# Physics of the pion liquid

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Excited hadronic matter in the temperature interval $T = 100-200$ MeV is not an ideal pion gas, but rather a liquid, in which attractive interaction among particles plays an important role. The pion dispersion curve is in this case essentially modified by a kind of collective momentum-dependent potential, which becomes important as the "quasipion" comes to the boundary of the system. We show that these effects can provide an explanation for a number of recent experimental puzzles, in particular, for the observed copious production of soft pions and soft photons in high-energy hadronic reactions.

## I. INTRODUCTION

During the past few years much attention has been paid to the production of high-multiplicity hadronic systems both in *pp* and nuclear collisions at high energies (see Ref. 1). The main inspiration of this activity was related to the possible manifestation of a new phase of matter at high energy density, the so-called quark-gluon plasma.[2] Ironically enough, the usual hadronic matter consisting mainly of pions has attracted much less attention. In most cases its properties were assumed to be well represented by simple formulas corresponding to the ideal pion gas.

In this paper we discuss properties of such excited hadronic matter at high enough temperatures, when the interparticle interaction cannot be neglected. This important feature is emphasized in the title of this work, where the notion of the "pion liquid" (in contrast with the pion gas) is introduced. Roughly speaking, such conditions take place at temperatures above approximately 100 and below 200 MeV, presumably the critical temperature of the transition to the plasma phase. Although this interval of temperatures is not that great, it corresponds to about 1.5 orders of magnitude in terms of the energy density. Actually, the excited hadronic system being considered exists in just such a liquid phase. As we show below, the observables are strongly connected to the properties of this liquid.

The main qualitative difference between any liquid and a gas is the existence of an attractive interaction between its constituents, which creates a kind of a surface. This, in turn, makes it more difficult for the constituents to leave the system. We present theoretical and experimental indications that this is also true for the pion liquid under consideration. As far as we know, surface phenomena of this kind have not yet been discussed in the literature (apart from a preliminary version of this work[3]).

Our discussion below develops into two different directions. The first one is an attempt to create a simple theoretical framework for our discussion. In doing this we are mainly inspired by the experience of low-temperature physics, especially of the classical Landau works on properties of liquid $^4$He. We make use of a similarity between the so-called "quasipions" (or pions modified by the matter) and phonons in liquid helium. In particular, both of them are Goldstone modes, which result in similar features of their interaction. The most important part of the interpion interaction is known to be well described by the famous Weinberg nonlinear Lagrangian;[4] therefore, such a pion liquid study is also interesting for condensed-matter theory, as an example of a quantum liquid made of particles with a well-known interaction.

But even more important is that rather macroscopic "drops" of such liquid are now experimentally available. For example, in CERN nuclear collision experiments the produced hadronic systems contain up to several hundreds of pions, and this number will be at least one order of magnitude larger in the future experiments with Pb beams. This large "fireball" comes through different stages of its evolution, with particle densities varying from several units per fm$^3$ (at the phase transition) to those one order of magnitude smaller at which final breakup into noninteracting secondaries takes place. Pion interferometry tells us that the radius at the breakup is about 6–8 fm, so it is already larger than the radii of the heaviest nuclei.

Another goal of this work is an attempt to understand few unexpected phenomena observed recently in various high-energy reactions. Their list includes, in particular, (1) enhanced production of pions at small $p_t$ found in nuclear collisions[5] and (2) enhanced production of soft photons and low-mass dileptons in various hadronic reactions.[6] In this paper we develop a semiquantitative model which is used for some estimates of soft-pion and soft-photon production rates. We have found strong correlations between properties of the pion liquid and the corresponding spectra. There are two other sets of observations: namely, (1) an unexpectedly sharp two-pion annihilation threshold seen in the dilepton spectra at Berkeley;[7] (2) strong clustering of pions, produced in various hadronic reactions. Although the last two items are not studied in this work (see only comments in Sec. IX), we hope they probably also can be better understood if effects of collective interaction between particles are taken into account.

 

In order to complete this introduction, let us mention some important works, also aiming to explain these puzzles. Gale and Kapusta[8] have considered a pion dispersion curve modification in compressed nuclear matter. Their idea is that the secondary minimum of this curve can produce a peak in the dilepton invariant-mass distribution. However, as the recently observed peak is at $2m_\pi$, we think that in fact only some flattening of the dispersion curve at low momenta is actually needed to explain it, and such a shape of the dispersion curve leads to a simultaneous explanation of puzzles (1) and (2) as well.

Van Hove[9] has suggested another mechanism aiming to explain these phenomena (which he called the "ultrasoft" ones). His idea is to relate them to the possible production of long-lived metastable droplets of "overcooled" quark-gluon plasma. We know of similar phenomena in other systems with strong first-order transitions (e.g., formation of a fog). Previously, production of plasma droplets at the hadronization stage was discussed, e.g., in Ref. 10 (as one more reason for small hydrodynamical effects); see also Ref. 11 and especially Ref. 12, where detailed numerical studies of the production and decay of plasma clusters were made. In Ref. 13 it is argued that such plasma clusters probably have at least unit baryonic charge. However, in all these papers the plasma clusters were equilibrium ones, at critical temperature (about 200 MeV), while the idea of Van Hove is that they can be cooled down to much lower parton momenta and have a metastable lifetime of the order of tens of fm/c.

A soft-pion component was also ascribed to pions coming from resonance decays in Ref. 13, especially of Δ's at energies reached by the Alternating Gradient Synchrotron (AGS) at BNL. Actually, our modified pions are also a kind of $\rho$ or Δ resonance, but not a resonance with one particular pion, but with many pions (or nucleons) in the matter. Such treatment is obviously more consistent if the system lifetime is larger than of the production and decay of the resonances, as is the case for Δ's and $\rho$. (The long-lived resonances which decay much later than pions go out at the breakup stage, should indeed be treated separately.)

Another idea concerning soft pions was proposed in Ref. 14, where it introduced the nonzero chemical potential for pions, being numerically nearly as big as the pion mass. It makes good fit of the data, but the physical meaning of this chemical potential is unclear. It is argued that it is a consequence of specific kinetics of pion decoupling in which the equilibrium in pion number is violated, while thermal equilibrium is still there. If so, it is a transitory effect and for very large and long-lived systems this effect should disappear.

## II. THERMODYNAMICS OF THE PION GAS

Let us start with some general remarks about pions. They are special because they are the lightest hadrons, so that if temperature $T$ is small enough, only pions are excited. This fact is not occasional, but related to the specific nature of the pions, which is crucial for what follows. Unlike that of all other hadrons, the pion mass squared is proportional to $m_u+m_d$, the masses of light quarks. They are very small, of the order of 10 MeV in sum. That is why the real world is close to the so-called "chiral" one, in which $m_u+m_d=0$ and pions are massless. They are massless exactly for the same reason as, e.g., acoustical phonons in solids: they are Goldstone modes, a remnant of the spontaneously broken symmetry (translational symmetry for solids, the chiral one for the QCD vacuum.) General facts concerning this physics can be found, for example, in Ref. 15, while recent works on microscopic theory of chiral-symmetry breaking are in Ref. 16.

At small $T$ the mean interparticle distances are large, $1/T$, so interaction is small and the ideal-gas approach is justified. Also the discussion of massless pions makes the problem simpler, so we start with this case. At small temperatures we get then a simple formula for the energy density,

$$\epsilon(T)=(\pi^2/10)T^4 , \tag{2.1}$$

which differs only by the factor $\frac{3}{2}$ from that for blackbody radiation.

The interaction of pions is also quite specific and related to their Goldstone nature. In the chiral world, pions with momenta $k\to 0$ cannot interact with anything including themselves. (Exactly because of this feature acoustical phonons propagate distances much larger than molecule free paths; therefore we can hear each other.) This feature makes this gas even more ideal at small $T$.

Nonzero quark masses lead also to a nonzero interaction amplitude at $k=0$, the so-called scattering lengths for isospins $I=0,2$[22]. Their account[17] produces corrections to Eq. (2.1) of the order of $(m_\pi/F_\pi),^2$ which numerically are small, about $10^{-2}$, and we do not discuss them here.

