Constrained partition functions and the antisymmetric tensor field

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We consider the case when the spatial topology of the Universe is that of a torus with metric g_{ij}. We form an expression for the density of states under the constraint that the total momentum and winding number are zero. To accomplish this we introduce nonzero values of g_{00}, g_{0i}, and B_{0i} in the imaginary-time formalism and integrate over all such values.

I. INTRODUCTION

String theory on toroidal backgrounds is of special interest because the $R \rightarrow R^{-1}$ duality is evidently present there.[1-7] Point-particle theories are incapable of such a duality so this is a fundamental string result. String statistical mechanics provides one way of studying the dynamics on such toroidal backgrounds.[2] String statistical mechanics also provides us with fundamental string behavior in the way of the Hagedorn temperature.[8-25] Thus string statistical mechanics on toroidal backgrounds are fundamentally stringlike and an interesting system. String statistical mechanics on tori has been studied by Turok,[26] Bowick and Giddings,[27] and Deo, Jain, and Tan.[27] The main emphasis has been on the calculation of the density of states, computed either directly or by taking the inverse Laplace transform of the partition function. If one opts for the latter approach then the partition function for string theory is readily computed in the imaginary-time formalism, that is by compactifying the imaginary-time direction on a circle of radius $\beta/2\pi=(2\pi kT)^{-1}$ and computing the one-loop contribution to the vacuum energy. In flat space this method has been studied by many authors.[16-25] On toroidal backgrounds one must take into account momentum and winding-number constraints in computing the density of states.[26,27] In the imaginary-time formalism this is accomplished by forming a constrained partition function as was done in Ref. 27.

In gauge theories it is known that one can impose a charge or colorless constraint by introducing a Wilson line A_0^a in the imaginary-time direction and integrating the partition function over all such Wilson lines. Note that in the imaginary-time formalism the imaginary-time direction is compact so it makes sense to put a Wilson line through it. Similarly we find that momentum and winding-number constraints can be treated by turning on nonzero values of g_{0i} and B_{0i} in the imaginary-time direction and integrating over all such values. Here $g_{\mu\nu}$ and $B_{\mu\nu}$ represent the Euclidean metric and antisymmetric tensor field.

This paper is organized as follows. In Sec. II we form an expression for the density of states under the constraint that the total momentum and winding number are zero by introducing nonzero values of g_{00}, g_{0i}, and B_{0i} in the imaginary-time formalism and integrating over all such values. In addition by turning on g_{ij} and B_{ij} in the compactified spatial directions we effectively introduce nonzero values of $g_{\mu\nu}$ and $B_{\mu\nu}$. In Sec. III we discuss $SL(d,Z)$ and $O(d,d,Z)$ transformation properties of the thermal partition function. In Sec. IV we state our main conclusions and discuss open questions raised by this work.

II. WINDING NUMBER, THE ANTISYMMETRIC TENSOR FIELD, AND DUALITY

There are three globally conserved quantities present in any string theory: the total energy E, the total momentum P^i, and the total winding number L^i. The last quantity is peculiar to string theory and present when the spatial directions are not simply connected. We shall consider the special case when the spatial dimensions form a torus of radii R_i. We shall consider off-diagonal elements of the spatial metric shortly. We may count the number of states available to the string theory of total energy E, total momentum zero, and total winding number zero from the expression

$$\sigma_d(E)=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} d\beta \int d^{d-1}x \int d^{d-1}y \,\mathrm{tr}(e^{-\beta H+ix^iP^i+iy^iL^i})$$
$$=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} d\beta \int d^{d-1}x \int d^{d-1}y \, Z(\beta,x^i,y^i) \,. \tag{2.1}$$

