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Phenomenology of fermionic strings

Joel A. Schwartz
Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853
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We investigate the low-energy phenomenology of string models constructed in the fermionic formulation. Models are constructed in which the gauge group SU(3)×SU(2)×U(1)× ⋯ is obtained directly from free world-sheet fermions. We also construct models in which SU(3)×SU(2)×U(1)× ⋯ is obtained by symmetry breaking along completely flat directions in the effective potential; this corresponds to world-sheet interactions. In this category we construct models in which SU(5) is broken to SU(3)×SU(2)×U(1) by a Higgs boson in the adjoint representation and in which SO(4)×SO(6) is broken by the spinor representation. We analyze the effects of the spin structure, massless moduli, and Fayet-Iliopoulos D terms on low-energy phenomenology. In particular we consider the renormalization-group flow of the couplings, proton decay, mass matrices, Higgs-boson mass, and charge quantization. We briefly compare these results with the flipped SU(5) approach and other constructions such as orbifolds and Calabi-Yau manifolds.

I. INTRODUCTION

One of the greatest challenges to string theory is the extraction of predictions to compare with experiment.[1] Progress in this direction is made rather difficult by our inability to locate the true string vacuum. There exists a multitude of possible string vacua while the dynamics which selects the preferred vacuum remains unknown. The possible vacua are the known string models, or solutions to the string equations of motion. These, however, are degenerate in perturbation theory. The needed dynamics would presumably come from a second-quantized closed-string theory which, as yet, has not been formulated. As a result, we can only perform perturbative calculations around a given vacuum; we cannot determine how the degeneracy between the different vacua is lifted.

One possible approach to this problem is to understand the phenomenological consequences associated with particular known vacua, or classes of vacua, and use low-energy data to assess whether the models based on these vacua are viable. Given the multitude of possible vacua this seems at first too formidable a task. Indeed, in the restricted class of fermionic string models alone, over 100 000 classical vacua have been constructed.[2] Nevertheless, the great majority of these models can be ruled out by the simplest phenomenological criteria,[2] while the others rarely survive under closer scrutiny. Furthermore, it is possible to explicitly construct models exhibiting chosen phenomenological characteristics by understanding the relation of world-sheet phenomena to spacetime phenomena. This was accomplished in Refs. 3 and 4, for example, in constructing flipped SU(5) models. As a result it is possible to survey the more interesting vacua available for analysis. This allows the understanding of string phenomenology to progress in the absence of, and in preparation for, a second-quantized string theory.

In this paper we investigate the relation of the string construction to spacetime phenomena in the case of heterotic,[5] fermionic strings.[6,7] We show how string models with desirable phenomenological properties may be constructed and identify what difficulties arise in meeting all the physical constraints. Rather than advocate one particular scenario for unification, we survey a range of possibilities and discuss the merits and problems of each. Two ways of obtaining the gauge group SU(3)×SU(2)×U(1) are demonstrated; in one case SU(3)×SU(2)×U(1)× ⋯ is obtained in the free fermionic formulation, that is without using the massless moduli. We will call these 3-2-1 models. In the other case, a larger group is broken to SU(3)×SU(2)×U(1) × ⋯ along flat directions in the effective potential, corresponding to (conformal) world-sheet interactions. These are grand-unified-theory (GUT) models though the unification scale is typically the same as the Planck scale.

The case of 3-2-1 models has been investigated before for string models constructed via orbifolds[8–10] and Calabi-Yau manifolds.[11,12] The construction of fermionic 3-2-1 models with Weyl world-sheet fermions was discussed in Ref. 13 though no three- or four-generation models were found. Four-generation fermionic 3-2-1 models with Weyl world-sheet fermions were found in Ref. 2 using a computer search; the phenomenology of these has not yet been analyzed in detail. One three-generation 3-2-1 model was recently constructed in Ref. 14. In what follows we construct fermionic 3-2-1 models with four generations using both Weyl (complex) and Majorana-Weyl (real) world-sheet fermions. We find techniques for incorporating the basic phenomenological constraints into model construction; however, it remains a nontrivial problem to incorporate all the phenomenological constraints into a single model together. So none of the 3-2-1 models found as yet are truly candidates for describing nature. However it seems that there is no fundamental obstacle to meeting the most stringent requirements.

The case of GUT unification in the fermionic string has been investigated by Antoniadas *et al.* for flipped SU(5) grand unification[3,4] [SU(5)×U(1) broken by Higgs bosons in the $10, \overline{10}$ representations[15]]. Recent work has shown that this is a promising avenue.[4] Part of the motivation for investigating flipped SU(5) was the absence, in string models, of Higgs bosons in the adjoint (and higher) representations of grand unified groups; these Higgs bosons are absent in models with chiral fermions and world-sheet level-1 Kac-Moody algebras.[16] However, as recently shown by Lewellen,[17] fermionic strings with Majorana-Weyl world-sheet fermions may employ higher-level Kac-Moody algebras (at least level 2) and so contain adjoint Higgs-boson breaking. Here we investigate the phenomenology of such models, in particular minimal supersymmetric SU(5). We show how SU(5) unification may be obtained; however further work is needed to see if the correct electroweak scale can be preserved in such models.

In addition to GUT unification with adjoint Higgs bosons we also consider gauge groups such as SO(6)×SO(4) (Pati and Salam[18]) and SO(10)×SO(4) broken to SU(3)×SU(2)×U(1) by Higgs bosons in the spinor representations. We find that such models can be constructed and that some phenomenological constraints, such as the proton lifetime, can be met. However, these examples typically suffer from excess charged light particles; as a result, the running couplings below up before perturbative unification is possible. Some possible remedies for this feature of the construction are discussed below.

The outline of the paper is as follows. In Sec. II we discuss in general the principles used to analyze these models. In particular we consider the effects of massless moduli, Fayet-Iliopoulos D terms and nonrenormalizable terms in the effective action. We consider the phenomenological requirements of supersymmetry breaking, superunification, quark and lepton mass matrices, Higgs-boson masses, and electric charge quantization. In Sec. III we consider examples of 3-2-1 models in the fermionic construction. In Sec. IV we consider models in which a GUT group is broken along a flat direction in the effective potential. We analyze both spinor Higgs-boson breaking and adjoint Higgs-boson breaking. In Sec. V we summarize the results and discuss the prospects for string phenomenology. Two appendixes are devoted to discussing the fermionic formulation. In Appendix A we review the construction of fermionic string models[6,7] and discuss the calculation of interactions. In Appendix B we present the techniques for identifying the massless moduli and discuss some generic features of the moduli in the fermionic construction.

II. STRING PHENOMENOLOGY

Let us first consider stringy effects which constrain the choice of vacua; next we shall consider low-energy constraints. The string loop corrections are particularly important.

Cosmological constant. In classical vacua without spacetime supersymmetry the one-loop contribution to the cosmological constant (or, equivalently, the dilaton tadpole) is generically on the order of the Planck mass.[19] As a result these solutions are not true vacua. Henceforth we shall restrict our attention to classical vacua with $N=1$ spacetime supersymmetry. Of course, supersymmetry will have to be broken at low energies in such a way that the cosmological constant still vanishes. We will be forced to assume that supersymmetry is broken by a nonperturbative mechanism; this situation is discussed below.

Fayet-Iliopoulos D terms. In classical vacua with spacetime supersymmetry a U(1) gauge symmetry may apparently be anomalous.[20,21] This anomaly is canceled by the Green-Schwarz mechanism[22] while a Fayet-Iliopoulos D term[23] is generated in the effective action.[20,21] This corresponds to a dilaton tadpole at two loops in string perturbation theory. As a result of the D term the auxiliary field for the (anomalous) U(1) gauge boson has (in its equation of motion) a constant piece added to it proportional to the anomaly:

$$D \sim \mathrm{Tr}Q + \sum_i q_i \phi_i^* \phi_i \ , \tag{1}$$

where D is the auxiliary field, Q the charge, and ϕ_i are scalar fields. Thus these vacua are not true vacua either. However, as shown in Ref. 20, the constant TrQ may be canceled by giving appropriate vacuum expectation values (VEV's) to the scalar fields; if this can be done so that the auxiliary D fields and F fields have vanishing VEV's then supersymmetry is restored and the dilaton tadpole vanishes. In what follows it will be important to take into account such terms in order to find the true vacua.

Certain effects at the string tree level are also worth noting.

Massless moduli. Generically there exist some flat directions in the (massless) scalar potential in any classical vacuum. The completely flat directions are known as massless moduli of the theory. Giving a VEV to a scalar in a flat direction corresponds, in world-sheet language, to adding an exactly marginal operator to the Lagrangian, the marginal operator being the vertex operator for that scalar.

Since all points along a flat direction are degenerate, the physics that determines the true vacuum is not yet understood. Supersymmetry breaking is expected to provide corrections to the potential and so determine the vacuum; thus it is possible to get some idea of the corrected potential within the context of some assumptions about the supersymmetry-breaking parameters in the low-energy field theory.[24] However it is difficult to investigate VEV's on the order of the Planck mass this way since the low-energy field theory is cut off at the Planck scale. We can expect nevertheless that VEV's along flat directions obtain generic Planck scale values. If a VEV must be fine-tuned to produce a desired phenomenological result, we may consider this unnatural. Thus, naturalness in this sense is one way of assessing whether a model is viable. One particular flat direction will always be the dilaton VEV, which determines the string coupling constant. We expect we will always fine-tune this value until a better understanding of string dynamics is achieved. In the examples considered below, we shall choose a value which gives us the correct fine-structure constant.

It remains an open question whether the dynamical corrections to the dilaton potential fix the string coupling at a value which is nonzero and consistent with perturbation theory.[25,26] However recent results suggest that this is indeed possible in vacua where gaugino condensation occurs in more than one extra gauge group.[27] In the examples considered below, we shall see that multiple confining gauge groups arise quite easily in model construction. Although this is still just a necessary condition for the mechanism proposed in Ref. 27, it appears that the "dilaton problem" is not intractable.

Nonrenormalizable interactions. Nonrenormalizable terms suppressed by the Planck mass appear in the effective field theory as a result of integrating out the massive modes of the string.[12] These terms may play an important role if one or more fields appearing in a term acquire a large VEV, e.g., a Planck scale VEV. Then the term may provide in effect a new renormalizable interaction. This mechanism could contribute to the quark mass matrices, providing the needed Kobayashi-Maskawa (KM) mixings.[28] In addition the nonrenormalizable terms could be responsible for introducing intermediate scales into a model; as pointed out in Ref. 12, this could occur if nonrenormalizable terms are responsible for lifting a direction left flat by the renormalizable potential (see Ref. 12).

Next we consider the requirements of low-energy phenomenology.

Superunification. By superunification we mean the unification of all coupling constants somewhere near the Planck scale. Actually, as shown in Ref. 29, the effective field-theory couplings are expected to unify at about $g_{\text{string}} 5\times 10^{17}$ GeV. This allows us to make predictions for $\sin^2\theta_W$ and Λ_{QCD} in terms of the fine-structure constant, as long as perturbative unification is possible. We will do this for specific models using the lowest-order (one-loop) Georgi-Weinberg-Quinn equations.[30] It will be seen in the examples below that these calculations provide a stringent test of the string models, ruling out most of the examples we have.

As also shown in Ref. 29, string threshold effects add corrections of order unity to the values of the various couplings at the unification scale. This means that the different gauge couplings in a 3-2-1 or Pati-Salam model can have different threshold corrections. We will keep this effect in mind so we can properly assess the accuracy of the simple one-loop approach.

Supersymmetry breaking. Since we have confined our attention to classical vacua with spacetime supersymmetry, it is necessary to envision a mechanism which breaks supersymmetry at low energies. It is natural to think this breaking determines the Higgs-boson mass and explains the smallness of the electroweak scale compared to the Planck scale. Thus the supersymmetry-breaking scale in the visible sector will be called m_H (Higgs-boson mass). Its order of magnitude may be estimated only after assumptions are made regarding the manner of supersymmetry breaking.

The way in which supersymmetry is broken in string theory is not well understood. Most candidate mechanisms either produce an unacceptably large cosmological constant or a dilaton potential which drives the theory to zero coupling.[25] It seems that some kind of nonperturbative mechanism is needed such as a gaugino condensate in one (or more) of the extra gauge groups.[26] The Higgs-boson mass would then depend on how the extra gauge group couples to the visible sector (e.g., if it is hidden) and on the scale of the condensate. Faced with this lack of understanding we will mostly avoid this issue but, when necessary, assume that such a condensate is indeed responsible for the breaking. We may also estimate the scale of the condensate(s) from the renormalization-group flow of the gauge couplings.

Proton decay. The rate of proton decay is easy to estimate in most models and can be used to discard a great many candidates. In 3-2-1 models and GUT models with string scale unification, there is no danger of rapid proton decay due to gauge-boson (fermion) exchange; however, we must still consider the possibility of squark exchange or colored "Higgs-boson" ("Higgsino") exchange.

The squark exchange, shown in Fig. 1,[31] could be brought on by terms in the superpotential of the form $QL\bar{D}$ and $\bar{U}\,\bar{D}\,\bar{D}$ where Q is the quark doublet, L the lepton doublet, and $\bar{U}$ and $\bar{D}$ the conjugate quarks. However, in the examples constructed below in the fermionic formulation, it will be apparent that these terms are generically absent. This is due to the extra quantum numbers carried by the matter fields; these quantum numbers are either associated with discrete symmetries or charges that couple to a gauge group broken at higher energies. In either case the extra selection rules come from extra world-sheet fields in the quark vertex operator which are singlets of the world-sheet $SU(3)\times SU(2)\times U(1)$ current algebra.[32]

The possibility of colored "Higgs-boson" ("Higgsino") exchange,[33] shown in Fig. 2,[31] is more relevant here. In minimal or flipped SU(5) models the colored Higgs boson appears in the same multiplet as the electroweak Higgs boson. Thus some mechanism is needed to split the doublets and triplets while leaving light doublets. The GUT Higgs boson which breaks SU(5) in the minimal case can split the doublets and triplets if the VEV is in the right direction.[34] However the electroweak Higgs boson still acquires a GUT scale mass unless some additional mechanism is present (see Ref. 31 for a review). In the flipped case, if the U(1) charges (of the GUT and electroweak Higgs bosons) are properly assigned, the doublet-triplet splitting with light weak doublets occurs naturally.[15]

In 3-2-1 and Pati-Salam models a colored Higgs boson need not even be present. If it is present, there are

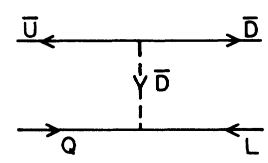

FIG. 1. Squark exchange leading to proton decay (Ref. 31). The solid lines are fermions; the dashed line is the squark.

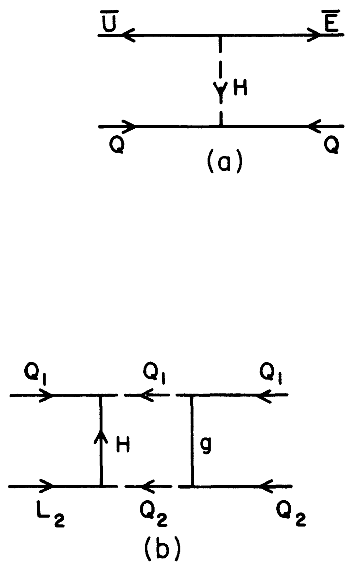

FIG. 2. (a) Colored "Higgs-boson" exchange (Ref. 31). (b) Colored "Higgsino" exchange together with gluino exchange (Ref. 31). The subscripts 1 and 2 refer to two different generations. This process can lead to the proton decay $p \to K^+ \bar{\nu}_\mu$ (Ref. 33).

several ways in which too rapid a proton decay may be avoided. An SU(3) triplet may have different couplings than the Higgs doublet; so the triplet may be prevented from coupling to the quarks due to the extra selection rules mentioned above. Alternatively, as in the GUT case, the triplet can become superheavy from a large VEV given to a 3-2-1 singlet (e.g., along a flat direction). Both of these mechanisms will be illustrated in the constructions of Secs. III and IV.