Accounting for momentum-dependent pion-pion interaction, one can use chiral perturbation theory, considering $T/F_\pi$ as a small parameter. In the chiral limit ($m_\pi=0$) the behavior of the energy density with temperature $\epsilon(T)$ is of the type

$$\epsilon(T)=(\pi^2/10)T^4[1+C_1T^2/F_\pi^2+C_2T^4/F_\pi^4+O(T^6)] , \tag{2.2}$$

where $C_1,C_2$ are some constants. The former was calculated in Ref. 17, and happens to be zero for the following reason. The amplitude for pion-pion scattering at low momenta can be expanded in powers of the (c.m.) momentum $q$ in the standard way,

$$T_l^I(q)=(q^{2l})(a_l^I+b_l^Iq^2+\cdots) , \tag{2.3}$$

and, after averaging over spin and isospin of the $O(q^2)$ terms, for Weinberg values of $a$ and $b$ parameters one gets an exact compensation:

$$b_{\text{average}}=b_0^0+9a_1^1+5b_0^2=0 . \tag{2.4}$$

Repulsion in the last channel is exactly as strong as attraction in $I=0,1$ ones taken together.

Further corrections are either second-order rescattering-type effects (which were also estimated in Ref. 17), or those due to new higher-order terms in the pion Lagrangian compared to the Weinberg term. Recently, a rather detailed analysis of these effects up to three-loops was performed by Gerber and Leutwyler.[18] They got a more accurate estimate of the $T^8$ term in $\epsilon(T)$.

We summarized results on energy density in Fig. 1, where $\epsilon(T)/T^4$ is plotted vs $T$. For the ideal gas this ratio is close to one, until interaction of pions makes some modifications (see the dotted line). Much more dramatic things happen if one takes into account the nonzero pion mass and, even more important, hadronic resonances (in Beth-Ulenbeck approximation; see discussion in Refs. 17 and 18). The ratio becomes much more temperature dependent (see the solid line in Fig. 1). We also show simple parametrization for the "pion+resonances" suggested by the author many years ago:[19]

$$\epsilon(T)=T^6/T_0^2, \quad T_0=100 \text{ MeV} , \tag{2.5}$$

which is shown by the dashed line. One can see that it approximately describes the energy-density dependence on temperature over the whole interval.

Summarizing, above $T$ around 100 MeV or so, corrections to the ideal pion gas energy become noticeable, and at critical $T_c$ around 200 MeV it is about a factor of 4 greater than the ideal-gas estimate (2.1). (However, this still may be essentially smaller than the energy density of the quark-gluon plasma at this temperature, so the first-order transition with large jump in $\epsilon$ still may be there.)

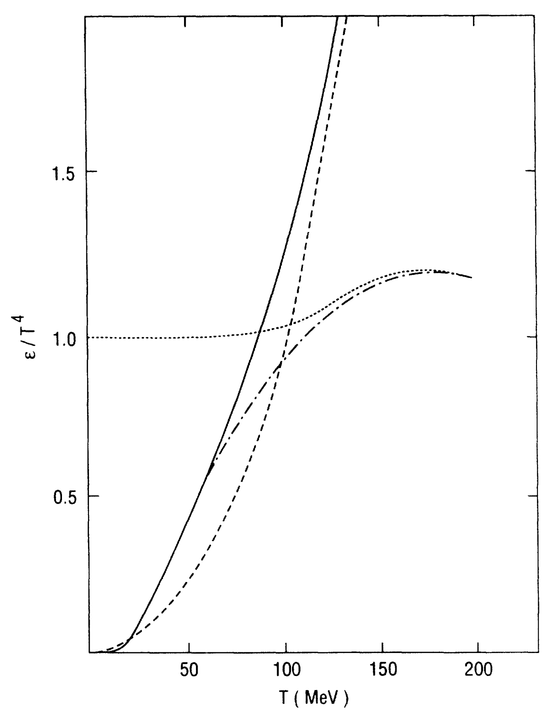


FIG. 1. Ratio of the energy density of excited hadronic matter $\epsilon(T)$ to $T^4$, vs temperature $T$. The dotted, dashed-dotted, and solid lines correspond to results of Ref. 18 for massless pions, massive pions, and pion together with resonances, respectively. The dashed curve corresponds to parametrization suggested in Ref. 9.

## III. KINETICS OF THE PION GAS

In this section we discuss how often the pion is scattered in the pion gas. The Goldstone nature of the pion plays an important role here: apart from small effects related with scattering lengths, the $\pi\pi$ cross section is essentially

$$\sigma_{\pi\pi}=5s/(48\pi F_\pi^4) , \tag{3.1}$$

where $s=(p_1+p_2)^2$ is the usual kinematic invariant. Therefore the mean time between collisions depends on $T$ as follows:[20]

$$\tau_{\text{coll}}=\text{const}\times F_\pi^4/T^5 . \tag{3.2}$$

Goity and Leutwyler have recently made extensive calculations[21] supporting this expression and getting more accurate const=12. Note, that at $T=200$ MeV $\tau_{\text{coll}}$ is small, 0.7 fm/$c$, but at other extreme $T=100$ MeV it is already about 25 fm/$c$, comparable to the lifetime of even the largest fireballs considered.

Another effect producing pion scattering is resonance formation. Using the detailed balance argument[20] we may say that production of a $\rho$ meson in equilibrium has the same rate as its decay, which is easier to calculate:

$$\tau_{\text{coll}}(\pi\pi\to\rho)^{-1}=\Gamma_\rho(2S+1)(2I+1)\times(m_\rho T/2\pi)^{3/2}\exp(-m_\rho/T) , \tag{3.3}$$

where $\Gamma_\rho$ and $m_\rho$ are the lifetime and mass of $\rho$ (in matter), $S$, $I$ are its spin and isospin, and the rest is just its density in matter, assuming ideal-gas thermodynamics. A resonant-induced scattering estimated like that becomes significant at $T$ above 150 MeV.

One should compare these numbers to the lifetime of the system and evaluate the expected breakup temperatures. The larger the system, the smaller is $T_{\text{breakup}}$, but due to very strong dependence on $T$, in practice the breakup parameters depend on multiplicity very little, for spherical fireballs only as $T\sim N_\pi^{1/15}$.

Another useful way to look at $\tau_{\text{coll}}$ is to compare its inverse, the imaginary part of the pion energy $\text{Im}E_\pi=1/\tau_{\text{coll}}$, with its real part. The corresponding ratio can be estimated as

$$\text{Im}E_\pi/\text{Re}E_\pi=0.03(T/F_\pi)^4 . \tag{3.4}$$

It is still only about 0.1 at $T=140$ MeV, but it is of the order 1 at $T_c=200$ MeV. Therefore, in the former case the pions are still propagating rather well, while in the latter case they are strongly absorbed.

## IV. PION MODIFICATION IN HADRONIC MATTER

Pion waves in matter are not only scattered in hot matter, but they also interact with many particles simultaneously and change their properties. In formal terms it means that the pion mass operator $\Pi$ has not only an imaginary part, but also a real one. The dispersion curve is defined as a solution for $\omega(k)$ of the equation

$$\omega^2=k^2+m_\pi^2+\Pi(\omega,k,T) . \tag{4.1}$$

In this section we argue that $\Pi$ leads to specific modifications of the $\omega(k)$ curve, and below we show that it has important consequences for various processes of experimental interest.

A problem of one particle interacting simultaneously with many was, for example, first met by Fermi in the 1930's when he considered propagation of slow neutrons in ordinary matter. He has introduced the notion of "pseudopotential," related to the scattering length $a$ at one nucleus and their density $n$:

$$U_{\mathrm{eff}}=2\pi na/m \tag{4.2}$$

(where $m$ is the mass of the particle). According to Weinberg,[22] the scattering lengths for the pion-pion interaction are such that their mean value is

$$a_{\mathrm{average}}=(a_0^0+5a_0^2)/9=0.067/m_\pi\ , \tag{4.3}$$

which means that repulsive $I=2$ channel wins by a small amount. As a result, the "pseudopotential" induced by the low-energy $\pi\pi$ scattering is approximately equal to

$$U_{\mathrm{eff}}=0.015T^3/F_\pi^2\ , \tag{4.4}$$

where we have substituted an approximate formula for the density of the ideal pion gas $n_\pi=0.36T^3$. It turns out to be very small, just few MeV, for all temperatures of interest. Let us repeat, it is so small because these scattering lengths are proportional to small quark masses.