The requirement of zero spatial momentum is familiar for any closed spatial topology. As momentum is quantized as mR^{-1} and winding number as nR clearly both momentum and winding-number constraints must be imposed if the density of states is to be invariant under $R \rightarrow R^{-1}$. For the case of free string theory the trace can be evaluated by standard methods with the result

$$\ln Z(\beta,x^i,y^i)=-\sum_{m_i}\sum_{n_i}\int \rho(m)dm\,\ln\left\{1-\exp\left[-\beta\left[\frac{n_in_i}{R_i^2}+\frac{m_im_i}{\alpha'^2}R_i^2+m^2\right]^{1/2}+i\frac{x^in^i}{R_i}+i\frac{y^im^i}{\alpha'}R_i\right]\right\}, \tag{2.2}$$

where $\rho(m)$ represents the density of single-string states of mass m. One can use the identity

$$\ln[1-\exp(-\beta E-\alpha)]=\tfrac{1}{2}\sum_r \ln\left[\left[\frac{2\pi(r+i\alpha)}{\beta}\right]^2+E^2\right]-\tfrac{1}{2}\beta E \tag{2.3}$$

to obtain

$$\begin{aligned}
-\beta&\left[\prod_i R_i\right]c_d(R_i)+\ln Z(\beta,x^i,y^i)\\
&=\tfrac{1}{2}\int_0^\infty \frac{d\tau_2}{\tau_2}\sum_r\sum_{m^i}\sum_{n^i}\int dm\,\rho(m)\exp\left\{-\pi\tau_2\left[\left[r-\frac{x^in^i}{R_i}-\frac{y^im^i}{\alpha'}R_i\right]^2\frac{1}{\beta^2}+\frac{n^{i2}}{R_i^2}+\frac{m^{i2}}{\alpha'^2}R_i^2+m^2\right]\right\}\\
&=\tfrac{1}{2}\int_{-1/2}^{1/2}d\tau_1\int_0^\infty\frac{d\tau_2}{\tau_2}\sum_r\sum_{m^i}\sum_{n^i}\exp\left\{-\pi\tau_2\left[\left[r-\frac{x^in^i}{R_i}-\frac{y^im^i}{\alpha'}R_i\right]^2\frac{1}{\beta^2}\right]\right\}\\
&\qquad\times\exp\left[i\pi(\tau_1+i\tau_2)\left[\frac{n^i}{R_i}+\frac{m^i}{\alpha'}R_i\right]^2-i\pi(\tau_1-i\tau_2)\left[\frac{n^i}{R_i}-\frac{m^i}{\alpha'}R_i\right]^2\right]\\
&\qquad\times\mathrm{tr}\{\exp[i\pi(\tau_1+i\tau_2)M_L^2]\exp[-i\pi(\tau_1-i\tau_2)M_R^2]\}\ .
\end{aligned} \tag{2.4}$$

Here $c_d(R_i)$ is the Casimir energy density. We have substituted the definition of $\rho(m)$,

$$\rho(m)=\int_{-1/2}^{1/2}d\tau_1\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}d\tau_2 e^{\pi\tau_2 m^2}\mathrm{tr}\{\exp[i\pi(\tau_1+i\tau_2)M_L^2]\exp[-i\pi(\tau_1-i\tau_2)M_R^2]\}\ , \tag{2.5}$$

and used the relation

$$\int_0^\infty dm\,\rho(m)e^{-\pi\tau_2 m^2}=\int_{-1/2}^{1/2}d\tau_1\mathrm{tr}\{\exp[i\pi(\tau_1+i\tau_2)M_L^2]\exp[-i\pi(\tau_1-i\tau_2)M_R^2]\}\ . \tag{2.6}$$

Now expression (2.4) is clearly related to the vacuum energy resulting from some sort of compactification. For $x^i=y^i=0$ we have the vacuum energy resulting from the compactification of the imaginary-time direction on a circle of radius $\beta/2\pi$.[16] The vacuum energy of a toroidal compactification in the presence of constant background g_{ij}, B_{ij}, and A_i^a fields in the compactified directions has been computed in Ref. 28. Comparison with those results indicates that $(\prod_i R_i)c_d(R_i)-\beta^{-1}\ln Z(\beta,x^i,y^i)$ represents the vacuum energy produced upon compactification of the imaginary-time direction on a circle of radius $\beta/2\pi$ with constant background fields $g^{0i}/g^{00}=x^iR_i^{-1}$ and $B_{0i}=B_i=-y^iR_i$ turned on in this direction. Furthermore the turning on of a constant vector potential A_0^a and subsequent integration would impose an overall colorless constraint on physical states[29] but we shall not do so here. In the limit $R\to\infty$ it is well known that expression (2.4) can be rewritten as an integral over the fundamental region of moduli space by the inclusion of a winding number in the imaginary-time direction and the definition of a suitable F function.[18,20]

