Model-independent determination of K-matrix poles and residues in the $\Delta(1232)$ region from the multipole data for pion photoproduction

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Using only the analytic structure of the K matrix and the assumption that the Compton-scattering amplitude is small compared to strong and electrostrong amplitudes, we extract the pole position and the residues for the resonant multipoles, in a model-independent and background-free fashion, directly from the experimentally obtained multipole data base in the $\Delta(1232)$ region, posing a challenge for QCD-inspired hadron models.

I. INTRODUCTION

Feynman,[1] in his widely admired Caltech lectures, *Photon-Hadron Interactions,* puts the advantage of probing hadrons with photons in his opening sentence: "One very powerful way of experimentally investigating the strongly interacting particles (hadrons) is to look at them, to probe them with a known particle; in particular the photon (no other is known as well)." He then goes on to contrast photons with hadronic probes, and his enthusiasm in favor of photons becomes evident. In his seventh lecture in this series, he is examining the difficulty of analyzing the resonance excitation, and poses the following question.

"How much is resonances and how much is background? Can the background below a resonance be simply tails of other resonances? ... How big is the tail of a resonance?" He adds as an answer to the *last* question: "Impossible to answer except arbitrarily." It is the *first* of these questions that we shall be concerned with in this paper. We shall try to answer it in one specific example, that of exciting the $\Delta(1232)$ resonance.

There is considerable topical interest[2] in the existing experimental data base on the photoproduction of mesons from nucleons in the resonance region and extraction of electromagnetic transition amplitudes for various nucleon resonances, in the wake of its qualitative improvements possible in newer accelerators under construction, such as the Continuous Electron Beam Accelerator Facility (CEBAF). The analyses of data are most complete for the first resonance region, $\Delta(1232)$, and many multipole sets are available[3] in this domain.

A fundamental issue connected with Feynman's earlier cited question, which has not been addressed in the literature so far, is whether we can extract any *model-independent* information from this vast electromagnetic multipole data base, in contrast with the *model-dependent* separations of resonance and background contributions[1] to the multipole amplitudes. It is this issue that we shall address here. The central result of this paper is to show that we can determine, from the available multipole data base, obtained by the direct energy-independent analyses of experiments on pion photoproduction on the nucleon targets from threshold through the $\Delta(1232)$ energy region, *the K-matrix pole position and residues for the resonant multipoles in a model-independent and background-free fashion.* The only assumptions that need to be made are on the analytic structure of the K matrix and the smallness of the Compton-scattering amplitude compared with strong and electrostrong amplitudes.

II. K-MATRIX FORMALISM

Consider the πN scattering and photoproduction of pions in the $\Delta(1232)$ region. Let the scattering amplitude be represented either by the K matrix or by the T matrix, the subscripts of which indicate the process it is describing. Assuming the K matrix to have a rapidly varying resonant piece and a smooth "background," indicated below by the first and second terms on the right-hand side of Eqs. (1a) and (1b), we have

$$K_{\gamma\pi}=\frac{A}{M-W}+B\ , \tag{1a}$$

$$K_{\pi\pi}=\frac{C}{M-W}+D\ , \tag{1b}$$

where W is the center-of-momentum (c.m.) energy, M gives the location of the K-matrix pole, and A,B,C,D are smooth functions of W. Model dependence will arise in determining A away from $W=M$, as is seen in Taylor expanding $A(W)$ about M:

$$K_{\gamma\pi}(W)=\frac{A(M)}{M-W}+\left[\frac{dA}{dW}\right]_{W=M}+B(M)+\cdots\ , \tag{2}$$

so what we call "background" in this paper may also include "resonant" contributions, as it has been stressed.[1,4]

The T and K matrices are related to each other by

$$T=K(1-iK)^{-1}=K(1+iT) \ . \qquad (3)$$

Thus, ignoring electromagnetic corrections, $T_{\pi\pi}$ is given by (1b) and (3):

$$T_{\pi\pi}=\frac{C+(M-W)D}{M-W-i[C+(M-W)D]} \ . \qquad (4)$$

So at the K-matrix pole of the resonance at $W=M$, we obtain the results[5]

$$\mathrm{Im}T_{\pi\pi}=i \ , \qquad (5a)$$

$$\frac{d}{dW}(\mathrm{Re}T_{\pi\pi})=-1/C \ . \qquad (5b)$$

From the second of Eqs. (3), we can write an expression for the pion-photoproduction T matrix:

$$T_{\gamma\pi}=K_{\gamma\pi}+iK_{\gamma\pi}T_{\pi\pi}+iK_{\gamma\gamma}T_{\gamma\pi}+\cdots \ , \qquad (6)$$

where $K_{\gamma\gamma}$ is the Compton-scattering K matrix. For the $\Delta(1232)$ resonance, the additional channels indicated by the ellipsis in Eq. (6) are absent. Neglecting the Compton-scattering amplitude $K_{\gamma\gamma}$, compared with strong and electrostrong amplitudes, Eq. (6) simplifies to

$$T_{\gamma\pi}\simeq K_{\gamma\pi}(1+iT_{\pi\pi}) \ . \qquad (7)$$

Denoting the denominator in the right-hand side of Eq. (4) by E, we get the following expression for $T_{\gamma\pi}$:

$$T_{\gamma\pi}=\frac{A}{E}+(M-W)\frac{B}{E} \ . \qquad (8)$$

We can now write down the expressions for the real and imaginary parts of the T matrix:

$$\mathrm{Re}T_{\gamma\pi}=\frac{[A+(M-W)B](M-W)}{(M-W)^2+[C+(M-W)D]^2} \ ,$$
$$\mathrm{Im}T_{\gamma\pi}=\frac{[A+(M-W)B][C+(M-W)D]}{(M-W)^2+[C+(M-W)D]^2} \ . \qquad (9)$$

In terms of the πN-scattering partial-wave amplitudes, $f_{l\pm}=T_{\pi\pi}/q$, and for the pion-photoproduction multipole amplitude, we can write the relation $T_{\gamma\pi}=\sqrt{qk}\,M_{\gamma\pi}$, q and k being pion and photon c.m. momenta, $M_{\gamma\pi}$ being the $\gamma\pi$ multipole amplitude. Additional angular momentum factors are also present in the relation between $T_{\gamma\pi}$ and $M_{\gamma\pi}$, but are of no importance here. We shall include these factors when comparing with amplitudes given by the Particle Data Group[6] (PDG).