Quark mass matrices. To agree with experiment, the quark mass matrices need to be sufficiently complicated to provide all the quarks with masses and provide the KM mixings. In many models the Yukawa couplings are not complete enough to give all the quarks a mass. In 3-2-1 models it seems advantageous in this respect to have multiple weak doublets which can combine to form the Higgs bosons; a linear combination could couple to more quarks than either doublet alone. However, the other linear combinations, if light, may then be problematic. They affect the coupling constant flow and so endanger unification. Also the extra weak doublets could result in flavor-changing neutral currents[35] unless their masses are sufficiently high (above the TeV scale). Indeed none of the models constructed below solve all of these problems simultaneously.

Neutrino masses. There are several ways in which the neutrinos may become extremely light. In either GUT or 3-2-1 models a seesaw mechanism can arise, originating from a large VEV given to a scalar (e.g., along a flat direction). The conjugate neutrino could acquire a large Majorana mass as in the usual seesaw mechanism or could be coupled via a large Dirac mass to another 3-2-1 singlet. Alternatively, the conjugate neutrinos could be absent from the spectrum. The neutrinos would then be barred from obtaining a Dirac mass. None of the examples constructed so far have this property. However, in one example (Sec. III) we will see the neutrinos couple to a different Higgs doublet than the other fermions. If this Higgs doublet is not part of the true Higgs doublet, that is, if it does not acquire a VEV, then the neutrinos could in principle be massless. We will discuss this situation in more detail in Sec. III.

Electric charge quantization. The presence of fractionally charged color singlets in the spectrum is a generic feature in string models[36,37] which, as we shall see, is avoided in certain cases. Of course, whether such fractional charges represent a problem or a most welcome prediction depends on the masses of the particular states (be they elementary or bound states) in a given model. As shown in Ref. 37, fractionally charged color singlets inevitably occur when $\sin^2\theta_W$ is $\frac{3}{8}$ at the unification scale (as in usual GUT's) and the gauge group $SU(3)\times SU(2)\times U(1)$ is not embedded in SU(5). In the examples discussed below we shall see three mechanisms by which the phenomenological problem of light fractional charges is avoided. One of which is indeed to have SU(5) unification which, as mentioned in Sec. I, is made possible here by incorporating level 2 Kac-Moody algebras on the world sheet.[17] Another mechanism is to have the fractionally charged particles acquire Planck scale masses via the massless moduli. A third mechanism, also discussed in Refs. 4 and 37, is for the fractionally charged particles to form heavy bound states as a result of carrying an extra "color" charge which confines at higher energies.

With these basic criteria in mind, we shall investigate the phenomenology of particular models. We next show how 3-2-1 models may be constructed.

III. FREE FERMIONIC 3-2-1 MODELS

In this section we construct fermionic string models with the gauge group $SU(3)\times SU(2)\times U(1)\times\cdots$ obtained directly from the spin structure. In the next section we consider models in which a larger group is broken along flat directions at the Planck scale. We will present the analysis of one model in detail and discuss the salient features of two others. We choose to present in detail the model with the best predictions for $\sin^2\theta_W$ and $\Lambda_{\rm QCD}$, since it requires more detail to see how these predictions emerge. The other examples illustrate the ways in which the proton lifetime and quark mass matrices may be incorporated in the construction.

We employ the notation of Kawai, Lewellen, and Tye[6] for the fermionic string. A brief review of this formulation is given in Appendix A; the references may be consulted for more details. A fermionic string model is specified by a set of 64-component vectors $\mathbf{W}_i$ and a matrix of constants k_{ij} meeting certain consistency conditions (Appendix A); the $\mathbf{W}_i$'s determine the allowed boundary conditions of the world-sheet fermions around the string loop; the k_{ij}'s fix certain phases appearing in the one-loop partition function.

A 3-2-1 model may be constructed with the following spin structures:

$$
\begin{aligned}
&\mathbf{W}_0=(\tfrac{1}{2}^{20}|\tfrac{1}{2}^{44})\ ,\\
&\mathbf{W}_1=(0^2(0\tfrac{1}{2}\tfrac{1}{2})^6|\tfrac{1}{2}^{44})\ ,\\
&\mathbf{W}_2=(0^2(0\tfrac{1}{2}\tfrac{1}{2})^2(\tfrac{1}{2}0\tfrac{1}{2})^2(\tfrac{1}{2}0\tfrac{1}{2})^2|0^{16}\tfrac{1}{2}^{28})\ ,\\
&\mathbf{W}_3=(0^2(\tfrac{1}{2}0\tfrac{1}{2})^2(\tfrac{1}{2}0\tfrac{1}{2})^2(0\tfrac{1}{2}\tfrac{1}{2})^2|0^{10}0^2\tfrac{1}{2}^40^4\tfrac{1}{2}^{24})\ ,\\
&\mathbf{W}_4=(0^2(\tfrac{1}{2}\tfrac{1}{2}0)^2(0\tfrac{1}{2}\tfrac{1}{2})^2(\tfrac{1}{2}\tfrac{1}{2}0)^2|0^{10}\tfrac{1}{2}^6\tfrac{1}{2}^40^6\tfrac{1}{2}^{18})\ ,\\
&\mathbf{W}_5=(0^2(000)^2(0\tfrac{1}{2}\tfrac{1}{2})(000)(000)^2|(\tfrac{1}{2}^40^6)(\tfrac{1}{2}0^5)0^4(0^2\tfrac{1}{2}^4)0^9\tfrac{1}{2}^9)\ ,\\
&\mathbf{W}_6=(0^2(0\tfrac{1}{2}\tfrac{1}{2})(000)(000)^2(0\tfrac{1}{2}\tfrac{1}{2})(000)|\tfrac{1}{4}{}_c^5(0^2\tfrac{1}{2}0\tfrac{1}{4}{}_c)(\tfrac{1}{4}{}_c\tfrac{1}{2}0)(\tfrac{1}{4}{}_c\tfrac{1}{2}^4)\tfrac{1}{4}{}_c^20^5(\tfrac{1}{4}{}_c^2\tfrac{1}{2}^40))\ .
\end{aligned}
\tag{2}
$$

We call this model A1. The subscript c on the $\frac{1}{4}$'s means that the $\frac{1}{4}$ refers to complex fermions; each complex fermion consists of two real fermions [cf. Eq. (A3)]. The k_{ij} matrix for model A1 is chosen to be

$$
\begin{aligned}
&k_{ij}=0 \ \text{ for } i<j\ ,\\
&\text{except } \ k_{25}=\tfrac{1}{2}\ ,\\
&k_{06}=k_{16}=k_{46}=\tfrac{1}{4}\ .
\end{aligned}
\tag{3}
$$

All other k_{ij}'s $(i\geq j)$ are determined by the constraints (Appendix A). The reasons for this choice shall be explained below.

The spin structures in (2) have a direct bearing on the particle spectrum. The $\mathbf{W}_0$ sector contains the graviton, axion, dilaton, and all the gauge bosons in this model; it also contains scalars to be discussed below. The vector $\mathbf{W}_1$ introduces spacetime supersymmetry. $\mathbf{W}_2$ contains one generation of fermions. $\mathbf{W}_3$ contains a second generation of fermions which couples to the $\mathbf{W}_2$ generation via Higgs bosons in $\overline{\mathbf{W}_0+\mathbf{W}_2+\mathbf{W}_3}$ (this structure is the same as the toy model in Appendix A). The vector $\mathbf{W}_4$ forces the $\mathbf{W}_2$ and $\mathbf{W}_3$ sector fermions to be chiral [cf. Eq. (A4)]. It also contains two generations of chiral fermions itself. At this stage the first ten left-moving Majorana-Weyl fermions provide (in the $\mathbf{W}_0$ sector) the gauge group SO(10) under which the matter generations transform as spinors; the choice $k_{02}+k_{24}=k_{03}+k_{34}$ ensures that all four generations have the same chirality. The vector $\mathbf{W}_5$ then cuts the SO(10) symmetry to SO(4)×SO(6) [cf. Eq. (A4)]. Finally, $\mathbf{W}_6$ reduces SO(4)×SO(6) to U(2)×U(3). Two combinations of the two U(1)'s are viable hypercharges with the standard assignments.

Let us remark briefly on the choice of k_{ij}'s [Eq. (3)]. The choices $k_{15}=k_{05}$, $k_{16}=k_{06}$, and $k_{12}+k_{13}+k_{14}=k_{02}+k_{03}+k_{04}$ (mod1) are needed to preserve $N=1$ spacetime supersymmetry. This is evident from considering the projection [Eq. (A4)] of gravitinos in the $\mathbf{W}_0$ sector. Then the choice $k_{25}\neq k_{35}$ forces the Higgs bosons in the $\overline{\mathbf{W}_0+\mathbf{W}_2+\mathbf{W}_3}$ sector to be weak doublets rather than color triplets.

Before considering the Fayet-Iliopoulos D terms and massless moduli, the gauge group for this model is

$$
\begin{aligned}
\mathrm{SU}(2)_L\times\mathrm{SU}(3)\times[\mathrm{U}(1)]^2\times[\mathrm{SU}(2)]^2&\\
\times[\mathrm{U}(1)]^3\ \ (\text{visible})\ ,&\quad\text{(4a)}\\
\times[\mathrm{SU}(2)]^2\times[\mathrm{U}(1)]^2\ \ (\text{semihidden})\ ,&\quad\text{(4b)}\\
\times[\mathrm{SU}(2)]^2\times\mathrm{SO}(5)\ \ (\text{semihidden})&\quad\text{(4c)}
\end{aligned}
$$

which has rank 18. By visible we mean gauge bosons which couple directly to matter. Semihidden means that these gauge fields couple to extra chiral superfields which couple in turn to the visible gauge group. After considering the D terms and moduli this situation will change quite a bit.

The complete massless spectrum of A1 before turning on D terms and moduli is given in Table I. The more relevant particles in the spectrum include

2 generations with extra U(1) charges ($\mathbf{W}_2$ and $\mathbf{W}_3$ sectors) ,

2 generations doublets of extra SU(2) with extra U(1) charges ($\mathbf{W}_4$ sector) ,

8 weak doublets $h_i,\bar{h}_i$, $i=1,\ldots,4$ ($\overline{\mathbf{W}_0+\mathbf{W}_2+\mathbf{W}_3}$ sector) ,

6 color triplets f_i, $\bar{f}_i$, $i=1,\ldots,3$ ($\mathbf{W}_0$ sector) ,

4 vectors of visible SO(4) {i.e., $[\mathrm{SU}(2)]^2$} $g_i,\bar{g}_i$, $i=1,2$ ($\overline{\mathbf{W}_0+\mathbf{W}_2+\mathbf{W}_3}$ sector) ,

6 particles with U(1) charges only $\phi_{12},\bar{\phi}_{12},\phi_{13},\bar{\phi}_{13},\phi_{23},\bar{\phi}_{23}$ ($\mathbf{W}_0$ sector) ,

2 singlets Φ_1,Φ_2 ($\mathbf{W}_0$ sector) ,

SO(5) vector X ($\mathbf{W}_0$ sector) .

TABLE I. Model A1 spectrum—"before." Gauge group $=SU(2)_L\times SU(3)\times[SU(2)]^6\times SO(5)\times[U(1)]^7$.

Superfield	Representation	U(1) charges[a]	Sector
Q_1	$(2,3,1,1,1,1,1,1,1)$	$(0,-\frac{1}{2},\frac{1}{2},0,0,0,0)$	$\mathbf{W}_2$
L_1	$(2,1,1,1,1,1,1,1,1)$	$(0,\frac{3}{2},\frac{1}{2},0,0,0,0)$	$\mathbf{W}_2$
$\bar{U}_1$	$(1,\bar{3},1,1,1,1,1,1,1)$	$(1,\frac{1}{2},\frac{1}{2},0,0,0,0)$	$\mathbf{W}_2$
$\bar{D}_1$	$(1,\bar{3},1,1,1,1,1,1,1)$	$(-1,\frac{1}{2},\frac{1}{2},0,0,0,0)$	$\mathbf{W}_2$
$\bar{e}_1$	$(1,1,1,1,1,1,1,1,1)$	$(-1,-\frac{3}{2},\frac{1}{2},0,0,0,0)$	$\mathbf{W}_2$
$\bar{\nu}_1$	$(1,1,1,1,1,1,1,1,1)$	$(1,-\frac{3}{2},\frac{1}{2},0,0,0,0)$	$\mathbf{W}_2$
Q_2	$(2,3,1,1,1,1,1,1,1)$	$(0,-\frac{1}{2},0,\frac{1}{2},0,0,0)$	$\mathbf{W}_3$
L_2	$(2,1,1,1,1,1,1,1,1)$	$(0,\frac{3}{2},0,\frac{1}{2},0,0,0)$	$\mathbf{W}_3$
$\bar{U}_2$	$(1,\bar{3},1,1,1,1,1,1,1)$	$(1,\frac{1}{2},0,\frac{1}{2},0,0,0)$	$\mathbf{W}_3$
$\bar{D}_2$	$(1,\bar{3},1,1,1,1,1,1,1)$	$(-1,\frac{1}{2},0,\frac{1}{2},0,0,0)$	$\mathbf{W}_3$
$\bar{e}_2$	$(1,1,1,1,1,1,1,1,1)$	$(-1,-\frac{3}{2},0,\frac{1}{2},0,0,0)$	$\mathbf{W}_3$
$\bar{\nu}_2$	$(1,1,1,1,1,1,1,1,1)$	$(1,-\frac{3}{2},0,\frac{1}{2},0,0,0)$	$\mathbf{W}_3$
Q_3,Q_4	$(2,3,2,1,1,1,1,1,1)$	$(0,-\frac{1}{2},0,0,-\frac{1}{2},0,0)$	$\mathbf{W}_4$
L_3,L_4	$(2,1,2,1,1,1,1,1,1)$	$(0,\frac{3}{2},0,0,-\frac{1}{2},0,0)$	$\mathbf{W}_4$
$\bar{U}_3,\bar{U}_4$	$(1,\bar{3},1,2,1,1,1,1,1)$	$(1,\frac{1}{2},0,0,-\frac{1}{2},0,0)$	$\mathbf{W}_4$
$\bar{D}_3,\bar{D}_4$	$(1,\bar{3},1,2,1,1,1,1,1)$	$(-1,\frac{1}{2},0,0,-\frac{1}{2},0,0)$	$\mathbf{W}_4$
$\bar{e}_3,\bar{e}_4$	$(1,1,1,2,1,1,1,1,1)$	$(-1,-\frac{3}{2},0,0,\frac{1}{2},0,0)$	$\mathbf{W}_4$
$\bar{\nu}_3,\bar{\nu}_4$	$(1,1,1,2,1,1,1,1,1)$	$(1,-\frac{3}{2},0,0,\frac{1}{2},0,0)$	$\mathbf{W}_4$
h_1,h_2	$2\times(2,1,1,1,1,1,1,1,1)$	$(1,0,\frac{1}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3$
$\bar{h}_1,\bar{h}_2$	$2\times(2,1,1,1,1,1,1,1,1)$	$(-1,0,-\frac{1}{2},-\frac{1}{2},0,0,0)$	$\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3$
h_3,h_4	$2\times(2,1,1,1,1,1,1,1,1)$	$(1,0,-\frac{1}{2},-\frac{1}{2},0,0,0)$	$\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3$
$\bar{h}_3,\bar{h}_4$	$2\times(2,1,1,1,1,1,1,1,1)$	$(-1,0,\frac{1}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3$
g_1,g_2	$2\times(1,1,2,2,1,1,1,1,1)$	$(0,0,\frac{1}{2},-\frac{1}{2},0,0,0)$	$\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3$
$\bar{g}_1,\bar{g}_2$	$2\times(1,1,2,2,1,1,1,1,1)$	$(0,0,-\frac{1}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3$
f_1	$(1,3,1,1,1,1,1,1,1)$	$(0,1,-1,0,0,0,0)$	$\mathbf{W}_1$
$\bar{f}_1$	$(1,\bar{3},1,1,1,1,1,1,1)$	$(0,-1,1,0,0,0,0)$	$\mathbf{W}_1$
f_2	$(1,3,1,1,1,1,1,1,1)$	$(0,1,0,-1,0,0,0)$	$\mathbf{W}_1$

In addition there are a number of particles with both semihidden and visible quantum numbers (Table I). The only asymptotically free group is the semihidden SO(5). After turning on the D terms and moduli, the SO(5) will be broken to SO(4) but will stay asymptotically free and become fully hidden. The semihidden $[SU(2)]^2$ in (4c) will also become asymptotically free and fully hidden. So we postulate that supersymmetry is broken by gaugino condensation in these (to be) hidden groups.