Now we generalize our previous results to include off-diagonal metrics g_{ij} and antisymmetric tensor fields B_{ij}. Together with the g_{0i} and B_{0i} introduced above this amounts to introducing a constant background $g_{\mu\nu}$ and $B_{\mu\nu}$ on a d-dimensional torus T^d. We parametrize the four-metric as

$$[g_{\mu\nu}]=\begin{bmatrix}\beta^2+g_kg_l\gamma^{kl} & -g_k\\ -g_l & \gamma_{lk}\end{bmatrix} \tag{2.7}$$

with inverse metric then

$$[g^{\mu\nu}]=\begin{bmatrix}\beta^{-2} & \beta^{-2}g_l\gamma^{il}\\ \beta^{-2}g_l\gamma^{jl} & \gamma^{ij}+\beta^{-2}g_kg_m\gamma^{ik}\gamma^{jm}\end{bmatrix}\ . \tag{2.8}$$

As bosonic theories at finite temperature can be formulated by compactifying the imaginary-time direction, one can extend our previous results for toroidal compactification by defining

$$F(\beta,g_i,B_i,\gamma_{ij},B_{ij})=\tau_2^{d/2}\frac{1}{\sqrt{g}}\sum_{(n)}\sum_{(m)}\exp[i\pi\tau_1(2n^\mu m_\mu)-\pi\tau_2(g^{\mu\nu}m_\mu m_\nu-2B^\mu{}_\nu m_\mu n^\nu+B_{\lambda\mu}B^\lambda{}_\nu n^\mu n^\nu+g_{\mu\nu}n^\mu n^\nu)]\ . \tag{2.9}$$

The insertion of F into the vacuum amplitude of a string theory yields the identity

$$-\frac{1}{2}\int_{-1/2}^{1/2}d\tau_1\int_{\sqrt{1-\tau_1^2}}^{\infty}\frac{d\tau_2}{\tau_2^2}P_B(\tau_1,\tau_2)F(\beta,g_i,B_i,\gamma_{ij},B_{ij})\sqrt{g}=\beta\sqrt{\gamma}\Lambda_d+\beta\sqrt{\gamma}c_d(\gamma_{ij},B_{ij})-\ln\tilde{Z}(\beta,g_i,B_i,\gamma_{ij},B_{ij})\ , \tag{2.10}$$

where Λ_d and $c_d(\gamma_{ij},B_{ij})$ are the vacuum and Casimir energy densities. We have defined

$$P_B(\tau_1,\tau_2)=\tau_2^{(2-d)/2}\mathrm{tr}\{\exp[-\pi\tau_2(M_L^2+M_R^2)+i\pi\tau_1(M_L^2-M_R^2)]\}\ .$$

Shapere and Wilczek[5] have shown that the inclusion of the $B_{\mu\nu}$ field extends the discrete symmetry from $SL(d,Z)$ to $O(d,d,Z)$ and this modular invariance is big enough to include the duality symmetries $\beta\to 1/\beta$ and $\gamma_{ij}\to\gamma^{ij}$. Our final expression for the density of states is given by

$$\begin{aligned}\tilde{\sigma}_d(E)&=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}d\beta\, dg_i dB_i e^{\beta E}\tilde{Z}(\beta,g_i,B_i,\gamma_{ij},B_{ij})\\&=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}d\beta\, dg_i dB_i e^{\beta E}\exp\left[\frac{1}{2}\int_{-1/2}^{1/2}d\tau_1\int_{\sqrt{1-\tau_1^2}}^{\infty}\frac{d\tau_2}{\tau_2^2}P_B(\tau_1,\tau_2)F(\tau,\beta,g_i,B_i,\gamma_{ij},B_{ij})\sqrt{g}\right]\\&\qquad\times\exp[\beta\sqrt{\gamma}\Lambda_d+\beta\sqrt{\gamma}c_d(\gamma_{ij},B_{ij})]\ .\end{aligned} \tag{2.11}$$