Let us now recall that $\mathrm{Re}T_{\gamma\pi}$ and $\mathrm{Im}T_{\gamma\pi}$ are given as data bases[3] from the analyses of many previous experiments involving differential cross sections, photon asymmetries, etc. Given these, what can we learn from them in a *model-independent* background-free way? To answer this basic question, the central objective of this paper, we first note that the quantity $\mathrm{Re}T_{\gamma\pi}$ *vanishes* at $W=M$, while $\mathrm{Im}T_{\gamma\pi}$ *cannot vanish* at $W=M$, unless $A=0$. We, thus, have the following two relations for the residue of the K-matrix pole for the $\gamma\pi$ reaction:

$$(\mathrm{Im}T_{\gamma\pi})_{W=M}=\frac{A}{C} \ , \qquad (10a)$$

$$-C^2\left[\frac{d}{dW}\mathrm{Re}T_{\gamma\pi}\right]_{W=M}=A \ . \qquad (10b)$$

Equations (10a) and (10b) are the *central relations* of this paper, both of which are free from the effective background contributions B and D.

It is worth stressing the fact that strong-interaction physics, represented here by the residue C, drops out in the ratio of two electromagnetic multipole residues. Thus, this ratio should have fundamental significance in comparing with hadronic models, which, in most cases, ignore the treatment of the final-state interaction of the resonance, responsible for its width. Thus far, all analyses of resonance multipoles are plagued by the model dependence of the treatment of strong interaction. This point is fundamental to the raison d'être of this analysis.

III. ANALYSIS OF DATA: PROCEDURE

In the $\Delta(1232)$ resonance region, the multipoles of interest are the E_{1+} and the M_{1+}, both for isospin $\frac{3}{2}$. It should now be easy to get the K-matrix residues for the E_{1+} and M_{1+}, using Eq. (10a). One merely determines $\mathrm{Im}E_{1+}$ and $\mathrm{Im}M_{1+}$ at $W=M$, which according to (2) or (5a) is just the energy at which the P_{33} phase shift passes through 90°. Recall that $\tan\delta=K_{\pi\pi}$, where δ is the P_{33} phase shift. Our knowledge of the K-matrix residue is currently limited by how well we can evaluate $\mathrm{Im}T_{\gamma\pi}$ or $(d/dW)\mathrm{Re}T_{\gamma\pi}$ at $W=M$. We are tacitly assuming here that we can neglect higher-order electromagnetic corrections to (10a) and (10b).

There are, in practice, a few difficulties in using (10a) or (10b) to get the K-matrix residues. First, the multipoles are not already available from experiments at $W=M$, and one must interpolate the data to $W=M$. This introduces additional uncertainties in the determination of the residues. The second problem is that the extant multipole data sets[3] are often in poor statistical agreement with one another, making an estimate of the error on the residues difficult. How we handle this problem will be discussed below.

We have used two different methods to analyze the multipole data sets[3] of Berends and Donnachie (BD), Pfeil and Schwela (PS), Grushin *et al.* (GRU), Miroshnichenko *et al.* (MIR), Get'man *et al.* (GET), and Suzuki *et al.* (SUZ). In the first method we use a Lagrange interpolating function[7] for both (10a) and (10b); in the second method we assume some energy dependence for A and B in (1a) and do a two-parameter fit to the $\mathrm{Re}E_{1+}$ ($\mathrm{Re}M_{1+}$) and to the $\mathrm{Im}E_{1+}$ ($\mathrm{Im}M_{1+}$), in separate fits. Although the real and imaginary parts of the multipoles are related by Watson's theorem,[8] fits to the real and imaginary parts will not necessarily give the same results, since the errors are *not* related by Watson's theorem, as is discussed below. We analyze BD's data that are in the range 320 MeV $\le K_L \le$ 360 MeV (5 points), K_L being the photon laboratory energy. For all other data sets, we analyze the data in the range $300 \leq K_L \leq 400$ MeV (6 points each for PS, MIR, and GET, and 5 points each for GRU and SUZ).

As a matter of completeness, we recall that the Lagrange interpolating function is an $(N-1)$ degree polynomial that passes through all N data points and has the form

$$L(W)=\sum_{i=1}^{N} L_i \left[\prod_{j\neq i=1}^{N}(W-W_j)\right] \Big/ \left[\prod_{j\neq i}^{N}(W_i-W_j)\right] , \tag{11}$$

and its derivative is

$$\frac{dL(W)}{dW}=\sum_{i=1}^{N} L_i \left[\sum_{k\neq i}\prod_{j\neq i\neq k}^{N}(W-W_j)\right] \Big/ \left[\prod_{j\neq i}^{N}(W_i-W_j)\right] . \tag{12}$$

Here L_i is the data point at energy W_i. In addition to the errors arising from the interpolation function, the multipoles also have an error σ_i from which we can estimate the error σ_L for L and $\sigma_{L'}$ for dL/dW:

$$\sigma_L^2=\sum_i \left[\frac{\partial}{\partial L_i}[L(W)]\sigma_i\right]^2 , \tag{13}$$

$$\sigma_{L'}^2=\sum_i \left[\frac{\partial}{\partial L_i}\left[\frac{dL(W)}{dW}\right]\sigma_i\right]^2 . \tag{14}$$

Other choices of interpolating functions are of course possible, and this is the reason for our second method of analyzing the data. If the K-matrix residue for the E_{1+} were zero, we find that (12) would not give zero, although it would be zero within the quoted error. This is tested by taking in (1a) some functions $a(W)$ and $b(W)$ that reproduce the shape of $\mathrm{Re}E_{1+}$, but has $a(M)=0$. In units where the error obtained from (14) is ~ 2, we find the residue obtained from the slope of $\mathrm{Re}E_{1+}$ [Eq. (12)] to be ~ 1. On the other hand, $\mathrm{Im}E_{1+}$ [Eq. (11)] gives $\sim 5\times 10^{-3}$ in the same units.