The main pion-pion interaction is proportional to pion momenta; consequently, our collective potential should grow with momentum as well. Such a situation is common for all Goldstone modes; let us mention two well-known examples of cases in which the dispersion curve is strongly modified at larger $k$, so that even secondary minimum is developed.

(1) In Fig. 2 we show the famous dispersion curve of elementary excitations in liquid $He^4$, measured directly by neutron scattering experiments. Note the secondary minimum which was suggested by Landau and called the "roton" minimum. Modern theory of this curve, with accounting for phonon nonlinear interactions, can be found, e.g., in Ref. 31.

(2) In Fig. 3 we show the pion dispersion curve in cold dense nuclear matter, evaluated theoretically (see, e.g., Ref. 8, and references therein).

Returning now to pions in the pion liquid we note that terms which are nonzero in the chiral limit are first those proportional to $b_{\mathrm{average}}$, Eq. (2.4), which is zero. The following terms are not yet calculated, but it is possible to get a simple estimate of the gross effect of the pion modification. In doing this we may benefit from the strong similarity between this problem and physics of liquid $^4He$.

Let us remember how Landau came to the "roton" idea. He studied the data of the energy density of liquid $^4He$ as a function of the temperature and had noticed that at not too small $T$ it grows with temperature more rapidly than $T^4$, as suggested by the phonon contribution. Then he assumed that the phonon spectrum is distorted downward and has another minimum at nonzero momentum. (It was a long story until neutron scattering

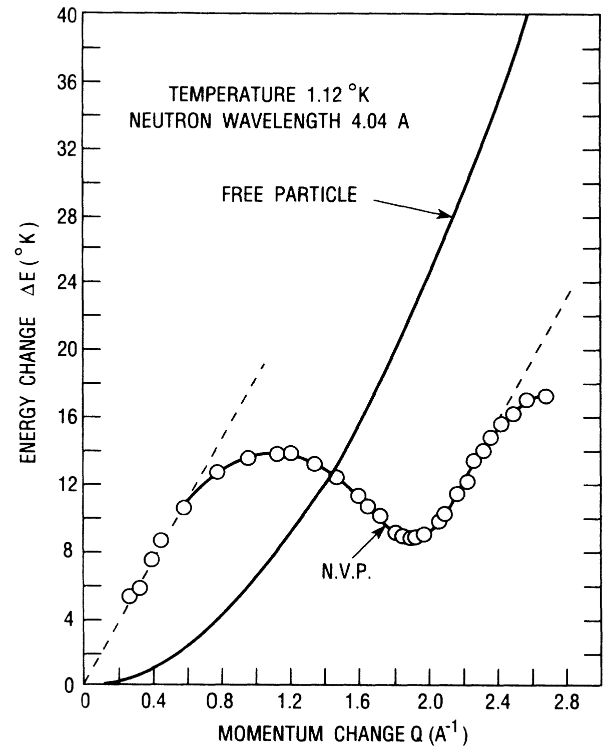


FIG. 2. Dispersion curve of quasiparticle in liquid $^4He$, taken from one of the first experiments on inelastic neutron scattering (Ref. 23).

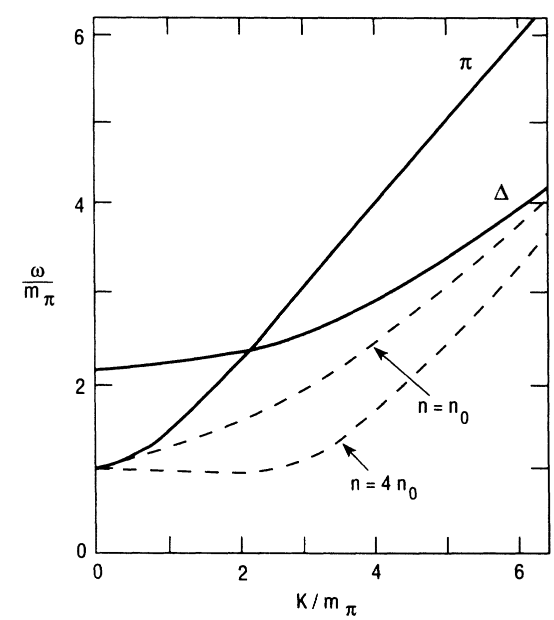


FIG. 3. Dispersion curve for pions in cold nuclear matter according to Ref. 8. Two dashed curves correspond to normal nuclear density $n=n_0$ and that four times larger.

experiments confirmed his guess.) He wrote the energy density of liquid $^4$He as the energy of the noninteracting (but distorted) phonons,

$$\epsilon(T)=\int d^3k\, \omega(k)/\{\exp[\omega(k)/T]-1\}\ , \tag{4.5}$$

and have found that the proposed distortion of $\omega(k)$ may explain the observed stronger growth of $\epsilon(T)$.

Before we proceed further, let us elucidate one question which has often confused people: does the modification considered in both cases correspond to attractive or repulsive interaction? The proposed shift of the dispersion curve down does imply that particles are attracted by each other. And still, it gives a positive contribution both to the pressure and to the energy density. Looking at the last formula it is easy to understand why this situation is different from the usual gas case. Diminishing in the exponent is more important than in the nominator. In a pion gas the number of particles is not fixed and attraction makes it possible to have more particles at the same temperature.

Now we are in the same position as Landau. We know that our $\epsilon(T)$ curve grows faster than that for the free pions. Suppose it is represented as gas of noninteracting "quasipions." What $\omega(k)$ curve can mimic that?

The expected qualitative behavior of the dispersion curve is shown in Fig. 4: the "pseudopotential" is slightly repulsive at small momenta but becomes attractive at larger ones. We do not want to have many free parameters, so we do not speculate about possible secondary

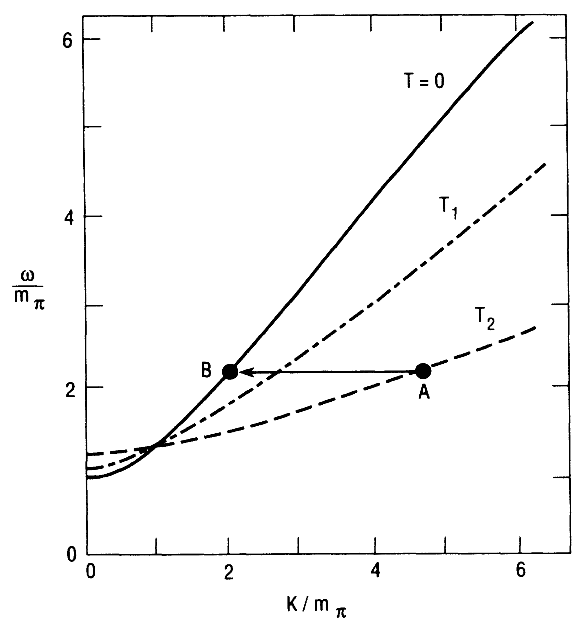


FIG. 4. Qualitative picture of the modification of the pion dispersion curve in hot pion liquid. It is assumed that $T_1 < T_2$. The "quasipion" with internal momentum corresponding to point $A$ moves to point $B$ as it goes out of the fireball.

minima and assume simple parametrization for the modified pion dispersion curve

$$\omega(k)=[m_\pi^2+u(T)^2k^2]^{1/2}\ , \tag{4.6}$$

introducing the temperature-dependent refraction index $u(T)$. The reason we took this form is that pion mass operator $\Pi(\omega,k)$ in the chiral limit starts with terms quadratic in $\omega$ and $k$, and in such an approximation Eq. (4.1) gives this simple form for the dispersion curve. Of course, there are further terms in $\omega$ and $k$, and, for example, secondary minimum may well be developed. With (4.6) momentum scales as $u(T)$, so the integral (4.5) gets extra $u(T)^{-3}$. This gives the following estimate for the mean quasipion velocity:

$$u(T)=[\epsilon(T)/\epsilon_{\rm gas}(T)]^{1/3}\ , \tag{4.7}$$

where $\epsilon_{\rm gas}$ stands for the noninteracting gas. Using now the information for the energy density discussed in Sec. II, we see that $u(T)$ is close to 1 at $T<100$ MeV, but at larger $T$ it starts decreasing approximately like $u\sim T^{-2/3}$ and at $T$ around 200 MeV it is about $u_c=\frac{1}{2}$. As we will show shortly, if this is the case there are important consequences for the observed spectra of secondaries and even for the whole space-time picture of the high-energy collisions.

