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Applications of QCD sum rules at finite temperature

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QCD sum-rule techniques are applied to the spectra of ρ and J/ψ mesons at finite temperature to investigate the relative importance of quark and gluon condensates and perturbative thermal effects in determining bound-state parameters. Of particular interest are the consequences of nonperturbative physics persisting above the deconfinement phase transition, which is implied by nonzero gluon condensates found in lattice calculations. For the ρ meson, the quark thermal bath induces only a smooth variation in the hadronic parameters as the temperature is increased; the quark condensate and its temperature dependence are the most important factors. For the J/ψ meson, perturbative thermal effects overwhelm the gluon condensate contribution at a temperature around 100 MeV, so that high-temperature charmonium physics is consistent with that expected in a weakly interacting quark-gluon plasma. Corrections to other plasma properties from nonperturbative physics are discussed.

I. INTRODUCTION

The description of hadronic matter under extreme conditions of temperature and density is an interesting and important theoretical problem, with direct relevance for experiments at present and future heavy-ion colliders. One facet of this description is the modification of hadronic spectra as the system is heated or compressed, which has been studied for light mesons using chiral models of QCD,[1,2] and for charmed mesons using potential models.[3] Instead of using such effective models, we would like to predict these modifications directly from quantum chromodynamics (QCD).

At sufficiently high temperature, hadronic matter is thought to undergo a phase transition to a quark-gluon plasma (QGP). While the nature of this transition is not well understood, its existence has been demonstrated by numerical simulations. Lattice calculations (without dynamical fermions) suggest that $T_c \sim 200$ MeV.[4] It is an exciting possibility that the energy densities believed necessary to produce such a QGP ($\sim 2-3$ GeV fm^{-3}) will be reached by existing and planned heavy-ion machines. Experiments in this direction are at present being conducted at the CERN SPS and are planned for the relativistic heavy-ion collider (RHIC) at BNL. To interpret these data and to identify signatures of QGP formation in ultrarelativistic heavy-ion collisions, the properties of both hot hadronic matter (including the vacuum) and the QGP must be known.

In this paper, we use QCD sum rules at finite temperature to study the spectra of ρ and J/ψ mesons. With the sum-rule approach, we can predict hadronic properties in terms of finite-temperature perturbation theory and long-distance nonperturbative physics, which is summarized by quark and gluon condensates. Since we can identify the contribution of each component, we learn about the relative importance of the various quark and gluon condensates in determining bound-state parameters and how thermal effects influence these parameters. Finally, we can directly explore the physical consequences of nonperturbative physics at high temperatures by relating nonzero condensates to observed resonance properties.

This last point is important because the widespread belief that the QGP is a gas of weakly interacting quarks and gluons has been questioned by several authors.[5-7] For example, an interesting lattice calculation[8] for pure SU(3) gauge theory suggests that nonperturbative physics persists at high temperature, above the phase-transition region. In this calculation, the area-law behavior of space-space Wilson loops was found to persist even above the deconfining temperature.[8] DeGrand and DeTar have argued that the area-law behavior of spacelike Wilson loops causes the static correlator to be screened by color-singlet modes.[9] If there exists a one-to-one correspondence between the real-time dispersion relation $f(p,\omega,t)=0$ and the static dispersion relation $f(\pm i\mu,0,T)=0$, the real-time excitations will also be color-singlet modes. In this scenario, a nonperturbative length scale $1/gT$ exists even above T_c so that, while the perturbative picture is correct for small distance scales, hadrons can only be color singlet at distances larger than $1/gT$.

Unfortunately, the situation is not entirely clear because spacelike Wilson loops cannot be directly interpreted as physical observables, or related to a parameter in an effective theory of QCD. However, lattice calculations [in pure SU(2) gauge theory] have found evidence for nonperturbative physics above T_c in the form of nonzero gluon condensates above T_c.[10,11] In this paper, we present additional lattice calculations and, using QCD sum rules at finite temperature, study the physical consequences of nonvanishing gluon condensates for hadronic parameters.

Finite-temperature QCD sum rules have been applied to a variety of mesonic resonances by Bochkarev and Shaposhnikov,[12–14] using the Matsubara (imaginary-time) formalism to extend the $T=0$ sum rules. In all cases (they do not consider heavy-quark mesons), they find that the spectra undergo rapid changes at temperatures around 130–150 MeV, which they interpret as evidence for a phase transition. Recently, Dosch and Narison have reanalyzed the calculation of the ρ meson, and conclude that only smooth changes in the spectra are predicted at these temperatures if proper sum-rule stability criteria are applied.[15] In this paper, we present a new analysis of the ρ channel, including temperature effects on the condensates not considered in Ref. 15, and give original results for the J/ψ, using numerical optimization techniques to extract spectral parameters from the sum rules.

The paper is organized as follows. In Sec. II, we present numerical lattice calculations of the gluon condensates above T_c and discuss how the area-law behavior of Wilson loops is related to nonvanishing condensates. In Sec. III, we review QCD sum rules at finite temperature. In Sec. IV, we study the ρ meson as an example of a light-quark meson. In accord with Ref. 15, we find that the quark thermal bath induces only a smooth variation in the hadronic parameters as a function of the temperature. The quark condensate and its temperature dependence are found to be the most important ingredients in determining the spectrum. In Sec. V, we study charmonium and the J/ψ in particular. In this case, the perturbative thermal effects overwhelm the gluon-condensate contribution at a temperature around 100 MeV, and hence the sum rule seems to break down. In Sec. VI, we summarize our results and discuss future investigations.