Fayet-Iliopoulos D term. The U(1) "anomaly" of A1 is in a linear combination of the latter three visible U(1)'s:

$$\mathrm{Tr}\mathbf{Q}=(0,0,24,24,-48,0,0)\ , \tag{5}$$

where the components of $\mathbf{Q}$ are just the U(1) charges. As discussed in Sec. II, the associated Fayet-Iliopoulos D term puts a constant in the equation of motion for the U(1) gauge auxiliary field:[20]

$$\mathbf{D}\sim m_{\mathrm{Pl}}^2(\mathrm{Tr}\mathbf{Q})+\sum_{\text{scalars}}\mathbf{q}_i\phi_i^*\phi_i\ . \tag{6}$$

A string vacuum is a supersymmetric minimum obtained where the expectation of $\mathbf{D}$ vanishes. This can be achieved if the fields with U(1) charges, ϕ_{13} and ϕ_{23}, acquire VEV's such that $|\langle\phi_{13}\rangle|^2=|\langle\phi_{23}\rangle|^2\sim m_{\mathrm{Pl}}^2$. It is clear from the methods of Appendix B that this is an F-flat direction so that supersymmetry is truly restored (and the dilaton tadpole vanishes). In the process, the superpotential couplings,

$$W\sim\phi_{12}\phi_{23}\bar{\phi}_{13}+\bar{\phi}_{12}\phi_{13}\bar{\phi}_{23}+\phi_{13}\bar{f}_1f_3+\phi_{23}\bar{f}_2f_3 \tag{7}$$

give masses to $\phi_{12},\bar{\phi}_{13},\bar{\phi}_{12},\bar{\phi}_{23},f_3$ and a linear combination $\bar{f}_1+(\text{phase})\bar{f}_2$; take $(\text{phase})=1$ for concreteness. There are additional superpotential couplings which make massive the entire $\overline{\mathbf{W}_2+\mathbf{W}_4+2\mathbf{W}_6}$ and $\overline{\mathbf{W}_3+\mathbf{W}_4+2\mathbf{W}_6}$ sectors (see Table I). Also half of the $\overline{\mathbf{W}_2+2\mathbf{W}_6}$ and $\overline{\mathbf{W}_3+2\mathbf{W}_6}$ sectors get massive. The U(1) in the TrQ direction is broken; also a second linear combination of the (latter) three visible U(1)'s is broken by the VEV's.

Massless moduli. The massless spectrum is further reduced via the massless moduli. The SO(5) vector has a

TABLE I. (*Continued*).

Superfield	Representation	U(1) charges[a]	Sector
$\bar{f}_2$	$(1,\bar{3},1,1,1,1,1,1,1)$	$(0,-1,0,1,0,0,0)$	$\mathbf{W}_1$
f_3	$(1,3,1,1,1,1,1,1,1)$	$(0,1,0,0,-1,0,0)$	$\mathbf{W}_1$
$\bar{f}_3$	$(1,\bar{3},1,1,1,1,1,1,1)$	$(0,-1,0,0,1,0,0)$	$\mathbf{W}_1$
$\phi_{12},\bar{\phi}_{12}$	$(1,1,1,1,1,1,1,1,1)$	$(0,0,\pm 1,\mp 1,0,0,0)$	$\mathbf{W}_1$
$\phi_{13},\bar{\phi}_{13}$	$(1,1,1,1,1,1,1,1,1)$	$(0,0,\pm 1,0,\mp 1,0,0)$	$\mathbf{W}_1$
$\phi_{23},\bar{\phi}_{32}$	$(1,1,1,1,1,1,1,1,1)$	$(0,0,0,\pm 1,\mp 1,0,0)$	$\mathbf{W}_1$
Φ_1,Φ_2	$2\times(1,1,1,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$\mathbf{W}_1$
S	$(1,1,1,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0)$	$\mathbf{W}_1$
X	$(1,1,1,1,1,1,1,1,5)$	$(0,0,0,0,0,0,0)$	$\mathbf{W}_1$
	$(2,1,1,1,1,1,1,1,4)$	$(\pm\frac{1}{2},\pm\frac{3}{4},\pm\frac{1}{4},\pm\frac{1}{4},\mp\frac{1}{4},\pm\frac{1}{2},\pm\frac{1}{2})$	$\mathbf{W}_4+\mathbf{W}_6$
	$(1,1,1,1,2,1,1,1,4)$	$(\mp\frac{1}{2},\pm\frac{3}{4},\pm\frac{1}{4},\pm\frac{1}{4},\mp\frac{1}{4},\mp\frac{1}{2},\pm\frac{1}{2})$	$\mathbf{W}_4+\mathbf{W}_6$
	$(1,1,2,1,1,1,2,1,1)$	$(\mp\frac{1}{2},\mp\frac{3}{4},\pm\frac{1}{4},\pm\frac{1}{4},\mp\frac{1}{4},\mp\frac{1}{2},\mp\frac{1}{2})$	$\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3+\mathbf{W}_6$
	$(1,1,2,1,1,1,2,1,1)$	$(\mp\frac{1}{2},\mp\frac{3}{4},\pm\frac{1}{4},\pm\frac{1}{4},\mp\frac{1}{4},\mp\frac{1}{2},\mp\frac{1}{2})$	$\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3+\mathbf{W}_6$
	$(1,1,1,1,1,1,2,2,4)$	$(0,0,\frac{1}{2},0,-\frac{1}{2},0,0)$	$\mathbf{W}_2+\mathbf{W}_4+2\mathbf{W}_6$
	$(1,1,1,1,1,1,2,2,4)$	$(0,0,0,\frac{1}{2},-\frac{1}{2},0,0)$	$\mathbf{W}_3+\mathbf{W}_4+2\mathbf{W}_6$
	$(1,1,1,1,1,1,1,1,1)$	$(0,0,0,\frac{1}{2},-\frac{1}{2},\pm 1,\pm 1)$	$\mathbf{W}_2+2\mathbf{W}_6$
	$(1,1,1,1,2,2,1,1,1)$	$(0,0,0,\frac{1}{2},-\frac{1}{2},0,0)$	$\mathbf{W}_2+2\mathbf{W}_6$
	$(1,1,1,1,1,2,1,1,1)$	$(0,0,0,-\frac{1}{2},-\frac{1}{2},\pm 1,0)$	$\mathbf{W}_2+2\mathbf{W}_6$
	$(1,1,1,1,2,1,1,1,1)$	$(0,0,0,-\frac{1}{2},-\frac{1}{2},0,\pm 1)$	$\mathbf{W}_2+2\mathbf{W}_6$
	$(1,1,1,1,1,1,1,1,1)$	$(0,0,\frac{1}{2},0,-\frac{1}{2},\pm 1,\pm 1)$	$\mathbf{W}_3+2\mathbf{W}_6$
	$(1,1,1,1,2,2,1,1,1)$	$(0,0,\frac{1}{2},0,-\frac{1}{2},0,0)$	$\mathbf{W}_3+2\mathbf{W}_6$
	$(1,1,1,1,1,2,1,1,1)$	$(0,0,-\frac{1}{2},0,-\frac{1}{2},\pm 1,0)$	$\mathbf{W}_3+2\mathbf{W}_6$
	$(1,1,1,1,2,1,1,1,1)$	$(0,0,-\frac{1}{2},0,-\frac{1}{2},0,\pm 1)$	$\mathbf{W}_3+2\mathbf{W}_6$
	$(1,1,2,1,1,1,1,1,1)$	$(0,0,\frac{1}{2},\frac{1}{2},0,\pm 1,\pm' 1)$	$\mathbf{W}_4+2\mathbf{W}_6$
	$(1,1,1,2,2,2,1,1,1)$	$(0,0,\frac{1}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_4+2\mathbf{W}_6$

[a]The first two U(1) charges, $U(1)_A$ and $U(1)_B$, contain two combinations which are viable hypercharges, $\pm U(1)_A-\frac{2}{3}U(1)_B$. With this normalization, the hypercharge coupling α_Y satisfies $\sin^2\theta_W=\frac{3}{8}$ at μ_U, as in usual GUT's.

flat direction $\langle X\rangle \sim m_{\rm Pl}$ which naturally breaks SO(5) to SO(4). Next, consider the doublets h_i, the singlets Φ_i,ϕ_i, and the visible SO(4) vectors g_i. These have the couplings

$$W\sim h_1\bar{h}_2\Phi_+ + h_2\bar{h}_4\Phi_+ + \bar{h}_1 h_2\Phi_- + \bar{h}_3 h_4\Phi_- + g_1\bar{g}_2\Phi_+ + \bar{g}_1 g_2\Phi_- + g_1 g_1\phi_{12} + g_2 g_2\phi_{12} + \bar{g}_1\bar{g}_1\bar{\phi}_{12} + \bar{g}_2\bar{g}_2\bar{\phi}_{12} \,, \tag{8}$$

where $\Phi_\pm=\Phi_1\pm\Phi_2$. The singlets $\Phi_\pm$ could each acquire a VEV, but this would make all the h's and g's heavy. Some h's are definitely needed for electroweak breaking while the g's can be used to break the horizontal SO(4). So, a more interesting region of moduli space is along the flat direction given by

$$\langle \Phi_+\rangle \sim m_{\rm Pl} \,, \quad |\langle \bar{g}_1\rangle|^2 = |\langle g_2\rangle|^2 \sim m_{\rm Pl}^2 \,. \tag{9}$$

This makes Φ_-, h_1, $\bar{h}_2$, h_3, and $\bar{h}_4$ massive. It also breaks the horizontal SO(4) to U(1) at the Planck scale. The VEV $\langle\Phi_+\rangle \sim m_{\rm Pl}$ gives mass to the entire $\mathbf{W}_4+\mathbf{W}_6$ and $\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3+\mathbf{W}_6$ sectors (see Table I). This makes the asymptotically free group from Eq. (4c), $[SU(2)]^4$, completely hidden. In addition, all the fractionally charged color singlets become superheavy; the potential culprits are either in the $\mathbf{W}_4+\mathbf{W}_6$ or $\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3+\mathbf{W}_6$ sectors, depending on the choice of hypercharge. Thus, the modulus $\langle\Phi_+\rangle$ removes these particles.

At this point we still have two more massless moduli; these are the VEV's for a linear combination of the triplets f and a linear combination of the doublets h. To preserve the standard model they must vanish; in particular,

$$\langle f_1-f_2\rangle = \langle \bar{f}_1-\bar{f}_2\rangle = 0 \,, \quad \langle \bar{h}_1+h_2+i\bar{h}_3+ih_4\rangle = 0 \,. \tag{10}$$

The choice for these two is unnatural in the sense described in Sec. II. However, taking the VEV's to vanish does give a point of enhanced symmetry; as a result the VEV's are natural at least in the technical sense. In prin-

TABLE II. Model A1 spectrum—"after." Gauge group $=SU(2)_L \times SU(3) \times [SU(2)]^6 \times [U(1)]^6$.

Superfield	Representation	U(1) charges	Sector
Q_1	(2,3,1,1,1,1,1,1)	$(0,-\frac{1}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_2$
L_1	(2,1,1,1,1,1,1,1)	$(0,\frac{3}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_2$
$\bar{U}_1$	$(1,\bar{3},1,1,1,1,1,1)$	$(1,\frac{1}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_2$
$\bar{D}_1$	$(1,\bar{3},1,1,1,1,1,1)$	$(-1,\frac{1}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_2$
$\bar{e}_1$	(1,1,1,1,1,1,1,1)	$(-1,-\frac{3}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_2$
$\bar{\nu}_1$	(1,1,1,1,1,1,1,1)	$(1,-\frac{3}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_2$
Q_2	(2,3,1,1,1,1,1,1)	$(0,-\frac{1}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_3$
L_2	(2,1,1,1,1,1,1,1)	$(0,\frac{3}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_3$
$\bar{U}_2$	$(1,\bar{3},1,1,1,1,1,1)$	$(1,\frac{1}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_3$
$\bar{D}_2$	$(1,\bar{3},1,1,1,1,1,1)$	$(-1,\frac{1}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_3$
$\bar{e}_2$	(1,1,1,1,1,1,1,1)	$(-1,-\frac{3}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_3$
$\bar{\nu}_2$	(1,1,1,1,1,1,1,1)	$(1,-\frac{3}{2},\frac{1}{2},0,0,0)$	$\mathbf{W}_3$
Q_3,Q_4	(2,3,1,1,1,1,1,1)	$(0,-\frac{1}{2},-\frac{1}{2},\pm\frac{1}{2},0,0)$	$\mathbf{W}_4$
L_3,L_4	(2,1,1,1,1,1,1,1)	$(0,\frac{3}{2},-\frac{1}{2},\pm\frac{1}{2},0,0)$	$\mathbf{W}_4$
$\bar{U}_3,\bar{U}_4$	$(1,\bar{3},1,1,1,1,1,1)$	$(1,\frac{1}{2},-\frac{1}{2},\pm\frac{1}{2},0,0)$	$\mathbf{W}_4$
$\bar{D}_3,\bar{D}_4$	$(1,\bar{3},1,1,1,1,1,1)$	$(-1,\frac{1}{2},-\frac{1}{2},\pm\frac{1}{2},0,0)$	$\mathbf{W}_4$
$\bar{e}_3,\bar{e}_4$	(1,1,1,1,1,1,1,1)	$(-1,-\frac{3}{2},-\frac{1}{2},\pm\frac{1}{2},0,0)$	$\mathbf{W}_4$
$\bar{\nu}_3,\bar{\nu}_4$	(1,1,1,2,1,1,1,1)	$(1,-\frac{3}{2},-\frac{1}{2},\pm\frac{1}{2},0,0,)$	$\mathbf{W}_4$
h_2	(2,1,1,1,1,1,1,1)	(1,0,1,0,0,0)	$\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3$
$\bar{h}_1$	(2,1,1,1,1,1,1,1)	$(-1,0,-1,0,0,0)$	$\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3$
h_4	(2,1,1,1,1,1,1,1)	$(1,0,-1,0,0,0)$	$\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3$
$\bar{h}_3$	(2,1,1,1,1,1,1,1)	$(-1,0,1,0,0,0)$	$\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3$
g	$2\times(1,1,1,1,1,1,1,1)$	(0,0,0,0,0,0)	$\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3$
f_1	(1,3,1,1,1,1,1,1)	$(0,1,-1,0,0,0)$	$\mathbf{W}_1$
f_2	(1,3,1,1,1,1,1,1)	$(0,1,-1,0,0,0)$	$\mathbf{W}_1$
$\bar{f}_1-\bar{f}_2$	$(1,\bar{3},1,1,1,1)$	$(0,-1,1,0,0,0)$	$\mathbf{W}_1$
$\bar{f}_3$	$(1,\bar{3},1,1,1,1)$	$(0,-1,1,0,0,0)$	$\mathbf{W}_1$
ϕ_{13}	(1,1,1,1,1,1,1,1)	(0,0,0,0,0,0)	$\mathbf{W}_1$
ϕ_{23}	(1,1,1,1,1,1,1,1)	(0,0,0,0,0,0)	$\mathbf{W}_1$
Φ_+	(1,1,1,1,1,1,1,1)	(0,0,0,0,0,0)	$\mathbf{W}_1$
S	(1,1,1,1,1,1,1,1)	(0,0,0,0,0,0)	$\mathbf{W}_1$
X	(1,1,1,1,1,1,2,2)	(0,0,0,0,0,0)	$\mathbf{W}_1$
	$2\times(1,1,1,2,1,1,1,1)$	$(0,0,-1,0,\pm1,0)$	$\mathbf{W}_2+2\mathbf{W}_6$
	$2\times(1,1,2,1,1,1,1,1)$	$(0,0,-1,0,0,\pm1)$	$\mathbf{W}_2+2\mathbf{W}_6$
	$2\times(1,1,1,2,1,1,1,1)$	$(0,0,-1,0,\pm1,0)$	$\mathbf{W}_3+2\mathbf{W}_6$
	$2\times(1,1,2,1,1,1,1,1)$	$(0,0,-1,0,0,\pm1)$	$\mathbf{W}_3+2\mathbf{W}_6$
	$8\times(1,1,1,1,1,1,1,1)$	$(0,0,1,\pm\frac{1}{2},\pm'1,\pm''1)$	$\mathbf{W}_4+2\mathbf{W}_6$
	$2\times(1,1,2,2,1,1,1,1)$	$(0,0,1,\pm\frac{1}{2},0,0)$	$\mathbf{W}_4+2\mathbf{W}_6$

ciple these VEV's will be zero for some values of the supersymmetry-breaking parameters. This information could be used to constraint the values of those parameters in a more detailed analysis. However, it would not be worthwhile to engage in such antics with this particular model as we will soon uncover more serious defects.