Finally, a few words about other hadrons in the pion liquid. Let us start with kaons, for their modification should be similar to that of the pions. There is $\phi$ resonance, which is very narrow because of very close $KK$ threshold. An interesting question is whether kaon modification can affect $\phi$ meson width. Unfortunately, for the $\pi$-$K$ case the $I=\frac{1}{2}$ and $\frac{3}{2}$ scattering lengths tend to compensate each other (inside experimental errors), so this effect is not large at zero momenta. However, the role of momentum-dependent corrections was not clarified.

Speaking about other hadrons such as nucleons, hyperons, or $\rho$ mesons, we should remember that these are not Goldstone modes and therefore their strong modifications in the pion gas can, in principle, take place even for zero momenta. This topic is discussed in the literature in a different framework including lattice calculations, but it is probably too early to draw any conclusions on these issues.

## V. SURFACE PHENOMENA

If the collective interaction between particles creates the (momentum-dependent) potential, its presence produces important effects at the surface of the system, where the force (gradient of this potential) is substantial.

First of all, there appears surface tension which was never included in hydrodynamical calculations of the system expansion. It may be responsible for the production of clusters (or "drops") of hadronic matter. This phenomenon is well seen if one analyzes data on particle correlations. However, we do not discuss these effects in this work.

Another (and less trivial) surface phenomena which we are going to discuss in detail is the so-called "surface impedance." It is well known in low-temperature physics since the Kapitza experiments with liquid $^4$He. In practice it strongly reduces heat transfer through phase boundaries. (Recently it was discussed, e.g., in connection with the so-called dark-matter detectors, consisting of small superconducting clusters. It creates problems in keeping all clusters at the same temperature.) The physics is simple: it is because it is difficult for quasiparticles to penetrate the phase boundary. (The usual heat transfer by scattering is not effective at low temperatures.)

More precisely, propagation through the boundary has two different regimes, depending on the relation between the characteristic width of the boundary $d$ and the wavelength $\lambda$. If $d \ll \lambda$, the quasiparticle is split into two, propagating into different phases. (This is what happens when, e.g., light comes out of the glass.) The opposite limit $d \gg \lambda$ makes it possible to make a wave packet and consider its motion as that of a classical particle climbing out of the potential well. The force acts on it at the boundary, so the momentum component normal to the boundary $k_t$ is diminished. We schematically show this transition in Fig. 4, the quasipion proceeds from point $A$ at temperature $T_2$ curve to point $B$.

Energy conservation reads as

$$\omega_{\text{inside}}[(k_{t,\text{inside}}^2+k_l^2)^{1/2}]=\omega_{\text{outside}}[(k_{t,\text{outside}}^2+k_l^2)^{1/2}] \quad (5.1)$$

(momentum components $k_l$ in the boundary plane are unchanged). For example, with the parametrization (4.6) one gets

$$k_{t,\text{outside}}^2=u^2k_{t,\text{inside}}^2-k_l^2(1-u^2) \ . \quad (5.2)$$

If the right-hand side (RHS) is negative, the particle is reflected back.

For the pion coming out of the system into vacuum $\lambda$ is probably comparable to the interparticle separations, so in this case we have intermediate case $d$ of the order of $\lambda$. However, for simplicity in what follows we use the last expression (5.2) for evaluation of the momentum changes at the surface.

## VI. ANALYSIS OF THE PION $p_t$ SPECTRA

In this section we switch from the theory of the pion gas to phenomenology, concentrating on inclusive $p_t$ spectra of the pions, produced in recent CERN experiments[5] with nuclear beams at 200 GeV/$N$.

At first glance, the $p_t$ spectra measured in nuclear collisions look nearly identical to those observed in $pp$ collisions. This fact was first pessimistically interpreted as "these experiments have not showed anything new." However, these observations are far from being trivial. In particular (as emphasized in our previous paper[20]), this similarity can hardly be explained by naive models based on the picture of independent $NN$ collisions because rescattering of secondaries is by no means negligible: each pion suffers more than one rescattering (see Table I and discussion below). Also no strong indications for the collective hydrodynamical transverse flow were found, in contrast with what was suggested by earlier cosmic-ray data of the JACEE group. This means that the equation of state is soft enough, which in turn means[10] that we have approached the phase transition region.

TABLE I. Some parameters of the expanding fireball model calculations described in the text. $N_\pi$ is the number of pions in the fireball, $u_0$ is the parameter of the pion dispersion curve at the initial (critical) temperature, its value equal to 1 means that pions are unmodified. $R_0$, $R$ are initial radius and rms radius of the last collision point, $\tau_{\text{life}}$ is the mean time of the last collision, $N_{\text{coll}}$, $N_{\text{refl}}$ are mean number of collisions and reflections from the boundary.

| $N_\pi$ | $u_0$ | $R_0$ | $R$ | $\tau_{\text{life}}$ | $N_{\text{coll}}$ | $N_{\text{refl}}$ |
|---|---|---|---|---|---|---|
| 4000 | 1.0 | 13.7 | 22.0 | 81.0 | 62.0 | |
| 4000 | 0.5 | 6.8 | 10.0 | 33.2 | 53.0 | 1.34 |
| 400 | 1.0 | 6.3 | 11.0 | 41.0 | 23.0 | |
| 400 | 0.5 | 3.2 | 4.8 | 16.6 | 35.6 | 1.08 |
| 10 | 1.0 | 1.8 | 1.9 | 3.3 | 2.2 | |
| 10 | 0.5 | 0.90 | 0.92 | 2.6 | 3.0 | 0.88 |

However, more accurate analysis of these data shows important deviations from the $pp$ ones. In this section we perform rather detailed discussion of these spectra, using several different methods. Although we mainly concentrate on the soft-pion component, we also point out some indications for hadron "evaporation" during the mixed phase, related with fixed critical temperature for all secondaries.

Our first approach is based on "effective slope" plot, which is motivated by the idea[24] that looking for discontinuities of the spectra (presumably indicating different physical effects, dominating in different $p_t$ regions), one should differentiate them. We therefore introduce the "local slope"

$$T_{\text{slope}}=[d\ln(d\sigma/dp_t^2)/dp_t]^{-1} \ , \quad (6.1)$$

and plot it vs $p_t$ in Fig. 5. The dashed line corresponds to $pp$ data,[25] while dots represent data on nuclear collisions.[5]

Two observations can be made based on this figure. First, although both sets of data are not quite the same at $p_t$ less than 0.7 GeV (which still may be related to somewhat different rapidity intervals of these detectors), both show significantly smaller $T_{\text{slope}}$ in this region.