II. GLUON CONDENSATE AT HIGH TEMPERATURE

Finite-temperature lattice calculations of gluon condensates in pure SU(2) gauge theory have indicated that both magnetic and electric condensates are nonzero above the phase-transition region.[10,11] Here we supplement the results of Ref. 11 with additional calculations that feature better statistics. To extract the magnetic or the electric condensate, we calculate the average plaquette in the space-space direction or the space-time direction. In the continuum limit ($a \to 0$),

$$\langle P_{12} \rangle = 1 - \left\langle \frac{\alpha_s}{\pi} G_{12}^a G_{12}^a \right\rangle \frac{a^4 \pi^2}{N} + O(a^6) , \quad (1)$$

where P_{12} means the elementary plaquette in the 1-2 direction.

To extract the nonperturbative condensate as defined by Shifman *et al.*,[16] we have to subtract out from the measured Monte Carlo data the contribution of the weak-coupling perturbation expansion of $\langle P \rangle$ on the lattice.[17] This weak-coupling expansion has been calculated on an asymmetric lattice by Heller and Karsch[18] to second order in the lattice coupling, $O(1/\beta^2)$. In our lattice calculation, we fitted higher-order corrections in the weak-coupling region to $O(1/\beta^5)$. If there is a nonvanishing condensate, the difference between the lattice and weak-coupling results would be proportional to $\langle G^2 \rangle a^4$ and hence would scale with β like

$$a^4 = \frac{1}{\Lambda_L^4} \exp(-12\pi^2\beta/11)(11/6\pi^2\beta)^{-204/121} . \quad (2)$$

Figure 1 (space-space plaquette) and Fig. 2 (space-time plaquette) show this difference for a $10^3 \times 3$ lattice in pure SU(2) lattice gauge theory, with 5000 iterations for each point. The results are essentially identical to those of Ref. 11, but with smaller uncertainties. The thermal Wilson lines were used as the order parameter for deconfinement and all these points were in the deconfined phase (smaller than 0.2).

The weak-coupling fit was done at β larger than 2.8 for 10 points, using a least-squares fit. If we include any points at β smaller than 2.8, the χ^2 value increases appreciably. For example, if we include two points at $\beta = 2.3$ and 2.5, the χ^2 value increases by as much as 10. Assuming $\Lambda_L = 5.2$ MeV, the temperature at $\beta = 2.3$ is about 295 MeV. Similar features have been observed in a symmetric lattice.[17] By comparing the absolute values, we find that the magnitude of the magnetic condensate is about 0.6 times that of the zero-temperature value and the magnitude of the electric condensate is about 0.7 times that of the zero-temperature value:

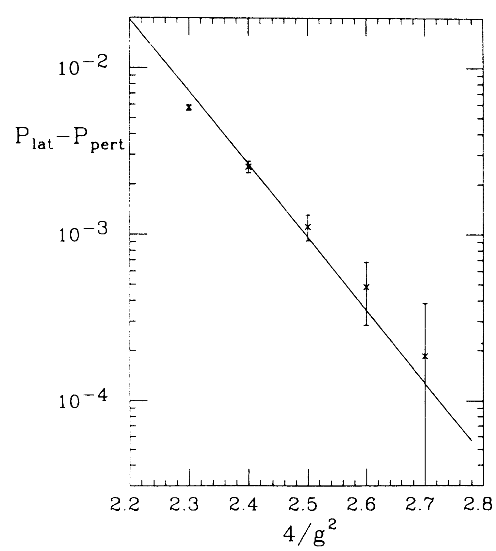

FIG. 1. Logarithmic plot of the difference between the lattice result and the weak-coupling result for a space-space elementary plaquette on a $10^3 \times 3$ lattice, as a function of $\beta = 4/g^2$.

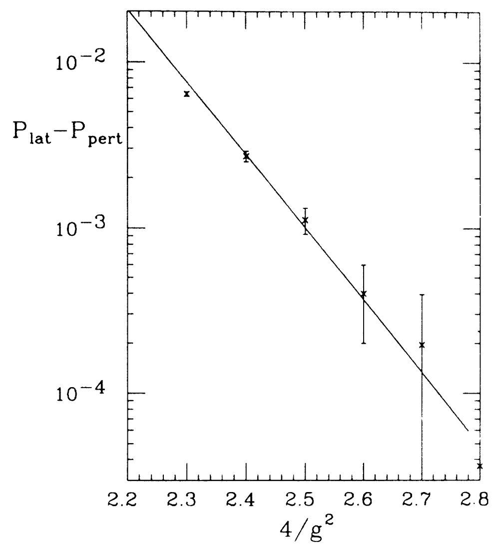

FIG. 2. Same as Fig. 1 for a space-time elementary plaquette.

$$2\left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle = -2\left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle = 5\times 10^{-3}\ \mathrm{GeV}^4 \quad \text{at } T=0 \tag{3}$$

and

$$\begin{aligned} \left\langle\!\left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle\!\right\rangle_{T=295\ \mathrm{MeV}} &\approx 0.6 \left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle_{T=0\ \mathrm{MeV}} , \\ \left\langle\!\left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle\!\right\rangle_{T=295\ \mathrm{MeV}} &\approx 0.7 \left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle_{T=0\ \mathrm{MeV}} . \end{aligned} \tag{4}$$

The magnitude of the $T=0$ condensate is comparable to, although somewhat smaller than, values used in QCD sum-rule applications.

Can we relate nonvanishing condensates above T_c to the area-law behavior of Wilson loops? Lattice gauge calculations tell us that Wilson loops at zero temperature can be parametrized in terms of a part that is proportional to the perimeter of the Wilson loop, a part that is proportional to the area of the Wilson loop, and a part that behaves like the Coulomb potential.[19] In a perturbative calculation, we get the Coulomb part and the perimeter law part, but not the area-law behavior, which is therefore linked to the nonperturbative nature of QCD.