The explicit functional form for $a(W)$ and $b(W)$ that we have used is

$$\zeta=\frac{qk}{M_N^2}\left[\frac{A}{M-W}+B\right]\cos\delta e^{i\delta} , \tag{15}$$

where M_N is the nucleon mass (938.9 MeV), ζ is either the M_{1+} or the E_{1+}, and A and B are to be determined from minimizing the χ^2. This form is particularly simple, and we may analytically solve for A and B that minimize the χ^2; in this case there is a unique minimum. Furthermore we may find analytic expressions for the χ^2+N contours in the A,B plane. Denoting

$$f_R(W_i)=\frac{qk}{M_N}\cos^2\delta/(M-W_i),\quad f_I(W_i)=\frac{qk}{M_N^2}\cos\delta\,\sin\delta/(M-W_i) ,$$

$$g_R(W_i)=\frac{qk}{M_N}\cos^2\delta,\quad \text{and } g_I=\frac{qk}{M_N^2}\cos\delta\,\sin\delta ,$$

we have, from the real part of the multipole,[3]

$$A=\frac{\left[\sum_{i=1}^{N}\frac{L_i g_R(W_i)}{\sigma_i^2}\right]\left[\sum_i\frac{f_R(W_i)g_R(W_i)}{\sigma_i^2}\right]-\left[\sum_i\frac{g_R^2(W_i)}{\sigma_i^2}\right]\left[\sum_i\frac{L_i f_R(W_i)}{\sigma_i^2}\right]}{D} , \tag{16}$$

$$B=\frac{\left[\sum_i\frac{L_i g_R(W_i)}{\sigma_i^2}\right]\left[\sum_i\frac{f_R(W_i)g_R(W_i)}{\sigma_i^2}\right]-\left[\sum_i\frac{f_R^2(W_i)}{\sigma_i^2}\right]\left[\sum_i\frac{L_i g_R(W_i)}{\sigma_i^2}\right]}{D} , \tag{17}$$

where L_i is the experimental multipole at energy W_i, σ_i is the error, and

$$D=\left[\sum_i\frac{f_R(W_i)g_R(W_i)}{\sigma_i^2}\right]^2-\left[\sum_i\frac{g_R^2(W_i)}{\sigma_i^2}\right]\left[\sum_i\frac{f_R^2(W_i)}{\sigma_i^2}\right] . \tag{18}$$

Similar expressions hold when evaluating the imaginary part of the multipole with $f_R \to f_I$ and $g_R \to g_I$.

Defining ΔA and ΔB as the changes in A and B that increase χ^2 from the minimum to χ^2+N, we find

$$N = \Delta A^2 \sigma_A^2 + 2\sigma_{AB} \Delta A \Delta B + \Delta B^2 \sigma_B^2 , \tag{19}$$

where

$$\sigma_A^2 = 1 \Big/ \sum_i \left[\frac{f^2(W_i)}{\sigma_i^2} \right] , \tag{20}$$

$$\sigma_B^2 = 1 \Big/ \sum_i \left[\frac{g^2(W_i)}{\sigma_i^2} \right] , \tag{21}$$

and

$$\sigma_{AB} = 1 \Big/ \sum_i \left[\frac{f(W_i) g(W_i)}{\sigma_i^2} \right] . \tag{22}$$

Here, f and g refer to either f_R and g_R or f_I and g_I.

Having described our methods used to analyze data, we now discuss the data we analyze. As we wish to obtain a reasonable estimate of the errors of the residues, we are faced with the problem that there is often poor statistical agreement among the multipole data sets. To give an extreme example, consider the value of the real part of the amplitude M_{1+} at $K_L = 350$ MeV: BD report $\mathrm{Re}M_{1+} = -4.33 \pm 0.02$, GET give $\mathrm{Re}M_{1+} = 4.43 \pm 0.012$, and MIR have $\mathrm{Re}M_{1+} = -2.307 \pm 0.004$. Some of this discrepancy can be traced to the different sets of phase shifts used by different authors.

The quantity that is normally[9] fitted to the data is

$$\zeta = N e^{i\delta} , \tag{23}$$

where N is real, but can be positive or negative, since Watsons's theorem only fixes the phase to within a multiple of π. Our assumption is that the listed values of the errors, $\Delta(\mathrm{Re}M_{1+})$ and $\Delta(\mathrm{Re}E_{1+})$, do *not* include any error from the phase shifts and thus $\Delta(\mathrm{Re}\zeta) = (\Delta N)\cos\delta$. Thus,

$$\Delta N = \frac{\Delta(\mathrm{Re}\zeta)}{\cos\delta} . \tag{24}$$

In our example above, we obtain, using the phase shifts in Table I, $N = 35.7 \pm 0.2$ for BD, $N = 33.401 \pm 0.09$ for GET, and $N = 33.92 \pm 0.06$ for MIR, which are in better agreement, in the sense defined below, than the corresponding values for $\mathrm{Re}M_{1+}$. There is, however, still large disagreement among these values.