To summarize the results of Fayet-Iliopoulos D terms and massless moduli, the remaining gauge group is

$$SU(2)\times SU(3)\times[U(1)]^2\times[U(1)]^2 \quad (\text{visible}) , \tag{11a}$$

$$\times[SU(2)]^2\times[U(1)]^2 \quad (\text{semihidden}) , \tag{11b}$$

$$\times[SU(2)]^4 \quad (\text{hidden}) . \tag{11c}$$

Of the remaining particles, the more relevant ones include 4 generations, 4 weak doublets, and 4 color triplets. The complete spectrum which remains is given in Table II.

$\sin^2\theta_W$ *and* Λ_{QCD}. Following Sec. II, the one-loop renormalization-group flow of the gauge couplings provides a prediction for the Weinberg angle and Λ_{QCD}. We use the Georgi-Quinn-Weinberg equations[30]

$$\alpha_i^{-1}(\mu)\simeq\alpha_U^{-1}+\frac{b_i}{2\pi}\ln\frac{\mu_U}{\mu} \tag{12}$$

for the couplings at the scale μ in terms of the unified coupling α_U at scale μ_U and the b coefficients. For SU(N), in the supersymmetric case, the b's are given by

$$b_i=-3N+\sum_r l_r , \tag{13}$$

where l_r is the quadratic Casimir of the representation r (e.g., $l_r=\frac{1}{2}$ for the fundamental representation). For the U(1) factors normalized to have coupling α_U at the string tree level the b_i's are

$$b_i=\tfrac{1}{4}\sum Q^2 \ , \tag{14}$$

where Q is the charge.

To calculate $\sin^2\theta_W$ and $\Lambda_{\rm QCD}$ we assume all the remaining particles not in the standard model have masses ~ 1 TeV. Let the superunification scale be of order $g_{\rm string}5\times10^{17}$ GeV. The Georgi-Weinberg-Quinn equations may then be solved for $\sin^2\theta_W$, $\Lambda_{\rm QCD}$, and α_U in terms of the fine-structure constant, $\alpha(100\ \text{GeV})=\frac{1}{128}$. For model A1 we obtain $\alpha_U^{-1}\simeq5.3$, $\sin^2\theta_W=0.20+O(\alpha)$ at 100 GeV and $\Lambda_{\rm QCD}\approx100$ MeV. These predictions are compatible with experiment, so A1 passes this test. Also the unification coupling is consistent with perturbation theory. So it is seen that after the dust settles, a seemingly complicated model can exhibit a grand desert scenario and meet these basic criteria.

Proton decay. The problem of proton decay is the main defect of this model. The first mechanism for proton decay mentioned in Sec. II, from the couplings $QL\bar{D}$ or $\bar{U}\,\bar{D}\,\bar{D}$ (Fig. 1) is avoided, because the extra visible U(1)'s exclude these couplings; it is also easy to check that the Planck scale VEV's given to moduli do not induce these couplings via nonrenormalizable terms. However, the second mechanism, colored Higgs-boson exchange (Fig. 2), is present at low energies. In particular, three of the four remaining triplets, f_1, f_2, and $\bar{f}_3$ couple to matter with the superpotential

$$\begin{aligned} W\sim &\sum_{i=1}^{2}(f_iQ_iQ_i+f_i\bar{D}_i\bar{\nu}_i+f_i\,\bar{U}_i\bar{e}_i)\\ &+\bar{f}_3Q_3L_4+\bar{f}_3\bar{U}_3\bar{D}_4+\bar{f}_3Q_4L_3+\bar{f}_3\bar{U}_4\bar{D}_3 \ . \end{aligned} \tag{15}$$

These couplings allow for rapid proton decay. The decay proceeds (at most) at the TeV scale so the resulting proton lifetime is clearly unacceptable. Below we shall illustrate the mechanisms for avoiding this problem in other models.

Mass matrices. The four remaining Higgs bosons in model A1 couple the quarks and leptons in the $\mathbf{W}_2$ sector to those in the $\mathbf{W}_3$ sector. As a result there are enough couplings to give masses to all up and down quarks (and leptons) in two generations. However, the two generations in the $\mathbf{W}_4$ sector do not get masses. Furthermore it is clear that none of the Planck scale VEV's induce the needed Yukawa couplings. So, a completely satisfactory mass matrix is lacking. A model which overcomes this difficulty shall be constructed below.

Model A2. Some of the defects of model A1 may be avoided in other models. The construction in Table III shows how rapid proton decay may be avoided. We call this model A2. The gauge group, again from the $\mathbf{W}_0$ sector, is $SU(2)\times SU(3)\times[SU(4)]^2\times SU(5)\times[U(1)]^9$ (rank 22). There are four generations which appear in the $\mathbf{W}_2$, $\mathbf{W}_3$, $\mathbf{W}_2+\mathbf{W}_4$, and $\mathbf{W}_3+\mathbf{W}_4$ sectors. The long life of the proton is saved because the colored triplets in the $\mathbf{W}_0$ sector, called f_i before, are quintuplets of SU(5) and do not couple to quarks. This structure results from the projection of $\mathbf{W}_5$ on the $\mathbf{W}_0$ sector [Eq. (A4)]; the projection forces the $\mathbf{W}_0$ sector scalars which couple to matter to be weak doublets rather than color triplets.

TABLE III. Model A2.

$\mathbf{W}_0=({\tfrac12}^{20}|{\tfrac12}^{44})$

$\mathbf{W}_1=(0^2(0\tfrac12\tfrac12)^6|{\tfrac12}^{44})$

$\mathbf{W}_2=(0^2(0\tfrac12\tfrac12)^2(\tfrac12 0\tfrac12)^4|0^{16}{\tfrac12}^{28})$

$\mathbf{W}_3=(0^2(\tfrac12 0\tfrac12)^4(0\tfrac12\tfrac12)^2|0^{12}{\tfrac12}^{4}0^4{\tfrac12}^{24})$

$\mathbf{W}_4=(0^2(\tfrac12\tfrac12 0)^2(0\tfrac12\tfrac12)^2(\tfrac12\tfrac12 0)^2|0^{10}{\tfrac12}^{6}{\tfrac12}^{6}0^{22})$

$\mathbf{W}_5=(0^2(000)^2(0\tfrac12\tfrac12)^2(000)^2|({\tfrac12}^{4}0^6)(0^4{\tfrac12}^{2})({\tfrac12}^{2}0^2)0^2{\tfrac12}^{12}0^{10})$

$\mathbf{W}_6=(0^2(000)^2(000)^2(0\tfrac12\tfrac12)^2|{\tfrac14}_c^{5}({\tfrac12}^{2}0^2\tfrac14{}_c)(\tfrac14{}_c0^2){\tfrac12}^{2}{\tfrac12}^{6}0^6{\tfrac14}_c^{5})$

$k_{ij}=0$ for $i>j$ except $k_{41}=k_{31}=k_{21}=k_{53}=\frac{1}{2}$

The other salient features of model A2 include a Fayet-Iliopoulos D term corresponding to the "anomaly"[38]

$$\mathrm{Tr}\mathbf{Q}\sim(0,0,4,0,3,-4,0,-1,0) \ . \tag{16}$$

Again a supersymmetric minimum is found by giving VEV's to $\mathbf{W}_0$ sector scalars, with U(1) charges $(0,0,-1,0,0,+1,0,0,0)$ and $(0,0,0,0,-1,0,0,\pm1,0)$. The massless moduli may be chosen to leave four light doublets, though some VEV's again must vanish leaving some enhanced symmetry. A main problem of the model is that the asymptotically free groups, $[SU(4)]^2\times SU(5)$, do not become fully hidden. They stay semihidden in the sense that supermultiplets with hypercharge and color have $[SU(4)]^2$ and SU(5) quantum numbers, respectively (these particles also carry fractional electric charges). It is expected that supersymmetry breaking in the semihidden sector would then feed into the visible sector at a much higher scale. A rough calculation, assuming $\alpha_U=5.3$ as in the model A1, gives that SU(4) (with $b_4=-8$) confines at $\sim10^{16}$ GeV. SUSY breaking would be expected a few orders of magnitude below 10^{16} GeV (since the auxiliary fields in supergravity go as $\sim1/m_{\rm Pl}$ times the terms bilinear in the fermions), but hardly low enough to protect the mass of the Higgs boson. If we assume the dynamics of supersymmetry breaking somehow avoid this conclusion we may calculate $\sin^2\theta_W$ and $\Lambda_{\rm QCD}$; a rough calculation gives $\sin^2\theta_W=0.20+O(\alpha)$ and $\Lambda_{\rm QCD}\approx3$ GeV. $\Lambda_{\rm QCD}$ appears a bit high in this scheme. In any case, we prefer to think of A2 as an illustration of how to preserve the proton.

It is worth noting that the particles with fractional electric charge are confined into bound states by the semihidden $SU(5)\times[SU(4)]^2$ (these particles are in $\mathbf{W}_2+\mathbf{W}_3+\mathbf{W}_5+2\mathbf{W}_6$ and sectors of the form $\cdots+\mathbf{W}_5+\mathbf{W}_6$). Thus, the possibility exists, as in Ref. 4, that these may be quite massive. This illustrates a second mechanism for avoiding light fractional charges, though

TABLE IV. Model A3.

$$\mathbf{W}_0=(\tfrac{1}{2}^{20}|\tfrac{1}{2}^{44})$$
$$\mathbf{W}_1=(0^2(0\tfrac{1}{2}\tfrac{1}{2})^6|\tfrac{1}{2}^{44})$$
$$\mathbf{W}_2=(0^2(0\tfrac{1}{2}\tfrac{1}{2})^2(\tfrac{1}{2}0\tfrac{1}{2})^4|0^{16}\tfrac{1}{2}^{28})$$
$$\mathbf{W}_3=(0^2(\tfrac{1}{2}0\tfrac{1}{2})^4(0\tfrac{1}{2}\tfrac{1}{2})^2|0^{12}\tfrac{1}{2}^4 0^4\tfrac{1}{2}^{24})$$
$$\mathbf{W}_4=(0^2(0\tfrac{1}{2}\tfrac{1}{2})^2(\tfrac{1}{2}0\tfrac{1}{2})^2(\tfrac{1}{2}\tfrac{1}{2}0)^2|0^{10}(0^2 0^2\tfrac{1}{2}^2)\tfrac{1}{2}^4 0^2\tfrac{1}{2}^{22})$$
$$\mathbf{W}_5=(0^2(\tfrac{1}{2}\tfrac{1}{2}0)^2(\tfrac{1}{2}0\tfrac{1}{2})^2(0\tfrac{1}{2}\tfrac{1}{2})^2|0^{10}(0^2\tfrac{1}{2}^4)0^2\tfrac{1}{2}^2\tfrac{1}{2}^2 0^2\tfrac{1}{2}^{20})$$
$$\mathbf{W}_6=(0^2(000)^2(0\tfrac{1}{2}\tfrac{1}{2})^2(000)^2|0^{10}(\tfrac{1}{2}^2 0^4)0^4 0^2 0^2\tfrac{1}{2}^{10}0^{10})$$
$$\mathbf{W}_7=(0^2(000)^2(0\tfrac{1}{2}\tfrac{1}{2})(000)(000)^2|(\tfrac{1}{2}^4 0^6)(\tfrac{1}{2}0^5)0^4 0^2 0^2(\tfrac{1}{2}^5 0^5)\tfrac{1}{2}^8 0^2)$$
$$\mathbf{W}_8=(0^2(0\tfrac{1}{2}\tfrac{1}{2})(000)(000)^2(0\tfrac{1}{2}\tfrac{1}{2})(000)|\tfrac{1}{4}{}_c^5(0^2\tfrac{1}{4}{}_c\tfrac{1}{2}0)\tfrac{1}{4}{}_c\tfrac{1}{2}0\tfrac{1}{2}0\tfrac{1}{2}0(0\tfrac{1}{4}{}_c^2\tfrac{1}{4}{}_c^2 0)\tfrac{1}{2}^6 0^2\tfrac{1}{4}{}_c)$$

$k_{ij}=0$ for $i>j$ except $k_{21}=k_{31}=k_{41}=k_{51}=\frac{1}{2}$ and $k_{73}=k_{75}=k_{86}=k_{87}=\frac{1}{2}$

this mechanism is perhaps more dependent on assumptions regarding nonperturbative effects (i.e., confinement).

Although A2 avoids proton decay the real test is whether acceptable quark and lepton masses can be obtained at the same time we protect the proton lifetime. In other words, we need to know if the required Yukawa couplings of quarks and leptons to SU(2) doublets do not imply the existence of dangerous couplings to SU(3) triplets. In this sense model A2 is not quite adequate since the mass matrices have several defects. For example, there is one massless generation. Also there are no KM mixings between the first two generations ($\mathbf{W}_2$ and $\mathbf{W}_3$) and the second two ($\mathbf{W}_2+\mathbf{W}_4$ and $\mathbf{W}_3+\mathbf{W}_4$); this is true even after the nonrenormalizable terms are included. It should also be noted that the neutrino mass eigenvalues are the same as the up-quark mass eigenvalues; then the appropriate splitting, $m_\nu \ll m_u$, does not occur.

Model A3. We can, however, construct a model which incorporates both the proton lifetime and acceptable mass matrices. This model, called A3, is given in Table IV. Model A3 will have another problem; the spectrum contains so many particles that perturbative unification will not be possible. Nevertheless, it is useful to see how the mass matrices arise and how the problems of A3 might be avoided.