Shifman[20] showed that the area-law behavior of Wilson loops *at zero temperature* was related to the existence of nonvanishing gluon condensates. Unfortunately, Shifman's argument cannot be extended to finite temperature and so no direct relation is known between condensates and the behavior of Wilson loops above T_c. In fact, space-time Wilson loops do not have an area-law behavior above T_c. It is possible that the static space-space Wilson loops are less affected by temperature than the space-time Wilson loops. If so, the area-law behavior of space-space Wilson loops would still suggest nonperturbative physics, related to nonvanishing gluon condensates, at finite temperature (and above T_c). At present, we do not know of a rigorous proof of this conjecture.

In any case, lattice calculations indicate nonvanishing magnetic and electric condensates at high temperature. Since gluon condensates at high T are a manifestation of nonperturbative physics in QCD, we would like to know their physical consequences. In the following sections, we study resonance parameters using QCD sum rules at finite temperature to see how the persisting gluon condensate can affect the physics.

III. QCD SUM RULES AT FINITE TEMPERATURE

Let us first review QCD sum rules at zero temperature[16,21] by considering the time-ordered product of two vector currents $J_\mu(x)$, such as $(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/2$ or $\bar{c}\gamma_\mu c$:

$$i\int d^4x\, e^{ipx} T[J_\mu(x) J_\nu(0)] . \tag{5}$$

The vacuum expectation value of this operator is the time-ordered correlation function $\Pi_{\mu\nu}(q)$:

$$\begin{aligned} \Pi_{\mu\nu}(q) &= i\left\langle 0 \left| \int d^4x\, e^{iqx} T[J_\mu(x) J_\nu(0)] \right| 0 \right\rangle \\ &= (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2) . \end{aligned} \tag{6}$$

The function $\Pi(q)$, for spacelike $Q^2=-q^2$, satisfies the dispersion relation

$$-\frac{d}{dQ^2}\Pi(Q^2) = \frac{1}{\pi} P \int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{(s+Q^2)^2} . \tag{7}$$

The theoretical side of the sum rule is derived from an operator-product expansion of Eq. (5):

$$\begin{aligned} i\int d^4x\, e^{iqx} T[J_\mu(x) J_\nu(0)] \\ = C_1 \times 1 + C_m {:}m\bar{q}q{:} + C_G {:}G^2_{\mu\nu}{:} + \cdots . \end{aligned} \tag{8}$$

The nonperturbative nature of the QCD vacuum enters through the nonvanishing vacuum expectation values of the normal-ordered operators; these expectation values are the condensates. The condensate terms (those not involving the identity operator) are called the power corrections. The operator-product expansion (OPE) incorporates long-distance physics into the vacuum expectation values and short-distance physics into the Wilson coefficients. Corrections due to explicit dependencies on the separation scale are expected to be small in QCD sum-rule applications.[21]

The phenomenological side of the sum rule follows upon inserting a parametrized model of the spectral density, which is proportional to $\mathrm{Im}\Pi$, into the right-hand side of Eq. (7). We use the well-established model of the spectral density consisting of a narrow resonance and a smooth continuum with a sharp threshold:[16,21]

$$\mathrm{Im}\Pi(s) = f M_R^2 \delta(s - M_R^2) + \theta(s - S_0) \mathrm{Im}\Pi(s)_{\mathrm{pert}} . \tag{9}$$

As indicated, the continuum contribution is evaluated perturbatively.

The theoretical side can be calculated reliably for large $Q^2=-q^2$ with only a few power corrections, and using perturbation theory for the Wilson coefficients. On the other hand, we want the phenomenological side to be largely saturated by the lowest resonance, which happens at small Q^2. The region of overlap between the two sides can be enlarged by taking additional derivatives with respect to Q^2 in Eq. (7); these are the moment sum rules, which are typically used for heavy-quark bound states. Taking Q^2 and the number of derivatives n to infinity, with Q^2/n fixed, leads formally to the Borel-transformed sum rules.

To formulate a finite-temperature sum rule, we take the thermal (ensemble) average of Eq. (5) instead of a vacuum expectation value. At finite temperature this polarization tensor can be decomposed into transverse and longitudinal parts (following the notation of Ref. 13):

$$\Pi_{00}=\mathbf{q}^2\Pi_l \ , \qquad \Pi_{ij}=(\delta_{ij}-q_iq_j/\mathbf{q}^2)\Pi_t+(q_iq_j\omega^2/\mathbf{q}^2)\Pi_l \ . \tag{10}$$

Here we work in the rest frame of the thermal baths, with $q^\mu=(\omega,\mathbf{q})$. As in Ref. 13, we consider the formation of a resonance at rest, so we are interested in the $\mathbf{q}\to 0$ limit. For $\mathbf{q}\to 0$, $\Pi_t=(\omega^2-\mathbf{q}^2)\Pi_l$ and Π_l satisfies a dispersion relation similar to Eq. (7):

$$\frac{d}{d\omega^2}\mathrm{Re}\Pi_l(\omega^2)=P\int_0^\infty du^2\frac{\rho(u)}{(u^2-\omega^2)^2} \ , \tag{11}$$

where $\rho(u)$ is the spectral function given by

$$\rho(u)=\frac{1}{\pi}\tanh(u/2T)\mathrm{Im}\Pi_l(u) \ . \tag{12}$$

To calculate the theoretical side of the sum rule in the asymptotically free region ($\omega^2\to-\infty$), we evaluate the Wilson coefficients using finite-temperature Green's functions and again incorporate nonperturbative physics through nonzero expectation values of the normal-ordered operators. At finite temperature, the thermal expectation values of the normal-ordered operators are not equal to zero even in perturbation theory; in our approach, these perturbative thermal corrections are incorporated by making the Wilson coefficients temperature dependent. (We assume that perturbation theory is adequate at temperatures $T\sim 100$ MeV. Some supporting arguments in favor of this assumption are given in Ref. 13, but we have no rigorous demonstration.[22]) The temperature dependencies of the condensates themselves are taken from simple models or from lattice calculations.