Our strategy for dealing with the errors is the following. We first obtain N_E and N_M from (23) and ΔN_M from (24). We then determine the scaling factors S_E and S_M as defined by the PDG (Ref. 10) (Appendix). The errors ΔN_E and ΔN_M for all data sets[11] are then multiplied by S_E and S_M, respectively. Finally, we obtain $\Delta(\mathrm{Re})$ and $\Delta(\mathrm{Im})$ by taking an error of one degree for the phase shifts. Although this error is somewhat larger than errors quoted in an individual phase-shift analysis,[12] it is somewhat smaller than the spread of phase shifts in Table I. Thus, we have

$$\Delta(\mathrm{Re}\zeta) = [(\Delta N)^2 \cos^2\delta + N^2 \sin^2\delta (\Delta\delta)^2]^{1/2} , \tag{25}$$

$$\Delta(\mathrm{Im}\zeta) = [(\Delta N)^2 \sin^2\delta + N^2 \cos^2\delta (\Delta\delta)^2]^{1/2} , \tag{26}$$

where ΔN includes the scaling factor and $\Delta\delta \simeq 0.017$ rad. Note that in general $\Delta(\mathrm{Im}\zeta)/\Delta(\mathrm{Re}\zeta) \neq (\mathrm{Im}\zeta)(\mathrm{Re}\zeta)$, and thus fitting the real and imaginary parts will in general

TABLE I. Phase shifts used in this analysis for different data sets at various photon laboratory energies K_L are given here. A blank entry means that the authors do not give the multipole at this energy. GET have used the phase shifts of Ref. 5 and MIR and SUZ have used the phase shifts of Ref. 16. BD, PS, and GRU provide either the phase shifts used in their analysis or both the real and imaginary parts of the multipole. It should be stressed that consistency requires the use of phase shifts specific to each data set, and not the latest phase shifts available.

K_L (MeV)	BD	PS	GET (Ref. 5)	MIR (Ref. 16)	GRU (Ref. 9)	SUZ (Ref. 16)
300	57.45	56.49	56.70	53.6	62.91	53.6
310	66.64					
320	75.81	73.12	74.51	69.5	75.51	
325						74.6
330	81.98					
340	90.38					
350	96.97	96.00	97.62	93.9	97.76	
355						97.3
360	105.49	103.72	103.7	100.7		
370	110.69					
375						109.5
380	115.14	114.78	113.5	111.8	107.29	
390	118.95					
400	122.23	122.34	120.7	120.5	123.30	120.5

TABLE II. Weighted averages for the $E_{1+}(\frac{3}{2})$, $\bar{N}_E$, and the $M_{1+}(\frac{3}{2})$, $\bar{N}_M$, at different photon laboratory energies K_L. S_E and S_M are the scale factors defined in the Appendix. $\bar{N}_E$ and $\bar{N}_M$ are in units of $10^{-3} m_\pi^{-1}$, m_π being the pion mass.

K_L (MeV)	$\bar{N}_E$	S_E	$\bar{N}_M$	S_M
300	-1.950 ± 0.035	2.9	38.446 ± 0.048	2.1
320	-1.005 ± 0.048	2.1	40.640 ± 0.044	9.1
350	-0.215 ± 0.036	1.7	34.119 ± 0.046	9.7
360	-0.026 ± 0.042	2.1	33.814 ± 0.048	8.3
380	$+0.360\pm0.045$	2.4	29.107 ± 0.051	3.5
400	$+0.632\pm0.046$	2.4	25.646 ± 0.042	3.8

give different results.

The values of S_E and S_M are given in Table II. We have calculated S_E and S_M where the data sets overlap the most. Thus, at $K_L=300$ and 400 MeV, all data sets have been used; at $K_L=320$, 350, and 380 MeV, all data sets except SUZ have been used, and at $K_L=360$ MeV all data sets except SUZ and GRU have been used. At other energies we simply linearly interpolate the values of S_E and S_M in Table II. It is worth pointing out that $S_M=9.7$ at $K_L=350$ MeV, but calculating the scaling factor for $\mathrm{Re}M_{1+}$ at $K_L=350$ MeV gives 99.7.

IV. RESULTS OF OUR ANALYSIS

In Table III we give the results for χ_E and χ_M defined by

$$\chi_M = \left[\frac{d}{dW}\mathrm{Re}M_{1+}\right]_{W_M}, \tag{27a}$$

$$\chi_E = \left[\frac{d}{dW}\mathrm{Re}E_{1+}\right]_{W_E}, \tag{27b}$$

where χ is in units of $10^{-3}\ \mathrm{MeV}^{-1} m_\pi^{-1}$ (we use $\hbar=c=1$, $m_\pi=139.6$ MeV), and $W_E(W_M)$, in MeV, is the energy at which $\mathrm{Re}E_{1+}$ ($\mathrm{Re}M_{1+}$) goes to zero according to the interpolating function, Eq. (11). In principle we should expect W_E and W_M to be the energy at which $\delta=90°$, and therefore a comparison of W_E and W_M is a test of the accuracy of the interpolating procedure. We find that W_M agrees with the resonant energy (≡energy at which $\delta=90°$) to within 0.5 MeV for all data sets except SUZ where the agreement is within 2 MeV. Also, W_E and W_M agree to within two MeV, except for GET and PS, where there is a 5-MeV difference. Since $\Delta W/W \simeq 0.5\%$, we expect ΔW to have a negligible effect on $\Delta\chi_E$ and $\Delta\chi_M$, the errors of χ_E and χ_M, respectively.

In the tests we have done with the interpolating function, we have found that interpolating χ_E is accurate to within $0.3\times10^{-6}\ \mathrm{MeV}^{-1} m_\pi^{-1}$ and χ_M is accurate to within $0.02\times10^{-3}\ \mathrm{MeV}^{-1} m_\pi^{-1}$. In both cases these uncertainties are small compared to the errors propagated from the data via the error of the derivative in Eq. (14). Thus the errors listed in Table III include only the error propagated from the data. Except for the data set of SUZ, the values of χ_M are in good agreement. The values of χ_E obtained from the different data sets are also in good agreement, but the errors are very large. We find *weighted* averages of $\chi_E=(7.85\pm2.55)\times10^{-3}$ and $\chi_M=-0.743\pm0.037$; if we exclude SUZ from the averages, we obtain $\chi_E=(6.88\pm2.68)\times10^{-3}$ and $\chi_M=-0.667\pm0.047$, all in units of $10^{-3}\ \mathrm{MeV}^{-1} m_\pi^{-1}$.

