{-i(\mathbf{k}\cdot\mathbf{x}-kt)}]\right.$$
$$+\frac{k_i}{2k^{3/2}}[a_R(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{x}-k_3t)}+a_R^*(\mathbf{k})e^{-i(\mathbf{k}\cdot\mathbf{x}-k_3t)}]\eta(\mathbf{k})$$
$$\times\left[\frac{\lambda k_i}{2k^{3/2}}\xi(\mathbf{k})\bar\xi(\mathbf{k})-\frac{\delta_{i,3}k^{3/2}}{k^2-k_3^2}\right][a_Q(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{x}-k_3t)}+a_Q^*(\mathbf{k})e^{-i(\mathbf{k}\cdot\mathbf{x}-k_3t)}]\xi(\mathbf{k})$$
$$\left.-it\frac{\lambda k_i\eta(\mathbf{k})\bar\xi(\mathbf{k})\sqrt{k}}{2\alpha k_3}[a_Q(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{x}-k_3t)}-a_Q^*(\mathbf{k})e^{-i(\mathbf{k}\cdot\mathbf{x}-k_3t)}]\right]\; . \tag{3.6}$$

The modified propagator $D_{\mu\nu}(x[1],x[2])_\delta$ has two alternate forms depending on whether the PV or the ML sets of values are chosen for the functions $\eta(\mathbf{k})$, $\xi(\mathbf{k})$, $\bar{\eta}(\mathbf{k})$, and $\bar{\xi}(\mathbf{k})$. For the PV set of values, for the case $\lambda=0$, $D_{\mu\nu}(x[1],x[2])_\delta$ is given by

$$[D_{i,j}(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2)_\delta]^{\mathrm{PV}}=\sum_{\mathbf{k}}\frac{e^{i\mathbf{k}\cdot\mathbf{X}}}{2k}\left\{\left[\left[\delta_{i,j}+\frac{\delta_{i,3}k_j+\delta_{j,3}k_i}{k-k_3}\right]e^{ik(t_2-t_1)}-\frac{\delta_{i,3}k_j+\delta_{j,3}k_i}{k^2-k_3^2}ke^{ik_3(t_2-t_1)}\right]\Theta(t_1-t_2)\right.$$
$$\left.+\left[\left[\delta_{i,j}-\frac{\delta_{i,3}k_j+\delta_{j,3}k_i}{k+k_3}\right]e^{-ik(t_2-t_1)}+\frac{\delta_{i,3}k_j+\delta_{j,3}k_i}{k^2-k_3^2}ke^{ik_3(t_2-t_1)}\right]\Theta(t_2-t_1)\right\} \quad (3.7)$$

and agrees with the Fourier transform of $D_{\mu\nu}(k)$ in Eq. (1.1) when the PV prescription is used in the k_0 integration. The extension of this result to nonvanishing values of λ is trivial to evaluate, but will not be reported here. For the ML set of values of $\eta(\mathbf{k})$, $\xi(\mathbf{k})$, $\bar{\eta}(\mathbf{k})$, and $\bar{\xi}(\mathbf{k})$, $D_{\mu\nu}(x[1],x[2])_\delta$ is given by

$$[D_{i,j}(x[1],x[2])_\delta]^{\mathrm{ML}}=\sum_{\mathbf{k}}\frac{e^{i\mathbf{k}\cdot\mathbf{X}}}{k}\left\{\left[\left[\delta_{i,j}+\frac{\delta_{i,3}k_j+\delta_{j,3}k_i}{k-k_3}\right]\frac{e^{ik(t_2-t_1)}}{2}-\Theta(k_3)\frac{\delta_{i,3}k_j+\delta_{j,3}k_i}{k^2-k_3^2}ke^{ik_3(t_2-t_1)}\right]\Theta(t_1-t_2)\right.$$
$$+\left[\left[\delta_{ij}-\frac{\delta_{i,3}k_j+\delta_{j,3}k_i}{k+k_3}\right]\frac{e^{-ik(t_2-t_1)}}{2}\right.$$
$$\left.\left.+\Theta(-k_3)\frac{\delta_{i,3}k_j+\delta_{j,3}k_i}{k^2-k_3^2}ke^{ik_3(t_2-t_1)}\right]\Theta(t_2-t_1)\right\} \quad (3.8)$$