To simply illustrate how our prescription for the finite-temperature sum rule arises, we can consider a time-ordered product of fermion fields $\Psi(x)$. Using Wick's theorem,

$$\int d^4x\, e^{iqx}T[\Psi(0)\overline{\Psi}(x)] = \int d^4x\, e^{iqx}{:}\Psi(0)\overline{\Psi}(x){:}+iS_F^{T=0}(q) \ . \tag{13}$$

If we take the perturbative ensemble average of this operator, the normal-ordered term contributes $2\pi\delta(q^2-m^2)n_F$ and so gives, along with the (lowest-order) zero-temperature Green's function, the usual noninteracting Dolan-Jackiw finite-temperature Green's function. (To calculate the higher orders, one must follow the perturbation theory of the real-time matrix formalism,[23] but this distinction is immaterial for the calculations in this paper since we limit ourselves to one-loop order.) As indicated above, the nonperturbative nature of QCD is taken into account by assuming that, in addition to the perturbative temperature-dependent piece, the normal-ordered part has nonperturbative pieces, which will contribute to an ensemble average.

To include the nonperturbative physics via the condensates, we expand ${:}\Psi(0)\overline{\Psi}(x){:}$ for small x, as in Ref. 21, to obtain a series in x^n multiplying dimension $3+n$ operators. Then the lowest-order term in x gives rise to the quark condensate part $2\pi\delta^4(q)\langle\langle\bar{q}q\rangle\rangle$, from which the perturbative part has been subtracted. While the nonperturbative operator matrix elements are to be included only up to some finite dimension operator, perturbative parts of the matrix elements are included from all orders and summed to recover the temperature-dependent part of the perturbative finite-temperature Green's function. The net result is an expansion in terms of temperature-dependent condensates and temperature-dependent Wilson coefficients.

Applying an analogous procedure to the operator product of Eq. (5) and taking the thermal average, one obtains the expansion

$$i\int d^4x\, e^{iqx}\langle\langle T[J_\mu(x)J_\nu(0)]\rangle\rangle = C_1^T\times 1+C_m^T\langle\langle m\bar{q}q\rangle\rangle+2C_E^T\langle\langle E^2\rangle\rangle +2C_B^T\langle\langle B^2\rangle\rangle+\cdots \ , \tag{14}$$

where the perturbative temperature dependence is subtracted away in $\langle\langle\mathcal{O}\rangle\rangle$ and the Wilson coefficients have acquired temperature dependence. Again, if one starts from the OPE [Eq. (8)] and directly takes its thermal average, one needs contributions to infinite order in the expansion to get the C^T's in the above expression. Thus, although the scale is now set by T as well as the condensates (or Λ_{QCD}), Eq. (14) is an expansion only in Λ_{QCD}/Q; the expansion in T/Q is already summed.

On the phenomenological side of the sum rule, three types of terms contribute to the spectral density at $T\neq 0$, as shown in Appendix A: the resonance part, the annihilation part, and the scattering part, which have support in different regions of the energy plane:

$$\rho(u)=fM_R^2\delta(u^2-M_R^2) +\theta(u^2-S_0)\rho_g(u)+\delta(u^2)\rho_s \ . \tag{15}$$

The scattering term ρ_s, which arises only at finite temperature (or density), describes the absorption of the external current by thermally excited particles.

Just as at $T=0$, we must calculate moment or Borel-transformed sum rules to suppress higher-dimensional power corrections and to enhance the resonance contribution to the dispersion integral. The Borel sum rule takes the form

$$\int_0^\infty d\omega^2 e^{-\omega^2/M^2}\rho(\omega)_{\mathrm{pert}}+\sum_{n\geq 1}\frac{a_n}{M^{2n}} =\int_0^\infty d\omega^2 e^{-\omega^2/M^2}\rho(\omega)_{\mathrm{hadron}} \ , \tag{16}$$

where we have applied the Borel operator L_M to both sides of Eq. (11):

$$L_M=\frac{1}{(n-1)!}(-\omega^2)^n\left[\frac{\partial}{\partial\omega^2}\right]^n\Bigg|_{n\to\infty,\,-\omega^2/n=M^2} . \qquad (17)$$

IV. ρ MESON AT FINITE TEMPERATURE

In this section, we examine the relevance of the quark and gluon condensates to the spectrum of light mesons. We focus our attention on the ρ meson, since QCD sum rules work well at $T=0$ in this channel.[21] The temperature dependence of the ρ-meson mass $m_\rho(T)$ comes both from the nonperturbative condensates $\langle\langle\bar{q}q\rangle\rangle$ and $\langle\langle GG\rangle\rangle$ and from the perturbative Wilson coefficients. Dosch and Narison analyzed only the latter effect and concluded that it is important only at quite high temperature, $T>350$ MeV.[15] Our calculation takes into account both effects and shows that $\langle\langle\bar{q}q\rangle\rangle$ governs the mass shift of the ρ meson and the other factors are less important. This observation is consistent with the picture that chiral symmetry is responsible for the ρ meson properties. Such a picture is seen in studies of the linear σ model, where the ρ meson appears as a dynamically generated pole,[24] and in the theory of hidden local symmetry, where the ρ meson arises as a gauge boson of the symmetry.[25]