In Table IV we give our results obtained from the imaginary part of the multipoles using (11) and (13). $\mathrm{Im}E_{1+}$ (M_{1+}), in units of $10^{-3} m_\pi^{-1}$, is evaluated at W_E (W_M) taken from the appropriate row in Table III. We also give the energy $\tilde{W}$ (in MeV) at which $\mathrm{Im}E_{1+}$ is estimated to be zero. We always obtain $\tilde{W} > W_E$, suggest-

TABLE III. The slopes of E_{1+}, χ_E, and the M_{1+}, χ_M, at the resonant energy in units of 10^{-3} $\mathrm{MeV}^{-1} m_\pi^{-1}$. These results are obtained from Eqs. (12) and (14). The data sets are designated in a manner explained in the text. W_E (W_M) is the energy at which $\mathrm{Re}E_{1+}$ ($\mathrm{Re}M_{1+}$) vanishes according to the interpolating function.

Data set	χ_E (10^{-3})	W_E	χ_M	W_M
BD	4.91 ± 5.71	1232.58	-0.667 ± 0.084	1232.49
MIR	6.83 ± 3.86	1236.63	-0.662 ± 0.100	1236.83
GET	7.68 ± 5.69	1237.51	-0.635 ± 0.142	1232.63
PS	5.89 ± 18.59	1238.44	-0.693 ± 0.090	1234.77
GRU	12.06 ± 11.13	1233.77	-0.642 ± 0.147	1231.49
SUZ	17.35 ± 8.39	1236.10	-0.882 ± 0.063	1234.63

TABLE IV. ImE_{1+} and ImM_{1+} at resonance in units of $10^{-3}m_\pi^{-1}$ for the various data sets as abbreviated in the text, obtained from Eqs. (11) and (13). $\tilde{W}$ is the energy, in MeV, at which ImE_{1+} vanishes according to the interpolating function.

Data set	ImE_{1+}	$\tilde{W}$	ImM_{1+}
BD	-0.219 ± 0.328	1238.2	38.2 ± 1.4
MIR	-0.377 ± 0.162	1248.8	33.8 ± 0.9
GET	-0.143 ± 0.202	1253.9	35.2 ± 1.8
PS	-0.166 ± 0.243	1247.6	38.1 ± 3.1
GRU	-0.643 ± 0.155	1257.8	38.3 ± 1.8
SUZ	-1.037 ± 0.443	>1260	57.2 ± 1.7

ing that Im$E_{1+}(W_E)\neq0$. For some data sets, however, Im$E_{1+}(W_E)$ has a big error, and is consistent with zero. The values of ImE_{1+} are in good agreement, but the errors are large. We find that ImM_{1+} for SUZ is in disagreement with the other data sets. Even if we exclude SUZ, there is still some inconsistency among the remaining data sets for both the ImE_{1+} and ImM_{1+}. Excluding SUZ, we obtain weighted averages Im$E_{1+}=-0.379\pm0.097$ and Im$M_{1+}=35.79\pm0.99$, where the error on ImE_{1+} has been scaled by 1.1 and the error on ImM_{1+} scaled by 1.6. Including SUZ, we get for ImE_{1+} -0.414 ± 0.103, with a scaling factor of 1.2. Our tests show that the interpolation of ImE_{1+} is accurate to within 10% and ImM_{1+} is accurate to within about 1%. $\tilde{W}$ is accurate to within 0.5 MeV.

We now turn to our results of the fits using (15). In Table V we show the results for A_E (10^{-3} MeV m_π^{-1}) and B_E ($10^{-3}m_\pi^{-1}$), obtained by fitting the real and imaginary parts of the E_{1+}. In Table VI the results for A_M and B_M, obtained from the M_{1+}, are given. Also given in Tables V and VI are the parameters σ_A (10^{-3} MeV m_π^{-1}), σ_B ($10^{-3}m_\pi^{-1}$), and σ_{AB} (10^{-6} MeV m_π^{-2}), which define the constant χ^2 contours in the A,B plane [Eqs. (20)–(22)]. The additional scripts E or M denote the results for the E_{1+} or M_{1+}, respectively. σ_A is the amount A needs to be changed in order to increase the χ^2 from its minimum value, $\chi^2_{\rm min}$, to $\chi^2_{\rm min}+1$, while keeping B fixed, and is taken to be the error on A. The quantity given in the last columns of Tables V and VI is the χ^2 per degree of freedom, $\chi^2_{\rm DF}$.

The $\chi^2_{\rm DF}$ in Table V are generally quite small indicating that (15) is a good parametrization of the E_{1+} in this energy region. The results obtained by fitting ReE_{1+}, Table V (top), are very close to those obtained by fitting ImE_{1+}, Table V (bottom). From ReE_{1+}, we get an average of A_E, $\overline{A}=-504.7\pm89.8$, where the error has been scaled by 2.5, and the ImE_{1+} gives $\overline{A}_E=-515.9\pm92.0$ (scaling factor $S=2.7$). The background parameter ob-

TABLE V. Results for the resonance parameter A_E and the "background" B_E for the E_{1+} obtained (a) by fitting ReE_{1+} and (b) by fitting ImE_{1+}. A_E and σ_{A_E} are in units of 10^{-3} MeV m_π^{-1}; B_E and σ_{B_E} are in units of $10^{-3}m_\pi^{-1}$ and σ_{AB_E} is in units of 10^{-6} MeV m_π^{-2}. σ_{A_E}, σ_{B_E}, and σ_{AB_E} define the χ^2 contours in the (A_E,B_E) plane (see text). $\chi^2_{\rm DF}$ is the χ^2 per degree of freedom.