The gauge group for model A3 is $SU(2)_L\times SU(3)\times[U(1)]^7\times SU(6)\times SU(2)$ (rank 16), consisting of gauge bosons in the $\mathbf{W}_0$ and $\mathbf{W}_0+\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3+\mathbf{W}_4+\mathbf{W}_5+\mathbf{W}_6+2\mathbf{W}_8$ sectors. There are four generations in the $\mathbf{W}_2$, $\mathbf{W}_3$, $\mathbf{W}_4$, and $\mathbf{W}_6$ sectors and a total of 16 Higgs doublets in $\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_3$, $\mathbf{W}_1+\mathbf{W}_3+\mathbf{W}_4$, $\mathbf{W}_1+\mathbf{W}_4+\mathbf{W}_5$, and $\mathbf{W}_1+\mathbf{W}_2+\mathbf{W}_5$. This structure results (following the discussion of interactions in Appendix A) in Yukawa couplings between the sectors $\mathbf{W}_2$ and $\mathbf{W}_3$, $\mathbf{W}_3$ and $\mathbf{W}_4$, $\mathbf{W}_4$ and $\mathbf{W}_5$, and $\mathbf{W}_5$ and $\mathbf{W}_2$. All the needed Yukawa couplings can be found in the renormalizable action. The relevant terms in the superpotential have the form

$$W\sim(\bar{U}_1\bar{U}_2\bar{U}_3\bar{U}_4)\begin{pmatrix}0 & h_1 & 0 & h_7\\ h_1 & 0 & h_3 & 0\\ 0 & h_3 & 0 & h_5\\ h_7 & 0 & h_5 & 0\end{pmatrix}\begin{pmatrix}Q_1\\ Q_2\\ Q_3\\ Q_4\end{pmatrix}$$
$$+(\bar{D}_1\bar{D}_2\bar{D}_3\bar{D}_4)\begin{pmatrix}0 & \bar{h}_2 & 0 & \bar{h}_7\\ \bar{h}_1 & 0 & \bar{h}_4 & 0\\ 0 & \bar{h}_3 & 0 & \bar{h}_6\\ \bar{h}_8 & 0 & \bar{h}_5 & 0\end{pmatrix}\begin{pmatrix}Q_1\\ Q_2\\ Q_3\\ Q_4\end{pmatrix}$$
$$+(\bar{\nu}_1\bar{\nu}_2\bar{\nu}_3\bar{\nu}_4)\begin{pmatrix}0 & h_2 & 0 & h_8\\ h_2 & 0 & h_4 & 0\\ 0 & h_4 & 0 & h_6\\ h_8 & 0 & h_6 & 0\end{pmatrix}\begin{pmatrix}L_1\\ L_2\\ L_3\\ L_4\end{pmatrix}$$
$$+(\bar{e}_1\bar{e}_2\bar{e}_3\bar{e}_4)\begin{pmatrix}0 & \bar{h}_1 & 0 & \bar{h}_8\\ \bar{h}_2 & 0 & \bar{h}_3 & 0\\ 0 & \bar{h}_4 & 0 & \bar{h}_5\\ \bar{h}_7 & 0 & \bar{h}_6 & 0\end{pmatrix}\begin{pmatrix}L_1\\ L_2\\ L_3\\ L_4\end{pmatrix}, \quad (17)$$

where Q_i (L_i), $i=1,\ldots,4$ are the quark (lepton) doublets, $\bar{U}_i,\bar{D}_i$ ($\bar{\nu}_i,\bar{e}_i$) the quark (lepton) singlets, and h_j, $\bar{h}_j$ the Higgs doublets. Of course, in order for this scheme to work and still permit perturbative unification, a linear combination of the h_i's must become the true Higgs doublet while most of the others must be superheavy; in particular at most seven of these doublets can stay light. This could result, in principle, from couplings of a linear combination of h_i's to a combination of $\bar{h}_i$'s and a singlet. We return to this point momentarily.

The mass matrices resulting from the couplings in (17) have several interesting properties. First, the down quarks and the down leptons have the same mass eigenvalues so that the relation $m_b=m_\tau$ at the unification scale is obtained. Yet, the up quarks and the neutrinos have completely different couplings, so there is no analogous relation for up quarks and neutrinos. It is even possible to have massless neutrinos if the h's which couple to neutrinos do not acquire a VEV. Ideally, one would like to see these doublets become superheavy.

The question of whether the appropriate weak doublets acquire large masses can be answered by looking at the Fayet-Iliopoulos D terms, massless moduli, and nonrenormalizable terms. We find, alas, that the needed couplings are not in fact present in model A3. However, there seems to be no fundamental obstacle to obtaining such couplings in a model incorporating this structure.

Let us briefly examine the other salient features of this model, in particular to see how the proton lifetime is saved. The U(1) "anomaly" is

$$\mathrm{TrQ} \sim (0,0,-1,-1,0,0,-2)\ , \tag{18}$$

where the first two (nonanomalous) U(1) charges are, at this stage, viable hypercharges. A supersymmetric minimum may be found by giving Planck scale VEV's to three scalars in the $\overline{\mathbf{W}_2+\mathbf{W}_3+\mathbf{W}_6+\mathbf{W}_8}$ and $\mathbf{W}_2+\mathbf{W}_3+\mathbf{W}_6+3\mathbf{W}_8$ sectors having the following U(1) charges:

$$\begin{aligned} \phi_1&: \ (0,-1,\tfrac{1}{4},\tfrac{1}{4},0,0,\tfrac{3}{4})\ , \\ \phi_2&: \ (0,1,\tfrac{3}{4},-\tfrac{1}{4},0,0,\tfrac{1}{4})\ , \\ \phi_3&: \ (0,1,-\tfrac{1}{4},\tfrac{3}{4},0,0,\tfrac{1}{4})\ . \end{aligned} \tag{19}$$

The VEV's $|\langle\phi_1\rangle|^2=2|\langle\phi_2\rangle|^2=2|\langle\phi_3\rangle|^2\sim m_{\mathrm{Pl}}^2$ then give a supersymmetric minimum ($\langle D\rangle=\langle F\rangle=0$). It is interesting that these VEV's break one of the viable hypercharges. The only remaining choice for the hypercharge fixes which states are up quarks and which are down quarks. This leads to the mass matrices discussed above and the relations $m_b=m_\tau$ and $m_\nu\neq m_u$.

The spectrum includes two $3,\overline{3}$ pairs of SU(3) whose Yukawa couplings could result in proton decay. These states are essentially the same as $f_i,\overline{f}_i$ ($i=1,2$) in model A1, in the $\mathbf{W}_0$ sector. However, here there is a massless modulus which removes these particles. In particular, there are 3-2-1 singlets in $\overline{\mathbf{W}_2+\mathbf{W}_6+2\mathbf{W}_8}$ and $\mathbf{W}_3+\mathbf{W}_6+2\mathbf{W}_8$ with U(1) charges:

$$\begin{aligned} \Phi_{\pm}^{A}&: \ (0,0,\pm1,-\tfrac{1}{2},0,0,-\tfrac{1}{2}) \quad (\overline{\mathbf{W}_2+\mathbf{W}_6+2\mathbf{W}_8}\ \text{sector})\ , \\ \Phi_{\pm}^{B}&: \ (0,0,-\tfrac{1}{2},\pm1,0,0,-\tfrac{1}{2}) \quad (\overline{\mathbf{W}_3+\mathbf{W}_6+2\mathbf{W}_8}\ \text{sector})\ . \end{aligned} \tag{20}$$

These fields can acquire Planck scale VEV's, $\langle\Phi_{+}^{A}\rangle=\langle\Phi_{-}^{A}\rangle=\langle\Phi_{+}^{B}\rangle=\langle\Phi_{-}^{B}\rangle$ which, along with the VEV's $\langle\phi_i\rangle$ above, lie along a completely flat direction. The dangerous triplets $f_i,\overline{f}_i$ then become superheavy via couplings to $\Phi_{\pm}^{A,B}$ and another, less dangerous $3,\overline{3}$ pair. This demonstrates the other mechanism for preserving the proton discussed in Sec. II: rescue by massless moduli. The proton will still decay; however, the decay will proceed at the Planck scale. The dominant mechanism would be decay via dimension five operators[33] with a lifetime (orders of magnitude) above the value obtained for usual GUT's [$\sim 10^{30}$ yr for $M_{\mathrm{GUT}}\sim 2\times10^{16}$ GeV (Ref. 30)].

Finally, let us remark on the main problem with A3. In addition to the 16 extra weak doublets, the spectrum includes a plethora of doublets, triplets, and particles with hypercharge which have no Yukawa couplings to matter. In fact there are 8 more remaining doublets, 12 more triplets, and 96 more particles with only hypercharge. The hypercharge coupling constant has a Landau pole at $\sim 10^9$ GeV, so that perturbative unification is not possible. These extra particles also carry fractional electric charges. As before, this model is only an illustration of how certain desirable features may be built into the construction.

To summarize Sec. III, we have analyzed three models with gauge groups $\mathrm{SU(3)\times SU(2)\times U(1)}\times\cdots$. The first example (A1) produced acceptable predictions for $\sin^2\theta_W$ and Λ_{QCD} but suffered from rapid proton decay. The second model (A2) incorporated a mechanism for suppressing proton decay. The third example (A3) was able to produce acceptable mass matrices and long-lived protons but could not provide perturbative unification.

IV. MODELS WITH BROKEN GUT SYMMETRY

In this section we consider fermionic string models in which the massless moduli break the gauge symmetry to $\mathrm{SU(3)\times SU(2)\times U(1)}\times\cdots$. We construct an example for each of the following breaking patterns:

$$[\mathrm{SU(2)}]^2\times\mathrm{SU(4)}\to\mathrm{SU(3)\times SU(2)\times U(1)}\ , \tag{21a}$$

$$\mathrm{SU(5)}\to\mathrm{SU(3)\times SU(2)\times U(1)}\ , \tag{21b}$$

$$\mathrm{SO(10)}\times[\mathrm{SU(2)}]^2\to\mathrm{SU(3)\times SU(2)}\times[\mathrm{U(1)}]^2\ . \tag{21c}$$

We shall emphasize the first two examples; the latter example just illustrates how the breaking pattern may be achieved. The SU(5) model will have a single GUT Higgs boson in the adjoint representation. The others will be broken by spinor representations. It will be apparent that the GUT symmetry breaking will occur naturally at

TABLE V. Model B1.

$$\mathbf{W}_0=(\tfrac{1}{2}^{20}|\tfrac{1}{2}^{44})$$
$$\mathbf{W}_1=(0^2(0\tfrac{1}{2}\tfrac{1}{2})^6|\tfrac{1}{2}^{44})$$
$$\mathbf{W}_2=(0^2(0\tfrac{1}{2}\tfrac{1}{2})^2(\tfrac{1}{2}0\tfrac{1}{2})^4|0^{16}\tfrac{1}{2}^{28})$$
$$\mathbf{W}_3=(0^2(\tfrac{1}{2}0\tfrac{1}{2})^4(0\tfrac{1}{2}\tfrac{1}{2})^2|0^{12}\tfrac{1}{2}^{4}0^4\tfrac{1}{2}^{24})$$
$$\mathbf{W}_4=(0^2(\tfrac{1}{2}\tfrac{1}{2}0)^2(0\tfrac{1}{2}\tfrac{1}{2})^2(\tfrac{1}{2}\tfrac{1}{2}0)^2|0^{10}\tfrac{1}{2}^{6}\tfrac{1}{2}^{4}\tfrac{1}{2}^{10}0^{14})$$
$$\mathbf{W}_5=(0^2(\tfrac{1}{2}\tfrac{1}{2}0)(\tfrac{1}{2}0\tfrac{1}{2})(0\tfrac{1}{2}\tfrac{1}{2})^2(\tfrac{1}{2}\tfrac{1}{2}0)(\tfrac{1}{2}0\tfrac{1}{2})|0^{10}(\tfrac{1}{2}^{5}0)(0\tfrac{1}{2}^{3})\tfrac{1}{2}^{10}\tfrac{1}{2}^{10}0^4)$$
$$\mathbf{W}_6=(0^2(000)^2(0\tfrac{1}{2}\tfrac{1}{2})(000)(000)^2|(\tfrac{1}{2}^{4}0^6)(\tfrac{1}{2}0^5)0^4(\tfrac{1}{2}^{5}0^5)\tfrac{1}{2}^{8}0^20^4)$$
$$k_{ij}=0 \quad \text{for } i<j \text{ except } k_{26}=\tfrac{1}{2}$$

the Planck scale. As a result these models will have many of the features of a 3-2-1 model. However, some differences remain; in particular the running coupling constants will be altered so as to endanger perturbative unification.

For example, the first model, given in Table V, shall be called B1. The choice of k_{ij} (Table V) leads, as in the previous section, to four generations, in $\mathbf{W}_2$ and $\mathbf{W}_3$, and Higgs doublets in the $\overline{\mathbf{W}_0+\mathbf{W}_2+\mathbf{W}_3}$ sector. The gauge group, again from the $\mathbf{W}_0$ sector, is

$$[SU(2)]_W^2 \times SU(4) \times [SU(2)]^4 \text{ (visible)}, \tag{22a}$$

$$\times SO(8) \text{ (semihidden)}, \tag{22b}$$

$$\times [SO(5)]^2 \times U(1) \text{ (hidden)}, \tag{22c}$$

which has rank 18. Two generations in the $\mathbf{W}_2$ sector form a doublet of a horizontal SU(2) while two generations in $\mathbf{W}_3$ form a doublet of a different horizontal SU(2) [the SU(2)'s will be broken by moduli]. A brief summary of the spectrum is

4 generations ($\mathbf{W}_2$ and $\mathbf{W}_3$ sectors),

16 weak doublets h_i, $i=1,2$ [two $(2,2,1,2,2,1,1)$ representations of the visible group (22a)] ($\overline{\mathbf{W}_0+\mathbf{W}_2+\mathbf{W}_3}$ sector),

4 GUT Higgs bosons $H_i,\overline{H}_i$, $i=1,2$ [$(1,2,4,1,1,1,2)$, $(1,2,\overline{4},1,1,1,2)$, $(1,2,4,1,1,2,1)$, and $(1,2,\overline{4},1,1,2,1)$ representations of the visible group] ($\mathbf{W}_5$ sector),

10 **6**'s of SU(4) f_1,f_2,f [$(1,1,6,3,1,1,1)$, $(1,1,6,1,3,1,1)$, and $(1,1,6,1,1,2,2)$ representations of the visible group] ($\mathbf{W}_0$ sector),

1 adjoint of both horizontal SU(2)'s ϕ [$(1,1,1,3,3,1,1)$ representation] ($\mathbf{W}_0$ sector),

2 vectors of SO(8)/doublets of both horizontal SU(2)'s g_i, $i=1,2$ [two $(1,1,1,2,2,1,1,8)$ representations of (22a) and (22b)] ($\overline{\mathbf{W}_0+\mathbf{W}_2+\mathbf{W}_3}$ sector),

vectors of each SO(5) X_i, $i=1,2$ ($\mathbf{W}_0$ sector), singlet Φ ($\mathbf{W}_0$ sector).

The U(1) produces no Fayet-Iliopoulos D term in this model. A flat direction of ϕ and g may be chosen to break the (two) horizontal SU(2)'s to [U(1)], give Planck scale masses to f_1,f_2 and leave four weak doublets. None of the remaining triplets couple to quarks; as a result, proton decay (mediated by f_1 and f_2) is suppressed.

To see how this comes about in more detail, consider the terms in the superpotential

$$W \sim f_1 f_2 \phi + h_1 h_1 \phi + h_2 h_2 \phi + g_1 g_1 \phi + g_2 g_2 \phi + g_1 g_2 \Phi . \tag{23}$$

The VEV for ϕ_{ij} [where $i,j=x,y,z$ denote the SO(3) vector components], $\langle a\phi_{xx}+b\phi_{yy}+c\phi_{zz}\rangle \sim m_{\rm Pl}$ is a flat direction for all a,b,c. Choosing a, b, and c all nonzero gives masses to all components of f_1 and f_2. Choosing furthermore $c^2=(a\pm b)^2$ leaves a massless **4** of $SO(4)_W$ in h_1 and h_2. It also leaves a massless 8 of SO(8) in g_1 and g_2; then the VEV $\langle g_1+ig_2\rangle \sim m_{\rm Pl}$ [with different SO(8) components of g_1 and g_2 getting the VEV] gives a mass to the $c^2 \neq (a\pm b)^2$ mode of ϕ_{ij}.