This formula should be compared from the sum over the point-particle expression discussed previously:

$$\begin{aligned}\sigma_d(E)&=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}d\beta\, dg_i dB_i e^{\beta E}Z(\beta,g_i,B_i,\gamma_{ij},B_{ij})\\&=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}d\beta\, dg_i dB_i e^{\beta E}\exp\left[\frac{1}{2}\int_{-1/2}^{1/2}d\tau_1\int_{0}^{\infty}\frac{d\tau_2}{\tau_2^2}P_B(\tau_1,\tau_2)F'(\tau,\beta,g_i,B_i,\gamma_{ij},B_{ij})\sqrt{g}\right]\exp[\beta\sqrt{\gamma}c_d(\gamma ij,B_{ij})]\ .\end{aligned} \tag{2.12}$$

In this formula F' is identical with F except the winding number n^0 in the imaginary-time direction is fixed to be zero.

We seek a generalization of the McClain–Roth–O'Brien–Tan theorem including the constraints of momentum and winding number. In Ref. 30, Shiraishi has derived a generalized McClain–Roth–O'Brien–Tan Theorem for the special case of a one-dimensional circle compactification. In the Appendix we extend his result to the completely compactified case considered here. This in turn tells us that

$$Z(\beta,g_{ij},B_{ij},g_{0i},B_{0i})=\tilde{Z}(\beta,g_{ij},B_{ij},g_{0i},B_{0i})$$

and $\sigma_d(E)=\tilde{\sigma}_d(E)$. Again such constrained partition functions are of interest in studying the high-temperature limit of string theory.[26,27]

III. SL(d,Z) AND O(d,d,Z) INVARIANCE OF THE THERMAL PARTITION FUNCTION

$SL(2,R)$ and $SL(2,Z)$ invariance have proved to be useful symmetries in $(1+1)$-dimensional gravity. Indeed the inclusion of winding and twisted sectors arises from the desire to maintain these symmetries. We thus suspect that $SL(d,R)$ and $SL(d,Z)$ will prove useful in studying d-dimensional gravity.

The d-dimensional partition function $Z_d(\beta,g_i,\gamma_{ij})=Z_d(g_{\mu\nu})$ is one quantity which possesses these symmetries (for the time being we set $B_{\mu\nu}=0$). It is convenient to define $\tilde{g}_{\mu\nu}$ through $g_{\mu\nu}=g^{1/d}\tilde{g}_{\mu\nu}$ so that $\det(\tilde{g}_{\mu\nu})=1$. It is well known that $\tilde{g}_{\mu\nu}$ parametrizes $SL(d,R)/SO(d)$ and is transformed by $SL(d,R)$ and $SL(d,Z)$ according to

$$\tilde{g}''_{\mu\nu}=L_{\mu\lambda}\tilde{g}_{\lambda\sigma}L^T_{\sigma\nu}\ , \tag{3.1}$$

where $L\in SL(d,Z)$. If we define $C_d(g_{\mu\nu})=g^{1/2}c_d(\gamma_{ij})$ as the total Casimir energy times β then the partition function obeys

$$\begin{aligned}Z_d(g_{\mu\nu})e^{-C_d(g_{\mu\nu})}&=Z_d(g^{1/d}\tilde{g}_{\mu\nu})e^{-C_d(g^{1/2}\tilde{g}_{\mu\nu})}\\&=Z_d(g^{1/d}\tilde{g}''_{\mu\nu})e^{-C_d(g^{1/d}\tilde{g}''_{\mu\nu})}\ .\end{aligned} \tag{3.2}$$