Let us start with the Borel-improved sum rule at $T\neq 0$ discussed in Sec. III, and take the ρ-meson current $(\bar{u}\gamma_\mu u-\bar{d}\gamma_\mu d)/2$ as $J_\mu(x)$. In the asymptotic region ($\omega^2\to-\infty$), one can estimate $\mathrm{Re}\Pi_l$ reliably:

$$(M^2)L_M\mathrm{Re}\Pi_l=\int_0^\infty d\omega^2 e^{-\omega^2/M^2}[\theta(\omega^2-4m_q^2)\rho_g+\delta(\omega^2)\rho_s]+\frac{1}{8\pi^2}\left[\frac{a_1}{M^2}+\frac{a_2}{M^4}\right] , \qquad (18)$$

where ρ_s and ρ_g are given in Appendix A, and the Borel operator L_M is applied to improve the perturbation series. The first term in Eq. (18) is the perturbative one-loop contribution and the second (third) term corresponds to the quark (gluon) condensate

$$a_1(T)=\tfrac{1}{3}\pi\alpha_s\langle\langle GG\rangle\rangle ,$$
$$a_2(T)=-\tfrac{448}{81}\pi^3\alpha_s\langle\langle\bar{q}q\rangle\rangle^2 . \qquad (19)$$

Here we have assumed $\langle\langle E^2\rangle\rangle\approx-\langle\langle B^2\rangle\rangle$, in accord with the result from the lattice calculation (4), and we have neglected the small contribution of the dimension-four quark condensate $\langle\langle m_q\bar{q}q\rangle\rangle$. One should also note that α_s corrections to the perturbative part and the temperature dependence of the Wilson coefficients for the power corrections are neglected as well. The former approximation does not affect the result qualitatively (based on $T=0$ calculations). For the latter, we treat the a_1 and a_2 as phenomenological parameters that effectively include the T dependence of both condensates and Wilson coefficients. (As for the Wilson coefficient of $\langle\langle\bar{q}q\rangle\rangle^2$, one can easily see that it does not have T dependence.)

The phenomenological side of the sum rule, including an ansatz for the spectral function $\rho(\omega)_{\mathrm{hadron}}$, reads

$$\int_0^\infty d\omega^2 e^{-\omega^2/M^2}[\rho_{\mathrm{pole}}+\theta(\omega^2-S_0)\rho_{\mathrm{cont}}+\delta(\omega^2)\rho_\pi] , \qquad (20)$$

where the first term is the ρ-pole contribution, with the form $\rho_{\mathrm{pole}}=fm_\rho^2\delta(\omega^2-m_\rho^2)$, and the second term is the phenomenological continuum contribution, which starts from the threshold $\sqrt{S_0}$. The third term, which arises only at finite T, describes the Landau damping caused by the thermal pions. We follow Ref. 13 and approximate the thermal hadronic contribution by that of a free pion gas, which is certainly a valid approximation at low temperatures. The importance of pion interactions and higher mass states at temperatures above the pion mass is currently an open question. (See Ref. 26 for a discussion of these issues.) Explicit expressions for ρ_{cont} and ρ_π are given in Appendix A.

Equating (18) with (20) and eliminating the parameter f, one finds

$$\frac{m_\rho^2}{M^2}=\left[\int_{4m_q^2}^{S_0}d\omega^2\left[\frac{\omega}{M}\right]^2 e^{-\omega^2/M^2}\rho_g-\frac{1}{8\pi^2}\left[\frac{a_1}{M^2}+\frac{2a_2}{M^4}\right]\right]\left[\int_{4m_q^2}^{S_0}d\omega^2 e^{-\omega^2/M^2}\rho_g+\rho_s-\rho_\pi+\frac{1}{8\pi^2}\left[\frac{a_1}{M^2}+\frac{a_2}{M^4}\right]\right]^{-1} . \qquad (21)$$

Choosing M^2 small enhances the pole contribution while large M^2 suppresses the contributions of higher-dimensional operators. Following Dosch and Narison,[15] we have plotted m_ρ^2 as a function of M^2 and sought a stable region that is insensitive to changes in the threshold parameter. As such a region is found only when $\sqrt{S_0}$ is larger than 1.75 GeV irrespective of T, we fix $S_0=4.0\ \mathrm{GeV}^2$ and search for the minimum of $m_\rho(M^2;T)$ at each temperature. For the parameters at $T=0$, we take $m_q=5.5$ MeV, $m_\pi=137$ MeV, and $\{a_1(0),a_2(0)\}=\{0.05\ \mathrm{GeV}^4,-0.03\ \mathrm{GeV}^6\}$.

In Fig. 3, we show $m_\rho(T)$ for three cases of interest: (i) $a_1(T)$ and $a_2(T)$ are assumed to be T independent, (ii) $a_2(T)=a_2(0)$ but $a_1(T)$ is assumed to be zero, and (iii) $a_1(T)=a_1(0)$ while $a_2(T)$ varies as

$$\frac{a_2(T)}{a_2(0)}=1-\left[\frac{T}{T_c}\right]^2 . \qquad (22)$$

This parametrization is deduced from a simple mean-field assumption

$$\langle\langle\bar{q}q\rangle\rangle=\langle\langle\bar{q}q\rangle\rangle_{T=0}[1-(T/T_c)^2]^{1/2} , \qquad (23)$$

whose behavior is shown in Fig. 4, assuming $T_c=200$ MeV. From the line (i) in Fig. 3, one sees that the T dependence of the perturbative part is significant only at