and agrees with the Fourier transform of $D_{\mu\nu}(k)$ in Eq. (1.1) when the ML prescription is used in the k_0 integration. In both cases the subscript 0 may be substituted for 3 in $D_{\mu\nu}(x[1],x[2])_\delta$. Equations (3.7) and (3.8) demonstrate that two canonical formulations based on the same Lagrangian, the same equations of motion, the same equal-time commutation rules, and the same weak constraints can lead to two different propagators, corresponding to the PV and ML prescriptions, respectively. Only the explicit dependence of the longitudinal and timelike components of the gauge fields on the ghost excitations accounts for the difference between the two propagators.

We now show that the *ad hoc* substitution of $\mathcal{U}|0\rangle$ for $|0\rangle$ leaves scattering amplitudes unchanged except for renormalization constants. For purposes of this argument we express H as $H=(H_0)_\delta+\underline{H}_1$, where $\underline{H}_1=H_1+h_0$ and h_0 is given by $h_0=H_0-(H_0)_\delta$. h_0 includes all those parts of H_0 that create gluon (or photon) pairs from the vacuum. We write $S_{f,i}=\delta_{f,i}-2\pi i\delta(E_f-E_i)T_{f,i}$ and compare $T_{f,i}=\langle f|\underline{H}_1+\underline{H}_1(E_i-H+i\epsilon)^{-1}\underline{H}_1|i\rangle$, the transition amplitude in the Fock space constructed on the $|0\rangle$ vacuum, with

$$\bar{T}_{f,i}=\langle f|(H_1)_\delta+(H_1)_\delta[E_i-(H)_\delta+i\epsilon]^{-1}(H_1)_\delta|i\rangle ,$$

the transition amplitude in the Fock space constructed on the $\mathcal{U}|0\rangle$ vacuum, where $(H)_\delta=\mathcal{U}H\mathcal{U}^{-1}$ and $(H)_\delta=(H_0)_\delta+(H_1)_\delta$. The demonstration that $\bar{T}_{f,i}$ may safely be substituted for $T_{f,i}$ in S-matrix elements, is based on a proof, used in earlier work,[22,27] that

$$\bar{T}_{f,i}=T_{f,i}+(E_f-E_i)\mathcal{T}^{\alpha}_{f,i}+i\epsilon\mathcal{T}^{\beta}_{f,i} . \quad (3.9)$$

Since $E_i=E_f$ in S-matrix elements, $\mathcal{T}^{\alpha}_{f,i}$ and $\mathcal{T}^{\beta}_{f,i}$ do not contribute to the latter unless $\mathcal{T}^{\alpha}_{f,i}$ or $\mathcal{T}^{\beta}_{f,i}$ exhibit $(E_i-E_f)^{-1}$ or $(i\epsilon)^{-1}$ singularities, respectively. Such singularities can develop only in self-energy corrections to external lines, and the resulting contributions are absorbed into wave-function renormalization constants. When we consistently use the vacuum state $\mathcal{U}|0\rangle$ and the n-particle states built on it to derive Feynman rules, we obtain, in addition to the gauge field propagator, the three-gluon vertex rule

$$ie\{[(p_i-q_i)-(p_0-q_0)\delta_{i,3}]\delta_{m,n}$$
$$+[(q_m-k_m)-(q_0-k_0)\delta_{m,3}]\delta_{i,n}$$
$$+[(k_n-p_n)-(k_0-p_0)\delta_{n,3}]\delta_{i,m}\}\epsilon_{\alpha\beta\gamma} ,$$

where the three gluon lines correspond to momentum p, spatial component m, and isospin index α, to q, n, and β and to k, i, and γ, respectively, and all have momenta directed towards the vertex. The four-gluon vertex is the obvious generalization of the three-gluon case; and the projection operator for external incident and scattered gluon (and photon) lines is obtained from Eq. (3.6) and has the form

$$\frac{1}{\sqrt{2k}}\left[\epsilon_i^n(\mathbf{k})-\frac{k_i}{k_3-k}\epsilon_3^n(\mathbf{k})\right]e^{i(\mathbf{k}\cdot\mathbf{x}-kt)} .$$