Data	A_E	σ_{A_E}	B_E	σ_{B_E}	σ_{AB_E}	$\chi^2_{\rm DF}$
			(a)			
BD	−174.4	137.7	−33.4	11.8	−4903	0.001
MIR	−468.8	55.7	−34.4	2.8	−438.7	2.2
GET	−534.4	64.1	−32.6	3.1	−630.6	2.4
PS	−267.9	125.5	−23.1	9.9	−2586	0.72
GRU	−720.3	126.5	−26.3	3.8	−575.8	0.11
SUZ	−1258.0	179.7	−35.2	5.8	1633	0.56
			(b)			
BD	−177.1	126.7	−33.4	11.8	−4893	0.002
MIR	−456.0	52.3	−34.3	2.8	−431.1	2.4
GET	−546.6	66.6	−32.6	3.1	−757.5	2.4
PS	−281.9	109.2	−23.3	9.5	−1993	0.69
GRU	−797.5	95.9	−29.9	3.6	−688.8	0.77
SUZ	−1138	157.5	−38.3	5.6	1772	0.81

TABLE VI. (a) Results for the M_{1+} resonance, parameter and background, A_M and B_M, respectively, obtained by fitting ReM_{1+}. Symbols are as in Table V, except they now refer to the M_{1+} multipole. (b) Results for the M_{1+} resonance parameter and background obtained by fitting ImM_{1+}. Symbols as in (a), but referring to the M_{1+} multipole, hence M replacing E.

Data	A_M	σ_{A_M}	B_M	σ_{B_M}	σ_{AB_M}	$\chi^2_{\rm DF}$
			(a)			
BD	33 150	1 431	434.0	101.6	−361 500	0.57
MIR	32 930	483	250.6	13.8	−15 800	0.79
GET	34 250	527	191.4	14.5	−15 170	0.50
PS	33 760	1 649	261.7	14.5	−122 550	2.58
GRU	39 480	1 456	290.9	33.3	−53 160	0.90
SUZ	33 670	565	255.9	14.9	−21 740	5.47
			(b)			
BD	32 570	534	367.5	48.3	−143 300	5.85
MIR	31 810	168	237.9	6.0	−1 728	12.9
GET	34 100	206	197.1	6.5	−2 901	4.37
PS	33 200	300	257.8	8.8	−4 727	12.4
GRUP	37 060	582	150.8	30.2	−43 080	7.4
SUZ	36 350	270	293.1	17.1	−5 273	74.9

tained from ReE_{1+} gives $\bar{B}_E=-32.0\pm1.7$ ($S<1$, all data sets) and ImE_{1+} gives $B_E=-32.9\pm1.7$ ($S<1$, all data sets).

In Table VI (top), we see that $\chi^2_{\rm DF}$ is less than one except for the data of SUZ and PS, casting doubt on the use of (15) for fitting ReM_{1+} with these data sets. There is no conflict with the fact that (15) does not give a low $\chi^2_{\rm DF}$ to the ReM_{1+}; the relative errors are larger for ReM_{1+} than for ImM_{1+}. The fits to ReM_{1+} give the averages $\bar{A}_M=33\,786\pm570$ ($S=2$) and $\bar{B}_M=243.2\pm14.4$ ($S=2$), where all data sets have been included. Despite the large $\chi^2_{\rm DF}$, the results in the lower part of Table VI are generally in good agreement with the corresponding results in the upper part of Table VI. The averages in the lower part of Table VI, including all data sets, are $\bar{A}_M=33\,442\pm762$ ($S=7.2$) and $\bar{B}_M=229.8\pm13.8$ ($S=3.6$).

In Table VII we give a summary of our results in a form that makes a comparison of the different methods easier. We first define

$$\tilde{M}=-\chi_M(C)^{3/2}\left[\frac{16Mq\pi}{3M_N k}\right]^{1/2}, \tag{28}$$

$$\tilde{E}=-\chi_E(C)^{3/2}\left[\frac{16Mq\pi}{3M_N k}\right]^{1/2}; \tag{29}$$

also

$$\tilde{M}=\mathrm{Im}M_{1+}(M)\sqrt{C}\left[\frac{16Mq\pi}{3M_N k}\right]^{1/2}, \tag{30}$$

$$\tilde{E}=\mathrm{Im}E_{1+}(M)\sqrt{C}\left[\frac{16Mq\pi}{3M_N k}\right]^{1/2}; \tag{31}$$

and finally

$$\tilde{M}=\frac{1}{\sqrt{C}}\left[\frac{16\pi q^3 Mk}{3M_N^5}\right]^{1/2}A_M\ , \tag{32}$$

$$\tilde{E}=\frac{1}{\sqrt{C}}\left[\frac{16\pi q^3 Mk}{3M_N^5}\right]A_E\ . \tag{33}$$

The values we use for C are obtained by using (12) to evaluate (5b) for each set of phase shifts. We find $C=57.17$ for BD, $C=57.67$ for MIR and SUZ, $C=59.13$ for GET, $C=54.12$ for PS, and $C=61.24$ for GRU,[9] all in MeV^{-1}. The uncertainty in C, which from the spread of these numbers is about 5%, has not been included in the errors listed in Table VII. This additional error would have negligible effects on the results for $\tilde{E}$, and would have a small effect on the error of $\tilde{M}$ except in those cases where the error is on the order of 5%. In those cases the error would increase roughly a factor of $\sqrt{2}$.