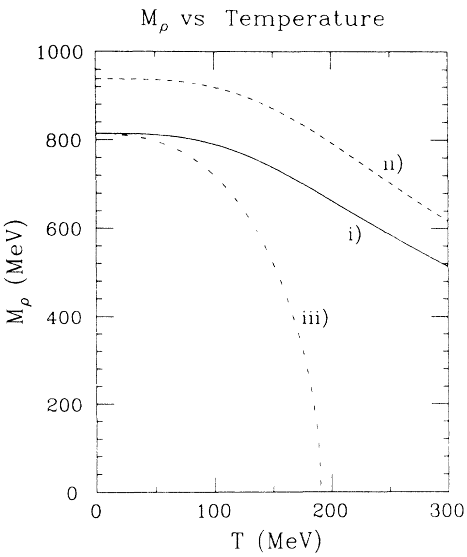

FIG. 3. Sum-rule predictions for m_ρ, by fixing $S_0=4$ GeV^2 and minimizing with respect to the Borel mass M. Three cases are considered: (i) $a_1(T)$ and $a_2(T)$ independent of T, (ii) $a_2(T)=a_2(0)$ but $a_1(T)=0$, and (iii) $a_1(T)=a_1(0)$ and $a_2(T)=[1-(T/T_c)^2]^{1/2}a_2(0)$.

high temperature, which is consistent with the observations of Dosch and Narison (but which contradicts Bochkarev and Shaposhnikov). The line (ii) shows that the gluon condensate is not at all important for the mass shift. The line (iii) shows that $m_\rho(T)$ decreases and vanishes around T_c as $a_2(T)$ tends to zero, which means that $\langle\langle \bar{q}q \rangle\rangle$ is essential to forming the ρ meson and so the restoration of the chiral symmetry is correlated with its mass shift. In the real world, the chiral transition is expected to be first order for the light-quark system, based on numerical lattice simulations. In this case, we have to use a more sophisticated parametrization for a_2 to be realistic, but the qualitative feature of our result should not change.

Here we remark that the Borel parameter M, found by minimizing m_ρ at each T in case (iii), is always at least 400 MeV, as shown in Fig. 4. Using this curve, we can evaluate the ratio of the power correction and the perturbative part in the OPE left-hand side (LHS) [Eq. (16)]. Naively, one might expect the ratio to increase significantly since the power correction is proportional to M^{-2} and M^{-4}. However, the scattering term in the perturbative part also increases, which gives the ratio 20% (36%) for $T=180$ MeV (190 MeV). Thus, the qualitative behavior of $m_\rho(T)$ seems to be reliable even around T_c.

Finally, we repeat our analysis of the ρ-meson sum rule for case (i) but now using a numerical optimization procedure to extract the spectral parameters. In particular, we minimize the relative difference squared of the theoretical and phenomenological sides of the sum rule with respect to the parameters m_ρ, f, and S_0, as summed over a set of M^2 points. We will use analogous procedures in the next section. The idea is to identify a region of M^2 in which we expect the sum rule to be valid, and then to choose the spectral parameters so that the two sides of the sum rule agree most closely in that region. This approach agrees in spirit, at least, with that used by Bochkarev and Shaposhnikov.[13]

Figure 5 shows the temperature dependence of the spectral parameters predicted by this method, with M^2 points taken between 0.5 and 0.8 GeV^2. (This is the region of M^2 where the $T=0$ ρ-meson sum rule is thought to be reliable.[21]) The mass m_ρ and the continuum threshold $\sqrt{S_0}$ are plotted with respect to the left-hand axis and the resonance strength with respect to the right-hand axis. Changes in the values of input parameters or the region of M^2 used lead to somewhat different quantitative predictions for the spectral parameters at $T=0$, with the continuum threshold being most sensitive to details. However, the same qualitative behavior with temperature illustrated by Fig. 5 is found in all cases. The behavior of

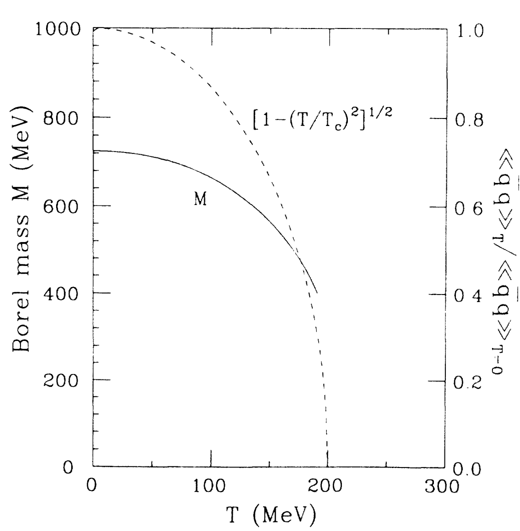

FIG. 4. The minimized Borel mass M from case (iii) of Fig. 3 and the mean-field model of the quark condensate, as functions of temperature.

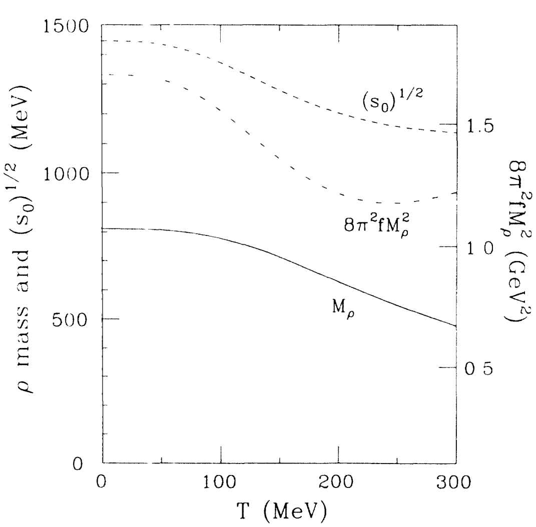

FIG. 5. Resonance parameters of the ρ meson for case (i) of Fig. 3 as functions of temperature, by optimizing the Borel sum rule for M^2 between 0.5 and 0.8.

m_ρ is consistent with curve (i) of Fig. 3 and with Ref. 15, but contradicts the sudden decline around $T=140$ MeV reported in Ref. 12. Similarly, the threshold and strength show only a smooth decline as a result of the thermal scattering terms; one would not interpret their behavior as signaling a phase transition.