Next we consider GUT symmetry breaking. The flat directions for the GUT Higgs bosons may be deduced from the superpotential

$$W \sim H_1\overline{H}_1\Phi + H_2\overline{H}_2\Phi + H_1H_2 f + \overline{H}_1\overline{H}_2 f . \tag{24}$$

The direction given by $\langle H_1+\overline{H}_1+iH_2+i\overline{H}_2\rangle \sim m_{\rm Pl}$ is flat according to (24); specifically, we give VEV's to the states with fermionic charge vectors:

$$\begin{aligned} &H_1 \quad (\cdots -\tfrac12-\tfrac12-\tfrac12-\tfrac12-\tfrac12 \cdots -\tfrac12-\tfrac12),\\ &\overline{H}_1 \quad (\cdots +\tfrac12+\tfrac12+\tfrac12+\tfrac12+\tfrac12 \cdots +\tfrac12+\tfrac12),\\ &H_2 \quad (\cdots -\tfrac12-\tfrac12-\tfrac12-\tfrac12-\tfrac12 \cdots -\tfrac12+\tfrac12),\\ &\overline{H}_2 \quad (\cdots +\tfrac12+\tfrac12+\tfrac12+\tfrac12+\tfrac12 \cdots +\tfrac12-\tfrac12) \end{aligned} \tag{25}$$

in the $[SU(2)]_W^2\times SU(4)\times[SU(2)]^2$ subspace of the charge lattice. An analysis using the method of Appendix B reveals that this direction is completely flat. Thus $[SU(2)]_W^2\times SU(4)$ is naturally broken at the Planck scale; more correctly, since H is in the spinor of $[SU(2)]_R\times SU(4)\times[SU(2)]^2$, the breaking is

$$[SU(2)]_W^2\times SU(4)\times[SU(2)]^2 \to SU(2)_L\times SU(3)\times[U(1)]^2 . \tag{26}$$

The superpotential (23) gives masses to six of the eight (color) triplets in f. Ideally we are then left with four generations, four weak doublets, and two color triplets; the remaining color triplets, components of f, do not mediate proton decay [because they carry extra SU(2) quantum numbers]. However, there are extra components of $H_i,\overline{H}_i$ which are neither massive from (23) nor eaten by gauge bosons. These include the GUT Higgs boson, eight triplets and six hypercharge ± 2 particles. An analysis according to Appendix B shows that these stay light along the flat direction. Luckily there is no QLH coupling that would induce proton decay. However, these extra particles do contribute to the renormalization-group flow. A rough calculation then shows that the hypercharge coupling α_Y has a Landau pole at $\sim 10^{11}$ GeV. So perturbative unification is not possible.

It seems that this problem is endemic to the scheme used here of breaking at the Planck scale. The fact that there are four generations hardly makes a difference; if there were only three generations, the Landau pole would only move up to $\sim 10^{12}$ GeV. The only solution to this problem is to get the extra components of $H_i, \bar{H}_i$ out of the spectrum. In principle this could be achieved in a model if there was only one pair of GUT Higgs bosons, $H, \bar{H}$, without the coupling $H\bar{H}\Phi$. So far, we have not found how to construct such a model with spinor Higgs boson.

The model B2, defined in Table VI, has the GUT group SU(5) with one Higgs boson in the adjoint representation. The appearance of a single GUT Higgs boson suggests that such a construction may permit perturbative unification. As remarked in Sec. I, the adjoint Higgs boson together with chiral fermions is allowed because the world-sheet SU(5) current algebra is really level 2.[17] In fact, the model B2 is simply a variation of the SO(10) example given by Lewellen.[17] The first eight **W** vectors are the same as Lewellen's eight-generation SO(10) model; the ninth vector cuts the gauge symmetry to SU(5), halves the number of generations to four and reduces the number of adjoint Higgs bosons to one [Lewellen also constructs a four generation SO(10) model,[17] but we choose to concentrate on this SU(5) variation since it only requires a single GUT Higgs boson].

In B2 the gauge bosons are contained in the $\mathbf{W}_0+\{\mathbf{W}_2,\mathbf{W}_3,\mathbf{W}_4\}$ sectors, where $\{\mathbf{W}_2,\mathbf{W}_3,\mathbf{W}_4\}$ means any linear combination of $\mathbf{W}_2,\mathbf{W}_3,\mathbf{W}_4$. The full gauge group is

$$\mathrm{SU}(5)\times[\mathrm{U}(1)]^3 \ \text{(visible)}, \qquad \times[\mathrm{SU}(2)]^2\times\mathrm{SO}(7) \ \text{(semihidden)} \tag{27}$$

which has rank 12. The adjoint Higgs boson is in the $\mathbf{W}_0+\mathbf{W}_7+\{\mathbf{W}_2,\mathbf{W}_3,\mathbf{W}_4\}$ sectors while the four generations are in $\mathbf{W}_5+\{\mathbf{W}_2,\mathbf{W}_3,\mathbf{W}_4\}$. The spectrum may be summarized as follows:

4 generations (5 and $\overline{10}$) $\psi_i, \bar{\chi}_i$, $i=1,\ldots,4$ ($\mathbf{W}_5$ sector),

1 adjoint Higgs boson H ($\mathbf{W}_1+\mathbf{W}_7$ sector),

10 and $\overline{10}$ $F_1, \bar{F}_1$ ($\mathbf{W}_1+\mathbf{W}_7$ sector),

10 and $\overline{10}$ $F_2, \bar{F}_2$ ($\mathbf{W}_1$ sector),

12 $5,\bar{5}$ electroweak Higgs boson pairs $h_i, \bar{h}_i$ ($\mathbf{W}_1$, $\mathbf{W}_1+\mathbf{W}_7$, $\mathbf{W}_6$, and $\mathbf{W}_6+\mathbf{W}_7$ sectors),

SO(7) vector/SU(2)$_A$ doublet Φ_1 [(1,2,1,7) representation of (25)] with U(1) charges (1,0,0) ($\mathbf{W}_6$ sector),

SU(2)$_A$ doublet/SU(2)$_B$ adjoint Φ_2 [(1,2,3,1) representation of (25)] with U(1) (1,0,0) ($\mathbf{W}_6$ sector)

more SU(5) singlets ϕ_i .

TABLE VI. Model B2.

$$\mathbf{W}_0=(\tfrac{1}{2}^{20}|\tfrac{1}{2}^{44})$$
$$\mathbf{W}_1=(0^2(0\tfrac{1}{2}\tfrac{1}{2})^6|\tfrac{1}{2}^{44})$$
$$\mathbf{W}_2=(0^{20}|\tfrac{1}{2}^{8}0^4\tfrac{1}{2}^{8}0^{24})$$
$$\mathbf{W}_3=(0^{20}|0^8 0^4 0^4\tfrac{1}{2}^{4}\tfrac{1}{2}^{4}0^{20})$$
$$\mathbf{W}_4=(0^{20}|(\tfrac{1}{2}^{2}0^4\tfrac{1}{2}^{2})\tfrac{1}{2}^{4}(\tfrac{1}{2}^{2}0^2)^4 0^{16})$$
$$\mathbf{W}_5=(0^2(0\tfrac{1}{2}\tfrac{1}{2})^2(\tfrac{1}{2}0\tfrac{1}{2})^4|(\tfrac{1}{2}^{4}0^4)\tfrac{1}{2}^{2}0^2(\tfrac{1}{2}0)^8 0^2\tfrac{1}{2}^{14})$$
$$\mathbf{W}_6=(0^2(\tfrac{1}{2}0\tfrac{1}{2})^2(0\tfrac{1}{2}\tfrac{1}{2})^2(\tfrac{1}{2}\tfrac{1}{2}0)^2|\tfrac{1}{2}^{8}0^4\tfrac{1}{2}^{16}(0^4\tfrac{1}{2}^{12}))$$
$$\mathbf{W}_7=(0^2(000)^2(\tfrac{1}{2}0\tfrac{1}{2})^4|\tfrac{1}{2}^{8}0^{36})$$
$$\mathbf{W}_8=(0^2(\tfrac{1}{2}\tfrac{1}{2}0)(\tfrac{1}{2}0\tfrac{1}{2})(\tfrac{1}{2}0\tfrac{1}{2})^2(000)^2|(\tfrac{1}{2}^{2}0\tfrac{1}{2}^{2}0^2)0^2\tfrac{1}{2}^{2}\tfrac{1}{2}^{8}0^4 0^4\tfrac{1}{2}^{2}(\tfrac{1}{2}0^3\tfrac{1}{2}^{7}0^3))$$

$k_{ij}=0$ for $i<j$ except $k_{37}=k_{38}=\tfrac{1}{2}$

There is a U(1) "anomaly,"

$$\mathrm{Tr}\mathbf{Q} \sim (-1,0,0) , \tag{28}$$

which may be handled by giving VEV's to Φ_1 and Φ_2. The supersymmetric minimum is given by $\langle \Phi_1^u \rangle = \langle \Phi_2^d \rangle \sim m_{\mathrm{Pl}}$ where u and d are the "up" and "down" components of the respective $\mathrm{SU}(2)_A$ doublets.

The analysis of this model has not yet been completed (since this is a relatively new construction); in particular the nonrenormalizable couplings are not yet computed. However certain things are clear: In order to permit perturbative unification, the $\mathbf{10},\overline{\mathbf{10}}$ pairs should get superheavy as well as most of the $\mathbf{5},\overline{\mathbf{5}}$ pairs. The singlets could in principle accomplish this via couplings such as $\phi F\overline{F}, \phi h\overline{h}$ and also nonrenormalizable terms. The couplings will have to be worked out carefully to determine if this is possible (this work is in progress). In addition, the only renormalizable coupling of the electroweak Higgs boson to matter occurs for the $\mathbf{5},\overline{\mathbf{5}}$ pair in the $\mathbf{W}_1$ sector:

$$W \sim h_1 \psi_i \overline{\chi}_i + \overline{h}_1 \psi_i \overline{\nu}_i . \tag{29}$$

This electroweak Higgs boson couples to the GUT Higgs boson,

$$W \sim H h_1 \overline{h}_2 + H \overline{h}_1 h_2 , \tag{30}$$

so that the Higgs doublets and triplets can be split. The VEV

$$\langle H \rangle \sim \begin{bmatrix} 3 & & & & \\ & 3 & & & \\ & & -2 & & \\ & & & -2 & \\ & & & & -2 \end{bmatrix} \tag{31}$$

is in a completely flat direction. In this region of moduli space the doublets and triplets in $h_1, \overline{h}_1$ are split but they are both superheavy. To obtain light doublets some other mechanism is needed. The most universal such mechanism, the "missing partner" mechanism,[39] relies on higher representations (e.g., **50** and **75**) and so is not applicable in this model. Higher-level representations are allowed in the case of higher-level Kac-Moody algebras,[40] though further work is needed to determine if these can be incorporated in such a model. **75** and **50** representations may occur but the **50** would be massive. Also, before we can say definitively that proton decay is suppressed it will be necessary to compute whether the singlets that should acquire VEV's do not induce proton decay through nonrenormalizable interactions (this work is in progress).

TABLE VII. Model B3.

$\mathbf{W}_0 = (\tfrac{1}{2}^{20} \vert \tfrac{1}{2}^{44})$
$\mathbf{W}_1 = (0^2(0\tfrac{1}{2}\tfrac{1}{2})^6 \vert \tfrac{1}{2}^{44})$
$\mathbf{W}_2 = (0^2(0\tfrac{1}{2}\tfrac{1}{2})^2(\tfrac{1}{2}0\tfrac{1}{2})^4 \vert 0^{16}\tfrac{1}{2}^{28})$
$\mathbf{W}_3 = (0^2(\tfrac{1}{2}0\tfrac{1}{2})^4(0\tfrac{1}{2}\tfrac{1}{2})^2 \vert 0^{12}\tfrac{1}{2}^4 0^4 \tfrac{1}{2}^{24})$
$\mathbf{W}_4 = (0^2(\tfrac{1}{2}\tfrac{1}{2}0)^2(0\tfrac{1}{2}\tfrac{1}{2})^2(\tfrac{1}{2}\tfrac{1}{2}0)^2 \vert 0^{10}\tfrac{1}{2}^6\tfrac{1}{2}^4\tfrac{1}{2}^{10}0^{14})$
$\mathbf{W}_5 = (0^2(\tfrac{1}{2}\tfrac{1}{2}0)(\tfrac{1}{2}0\tfrac{1}{2})(0\tfrac{1}{2}\tfrac{1}{2})^2(\tfrac{1}{2}\tfrac{1}{2}0)(\tfrac{1}{2}0\tfrac{1}{2}) \vert 0^{10}(\tfrac{1}{2}^5 0)(0\tfrac{1}{2}^3)\tfrac{1}{2}^{10}\tfrac{1}{2}^{10}0^4)$
$\mathbf{W}_6 = (0^2(000)^2(0\tfrac{1}{2}\tfrac{1}{2})(000)(000)^2 \vert 0^{10}(\tfrac{1}{2}0^5)0^4(\tfrac{1}{2}^5 0^5)\tfrac{1}{2}^4 0^6 0^4)$

Nevertheless, it is apparent from the preliminary analysis that this type of construction opens a range of new possibilities. On one hand, the problem of perturbative unification in GUT models seems manageable in this construction; also, integer electric charge quantization is guaranteed by SU(5) unification. On the other hand, the range of grand unified scenarios is greatly increased once we consider Higgs bosons in higher representations. So this makes it more difficult to extract a true prediction from string theory.

Let us briefly illustrate how the breaking (21c) may be obtained. The spin structure in Table VII, called B3, exhibits this pattern of breaking. The spin structure is the same as model B1 except that $\tfrac{1}{2}^4$ in $\mathbf{W}_6$ is removed from ψ_{21} through ψ_{24} and ψ_{52} through ψ_{56}. As a result the gauge group becomes

$$\begin{aligned} &\mathrm{SO}(10)\times[\mathrm{SU}(2)]^2 \quad (\text{visible}) \\ &\quad\times[\mathrm{SU}(2)]^4 \quad (\text{semihidden}) \\ &\quad\times[\mathrm{SO}(5)]^2\times\mathrm{SU}(4) \quad (\text{hidden}) . \end{aligned} \tag{32}$$

The group $\mathrm{SO}(10)\times[\mathrm{SU}(2)]^2$ is broken along flat directions by two **16**'s and two $\overline{\mathbf{16}}$'s in the $\mathbf{W}_5$ sector. One $\mathbf{16},\overline{\mathbf{16}}$ pair breaks $\mathrm{SO}(10)\times[\mathrm{SU}(2)]^2$ to $\mathrm{SU}(5)\times\mathrm{SU}(2)\times\mathrm{U}(1)$; this contains what is essentially the flipped SU(5) group. The next pair breaks this to $\mathrm{SU}(3)\times\mathrm{SU}(2)\times[\mathrm{U}(1)]^2$. As before, the GUT scale is naturally $\sim m_{\mathrm{Pl}}$. As in B1, perturbative unification is precluded by the extra components of the GUT Higgs bosons.

V. DISCUSSION

In this paper we have constructed a series of fermionic string models with gauge group $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)\times\cdots$. Two kinds of construction were used. In one case the group $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ was obtained directly in terms of free world-sheet fermions; in the other case a larger group was broken to $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ via massless moduli at the Planck scale.

In the first case it was possible to obtain realistic predictions for $\sin^2\theta_W$ and Λ_{QCD}. In separate examples it was possible to suppress the decay of the proton and obtain realistic quark (and lepton) mass matrices. We identified the aspects of the construction that led to these phenomenological consequences. In the future we may investigate whether these features can be incorporated into a single model. In these models the massless moduli and Fayet-Iliopoulos D terms played a key role in truncating the massless spectrum. These mechanisms were even able to provide a hidden sector in which supersymmetry could be broken (model A1). Of course, the problem of whether supersymmetry is really broken remains a serious unresolved issue.