The values of $\tilde{E}$ and $\tilde{M}$ are in units of 10^{-3} GeV$^{-1/2}$, and $\tilde{E}/\tilde{M}$=EMR is given in percent. The EMR is independent of C. This is a very important point: *the uncertainties of the strong interaction in the pion-nucleon final state, arising from the Δ decay, drop out of the EMR, a situation unique to this analysis,* as we mentioned earlier. The rows in Table VII are labeled according to the method of analysis used, interpolation (Int) or fitting (Fit), and whether the real (R) or imaginary (I) part of the multipole has been analyzed. In most cases the errors and $\tilde{E}$ and $\tilde{M}$ have been scaled by the appropriate factors discussed above. In particular, for $\tilde{E}$, the scaling factors are 1.2 for (Int,I), 2.5 for (Fit,R), and 2.7 for (Fit,I). For $\tilde{M}$, the scaling factors are 1.6 for (Int,I), 2 for (Fit,R), and 7.2 for (Fit,I). The scaling factor is 1 for (Int,R), and SUZ has been excluded in calculating the scaling factor for (Int,I).

TABLE VII. Results for $\tilde{M}$ and $\tilde{E}$ [Eqs. (28)–(33)] in units of 10^{-3} $\mathrm{GeV}^{-1/2}$, and the ratio $\tilde{E}/\tilde{M}$, EMR. The results are classified first by author, then by the extraction method, interpolating (Int) or fitting (Fit), and finally by whether the real (R) or imaginary (I) part of the multipole was analyzed.

Data	Method	R or I	$\tilde{E}$	$\tilde{M}$	EMR (%)
BD	Int	R	-2.11 ± 2.46	287 ± 36	-0.74 ± 0.87
	Int	I	-1.65 ± 2.96	288 ± 17	-0.57 ± 1.02
	Fit	R	-1.53 ± 3.03	291 ± 25	-0.53 ± 1.98
	Fit	I	-1.56 ± 3.00	286 ± 34	-0.55 ± 1.06
MIR	Int	R	-2.97 ± 1.69	290 ± 44	-1.02 ± 0.60
	Int	I	-2.86 ± 1.48	256 ± 11	-1.12 ± 0.58
	Fit	R	-4.17 ± 1.25	293 ± 9	-1.42 ± 0.43
	Fit	I	-4.06 ± 1.27	283 ± 11	-1.43 ± 0.45
GET	Int	R	-3.49 ± 2.58	288 ± 64	-1.21 ± 0.93
	Int	I	-1.10 ± 1.86	270 ± 22	-0.41 ± 0.69
	Fit	R	-4.84 ± 1.45	300 ± 9	-1.61 ± 0.48
	Fit	I	-4.95 ± 1.62	299 ± 13	-1.66 ± 0.54
PS	Int	R	-2.34 ± 7.38	275 ± 36	-0.85 ± 2.68
	Int	I	-1.22 ± 2.14	280 ± 36	-0.44 ± 0.77
	Fit	R	-2.53 ± 2.95	310 ± 30	-0.82 ± 0.95
	Fit	I	-2.66 ± 2.78	305 ± 20	-0.87 ± 0.91
GRU	Int	R	-5.76 ± 5.27	306 ± 70	-1.88 ± 1.77
	Int	I	-5.02 ± 1.45	299 ± 22	-1.67 ± 0.49
	Fit	R	-6.17 ± 2.70	333 ± 24	-1.85 ± 0.82
	Fit	I	-6.83 ± 2.21	312 ± 35	-2.19 ± 0.75
SUZ	Int	R	-7.57 ± 3.66	385 ± 28	-1.97 ± 0.96
	Int	I	-7.85 ± 3.35	433 ± 21	-1.81 ± 0.78
	Fit	R	-11.31 ± 4.05	299 ± 10	-3.78 ± 1.36
	Fit	I	-10.23 ± 3.83	333 ± 17	-3.07 ± 1.16

The results obtained for $\tilde{E}$ and $\tilde{M}$ using the different methods on a given data set generally agree within the error bars. The notable exception is the $\tilde{M}$ obtained from SUZ's data. We see that the interpolation gives quite different results than the fitting procedure; however, the χ^2_{DF} is large for the fits. All data sets give $\tilde{E}<0$, although the $\tilde{E}$'s obtained from BD and PS are consistent with zero. If we had not scaled ΔN_E, the error on $\tilde{E}$ would be roughly a factor of 2 smaller. For $\tilde{M}$, the error would have been 3–6 times smaller depending on the data set.

In Table VIII we summarize the results of Table VII and provide final estimates of $\tilde{E}$ and $\tilde{M}$ for each data set, as well as some final average $\tilde{E}$ and $\tilde{M}$. We take the unweighted average of the values within a given data set, but do *not* include the fitted results if $\chi^2_{\mathrm{DF}}\gtrsim 1$. (See Tables V and VI.) The average is weighted and the average $\tilde{M}$ (and hence EMR) does not include the data of SUZ. Our final results are $\tilde{E}=(-3.2\pm1.0)\times10^{-3}$ $\mathrm{GeV}^{-1/2}$, $\tilde{M}=(290\pm13)\times10^{-3}$ $\mathrm{GeV}^{-1/2}$, $\mathrm{EMR}=(-1.07\pm0.37)\%$. We may also make a comparison with the PDG listings by defining

$$\tilde{A}_{1/2}=-\tfrac{1}{2}(\tilde{M}+3\tilde{E}),\quad \tilde{A}_{3/2}=-\frac{\sqrt{3}}{2}(\tilde{M}-\tilde{E})\ . \quad (34)$$

We obtain $\tilde{A}_{1/2}=(-140\pm7)\times10^{-3}$ $\mathrm{GeV}^{-1/2}$ and $\tilde{A}_{3/2}=(-254\pm11)\times10^{-3}$ $\mathrm{GeV}^{1/2}$, in good agreement with those entries. Note, however, that the entries in the PDG listing are not necessarily the same as $\tilde{A}_{1/2}$ and $\tilde{A}_{3/2}$ defined here. Also, the results in the PDG (Ref. 6) listings for $A_{1/2}$ and $A_{3/2}$ are from *energy-dependent* multipole analyses, whereas $\tilde{A}_{1/2}$ and $\tilde{A}_{3/2}$ obtained here are from an analysis of the *energy-independent* multipole analyses.