In summary, our analysis in this section shows that the ρ meson is a bad (good) indicator of the effects of the gluon (quark) condensate at finite T. In the following section, we will examine the heavy-quark system, where $\langle\langle GG \rangle\rangle$ plays a more important role.

V. J/ψ

Zero-temperature sum rules for charmonium have been extensively studied by Reinders *et al.*[21] using the moment method and by Bertlmann[27,28] using the Borel-transform method. In this section, we present results for finite-temperature sum rules with both approaches, using the numerical optimization technique described above to extract the spectral parameters.

In the heavy-quark system, the dimension-four condensates are most important (particularly for $Q^2>0$).[21] In addition, the contribution from the heavy-quark condensate vanishes to leading order in the heavy-quark-mass expansion.[21] Thus, the charmonium spectrum is primarily determined in the sum-rule approach only by perturbation theory and $\langle\langle G^2 \rangle\rangle$. (Again, we assume that the electric and magnetic condensates are equal.)

We start once more with the Borel-transformed sum rule at $T\neq 0$ from Sec. III and take the J/ψ current $\bar{c}\gamma_\mu c$ as J_μ. In constructing the theoretical side at finite temperature, we neglect the temperature dependence of the α_s correction to the perturbative part. In Appendix B, we estimate the correction coming from this part and show that it is negligible for $T \ll m_c$. We also neglect the T dependence of the Wilson coefficient for the gluon condensate, as in the ρ-meson case. With these assumptions, the theoretical side reads

$$(M^2)L_M \mathrm{Re}\Pi_l = \int_0^\infty d\omega^2 e^{-\omega^2/M^2}[\theta(\omega^2-4m_c^2)\rho_g+\delta(\omega^2)\rho_s] + \frac{1}{2}e^{-4m_c^2/M^2}A(M^2)[\alpha_s a(M^2)+\phi b(M^2)] , \tag{24}$$

where

$$\phi = \frac{4\pi^2}{9(4m_c^2)^2}\left\langle\!\!\left\langle \frac{\alpha_s}{\pi}GG \right\rangle\!\!\right\rangle . \tag{25}$$

$A(M^2)$, $a(M^2)$, and $b(M^2)$ are given in Ref. 28 and summarized in Appendix C.

The phenomenological side of the sum rule will be modeled after Refs. 12 and 13. We include scattering contributions from thermal D mesons, which are analogous to the pion contribution in Sec. IV:

$$\int_0^\infty d\omega^2 e^{-\omega^2/M^2}[\rho_{\rm pole}+\theta(\omega^2-S_0)\rho_g'+\delta(\omega^2)(\rho_D'+\rho_{D_s}')] , \tag{26}$$

where $\rho_{\rm pole}=fm_{J/\psi}^2\delta(\omega^2-m_{J/\psi}^2)$, ρ_g' is the same as ρ_g with $\sqrt{S_0}/2$ replacing m_q and a multiplicative factor of $(1+\alpha_s/\pi)$, and ρ_D' and ρ_{D_s}' are the D-meson contributions ρ_D and ρ_{D_s} with a multiplicative factor of $(1+\alpha_s/\pi)$ (see Appendix A for further details).

After equating the theoretical and phenomenological sides, the logarithmic derivative with respect to $1/M^2$ gives us the following expression for the J/ψ mass:

$$m_{J/\psi}^2 = \left[\int_{4m_c^2}^\infty d\omega^2\omega^2 e^{-\omega^2/M^2}\rho_g - \int_{S_0}^\infty d\omega^2\omega^2 e^{-\omega^2/M^2}\rho_g' + 2m_c^2 e^{-4m_c^2/M^2}A(\alpha_s a+\phi b)\left[1-\frac{A'}{A}-\frac{\alpha_s a'+\phi b'}{\alpha_s a+\phi b}\right]\right] \times\left[\int_{4m_c^2}^\infty d\omega^2 e^{-\omega^2/M^2}\rho_g - \int_{S_0}^\infty d\omega^2 e^{-\omega^2/M^2}\rho_g' + \rho_s - \rho_D' - \rho_{D_s}' + \tfrac{1}{2}e^{-4m_c^2/M^2}A(\alpha_s a+\phi b)\right]^{-1} . \tag{27}$$

In Ref. 28, the denominator is expanded to first order in α_s and ϕ. Here, we maintain the ratio to study the effect on the continuum. In this ratio method, we have to make sure that we are studying the range of M^2 where the power correction is small. In our work, we keep the gluonic power correction less than 30%. The assumption is that neglected power corrections are of the order of the square of the power corrections that are kept, and so are small.[29] To determine the resonance mass and continuum threshold, we search for the range of M^2 for which the power correction is smaller than 30% using the ratio method and then minimize the relative difference of the theoretical side and the phenomenological side in this range. Using the new parameters, a new range of M^2 is determined and the process is iterated until the range is self-consistent.

We use one of the parameter sets from Ref. 28:

$$m_c = 1.42 \text{ GeV} \quad \text{or} \quad m=1.28 \text{ GeV} , \qquad \alpha_s = 0.27, \quad \phi = 1.23\times 10^{-3} . \tag{28}$$

We follow the prescription of Bertlmann in choosing the renormalization point to be at the physical mass $m_c^2 = m^2(p^2=m^2)$. As noted in Ref. 28, the appropriate scale for M^2 is set by the level splittings rather than by the particle mass. Thus, we expect $M^2 \sim 1$ GeV^2. Indeed, applying the criteria discussed above, the M^2 range is from about 0.7 to 1.4 GeV^2.