IV. UNITARITY IN THE PHYSICAL SUBSPACE

In this section we will demonstrate that the S matrix in this theory saturates unitarity in the space of states that contains quarks and transverse gluons only (the quotient space). This result applies to QCD as well as to QED, and follows from the fact that S-matrix elements to final states that include even a single R-ghost vanish, since ghost states have zero norm unless they contain at least one mixed pair (Q and R). We show that a single R ghost in the final state is sufficient for the S-matrix element to vanish, by choosing a final state $|f\rangle$ of the form $a_R^*(\mathbf{k})|f'\rangle$, where $|f'\rangle$ designates a state that may contain quarks, transverse gluons, or either variety of gluon ghost, R type as well as Q type. This makes the transition amplitude $T_{f,i}$ for transitions from states $|i\rangle$ to $|f\rangle$,

$$T_{f,i}=\langle\bar{f}'|a_Q(\mathbf{k})[\underline{H}_1+\underline{H}_1(E_i-H+i\epsilon)^{-1}\underline{H}_1]|i\rangle , \quad (4.1)$$

since it is the combination $a_R^*(\mathbf{k})|f'\rangle\langle\bar{f}'|a_Q(\mathbf{k})$ that appears in the unit operator. The identities $[a_Q(\mathbf{k}),H_1]=[a_Q(\mathbf{k}),\underline{H}_1]$ and

$$\begin{aligned}a_Q(\mathbf{k})(E_i-H+i\epsilon)^{-1}=&(E_i-k_3-H+i\epsilon)^{-1}a_Q(\mathbf{k})\\&+(E_i-k_3-H+i\epsilon)^{-1}\\&\times[a_Q(\mathbf{k}),H_1](E_i-H+i\epsilon)^{-1}\end{aligned} \quad (4.2)$$

lead to

$$T_{f,i}=\langle\bar{\psi}_{f'}{}^{(-)}{}_{(E_i-k_3)}|[a_Q(\mathbf{k}),H_1]|\psi_i{}^{(+)}{}_{(E_i)}\rangle , \quad (4.3)$$

where $\psi_i{}^{(+)}{}_{(E_i)}$ represents the scattering state given by

$$\psi_i{}^{(+)}{}_{(E_i)}=[1+(E_i-H+i\epsilon)^{-1}\underline{H}_1]|i\rangle \quad (4.4)$$

and $\psi_{f'}{}^{(-)}{}_{(E)}$ represents the scattering state with "incoming" boundary conditions whose asymptotic limit as $t\to+\infty$ is the state $|f'\rangle$, and is given by

$$\psi_{f'}{}^{(-)}{}_{(E)}=[1+(E-H-i\epsilon)^{-1}\underline{H}_1]|f'\rangle , \quad (4.5)$$

where $(H_0-E)|f'\rangle=0$. We can also verify that

$$[a_Q(\mathbf{k}),H_1]=\zeta(k)\{[H,J_0(\mathbf{k})]+k_3J_0(\mathbf{k})\} , \quad (4.6)$$

where the Lie group index has been suppressed and $\zeta(\mathbf{k})=\eta(\mathbf{k})\frac{1}{2}k^{-(3/2)}$. From Eqs. (4.2)–(4.6) we obtain the result that

$$\begin{aligned}T_{f,i}=\zeta(k)[&\langle\bar{\psi}_{f'}{}^{(-)}{}_{(E)}|J_0(\mathbf{k})|\psi_i{}^{(+)}{}_{(E_i)}\rangle(E-E_i+k_3)+i\epsilon\langle\bar{f}'|\underline{H}_1(E-H+i\epsilon)^{-1}J_0(\mathbf{k})|\psi_i{}^{(+)}{}_{(E_i)}\rangle\\&-i\epsilon\langle\bar{\psi}_{f'}{}^{(-)}{}_{(E)}|J_0(\mathbf{k})(E_i-H+i\epsilon)^{-1}\underline{H}_1|i\rangle]\end{aligned} \quad (4.7)$$