Second, above this $p_t$ region there is a sharp transition to stable $T_{\text{slope}}$ at the value around 200 MeV. A similar "plateau" was noticed long ago by Zhirov[24] for different $pp$ reactions, but it was less prominent and was seen only for larger $p_t=3-4$ GeV. Introducing "effective temperature," we correct (6.1) for the preexponential factor in the thermal $p_t$ spectrum, which is $m_t^{1/2}\exp(-m_t/T)$ above $m_t=3T$ [$m_t=(m^2+p_t^2)^{1/2}$ is the so-called transverse mass]. The corrected version looks then

$$T_{\text{eff}}=T_{\text{slope}}/(1-T_{\text{slope}}/2*m_t) \ . \quad (6.2)$$

The value of corrected $T_{\text{eff}}$ at the "plateau" happens to be the same for $pp$ and nuclear data, something around 200 MeV, in striking correlation to the expected critical

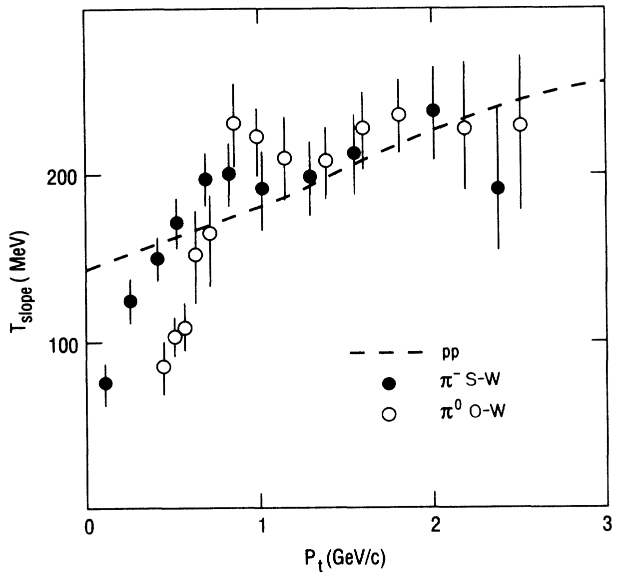


FIG. 5. "Effective slope" (6.1) vs transverse momentum $p_t$. The dashed line stands for the $pp$ data (Ref. 25), while the open and solid points represent data from nuclear collisions: neutral pions from O-Au (WA80) and negative secondaries in $S$-$W$ collisions (HELIOS), respectively (Ref. 5).

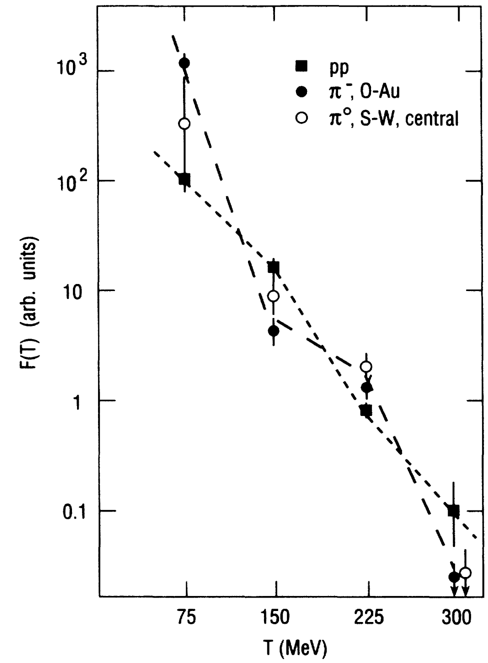


FIG. 6. The temperature profile function $F(T)$ vs temperature (for comparison, each spectrum is normalized to one). The open and solid points correspond (as in Fig. 4) to data (Ref. 5), for neutral pions (WA80) and negative secondaries (HELIOS), respectively. The stars represent the $pp$ data (Ref. 25). The curves are drawn simply to guide the eye.

temperature value.

The second straightforward way to analyze data is to represent them as a sum of the thermal spectra:

$$d\sigma/dp_t^2=\int dT\,F(T)f_{\text{therm}}(p_t,T)\ ,$$
$$f_{\text{therm}}(p_t,T)\sim m_t\sum_{n=1} K_1(nm_t/T) \tag{6.3}$$

(where $K_1$ is the Bessel function). The function $F(T)$, the so-called "temperature profile,"[17] contains all the history of the system in terms of temperature. Such representation may reveal some information hidden in the data. For example, assuming that the expected phase transition is of the first order and that system spends much time in the mixed phase, one expects a peak in $F(T)$ at the critical temperature $T_c$.

In Fig. 6 we show results of numerical conversion of the data[5] into $F(T)$. The data are good enough to allow a few parameter fits. We take them to be coefficients of the thermal spectra with $T$ distributed with a constant step 75 MeV. (At smaller step we get more parameters, and error bars become larger, making conclusions less clear.) The same two phenomena are seen. First, the smallest $T$ bin shows strong deviation of nuclear collision data from the $pp$ ones. Now the effect is much more spectacular: the coefficient for $T=75$ MeV jumps nearly by one order of magnitude. There is a smaller weight in the bin corresponding to $T=150$ MeV as one passes from $pp$ to nuclear collisions, which is not a big effect, but still it is statistically a significant one, reproduced by different experiments. We will see below, that this effect seems to be reproduced by the quasipion cascade model developed in this work.

Coming to the bin $T=225$ MeV, we see from Fig. 6 that nuclear spectra have larger weight here. Comparing this bin to its neighbors, one may speculate that it looks like a peak. Indeed, if one makes a smooth fit through three other points, one finds several standard deviations from it in the 225-MeV bin.

Two explanations of such a peak are possible: (1) we indeed start observing evaporation from the mixed phase, or (2) it is not really a temperature but instead a manifestation of the transverse flow. Let us present two arguments in favor of the former one. First of all, theoretical estimates point to this value of the temperature as the most probable one for the critical temperature. Second, the same value of $T_{\text{eff}}$ is seen for various secondaries, which should not be the case if flow velocity is noticeable; see recent NA35 data on $\pi$, $K_s$, $p$, and $\Lambda$ production in nuclear collisions in Fig. 7.

I think that experimentalists should be encouraged to obtain more accurate data and to examine more sharply the "temperature profile" of their spectra: it may be that this peak in $F(T)$ is really the first observed signal of the existence of a phase transition into the quark-gluon plasma.

Now we return to the "soft-pion puzzle." If one takes the thermodynamical fit of the $p_t$ literally, he finds low $p_t$ component with temperature as low as 50 MeV or so. Can a pion system be indeed cooled to such low temperatures in these experiments?

Using estimates of the pion kinetics considered above we can definitely reject such a possibility. At such low

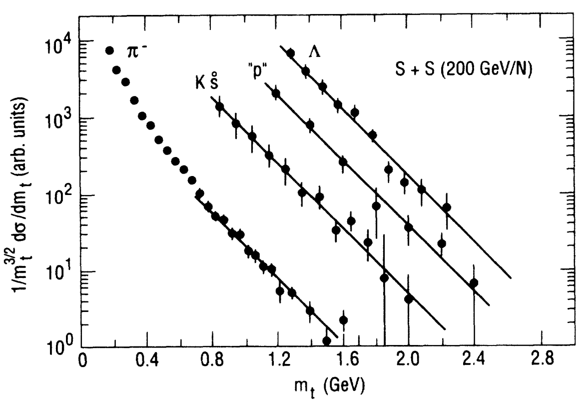


FIG. 7. Transverse mass spectra divided by $m_t^{3/2}$ for various types of secondaries, measured in SS collision by NA35 Collaboration (from Odyniec, Ref. 5). All solid lines correspond to the same temperature 200 MeV.

$T=50$ MeV they have $\tau_{\rm coll}$ of the order of hundreds of fm.

And even if some unknown scattering mechanism will create scatterings at such low $T$, the system size at breakup would be much larger than that observed by pion interferometry. For example, the pion density at $T=50$ MeV is so low, that for 400 secondary pions one needs the fireball radius of about 30 fm, much larger than that observed experimentally.

But how are these soft pions produced? This "soft-pion puzzle" is especially puzzling because, due to Goldstone theorem, pion interactions should be small for soft pions for all types of processes. Therefore, in any type of kinetic calculations one might expect at low momenta a dip in the experimental spectra compared to the thermal ones, because much more time is needed in order to equilibrate (or even produce) soft pions. Experiments show quite the opposite trend.

One simple idea which comes to one's mind in connection with this "cool component" is related with Bose-type interference effect. We recall that Bose gas occupation number can be written as a sum of the exponentials

$$n(E,T)=[\exp(E/T)-1]^{-1} = \sum_{n=1}^{\infty} \exp(-nE/T) , \tag{6.4}$$

where $n$ means the occupation number of a quantum state. As $E\sim m_\pi \sim T$, the exponent with $n=2$ is noticeable. However, as one integrates over the longitudinal momentum, he significantly washes away this effect in the $p_t$ spectrum.