In the case that $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ was obtained via massless moduli, we constructed models with spinor Higgs-boson breaking and adjoint Higgs-boson breaking. The models examined with spinor Higgs-boson breaking were plagued by excess particles; as a result, the running coupling constants were inconsistent with perturbative unification and the known couplings at present day energies. Modifications of the spectrum and the superpotential that would avoid this difficulty were pointed out in Sec. IV (in particular, some excess GUT Higgs bosons would have to be projected out while leaving behind a flat direction for the remaining GUT Higgs boson). Yet we were unable to obtain this feature in this construction.

The model with adjoint Higgs-boson breaking of SU(5) avoided this difficulty in that the spectrum contained only one GUT Higgs boson. This construction also guaranteed electric charge quantization. However the proper doublet-triplet splitting appears unlikely without some modifications of the spectrum. Further work is needed to determine how promising such models can be.

It is worth noting which aspects of these results are relevant to the case of three generations and which pertain only to four-generation models. The mechanisms described for suppressing proton decay, removing extra particles, obtaining various gauge groups, and removing fractional charges can be expected to apply in three-generation models. Also the problems of perturbative unification and obtaining acceptable mass matrices are found in three-generation cases as well;[10,41] the solution of the perturbative unification problem in the 3-2-1 models A1 and A2 may be useful in assessing other examples. On the other hand, the Yukawa coupling matrix of model A3 relies particularly on its four-generation structure. This is because the four spin structures containing each generation form combinations which contain the Higgs multiplets. It remains to be seen whether a variant of this structure can be constructed in the three-generation case.

Let us consider how these results relate to other work on the phenomenology of string models. First, with fermionic strings, a number of flipped SU(5) models have been constructed,[3,4] the most recent of which has promising phenomenology.[4] The spectrum contains three generations and just one GUT Higgs boson ($10,\overline{10}$ pair), which should allow for perturbative unification. At the level of renormalizable interactions, the colored Higgs bosons which couple to quarks are naturally heavy, thus suppressing proton decay; also at the level of renormalizable interactions, there is a seesaw mechanism for the neutrino in at least one generation.[4] As in the minimal SU(5) model discussed here, further calculation of the nonrenormalizable terms would be desirable. This would be useful to determine if proton decay (mediated by remaining light triplets) is not induced by the terms with large VEV's, if a realistic mass matrix is possible and to complete the analysis of $\sin^2\theta_W$ and Λ_{QCD}. It would also be desirable to see if the hidden sector is truly hidden or really semihidden, as in model A2 presented here in Sec. III. In any event, even if the particular model in Ref. 4 shows flaws, this type of construction shows promise.

The phenomenology of orbifold models[8] has been investigated in detail in Refs. 8–10 and 42 with results similar to those found here. In particular Z_3 orbifolds of the 3-2-1 type and also $SU(3)\times SU(2)_L\times SU(2)_R\times U(1)$ type were analyzed in Refs. 9 and 10. Some phenomenological constraints, such as the proton lifetime, could be met while others, such as perturbative unification, were more elusive without some intermediate scale in the model.[10] String models from Calabi-Yau manifolds were analyzed in Refs. 11, 12, and 41; the models based on Wilson line breaking of E_6 also have problems with perturbative unification.[41] However, more recent (2,0) compactifications may avoid this problem.[43] So, it is not at all possible to rule out any formulation of the heterotic string at this time.

In view of the generic problem of perturbative unification, it seems that the Antoniadas *et al.* model[4] which avoids this problem and the 3-2-1 model A1 (Sec. III) which has acceptable values for $\sin^2\theta_W$ and Λ_{QCD}, are significant. It seems that the coupling constant flow is a powerful test for the class of interesting models. Of course, the real problem is the incorporation of all the desirable features into a single model.

Though as yet no one model is a true candidate, the ingredients for meeting the phenomenological constraints are present. In a sense we are in an ideal situation (considering we do not yet have the dynamics for choosing the vacuum). The ingredients are there, yet to find them all in a single model is nontrivial. The number of truly acceptable models may then be rather small; there may be only one, or a few or none. Since we know some techniques for incorporating the desired features in constructing vacua, it should not be necessary to perform a random search. The more interesting constructions may be attempted, analyzed, and discarded as necessary. In this way we may, despite our lack of knowledge, obtain a good idea if string theory is really a viable description of nature.

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APPENDIX A: FERMIONIC STRINGS

In this appendix we briefly review the construction of fermionic strings. As discussed above we must also consider the space of string vacua continuously connected to these models via the massless moduli; the massless moduli are discussed in Appendix B. The advantage of considering fermionic strings is the relative ease with which renormalizable and nonrenormalizable terms in the effective action may be computed. The disadvantage, of course, is the loss of generality incurred in restricting attention to any one type of construction.

We employ the notation of Kawai, Lewellen, and Tye[6] for the fermionic formulation. The light cone degrees of freedom for the fermionic, heterotic string include 20 free right-moving Majorana-Weyl fermions, ψ_i, $i=1,\ldots,20$, and 44 free left-moving Majorana-Weyl fermions, ψ_i, $i=21,\ldots,64$. The first two right-moving fermions transform as two components of a spacetime (four-dimensional) vector. In addition there are two right- and left-moving free bosons X_i, $i=1,2$, which transform as a spacetime vector. There is a right-moving superconformal symmetry with supercurrent given by

$$T_F=i\sum_{i=1,2}\psi_i\partial X_i+\psi_3\psi_4\psi_5+\psi_6\psi_7\psi_8+\cdots+\psi_{18}\psi_{19}\psi_{20}\ . \tag{A1}$$

Other possible choices for the supercurrent exist;[44] in what follows we only consider the supercurrent of Eq. (A1). A string model is obtained by specifying the boundary conditions of the fermions around the closed loop of the string. More specifically, a model consists of a set of allowed sectors; in a given sector each fermion has a particular boundary condition. The boundary condition may be denoted by

$$\psi_i(\sigma=2\pi)=e^{2\pi i\mathbf{W}_j^i}\psi_i(\sigma=0) \tag{A2}$$

for the ith fermion in the $\mathbf{W}_j$ sector. Each $\mathbf{W}_j$ is then a 64-component vector. A set of $\mathbf{W}_j$'s meeting certain consistency conditions (Table VIII) is almost all that is needed to specify a model. A pair of Majorana-Weyl fermions may also be combined to form a Weyl (complex) fermion:

$$\lambda_k\sim\psi_i+i\psi_j\ . \tag{A3}$$

A Weyl fermion may have a boundary condition which is neither periodic nor antiperiodic, given by $\mathbf{W}_j^k=n/m$, with $n,m\in\mathbf{Z}$. We take all components $\mathbf{W}_j^i$ to lie in the interval [0,1).

Modular invariance requires that if the sectors $\mathbf{W}_i$ and $\mathbf{W}_j$ are present then the sector given by $\mathbf{W}_i+\mathbf{W}_j$ is present. So the vectors $\mathbf{W}_i$ form a basis for the allowed set of sectors. For those sectors given by linear combinations of $\mathbf{W}_i$'s we write $\overline{\mathbf{W}_i+\mathbf{W}_j}$ to indicate that each component is again between 0 and 1; $\overline{\mathbf{W}_i+\mathbf{W}_j}=\mathbf{W}_i+\mathbf{W}_j$ (mod1). The complete set of allowed sectors is then given by $\overline{\sum_j\alpha_j\mathbf{W}_j}$ where α_j's are non-negative integers less than the "order" (least common denominator) of the components of $\mathbf{W}_j$.

The rules for model construction are listed in Table VIII; see the references for a detailed discussion.[6,7] The spectrum of a given model consists of the various excitation in each sector, subject to a generalized Gliozzi–Scherk-Olive (GSO) projection.[45] In the $\overline{\sum\alpha\mathbf{W}}$ sector the physical states left by the projection are given by[6]

$$\mathbf{W}_i\cdot N_{\overline{\sum\alpha\mathbf{W}}}|\text{phys}\rangle=\left[\sum_j\alpha_jk_{ij}+k_{i0}+\mathbf{W}_i^1-\mathbf{W}_i\cdot\overline{\sum_j\alpha_j\mathbf{W}_j}\right]|\text{phys}\rangle \bmod 1 \tag{A4}$$

for all i where $N_{\overline{\sum\alpha\mathbf{W}}}$ is the fermion number operator in the $\overline{\sum_j\alpha_j\mathbf{W}_j}$ sector, the multidot products are Lorentzian (left movers minus right movers), $\mathbf{W}_0$ is the all-Neveu-Schwarz sector ($\mathbf{W}_0^k=\frac{1}{2}$ for all k), and k_{ij} is a matrix of rational numbers meeting the consistency conditions in Table VIII. A model is fully determined by specifying the $\mathbf{W}$ vectors and the k_{ij}'s. For an arbitrary choice of $\mathbf{W}_i^j$, a consistent set of k_{ij}'s may not exist; this is one of the main constraints on $\mathbf{W}_i^j$.

The definition of the number operators $N_{\overline{\sum\alpha\mathbf{W}}}$ in terms of the free fermion mode expansion is straightforward except for the Ramond zero modes in the case of real fermions. A precise definition for $N_{\overline{\sum\alpha\mathbf{W}}}$ in this case was worked out in Ref. 46. Since this case is relevant to the examples of Secs. III and IV, the definition is given in Table IX.

Let us illustrate the fermionic construction with some simple examples. The simplest model consists of just two sectors, one with all Neveu-Schwarz fermions and one with all Ramond fermions. The notation for this model is simply,

$$\mathbf{W}_0=(\tfrac{1}{2}^{20}|\tfrac{1}{2}^{44})\ , \tag{A5}$$

where $\frac{1}{2}^{20}$ refers to 20 right-moving Neveu-Schwarz fermions and $\frac{1}{2}^{44}$ refers to 44 left-moving Neveu-Schwarz fermions (right movers are on the left and vice versa due to obscure historical reasons). This model is not spacetime supersymmetric; it also contains tachyons. There are SO(44) gauge bosons in the $\mathbf{W}_0$ sector obtained by exciting one right-moving fermionic oscillator with a spacetime vector index and two left-moving fermionic oscillators. There are also $[\mathrm{SO}(3)]^6$ gauge bosons obtained by

TABLE VIII. Spin structure rules.

$k_{ij}+k_{ji}=\mathbf{W}_i\cdot\mathbf{W}_j \mod 1$

$m_jk_{ij}=0 \mod 1$

$k_{ii}+k_{i0}+\mathbf{W}_i^1-\frac{1}{2}\mathbf{W}_i\cdot\mathbf{W}_i=0 \mod 1$

$4\sum_{l:\ \text{real fermions}}\mathbf{W}_i^l\mathbf{W}_j^l\mathbf{W}_k^l=0 \mod 1$

$\mathbf{W}_i^1=\mathbf{W}_i^2=\mathbf{W}_i^3+\mathbf{W}_i^4+\mathbf{W}_i^5=\mathbf{W}_i^6+\mathbf{W}_i^7+\mathbf{W}_i^8=\cdots=\mathbf{W}_i^{18}+\mathbf{W}_i^{19}+\mathbf{W}_i^{20} \mod 1$

where m_j = least common denominator of $\mathbf{W}_j$

Modular invariance $\Longleftrightarrow \mathbf{W},k_{ij}$ exist

$\mathbf{W}_i\cdot N_{\overline{\sum\alpha\mathbf{W}}}|\text{phys}\rangle=\left[\sum_j\alpha_jk_{ij}+k_{i0}+\mathbf{W}_i^1-\mathbf{W}_i\cdot\overline{\sum_j\alpha_j\mathbf{W}_j}\right]|\text{phys}\rangle \bmod 1$

where $N_{\overline{\sum\alpha\mathbf{W}}}\equiv$ fermion number in the $\overline{\sum\alpha\mathbf{W}}$ sector

TABLE IX. Fermion number operator for real fermions.

$$\mathbf{W}_i\cdot N_{\sum\alpha\mathbf{W}}=\mathbf{W}_i\cdot N'_{\sum\alpha\mathbf{W}}+\tfrac{1}{4}(1-\Gamma_i^{\alpha\mathbf{W}})$$

where $N'_{\sum\alpha\mathbf{W}}\equiv$ fermion number vector excluding zero modes

and

$$\Gamma_i^{\alpha\mathbf{W}}\equiv g_i^0\prod_j (g_i^j)^{\alpha_j}$$

$$(g_i^j)\equiv(i)^{n/2}\gamma^{l_1}\gamma^{l_2}\cdots\gamma^{l_n}\quad\text{with } l_1<l_2<\cdots<l_n\quad\text{and } \mathbf{W}_i^{l_k}=\mathbf{W}_j^{l_k}=\tfrac{1}{2}$$

where $\gamma^l=\sqrt{2}$ (the Ramond zero-mode operator in ψ_l)

exciting a left-moving bosonic oscillator and a right-moving internal fermionic oscillator. The massless spectrum also includes SO(44) adjoint scalars, the graviton, axion, and dilaton. A spacetime supersymmetric model may be obtained by adding a second vector,

$$\mathbf{W}_1=(0^2(0\tfrac{1}{2}\tfrac{1}{2})^6|\tfrac{1}{2}^{44}) \tag{A6}$$

so that the allowed sectors are $\mathbf{0}$, $\mathbf{W}_0$, $\mathbf{W}_1$, and $\overline{\mathbf{W}_0+\mathbf{W}_1}$. The notation $(0\tfrac{1}{2}\tfrac{1}{2})^6$ means the components $0\tfrac{1}{2}\tfrac{1}{2}$ are repeated six times in a row. The $\mathbf{W}_1$ vector projects out the tachyons in the $\mathbf{W}_0$ sector [cf. Eq. (A4)] while the $\mathbf{W}_1$ sector contains the massless superpartners, e.g., gravitinos and gauginos. In general the superpartners of particles in an $\overline{\alpha\mathbf{W}}$ sector will be in the $\overline{\mathbf{W}_0+\mathbf{W}_1+\alpha\mathbf{W}}$ sector (because $\mathbf{W}_1$ contains the spacetime supersymmetry charge). This model has $N=4$ spacetime supersymmetry.

Consider adding two more vectors to the list of $\mathbf{W}$'s:

$$\begin{aligned}\mathbf{W}_2&=(0^2(0\tfrac{1}{2}\tfrac{1}{2})^2(\tfrac{1}{2}0\tfrac{1}{2})^4|0^{16}\tfrac{1}{2}^{28})\ ,\\ \mathbf{W}_3&=(0^2(\tfrac{1}{2}0\tfrac{1}{2})^4(0\tfrac{1}{2}\tfrac{1}{2})^2|0^{12}\tfrac{1}{2}^4 0^4\tfrac{1}{2}^{24})\ .\end{aligned} \tag{A7}$$

The model with vectors $\mathbf{W}_0$ through $\mathbf{W}_3$ has $N=1$ spacetime supersymmetry as a result of the new projections [Eq. (A4)] from $\mathbf{W}_2$ and $\mathbf{W}_3$. These projections in the $\mathbf{W}_0$ sector remove gauge bosons leaving the gauge symmetry SO(12)×SO(4)×SO(4)×SO(24). The $\mathbf{W}_2$ sector contains massless spacetime fermions, obtained with only Ramond zero-mode excitations and no Neveu-Schwarz excitations. Inspection of the zero modes reveals that these fermions transform in the (32,2,1,1), ($\overline{32}$,2,1,1), (32,$\overline{2}$,1,1), and ($\overline{32}$,$\overline{2}$,1,1) representations of SO(12)×[SO(4)]^2×SO(24). Likewise, fermions in the $\mathbf{W}_3$ sector are in the (32,1,2,1), ($\overline{32}$,1,2,1), (32,1,$\overline{2}$,1), and ($\overline{32}$,1,$\overline{2}$,1) representations.