with $E=E_i-k_3$. Equation (4.7) establishes the following result for an S-matrix element from a state $|i\rangle$, which consists of only quarks and transverse gluons, to a state $|f\rangle$ that contains at least one R-type ghost, but may contain anything else, including Q-type ghosts. The only contributions to that S-matrix element that survive are the ones in Eq. (4.7) that are proportional to $i\epsilon$, and these vanish in the limit $\epsilon\to 0$, except when the matrix elements $\langle\bar{f}'|\underline{H}_1(E-H+i\epsilon)^{-1}J_0(\mathbf{k})|\psi_i{}^{(+)}{}_{(E_i)}\rangle$ or $\langle\bar{\psi}_{f'}{}^{(-)}{}_{(E)}|J_0(\mathbf{k})(E_i-H+i\epsilon)^{-1}\underline{H}_1|i\rangle$ develop $(i\epsilon)^{-1}$ singularities; and that happens only in self-energy corrections to external lines. As we discussed previously, these contributions do not affect scattering cross sections, and in particular cannot keep these from vanishing.

It is worth mentioning how the argument given above fails for QCD in covariant gauges so that Faddeev-Popov ghosts are required in the latter. In the case of covariant gauges, Eq. (4.6) is replaced by[28]

$$[a_Q(\mathbf{k}),H_1]=\tfrac{1}{2}k^{-(3/2)}\{[H,J_0(\mathbf{k})]+kJ_0(\mathbf{k})\}+X(\mathbf{k}) . \quad (4.8)$$

After the on-shell condition has been imposed, and the terms proportional to $i\epsilon$ have been discounted, the transition matrix to the state $|f\rangle$ still contains the term $\langle\bar{\psi}_{f'}{}^{(-)}{}_{(E_i-k)}|X(\mathbf{k})|\psi_i{}^{(+)}{}_{(E_i)}\rangle$. $X(\mathbf{k})$ commutes with $a_Q(\mathbf{k})$, so that if the state $|f\rangle$ consists of quarks, transverse gluons, and Q-type gluon ghosts exclusively, then the S-matrix element will still vanish. But $X(\mathbf{k})$ does not commute with $a_R(\mathbf{k})$, so that if R-type ghosts are included in the final state, the final states that combine Q- and R-type ghosts do absorb probability, and unitarity in the physical subspace is not conserved. The structure of $X(\mathbf{k})$ allows us to construct the Faddeev-Popov ghost-gluon coupling necessary to restore that unitarity.[28] $X(\mathbf{k})$ vanishes identically in the light cone and other axial gauges, and that is why Faddeev-Povov ghosts are not necessary in axial gauges.

In the case of QED it is possible to demonstrate a more inclusive result, that is sufficient but not necessary to prove unitarity in the subspace of electrons and transverse photons. In that case states which obey the constraint equation, and operators, can be unitarily transformed to a representation in which QED in the

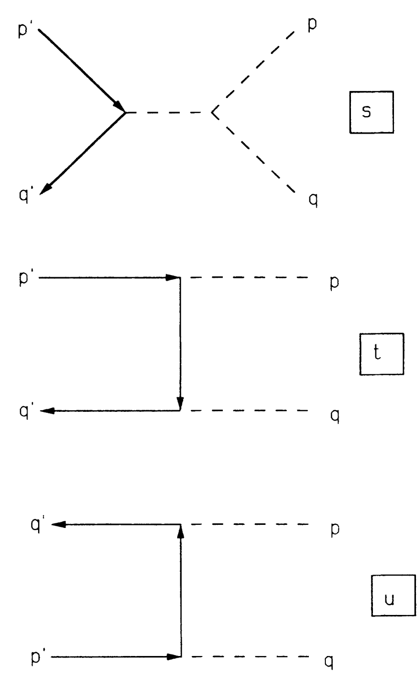

FIG. 1. s-, t-, and u-channel tree graphs for a gluon-gluon collision producing a quark-antiquark pair. The solid lines represent quarks, the dashed lines gluons. The graphs correspond to the expressions for $\mathcal{M}_s$, $\mathcal{M}_t$, and $\mathcal{M}_u$ given in the Appendix.

light cone gauge and QED in the Coulomb gauge are identical in the quotient space of electrons and transverse photons. In the entire subspace allowed by the constraint equation, the transformed Hamiltonian differs from the Coulomb gauge Hamiltonian, but in ways which can have no observable consequences for the time evolution of states vectors. The proof is virtually identical to a similar one for QED in the temporal gauge, which has been given elsewhere[22] and will not be repeated here.