One may then ask whether this "cool component" may be due to some rescattering of pions at nucleons, especially be a result of the isobar decay.[13] We give two arguments why we do not think this is the case, at least for CERN experiments. First of all, this effect was seen by the HELIOS group at rather large angles (rapidity roughly 1) and by NA35 experiment at small angles (central rapidity region, $y=2-3$). In the latter case there are about 10–20 pions per baryon at these rapidities, so even if all of them come from isobars it cannot give much. Our second argument: similar (although weaker) soft-pion component was also shown to develop even in pure $pp$ scattering,[26] if large multiplicity sample is selected. However, $p$-nuclear collisions also show a similar effect, especially close to the target-fragmentation region, which means that nucleons do play a role. All that deserves further study.

## VII. SPACE-TIME EVOLUTION OF THE PION LIQUID IN NUCLEAR COLLISIONS AND THE SOFT-PION PUZZLE

In this section we discuss whether surface phenomena discussed in Sec. V can provide at least a semiquantitative explanation of the soft-pion component at small $p_t$. There are two qualitative effects which we have in mind.

(i) Reflection from the boundary of some pions presumably explains why the pion radiation at $T$ around 150 MeV is relatively reduced, as the system becomes larger and our macroscopic considerations become more sound. It also makes the system live longer.

(ii) Climbing out of the attractive potential well, the outgoing particles become essentially softer (see Fig. 4). As explained above, in the soft boundary limit particles conserve energy but reduce their momenta. In principle, for the dispersion curve possessing wide flat region at low momenta, rather larger phase space in terms of internal momenta corresponds to extremely soft outgoing pions. It may produce very effective production of nearly stopping pions, which seems to be needed to explain the data.

In order to make these qualitative ideas more quantitative, we have used a simple model describing evolution of the "pion fireball" created in high-energy collisions. We neglect differences in transverse and longitudinal expansion and for simplicity consider a sphere of radius $R(t)$, expanding with time $t$. At any time moment we consider the system to be the homogeneous gas of quasipions, being in thermal equilibrium. We also ignore collective flow effect, and the (time-dependent) temperature $T(t)$ is fixed from the energy conservation

$$\epsilon(T)R^3=\epsilon_c(T_c)R_c^3 , \tag{7.1}$$

where the RHS corresponds to the initial conditions, just after the mixed phase has terminated and one gets pure hadronic matter. (Note that the entropy is then increasing during our expansion, in contrast with what is assumed in hydro calculations. However, with our parametrization, entropy is $T^5$ and energy $T^6$, so the difference is not that great.)

We simulated numerically quasipion paths, assuming that they are scattered by others with the rate

$$\tau_{\rm coll}=12F_\pi^4 u(T)^3/T^5 , \tag{7.2}$$

where $u$-dependent factor corrects the density of quasipions as compared to the ideal pion gas. Each collision results in "refreshed" momentum with random direction and magnitude, distributed with thermal spectrum at the temperature $T$, corresponding to the collision moment. As quasipions approach the surface, they either escape [changing momenta according to (5.2)], or are reflected back. The $u(T)$ dependence is parametrized as discussed above in Sec. IV. For comparison we also present calculations with unmodified pions, below denoted as the $u_0=1$ case.

We simply take linear expansion of the fireball, $R(t)=R_c+v\times t$. The spectra reported below are actually not that sensitive to $v$, provided it is small enough. In principle one may try to adjust $v$ so that the rms radius of the pion emission point (the point of the last scattering) is about 6–8 fm, as pion interferometry tells us[27] is the case for central collisions in CERN experiments considered. However, we found it to be instructive not to take different $v$'s for our two scenarios (modified and unmodified pions), but to take some fixed value of $v$, which is 0.15 for all results reported below.

We took the pion multiplicity to be 400, which roughly corresponds to central collisions in the experiments considered, but we also made similar calculations for 4000 pions (presumably corresponding to Pb-Pb collisions at CERN) and for much smaller fireball, containing only 10 pions. Some results are collected in Table I.

(We do not claim that the ten-pion fireball indeed represents a realistic $pp$ situation, but show them for comparison. We assume that phenomena under consideration are macroscopic by their nature and they should probably be essentially weaker in smaller systems. Therefore, for this small system the unmodified case $u_0=1$ is probably closer to reality than the modified one.)

One striking difference between the two scenarios is that even the initial and final radii of the fireball are significantly different. It happens because the latter case leads to much larger density of particles. Comparing the latter one with interferometry we notice for 400 pions the results for $u_0=\frac{1}{2}$ (modified case) are much closer to 6–8 fm, seen experimentally. (It becomes even closer if one corrects for smaller size in longitudinal direction.) The unmodified case gives rather large radius right from the start. (Note, also, that at 4000 pions the nonmodified case leads to such large size and lifetime that it nearly excludes the very possibility of the interferometric measurements. Resolution of the order of MeV will be needed, and enormous statistics.)

Now we return to $p_t$ spectra. It is interesting to make a systematic check of the multiplicity dependence of the low $p_t$ enhancement, preferably by the same apparatus. The best we could find is WA80 data on central/peripheral ratio, shown in Fig. 8. We also have plotted the ratio of HELIOS negatives to parametrization of CERN ISR $pp$ data, at least to show schematically the enhancement we discussed.

Our results are plotted in Fig. 9 also in the ratio form, but for spectra corresponding to modified and unmodified pion scenarios for 400 pions in the fireball. One can see that this enhancement in the $p_t$ spectrum due to modification of the pion dispersion curve is similar to that seen in experiments. Note, also, that the particular shape of this enhancement curve is sensitive to the shape

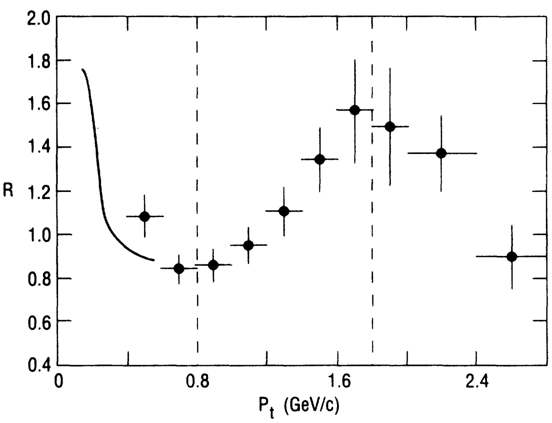


FIG. 8. Points are the ratio of the $p_t$ spectra for $\pi^0$ in central and peripheral OW collisions (WA80). The solid curve is the ratio of HELIOS SS data to $pp$ ones (we show no points or error bars, for the ratio of two spectra measured in completely different experiments is subject to possible systematic errors; we only give it for illustration of the trend).

of the pion dispersion curve. In particular, as we have emphasized above, a flat curve at small internal momenta leads to a strong peak at zero outgoing ones.

In Fig. 10 we also show our results on the corresponding temperature profiles, defined as the distribution over $T$ at the moment of last collisions. The unmodified pion scenario leads to a rather smooth profile in which nearly all temperatures between 100 and 200 MeV are represented. The modified one has a dip at $T=140-170$ MeV, demonstrating the effect (i) mentioned at the beginning of this section. There is also a peak at some breakup tem-

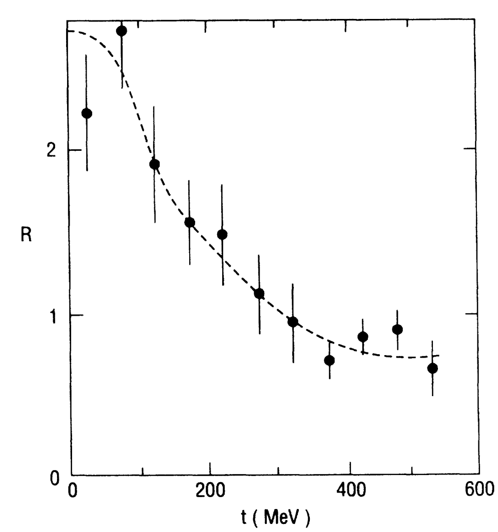


FIG. 9. Ratio of the calculated $p_t$ spectra for pions for two different scenarios described in the text, with and without modification of the pion dispersion curve. The total number of pions is 400.