Interactions between strings may be calculated using conformal techniques[47,48] with vertex operators obtained through bosonization. In cases where all world-sheet fermions are Weyl, these models are equivalent to lattice formulations[49,50] and the techniques of Ref. 49 may be applied directly. In the case of real fermions, some subtleties occur. These are discussed in Refs. 46 and 51. The bosonized vertex operators may be used; however, the physical states are not, in general, states of definite fermionic charge (momentum in the bosonic representation) but rather linear combinations of such states. Accordingly, the vertex operators are linear combinations. In addition, a consistent bosonization common to all sectors does not in general exist; this is because for a given real fermion, there may not be another fermion having the same boundary condition in all sectors. In other words, the real fermions in different sectors may not be simultaneously complexified. Nevertheless, when considering three-point interactions of massless states, the three sectors involved in any nonvanishing diagram may always be simultaneously complexified.

A useful selection rule in calculating interactions is apparent from the conservation of fermionic charge: In general it is true that the sum of the vectors, $\alpha_i\mathbf{W}_i-\mathbf{W}_0$, for the sector of each particle entering a Feynman graph must vanish (mod1) if the graph is nonvanishing.[46,51] That is,

$$\sum_n\left[\overline{\sum_i\alpha_i^n\mathbf{W}_i}-\mathbf{W}_0\right]=0\mod 1\ , \tag{A8}$$

where there are n external legs (see Fig. 3). This is because $\alpha_i\mathbf{W}_i-\mathbf{W}_0$ is the vacuum charge for the $\alpha_i\mathbf{W}_i$ sector; the vacuum charges must add up to an integer (for real fermions, this just means the number of incoming Ramond fermions must be even). In particular, for a three-point function, the three sectors must add up to $\mathbf{W}_0$ mod1. This is illustrated in Fig. 4. As an example, consider the model discussed above consisting of vectors $\mathbf{W}_0$ through $\mathbf{W}_3$. There will be Yukawa couplings between the scalars in the $\mathbf{W}_0$ sector and two fermions in $\mathbf{W}_1$. Likewise there will be Yukawa couplings between $\mathbf{W}_0$ and two fermions in $\mathbf{W}_2$ or two fermions in $\mathbf{W}_3$. There are also massless scalars in $\overline{\mathbf{W}_0+\mathbf{W}_2+\mathbf{W}_3}$; these will couple a fermion in $\mathbf{W}_2$ to a fermion in $\mathbf{W}_3$. This last structure is used in Secs. III and IV to provide the Yukawa couplings of the Higgs bosons to quarks and leptons.

APPENDIX B: MASSLESS MODULI

As mentioned in Sec. II the string models obtained from the free fermionic construction are string vacua degenerate with a much larger space of continuously connected vacua. This moduli space of vacua may be ex-

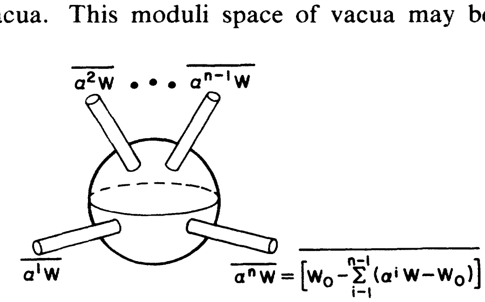

FIG. 3. Selection rule for string scattering.

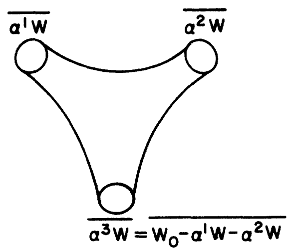

FIG. 4. Selection rule for three-point function.

plored by identifying the completely flat directions in the effective potential. In this appendix we show the methods by which the flat directions may be identified and discuss some generic features of these directions in the fermionic theories.

It is useful to address the identification of moduli in two parts. First we consider scalar fields of a restricted type for which flatness of the entire nonrenormalizable potential can be determined by the renormalizable terms. Next we find the methods which can be used outside this restricted class.

The first case arises if the vertex operator for the massless scalar in the zero ghost picture, at zero spacetime momentum, is a sum of products of currents. By currents we mean dimension-1 operators which are mutually local; their operator-product expansions (OPE's) contain no branch cuts. So, in this case the vertex operator has the form

$$\mathcal{O}(z,\bar{z})=\sum_{i,j} c_{ij}[\,A_i(z)\bar{B}_j(\bar{z})+\cdots\,]\ , \tag{B1}$$

where c_{ij} are real constants, A and $\bar{B}$ are left- and right-moving parts of the vertex operator, respectively, and the ellipsis refers to the piece involving world-sheet auxiliary fields (the world-sheet auxiliary fields provide the string contact interactions needed at zero spacetime momentum;[52] since this piece carries the same charges as the rest of the vertex operator, it does not affect the spacetime selection rules). If the A_i's are mutually local and the $\bar{B}_j$'s are mutually local, then it follows from the work of Ref. 53 that the scalar created by $\mathcal{O}(z,\bar{z})$ is in a completely flat direction if and only if the renormalizable potential is flat. So in this case it is not necessary to compute the nonrenormalizable terms. An example will be provided.

In the case where the operators A_i are not mutually local, Ref. 53 no longer applies. However, it is generally possible to identify the completely flat directions by invoking selection rules imposed by the world-sheet fermions on the nonrenormalizable terms. This is essentially the method used in the orbifold case.[10] What we do is consider the vertex operators for all the scalars obtaining VEV's and check if any combination can couple to the vertex operator for a spacetime auxiliary field. If no such coupling exists, then a spacetime auxiliary cannot acquire a VEV and the direction is flat. We illustrate the method for the spin structure construction below.

To illustrate how the flat directions are found in both cases, we consider the toy model of Appendix A:

$$\begin{aligned}
\mathbf{W}_0&=(\tfrac{1}{2}^{20}|\tfrac{1}{2}^{44})\ ,\\
\mathbf{W}_1&=(0^2(0\tfrac{1}{2}\tfrac{1}{2})^6|\tfrac{1}{2}^{44})\ ,\\
\mathbf{W}_2&=(0^2(0\tfrac{1}{2}\tfrac{1}{2})^2(\tfrac{1}{2}0\tfrac{1}{2})^4|0^{16}\tfrac{1}{2}^{28})\ ,\\
\mathbf{W}_3&=(0^2(\tfrac{1}{2}0\tfrac{1}{2})^4(0\tfrac{1}{2}\tfrac{1}{2})^2|0^{12}\tfrac{1}{2}^4 0^4\tfrac{1}{2}^{24})\ .
\end{aligned} \tag{B2}$$

For concreteness let $k_{ij}=0$ for $i<j$ (the constraints in Table VIII then fix the remaining k_{ij}'s). This model is equivalent to a covariant lattice construction[50] since all the sectors may be simultaneously complexified. In particular, for the left movers we may complexify adjacent pairs of fermions:

$$\lambda_{i/2}\sim\psi_i+i\psi_{i+1},\quad i \text{ odd},\quad 21\le i\le 44\ . \tag{B3a}$$

For the right movers, the pairing is

$$\begin{aligned}
\lambda_1&\sim\psi_1+i\psi_2\ ,\\
\lambda_2&\sim\psi_3+i\psi_6\ ,\\
\lambda_3&\sim\psi_4+i\psi_7\ ,\\
&\ \vdots\\
\lambda_{10}&\sim\psi_{17}+i\psi_{20}\ .
\end{aligned} \tag{B3b}$$

Bosons may be defined via $\lambda_j\sim e^{i\phi_j}$. The fermionic charge of a state is then equivalent to bosonic momentum.[6]

The flat directions of the first type, corresponding to vertex operators of the current-current form, are readily identified. For example, in the $\mathbf{W}_0$ sector there are scalars in the (12,4,1,1) representation of SO(12)×SO(4)×SO(4)×SO(24) with (zero picture) vertex operators such as $\psi_4\psi_5\psi_{21}\psi_{33}$. Since this operator is simply a Thirring interaction term,[54] it is clearly exactly marginal and corresponds to a completely flat direction (we dropped the world-sheet auxiliary piece since it cannot change the form of the effective action). Along this flat direction SO(12) is broken to SO(11) and SO(4) is broken to SO(3). There are other scalars in $\mathbf{W}_0$ which may be turned on simultaneously, in the (12,1,4,1), (12,1,1,24), (1,4,4,1), (1,4,1,24), and (1,1,4,24) representations. Again the vertex operators are just Thirring interactions.

Alternative flat directions, also of the current-current form are found, for example, in the $\overline{\mathbf{W}_0+\mathbf{W}_1+\mathbf{W}_2}$ sector (superpartners of the $\mathbf{W}_2$ sector fermions). Consider the particles in the (32,2,1,1) representation (32 is the spinor). It is clear that the renormalizable superpotential does not couple this representation to itself, since there are only vector and spinor massless representations. Thus the (32,2,1,1) is F flat in the renormalizable potential. Furthermore, since (32,2,1,1) is a real representation a D flat direction exists.[55] The lattice vectors in (32,2,1,1) have, among themselves, only integer multidot products, so the mutual locality condition of the "currents" is satisfied; therefore, we need not check any nonrenormalizable interactions to know that (32,2,1,1) has a completely flat direction. We can get the same answer by looking at the vertex operator; the fermionic charge vector for the scalar state (in the -1 picture) is

$$\mathbf{Q}=(0(000)(-\tfrac{1}{2}-\tfrac{1}{2}0)(-\tfrac{1}{2}-\tfrac{1}{2}0)|\mathbf{k}) . \tag{B4}$$

The vertex operator for a real scalar in the zero picture becomes [cf. Eqs. (A1) and (B3)],

$$\mathcal{O}\sim[e^{i(\phi_5+\phi_6-\phi_8-\phi_9)/2}(e^{i\phi_7}+e^{-i\phi_7}) + e^{-i(\phi_5+\phi_6-\phi_8-\phi_9)/2}(e^{i\phi_{10}}+e^{-i\phi_{10}})] \times(e^{i\mathbf{k}\cdot\boldsymbol{\phi}_L}+e^{-i\mathbf{k}\cdot\boldsymbol{\phi}_L})+\text{c.c.} , \tag{B5}$$

where $\boldsymbol{\phi}_L$ are the left-moving bosons. This operator is just a product of left- and right-moving currents. As such it is clearly exactly marginal. We can even define new fermions in terms of which $\mathcal{O}$ is a Thirring interaction. However, these fermions would not be the original fermions ψ_i but would be defined only transcendentally in terms of ψ_i.[56] In this sense such an operator is equivalent to a Thirring interaction in another fermionic basis.

It should be noted that the $\mathbf{W}_0$ sector scalar (12,4,1,1) couples this (32,2,1,1) to $(\overline{32},\overline{2},1,1)$ in the renormalizable superpotential. Hence the (12,4,1,1) and (32,2,1,1) directions together are not F flat.

Next consider the more general case, where the mutual locality condition is no longer satisfied. In particular consider giving a VEV to the $\mathbf{W}_0+\mathbf{W}_1+\mathbf{W}_2$ scalar in the (32,2,1,1) representation and also the $\overline{\mathbf{W}_0+\mathbf{W}_1+\mathbf{W}_2}$ scalar in the $(\overline{32},2,1,1)$ representation. Either VEV alone is in a flat direction; are the two VEV's together still flat? This can be answered by examining the world-sheet selection rules. The charge vector for the second scalar (in the -1 picture) is

$$\mathbf{Q}'=(0(000)(-\tfrac{1}{2}+\tfrac{1}{2}0)(-\tfrac{1}{2}-\tfrac{1}{2}0)|\mathbf{k}') , \tag{B6}$$

where $\mathbf{k}'\cdot\mathbf{k}$ is a half-integer; as a result the sum of the vertex operators for the two scalars does not satisfy the mutual locality condition [e.g., the left-moving factors, considered alone, have branch cuts $\sim(z-w)^{\pm\mathbf{k}\cdot\mathbf{k}'}$ in their OPE's]. Take $\mathbf{k}'\cdot\mathbf{k}=\frac{1}{2}$ for concreteness. If the direction under consideration was not F flat then some combination of the scalars with VEV's would couple to a spacetime auxiliary F field (to give that field a VEV). The only massless states which can be obtained from adding up combinations of $\pm\mathbf{k}$ and $\pm\mathbf{k}'$ are $\pm\mathbf{k}$ and $\pm\mathbf{k}'$ themselves; thus such a spacetime auxiliary must be in the $\overline{\mathbf{W}_0+\mathbf{W}_1+\mathbf{W}_2}$ sector. To get the charge of the vertex operator for a spacetime auxiliary field,[21] we use the spacetime supersymmetry charge

$$\mathbf{Q}_S=(\tfrac{1}{2}(\tfrac{1}{2}00)(\tfrac{1}{2}00)(\tfrac{1}{2}00)|0) . \tag{B7}$$

The charge of a spacetime auxiliary in the $\overline{\mathbf{W}_0+\mathbf{W}_2+\mathbf{W}_2}$ sector then has the form

$$\mathbf{Q}_F=(0(100)(\tfrac{1}{2}\pm\tfrac{1}{2}0)(\tfrac{1}{2}\pm\tfrac{1}{2}0)|\cdots) . \tag{B8}$$

There is no combination of states in (B4) and (B6) in any picture that can couple (add to zero) with the spacetime auxiliary $\mathbf{Q}_F$ (actually, two states would have to be in the -1 picture and the rest would have to be in the zero picture). Hence the direction in F flat. D flatness is again guaranteed by the reality of the representations. So the two VEV's are flat together.

It is interesting to note that the linear combination of vertex operators in the (32,2,1,1) and $(\overline{32},2,1,1)$ representations, which we now know is exactly marginal, is in no sense a Thirring interaction. This is because it cannot be fermionized; if one term such as $\mathcal{O}$ in Eq. (B5) is written as a four-Fermi interaction then the other term, with $\mathbf{k}'$ rather than $\mathbf{k}$, is still transcendental in terms of those fermions.

The same reasoning used to show that (32,2,1,1) and $(\overline{32},2,1,1)$ together give a flat direction can be used to find additional moduli. For example, the $\mathbf{W}_0$ sector scalar $(\underline{12},1,4,1)$ can acquire a VEV along with the (32,2,1,1) and $(\overline{32},2,1,1)$ while still remaining flat. Nevertheless, in cases where the mutual locality condition is not satisfied (i.e., multidot products such as $\mathbf{k}\cdot\mathbf{k}'$ are noninteger) it *is* necessary to check nonrenormalizable interactions. In other words, it is necessary to check that no combination of the scalars with VEV's couples to a spacetime auxiliary field. For example, the following four scalars couple to the spacetime auxiliary for (1,4,4,1):

$$\begin{aligned}&(32,2,1,1), \quad \overline{\mathbf{W}_0+\mathbf{W}_1+\mathbf{W}_2}\ \text{sector} ,\\ &(32,\overline{2},1,1), \quad \overline{\mathbf{W}_0+\mathbf{W}_1+\mathbf{W}_2}\ \text{sector} ,\\ &(32,1,2,1), \quad \overline{\mathbf{W}_0+\mathbf{W}_1+\mathbf{W}_3}\ \text{sector} ,\\ &(32,1,\overline{2},1), \quad \overline{\mathbf{W}_0+\mathbf{W}_1+\mathbf{W}_3}\ \text{sector} .\end{aligned} \tag{B9}$$

This means that there is a quintic term in the superpotential, coupling these four to (1,4,1,1); the quintic term is responsible for lifting a direction left flat by the renormalizable terms.

To summarize, the massless moduli of fermionic string models may be identified by considering the world-sheet selection rules. For moduli of the restricted current-current type, only renormalizable interactions in the effective field theory need be considered. For moduli of the more general type, nonrenormalizable terms can lift directions left flat by the renormalizable terms. As mentioned in Sec. II, this lifting provides the most viable mechanism for generating intermediate scales in the theory.[12] However, in the models constructed in Secs. III and IV, this mechanism was absent in the physically interesting regions of moduli space. The absence of intermediate scales led us to consider the grand desert scenario.

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