ACKNOWLEDGMENTS

This research was supported by the Department of Energy under Grant No. DE-AC02-79ER 10336A.

APPENDIX: S-MATRIX CALCULATIONS IN THE LIGHT-CONE GAUGE

In this appendix we will calculate the lowest-order tree contribution for the inelastic scattering of two gluons of momenta p and q, and polarizations $\epsilon(p)$ and $\epsilon(q)$, respectively, into a quark-antiquark pair of momenta p' and q', respectively. All of the diagrammatic rules for the light-cone gauge, the propagator developed in Sec. III, as well as the projection factor

$$\frac{1}{\sqrt{2k}}\left[\epsilon_i^n(\mathbf{k})-\frac{k_i}{k_3-k}\epsilon_3^n(\mathbf{k})\right]e^{i(\mathbf{k}\cdot\mathbf{x}-kt)}$$

for external transverse gluons, and the expression for the three-gluon vertex

$$ie\{[(p_i-q_i)-(p_0-q_0)\delta_{i,3}]\delta_{m,n}+[(q_m-k_m)-(q_0-k_0)\delta_{m,3}]\delta_{i,n}+[(k_n-p_n)-(k_0-p_0)\delta_{n,3}]\delta_{i,m}\}\epsilon_{\alpha\beta\gamma}$$

must be used. It is worth noting that each of the three graphs in Fig. 1 is frame dependent, and that frame independence is not restored until the s-, t-, and u-channel graphs are combined. We find that $\mathcal{M}_s$, $\mathcal{M}_t$, and $\mathcal{M}_u$, each designating the contribution to the S-matrix element made by the corresponding graph in Fig. 1, are given by

$$\mathcal{M}_s=ie^2\epsilon_{abc}\tau_c\left[\frac{\gamma\cdot(q-p)\epsilon(p)\cdot\epsilon(q)+\gamma\cdot\epsilon(p)p\cdot\epsilon(q)-\gamma\cdot\epsilon(q)q\cdot\epsilon(p)}{p\cdot q}+2\left[\frac{\gamma\cdot\epsilon(q)\epsilon_3(p)}{p_3-p}-\frac{\gamma\cdot\epsilon(p)\epsilon_3(q)}{q_3-q}-\frac{\gamma\cdot q\epsilon_3(q)\epsilon_3(p)}{(p_3-p)(q_3-q)}\right]\right] \tag{A1}$$

for the s-channel graph,

$$\mathcal{M}_t=e^2\tau_a\tau_b\left[\frac{\gamma\cdot p\gamma\cdot\epsilon(p)\gamma\cdot\epsilon(q)+2\gamma\cdot\epsilon(q)p'\cdot\epsilon(p)}{2p\cdot p'}-\frac{\gamma\cdot\epsilon(q)\epsilon_3(p)}{p_3-p}+\frac{\gamma\cdot\epsilon(p)\epsilon_3(q)}{q_3-q}+\frac{\gamma\cdot q\epsilon_3(q)\epsilon_3(p)}{(p_3-p)(q_3-q)}\right] \tag{A2}$$

for the t-channel graph, and

$$\mathcal{M}_u=e^2\tau_b\tau_a\left[\frac{\gamma\cdot\epsilon(q)\gamma\cdot\epsilon(p)\gamma\cdot p-2\gamma\cdot\epsilon(q)q'\cdot\epsilon(p)}{2q\cdot p'}+\frac{\gamma\cdot\epsilon(q)\epsilon_3(p)}{p_3-p}-\frac{\gamma\cdot\epsilon(p)\epsilon_3(q)}{q_3-q}+\frac{\gamma\cdot p\epsilon_3(q)\epsilon_3(p)}{(p_3-p)(q_3-q)}\right] \tag{A3}$$