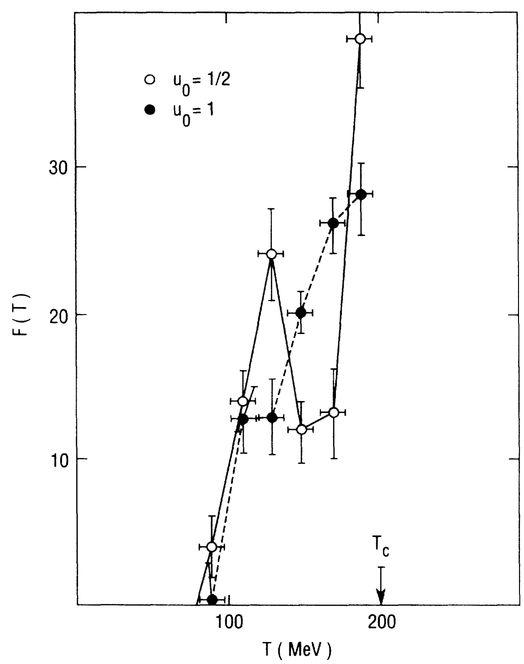


FIG. 10. Temperature profiles (distribution over temperatures at the moment of last collisions) for two scenarios discussed in the text. Open and solid points correspond to cases with and without modification of the pion dispersion curve. The curves are just shown to guide the eye.

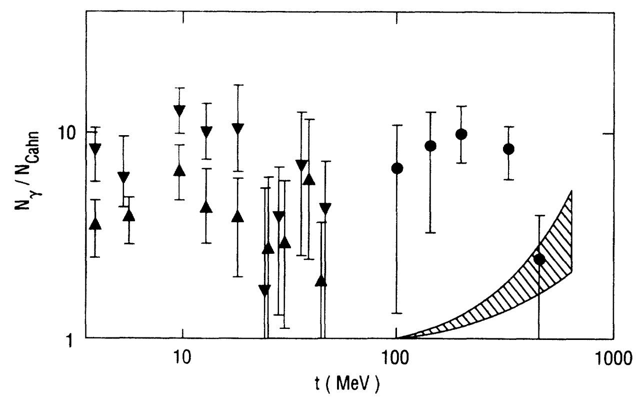


FIG. 11. The ratio of the observed photon $p_t$ spectra to predictions of the bremstrahlung, Ref. 28. Points ∇, △ are for $p$-Be and $p$-Al collisions (HELIOS) (Ref. 6), while solid points correspond to positron production (see discussion and references in Willis, Ref. 6). The shaded area shows predictions of the Lund model according to Andersson *et al.* (Ref. 29).

perature around 120 MeV. Of course, in both cases no scattering is seen at $T$ lower than, say, 70 MeV. [Therefore, there is no significant low-$T$ component, in contrast with what our naive analysis in Fig. 6 has shown. The reason is that we also have to include point (ii), modification of the pion momentum at the boundary. This makes the whole picture consistent.]

## VIII. THE SOFT-PHOTON PUZZLE

Another striking phenomenon observed in high-energy collisions during the past years is observation[6] of an excess of the photons with very small $p_t$ over the theoretical expectations. A sample of data is displayed in Fig. 11 as the ratio of the observed spectrum to that predicted by simple bremsstrahlung from the outgoing charged particles.[28] One can see that this ratio is much larger than one, and about the same at all $p_t$, even for the smallest values. Another argument against simple bremsstrahlung was raised by Willis and based on the compilation of the data:[6] the photon yield is not even proportional to the pion multiplicity, but seems to be instead proportional to its square. This also rules out the usual bremsstrahlung.

The reason this is disturbing is that emission of the soft photons is described by the classical electrodynamics, which demands the simple bremsstrahlung to be the main effect, provided photon energies $\omega$ are small compared to inverse lifetime of the radiating system. (By the way, one may find in literature the statement that the system size should be comparable to the photon wavelength: this is certainly incorrect if the radiating object is nonrelativistic, such as, e.g., atoms.) As the photon energy in the c.m. frame reaches few MeV in these experiments, one should ask why this "low-energy theorem" is not yet valid. And where should one expect a transition region, below which bremsstrahlung does dominate?

Attempts to describe these data in conventional models of the collisions has failed (see, e.g., Ref. 29), in which the Lund version of the string model was used. The results obtained with such a space-time picture more or less reproduce low-energy theorem in a wide range of momenta, up to those of the order of few hundreds MeV (see the shaded region in Fig. 11).

In this section we evaluate soft-photon radiation, using the model described above. We indeed found much stronger deviations from the bremsstrahlung than, say, in the Lund model. The main reason for that is that pions are rescattered several times before they come of the system. Roughly speaking, the excess factor compared to ordinary bremsstrahlung is the number of scatterings per particle. As the system becomes more macroscopic and more stable it is not surprising that each particle makes more collisions: therefore radiation should not be just proportional to the number of secondaries. Also, reflections from the boundary considered above make the system longer lived, which additionally increases the radiation. Moreover, our studies have shown that some specific paths (long acting "antennas") produce a lot of very soft photons. (Preliminary discussion of our calculations in a somewhat simpler model is already published.[20])

The main formula used (it is also the one used by Andersson *et al.*[29]) gives the emitted photon spectrum in terms of the pion path $x(t)$:

$$dN_\gamma/d\Omega\, d\omega=(\alpha/4\pi^2\omega)\sum_{i=1,2}\left|\int dt\,\frac{d}{dt}[\epsilon_i\cdot\mathbf{v}(t)/(1-\mathbf{v}\cdot\mathbf{n})]\exp\{i\omega[t-\mathbf{n}\cdot\mathbf{x}(t)]\}\right|^2, \quad (8.1)$$

where $\mathbf{v}(t)=d\mathbf{x}/dt$ is the velocity, $\epsilon_i$ is the photon polarization vector, and unit vector $\mathbf{n}$ shows the photon direction. As our paths are the sum of the straight segments, this integral is actually the sum over scattering points.

The low-energy theorem is formulated in these terms as follows: if all phase factors are equal to 1, the changes of the velocity in different collisions compensate each other, so that only the final velocity really matters. The opposite limit of large $\omega$ leads to random phase factors and radiation proportional to the number of collisions. In the transition region there is partial interference of the radiation coming from different scatterings.

Our results in terms of similar ratio are shown in Fig. 12 for two scenarios already discussed in the preceding section, and for the number of pions in the fireball equal 4000, 400, and 10. All curves go to unite ratio at small $p_t$, in agreement with the low-energy theorem mentioned. At large ones they saturate at some level, proportional to the mean number of scattering. One can see that a modified pion scenarion gives systematically more radiation than that for an unmodified one.

Comparing these results with experimental data shown in Fig. 11, we should first note that we definitely cannot describe observed excess for low $p_t<30$ MeV or so. However, we do observe strong effects of the pion modification on the photon radiation, especially in the transition region between the coherent and incoherent regimes of radiation. Moreover, it seems that we can naturally explain the virtual-photon (solid points in Fig. 11) by the $u_0=\frac{1}{2}$, $N_\pi=10$ case.

Certainly, this question deserves further study, both experimental and theoretical. Photon radiation is an interesting tool, allowing us to test better our understanding of the multiparticle system under consideration. It can provide additional text of space-time evolution of the system, measuring such important things as the number of rescatterings in the final stage.