In $1+1$ dimensions this formula has been applied to string theory. For a single bosonic scalar field we have

$$(\beta/L)^{-1/2}e^{-\beta Lc_2(L)}\prod_n(1-e^{-2\pi(|n|\beta+ing_1L^{-1})/L})^{-1}=(\beta''/L)^{-1/2}e^{-\beta''Lc_2(L)}\prod_n(1-e^{-2\pi(|n|\beta''+ing_1''L^{-1})})^{-1}\ . \tag{3.3}$$

In $1+1$ dimensions the Casimir energy of a massless scalar field is $c_2(L)=-4\pi/24L^2$. Usual treatments work with the variables $\tau_2=\beta/L$ and $\tau_1=g_1L^{-2}$. Finally the inclusion of the Casimir energy explains the $x^{1/24}$ prefactor in the Dedekind η function

$$\eta(x)=x^{1/24}\prod_{n=1}^{\infty}(1-x^n)$$

(Ref. 31).

It is known that $g_{\mu\nu}$ and $B_{\mu\nu}$ parametrize $O(d,d,R)/[O(d)\times O(d)]$ (Ref. 28) and again Wilczek and Shapere[5] have recently shown that the $SL(d,Z)$ invariance is extended to $O(d,d,Z)$ invariance by the inclusion of the antisymmetric tensor field. This invariance includes the duality transformation

$$\begin{aligned} g'_{\mu\nu} &= (g - Bg^{-1}B)^{-1}{}_{\mu\nu} , \\ B'_{\mu\nu} &= (B - gB^{-1}g)^{-1}{}_{\mu\nu} . \end{aligned} \tag{3.4}$$

If one works in the $\tilde{Z}(g_{\mu\nu},B_{\mu\nu})$ representation so that the integrand of the free energy is modular invariant, then

$$\tilde{Z}_d(g_{\mu\nu},B_{\mu\nu})e^{-[g^{1/2}\Lambda + C_d(g_{\mu\nu})]} = \tilde{Z}_d(g'_{\mu\nu},B'_{\mu\nu})e^{-[g'^{1/2}\Lambda + C_d(g'_{\mu\nu})]} . \tag{3.5}$$

Thus $SL(d,Z)$ invariance tells us that the exponential of the Casimir energy must be included as a prefactor to the partition function. $O(d,d,Z)$ invariance tells us that the exponential of the cosmological constant should also be included as a prefactor.

IV. CONCLUSION

We have formed an expression for the density of states for a bosonic string theory when the spatial topology of the Universe is that of a torus. We have done this by introducing nonzero values of g_{00}, g_{0i}, and B_{0i} in the imaginary-time formalism and integrating over these values. We have introduced two partition functions $\tilde{Z}(g_{\mu\nu},B_{\mu\nu})$ and $Z(g_{\mu\nu},B_{\mu\nu})$ depending on whether or not a winding number is introduced in the imaginary-time direction. In the Appendix we extend the result of Shiraishi[30] to show the equivalence of $\tilde{Z}(g_{\mu\nu},B_{\mu\nu})$ and $Z(g_{\mu\nu},B_{\mu\nu})$. It is necessary to explicitly check this as work by Burgess, Hambli, and Kshirsager[32] on superstring Casimir energies in exotic situations indicates that such generalizations do not always exist. If a winding number is introduced in the imaginary-time direction, then the integrand of the one-loop contribution to the free energy is modular invariant as in the unconstrained case. Interactions can then be included by working on higher-genus surfaces if an appropriate generalization of the Fishler Susskind mechanism exists. Recent work by Gribosky, Donoghue, and Holstein[33] on field theory at finite temperature in curved space indicates that such a generalization is possible. In the modular-invariant representation we have found that the exponential of the Casimir energy and cosmological constant should be included as a prefactor in order that the partition function be invariant under $SL(d,Z)$ and $O(d,d,Z)$. Finally this paper has been concerned with the statistical mechanics of bosonic string theories by compactifying the imaginary-time direction on a torus of radius $\beta/2\pi$. We can extend this analysis to more realistic string theories containing fermions (such as the heterotic string) by compactifying the imaginary-time direction on a twisted torus of radius $\beta/2\pi$ as was done in Refs. 20, 22, and 23.