for the u-channel graph. The sum of the contributions from the three graphs is given by $\mathcal{M}=\mathcal{M}_s+\mathcal{M}_t+\mathcal{M}_u$, and $\mathcal{M}$ takes the form

$$\mathcal{M}=e^2\left[\tau_a\tau_b\frac{-\gamma\cdot\epsilon(p)\gamma\cdot p\gamma\cdot\epsilon(q)+2\gamma\cdot\epsilon(q)p'\cdot\epsilon(p)}{t-m^2}-\tau_b\tau_a\frac{\gamma\cdot\epsilon(q)\gamma\cdot p\gamma\cdot\epsilon(p)+2\gamma\cdot\epsilon(q)q'\cdot\epsilon(p)}{u-m^2}\right.$$
$$\left.-2i\epsilon_{abc}\tau_c\frac{\gamma\cdot(q-p)\epsilon(p)\cdot\epsilon(q)+\gamma\cdot\epsilon(p)p\cdot\epsilon(q)-\gamma\cdot\epsilon(q)q\cdot\epsilon(p)}{s}\right]. \tag{A4}$$

[1]S. Weinberg, Phys. Rev. **150**, 1313 (1966).
[2]J. B. Kogut and D. E. Soper, Phys. Rev. D **1**, 2901 (1970).
[3]J. D. Bjorken, J. B. Kogut, and D. E. Soper, Phys. Rev. D **3**, 1382 (1971).
[4]S. J. Brodsky, R. Roskies, and R. Suaya, Phys. Rev. D **8**, 4574 (1973).
[5]G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).
[6]S. J. Brodsky, in *Quantum Chromodynamics,* proceedings of the Summer Institute on Particle Physics, Stanford, California, 1979, edited by Anne Mosher (SLAC Report No. 224, Conference No. 790774, Stanford, 1980).
[7]For an extensive review and list of references, see G. Leibbrandt, Rev. Mod. Phys. **59**, 1067 (1987).
[8]S. Caracciolo, G. Curci, and P. Menotti, Phys. Lett. **113B**, 311 (1982).
[9]S. C. Lim, Phys. Lett. **149B**, 201 (1984).
[10]H. P. Dahmen, B. Scholz, and F. Steiner, Phys. Lett. **117B**, 339 (1982).
[11]H. Yamagishi, Phys. Lett. **B189**, 161 (1987).
[12]P. V. Landshoff, Phys. Lett. **169B**, 69 (1986).

[13]H. Cheng and E. C. Tsai, Phys. Rev. D **36**, 3196 (1987).
[14]S. Mandelstam, Nucl. Phys. **B213**, 149 (1983).
[15]G. Leibbrandt, Phys. Rev. D **29**, 1699 (1984).
[16]A. Bassetto, M. Dalbosco, and R. Soldati, Phys. Lett. **159B**, 34 (1985); Phys. Rev. D **36**, 3138 (1987).
[17]G. Leibbrandt and S.-L. Nyeo, Nucl. Phys. **B276**, 459 (1986).
[18]S.-L. Nyeo, Phys. Rev. D **34**, 3842 (1986).
[19]P. Gaigg, M. Kreuzer, and G. Pollak, Phys. Rev. D **38**, 2559 (1988).
[20]G. Pollak, Phys. Rev. D **40**, 2027 (1989).
[21]A. Bassetto, M. Dalbosco, I. Lazzizzera, and R. Soldati, Phys. Rev. D **31**, 2012 (1985).
[22]K. Haller, Phys. Rev. D **36**, 1830 (1987).
[23]K. Haller, Phys. Rev. D **36**, 1839 (1987).
[24]I. Lazzizzera, Phys. Lett. B **210**, 188 (1988).
[25]P. V. Landshoff and J. C. Taylor, Phys. Lett. B **231**, 129 (1989).
[26]K. Haller, Phys. Lett. B **220**, 446 (1989).
[27]K. Haller and R. B. Sohn, Phys. Rev. A **20**, 1541 (1979).
[28]K. Haller and R. G. Brickner, Nuovo Cimento **73A**, 89 (1983).