TABLE VIII. Final estimates of $\tilde{E}$, $\tilde{M}$, the EMR, and the weighted average (SUZ has not been included in the average for $\tilde{M}$ and the EMR). $\tilde{E}$ and $\tilde{M}$ are in units of 10^{-3} $\mathrm{GeV}^{-1/2}$.

Data	$\tilde{E}$	$\tilde{M}$	EMR (%)
BD	-1.7 ± 2.9	289 ± 26	-0.59 ± 1.01
MIR	-2.9 ± 1.6	280 ± 30	-1.04 ± 0.58
GET	-2.3 ± 2.8	286 ± 30	-0.80 ± 0.98
PS	-2.2 ± 3.0	278 ± 38	-0.79 ± 1.08
GRU	-5.9 ± 2.7	313 ± 30	-1.88 ± 0.88
SUZ	-9.2 ± 4.1	409 ± 35	-2.25 ± 1.02
Average	-3.2 ± 1.0	290 ± 13	-1.07 ± 0.37

V. SUMMARY AND CONCLUSIONS

In summary we have extracted *in a model-independent fashion* the K-matrix residues for the $\gamma N \to \pi N$ reaction in the $\Delta(1232)$ resonance. We have tried to take into account the inconsistencies among the energy-independent multiple data sets, and have obtained results that are in good agreement with the energy-dependent results listed in the PDG review. We have taken care to examine the compatibility of different sets of data when comparing different sets.

The results obtained for $\tilde{E}$ and $\tilde{M}$ using different methods of analysis on a given data set generally agree within the estimated errors. The notable exception is the value of $\tilde{M}$ obtained from the data set SUZ. All data sets give together $\tilde{E}<0$, although the values of $\tilde{E}$ obtained from BD and PS are consistent with zero. Note that if we had not scaled the quantity ΔN_E, the error on $\tilde{E}$ would be roughly a factor of 2 smaller. For $\tilde{M}$, the error would have been 3–6 times smaller depending on the data set.

In Table VIII we provide the summary of estimates of $\tilde{E}$ and $\tilde{M}$ for each data set, as well as averages of $\tilde{E}$ and $\tilde{M}$. We take the average (*not* weighted) of the values within a given data set, but do *not* include the fitted results if $\chi^2_{\rm DF} \geq 1$. (See Tables V and VI.) The average is weighted and the average $\tilde{M}$ (and hence EMR) does not include the data of SUZ. Our final results are

$$\tilde{E} = (-3.2 \pm 1.0) \times 10^{-3}\ \text{GeV}^{-1/2}\ ,$$
$$\tilde{M} = (290 \pm 13) \times 10^{-3}\ \text{GeV}^{-1/2}\ ,$$

and

$$\text{EMR} = (-1.07 \pm 0.37)\%\ . \tag{35}$$

The most important point of this analysis is the elimination of the uncertainties coming from the theoretical attempts to model the contributions of the background, thus at least partially answering one of Feynman's questions[1] that we quoted at the onset for one specific case. *At the K-matrix pole, the residues* shown in (35) *are independent of the uncertainties coming from the effective "background,"* contributions B, D in Eq. (1), and thus contain information on properties of the three-point function involving the photon, nucleon, and the resonance. The ratio, called EMR in (35), is independent of the physics of the strong interaction in the final state into which the resonance can decay. Thus, the uncertainties coming from the treatment of the final-state interaction, which results in the extraction of the EMR, are avoided. The negative value of the ratio is predicted in a variety of models.[13] In quark model, this arises as the nucleon and the delta pick up "deformed" configurations. It remains to be understood if the conventional hadron models actually predict the K-matrix residues.

We can use Eq. (15) and extrapolate it to obtain the T-matrix residues. This can be done using the parameters of Tables V and VI, and extrapolating Eq. (15) to the T-matrix pole in the complex W plane. The K-matrix parameters, obtained in this paper, have the advantage over the T-matrix residues in that they are obtainable directly from the data, with no need for extrapolation. The question now arises if the K-matrix residues are simply related to the decay widths calculated in a hadron model. Such a comparison requires solving a scattering problem, to be examined in a forthcoming paper. It is interesting that many hadron models[13] predict electromagnetic nucleon-delta transition amplitudes numerically close to $\tilde{E}$ and $\tilde{M}$ obtained in this work.

Our analysis ignores the Compton contributions. There are reasons to believe that this is an excellent approximation,[14] but this point remains to be firmly established. Finally, our work here does not eliminate the need of the model-dependent analyses of the pion photoproduction from threshold through the resonance peak, as only these analyses have the ability to disentangle the complex interplay of diverse background mechanisms and the excitation of the resonance itself. Indeed, we are continuing to emphasize the importance of those analyses[15] to understand the rich physics content of the photoproduction of mesons. That richness also remains to be understood from the QCD standpoint.

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APPENDIX

The scaling factor[10] discussed in the text is determined in the following manner. First, we find the weighted mean using

$$\bar{X}_W = \left[\sum_i X_i/\sigma_i^2\right] \Big/ \sigma_W^2\ , \tag{A1}$$

where $1/\sigma_W^2 = \sum_i 1/\sigma_i^2$. Second, we calculate $\chi^2 \equiv \sum_i (\bar{X}_W - X_i)^2/\sigma_i^2$. The scaling factor is

$$S = \left[\frac{\chi^2}{N-1}\right]^{1/2}\ . \tag{A2}$$

If $S>1$ the data are inconsistent; we assume conservatively that all authors of the data base we are using have underestimated their errors by the same large factor. If $S>1$, the errors are not scaled. The procedure adopted here follows the method described by the Particle Data Group.

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