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APPENDIX: GENERALIZED McCLAIN–ROTH–O'BRIEN–TAN THEOREM

In this appendix we extend the result of Shiraishi[30] to the completely compactified case in order to show the equivalence of $\tilde{Z}(g_{\mu\nu},B_{\mu\nu})$ and $Z(g_{\mu\nu},B_{\mu\nu})$. Performing a Poisson resummation of (2.9) we obtain

$$F(\tau,\beta,g_i,B_i,\gamma_{ij},B_{ij}) = \sum_{(m)}\sum_{(n)} \exp\left[-\frac{\pi}{\tau_2} g_{\mu\nu}(m_\mu + \tau n^\mu)(m_\nu + \tau^* n^\nu) + 2\pi i B_{\mu\nu} m_\mu n^\nu\right] . \tag{A1}$$

Now as in the usual McClain–Roth–O'Brien–Tan theorem set

$$\tau' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tau = \frac{a\tau + b}{c\tau + d} ,$$

where $(m_0, n^0) = r(c,d)$ and the greatest common divisor of c and d is one:

$$\begin{aligned} F(\tau,\beta,g_i,B_i,\gamma_{ij},B_{ij}) = \sum_r \sum_{(m')} \sum_{(n')} \exp\Bigg[&-\frac{\pi}{\tau'_2} g_{00} r^2 - \frac{\pi}{\tau'_2} g_{0i} r (d + c\tau^*)^{-1}(m_i + \tau^* n^i) \\ &- \frac{\pi}{\tau'_2} g_{i0}(d + c\tau)^{-1}(m_i + \tau n^i) r - \frac{\pi}{\tau'_2} g_{ij} |d + \tau c|^{-2}(m_i + \tau n^i)(m_j + \tau^* n^j) \\ &+ 2\pi i B_{0i} r\, d n^i + 2\pi i B_{i0} m_i r c + 2\pi i B_{ij} m_i n^j \Bigg] . \end{aligned} \tag{A2}$$

The main device used in Ref. 30 is to define $m'_i = a m_i - b n^i$ and $n^{i\prime} = d n^i - c m_i$, where $ad - bc = 1$. This allows us to rewrite (A2) in terms of τ', m'_i, and $n^{i\prime}$ as

$$F(\tau,\beta,g_i,B_i,\gamma_{ij},B_{ij})=\sum_r \sum_{(m')} \sum_{(n')} \exp\left[-\frac{\pi}{\tau_2'}g_{00}r^2-\frac{\pi}{\tau_2'}g_{0i}r(m_i'+\tau'^*n^{i'})-\frac{\pi}{\tau_2'}g_{i0}(m_i'+\tau'n^{i'})r -\frac{\pi}{\tau_2'}g_{ij}(m_i'+\tau'n^{i'})(m_j'+\tau'^*n^{j'})+2\pi i B_{0i}rn^{i'}+2\pi i B_{ij}m_i'n^{j'}\right] . \quad \text{(A3)}$$

Identifying r as the momentum in the imaginary-time direction we see that the summand in (A3) is equivalent to fixing n^0 in (A1) to zero. This is by definition the summand of F' which was used to define $Z(g_{\mu\nu},B_{\mu\nu})$. The sum over c,d still exists in the integration over τ and sums up to yield the region $\tau_i \in [-\frac{1}{2},\frac{1}{2}]$, $\tau_2 \in [0,\infty]$ which as in the usual case is the integration region of $Z(g_{\mu\nu},B_{\mu\nu})$. The only new feature of this proof from the one-dimensional case is the presence of the last term in (A3) which is identically zero for a one-dimensional compactification.

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