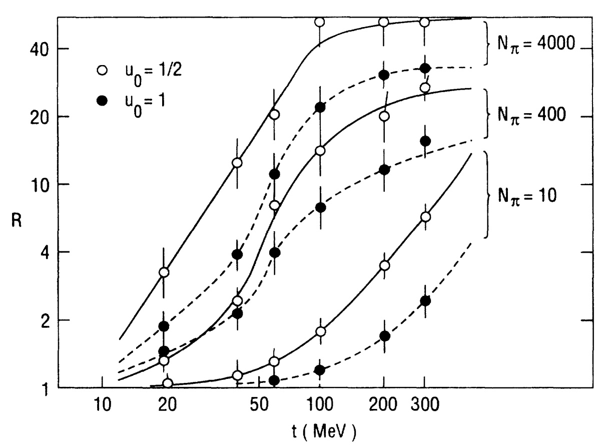


FIG. 12. The same ratio as plotted in Fig. 11, but calculated in the model discussed in the text. Open and solid points (and solid and dashed lines connecting them) correspond to two scenarios discussed in the text, with and without pion dispersion curve modification. We present results for the number of pions in the fireball 4000, 400, and 10, respectively.

## IX. THE SHAPE OF THE PION ANNIHILATION THRESHOLD

Dilepton production was suggested as the best signal for quark-gluon-plasma formation,[17] so most of the dilepton experiments study the highest available energies and dilepton invariant masses about 1–4 GeV. However, recently first dilepton experiments at Berkeley[7] have studied the lepton-pair production at energies as low as 4.9 GeV for proton-nucleus and about 1 GeV/$N$ in nuclear collisions. They have measured completely different invariant-mass region, and one of the spectrum, observed in this experiment, is shown in Fig. 13. One can see the $\rho$ meson peak, and also another peak at $M_{e^+e^-}$ just above $2m_\pi$. The data below it are well described by known backgrounds, but this peak has no obvious explanation.

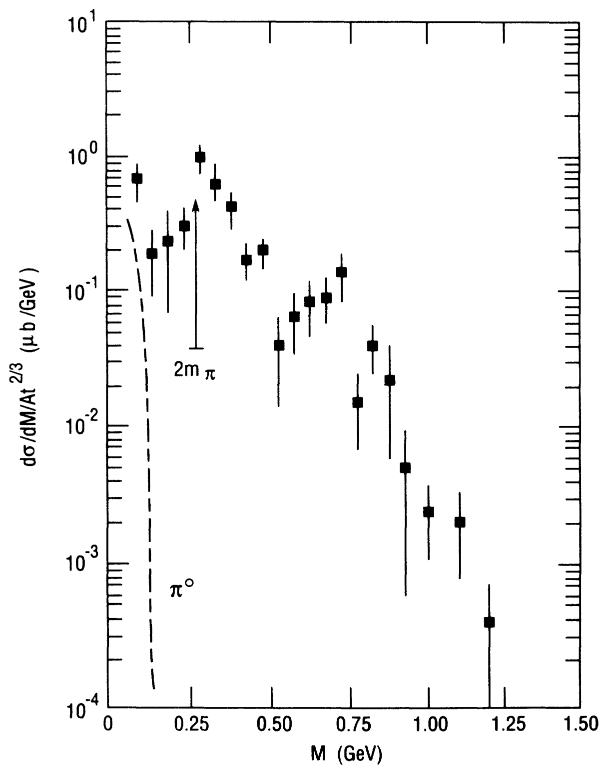


FIG. 13. The spectrum of effective dilepton masses measured in $p$-BE collisions in Berkeley (Ref. 7). An arrow shows position of the two-pion annihilation threshold, and the dashed curve shows the estimated background from the $\pi^0$ Dalitz decay.

One may think about $\pi^+\pi^-$ annihilation, and in fact $s$ dependence of this peak is consistent with that for pion-pair production. However, attempts to ascribe it to pion annihilation meet certain problems.

First of all, production of two soft pions is suppressed by smallness of the phase space. Second, the annihilation of soft pions into a virtual photon is suppressed at threshold because it takes place in the $P$ wave:

$$\sigma=(\pi\alpha^2/M^2)(1-4m_\pi^2/M^2)^{1/2} . \quad (9.1)$$

(The latter factor is the velocity; in normal cases such as $\bar{p}p$ annihilation it stands in the denominator.)

Let us emphasize that conditions of this experiment are quite different from nuclear collisions at CERN. There are no hundreds of pions in a fireball, but just one pair of them. However, kinematics are now such that pions are produced and annihilated in nuclear matter. Gale and Kapusta[20] have used this fact, suggesting that pion dispersion curve is modified so strongly in the compressed nuclear matter, that secondary minimum in $\omega(k)$ is developed. If so, annihilation of two "quasipions" leads to a peak in the dilepton spectrum, corresponding to the dilepton mass equal to twice the minimum value of $\omega(k)$. It may well be mixed with the double-pion threshold, and now there is no suppression mentioned above because momentum is now far from zero value.

It may well be that the peak seen in these experiments is indeed of this nature. We report no calculations for pion annihilation rate here, but just make few qualitative comments.

The first is that the secondary minimum suggested by Gale and Kapusta is not actually needed. The two puzzles discussed above were most naturally explained if $\omega(k)$ has a flat region at small $k$. This is actually what happens for the pion dispersion curve for densities around the nuclear one.

The second one deals with the fact that naive estimates of the annihilation rate (see Kapusta talk[8]) have produced numbers about one order of magnitude lower than observed. Pion motion was considered as that of a free classical particle, while it is a quantum particle moving in collective attractive potential. Some pions can even be reflected back from the surface of the nucleus, and all of them are somewhat concentrated inside the nucleus. This definitely increases the probability for the $\pi^+$ to meet the $\pi^-$, and therefore increases the annihilation rate into dileptons. We will report estimates of the magnitude of this effect elsewhere.

## X. CONCLUSIONS AND DISCUSSION

(1) The main point of the present work is that in the temperature region $T=100-200$ MeV excited hadronic matter is more like a liquid than an ideal gas: interparticle interaction is very important, and this interaction is attractive.

(2) For large and long-lived hadronic systems produced in high-energy nuclear collisions the traditional picture of production and decay of resonances is no longer adequate. Instead, it is logical to use the language of condensed-matter physics, and consider collective propagating modes, the quasiparticles, as the main element of such matter. Consequently, the coherently propagating pion becomes a "quasipion," if its interaction with surrounding matter is taken into account.

(3) We present first estimates of the modification of the pion dispersion curve $\omega(k)$, based on the same physical idea as used by Landau in his estimates of the phonon spectrum in liquid $^4$He. The distortion of the quasiparticle energy in both cases is negative and more pronounced at larger momenta, due to Goldstone nature of both quasiparticles.

(4) Such modification of $\omega(k)$, or the momentum-dependent potential, becomes important if these quasiparticles come to the surface of the system. The attractive interaction obviously makes it more difficult to leave the system. Quasipions may be reflected back at some angles, or if they come out their momenta may be reduced.

(5) We suggest that these phenomena may be the origin of the recently observed excess of soft pions in nuclear collisions. Reflections at the boundary also lead to a significant increase in soft-photon radiation. We have shown that some data above $p_t$ of the order of 30 MeV or so can well be explained by this effect. However, data at lower $p_t$ remain enigmatic.

(6) We hope that these effects will also shed some light on other puzzling observations. In particular, annihilation of quasipions concentrated inside the nuclei by collective potential presumably may explain why the observed two-pion annihilation threshold is so enhanced. Surface tension can lead to drop formation, and help to understand correlation data. Also this feature helps to compensate collective transverse flow.

(7) One of the most straightforward tests of these ideas is probably direct measurements of the lifetime distribution of the pion source by pion interferometry. But it is not a simple task: one needs resolution of the order of few MeV and large statistics in order to notice it.

(8) One more point, related to future experiments but unrelated to the pion liquid: possible signal of the mixed phase and critical temperature, indicated by our detailed analysis of the pion $p_t$ spectra. It is clear that better data can reveal much more, may be indeed a kind of a peak (or shoulder) in the temperature profile at the critical temperature $T_c$. Let us emphasize that this way of measurements of this fundamental quantity is the most straightforward one.

(9) Completing this paper, we should add a warning: although most of the phenomena under consideration were seen in several experiments, further checks and more systematic studies are badly needed. This is especially true for the soft-photon case, in which background problems are severe. Also the two-pion threshold in the dilepton spectra was only seen by one group, and it should be reproduced before any serious conclusions can be drawn.

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