In Fig. 6, we show how the resonance parameters change as we change the strength of the condensate at zero temperature. While the prediction for the J/ψ mass is fairly insensitive to the value of the condensate, the continuum threshold and the resonance strength decrease significantly as the condensate is reduced in magnitude. Thus the resonance becomes less prominent; this could indicate that the J/ψ is dissolving, but we cannot draw strong conclusions based on a sum-rule analysis once the threshold becomes close to the resonance mass. (Note that we do not have to know the initial condensate value precisely, because if we started with a different value we could adjust the other parameters to have the threshold around 4 GeV to start with.)

In Fig. 7, we show how the resonance parameters change as a function of the temperature, assuming a constant value for the condensate to isolate the perturbative thermal effects. Only small changes in the parameters are predicted until $T \sim 100$ MeV, at which point the resonance strength and the continuum threshold decrease suddenly. The exact position of this decrease depends somewhat on the parameters used and will shift if we include higher-order corrections and introduce the temperature dependence of the condensate. However, the sudden change in the threshold seems to indicate that the sum rule breaks down at approximately this temperature and that the bound state may disappear.

The sudden change is caused by the scattering term ρ_s, which incorporates the effects of the charm-quark thermal bath. It is surprising, at first, that this thermal bath should have an influence on the resonance properties, because $T \sim 100$ MeV $\ll m_c$ and so the thermal factor n_F is quite small. However, in the sum rule all terms except for ρ_s scale like $e^{-4m_c^2/M^2}$, as can be seen from Eq. (24), while ρ_s scales like $n_F \approx e^{-m_c/T}$. Thus, at a given temperature, the scattering contribution overwhelms the

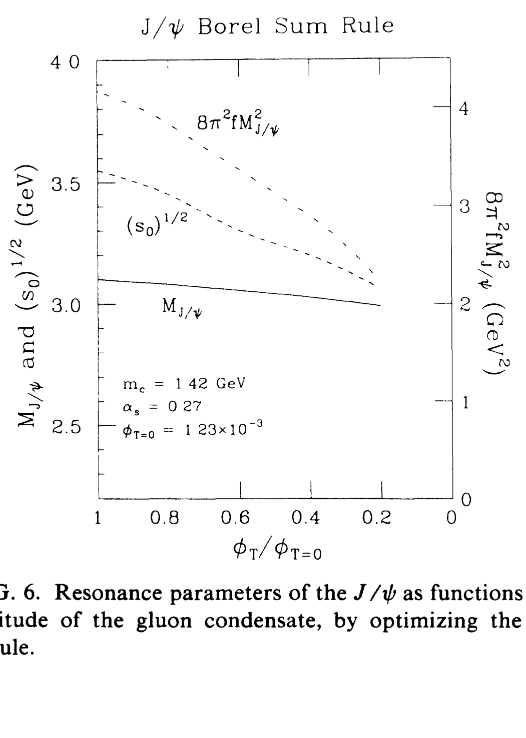

FIG. 6. Resonance parameters of the J/ψ as functions of the magnitude of the gluon condensate, by optimizing the Borel sum rule.

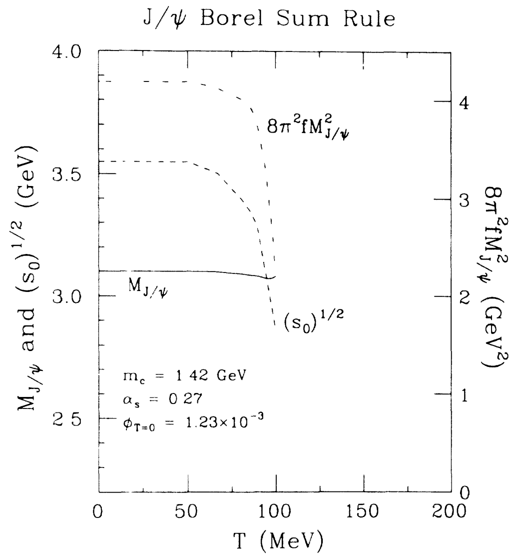

FIG. 7. Resonance parameters of the J/ψ as functions of temperature, by optimizing the Borel sum rule.

other terms for M^2 less than a characteristic value. As the temperature is increased, this value moves into the region where the power correction is less than 30%, at which point the scattering term dominates the sum rule and precipitates the sharp changes in the spectrum shown in Fig. 7.

Let us first explain that the exponential factor $e^{-4m_c^2/M^2}$ comes from the propagation of heavy intermediate states. For the correlator of $\bar{c}\gamma_\mu c$, the heavy-quark-mass expansion is possible except for the scattering term:

$$\frac{1}{n!}\left[-\frac{d}{dQ^2}\right]^n \langle\langle JJ\rangle\rangle \sim \int_0^\infty d\omega^2 \frac{1}{(\omega^2+Q^2)^{n+1}}\rho(\omega)$$
$$\sim \frac{c_1}{(4m_c^2)^n}+\frac{c_2}{(4m_c^2)^n}\frac{\langle\langle G^2\rangle\rangle}{(4m_c^2)^2}$$
$$+\frac{c_3}{Q^{2n}}\frac{4m_c^2}{Q^2}e^{-m_c/T} . \quad (29)$$

At small $Q^2/(4m_c^2)$, the last term, which comes from the scattering term, dominates the others. This is because the scattering spectral density is peaked at zero energy, so the "intermediate state" is almost on shell at small Q^2 and has a large propagation amplitude. The other terms come from heavy intermediate states whose propagation is suppressed; after the Borel transformation this suppression shows up as the exponential decay factor.

Now recall the Borel sum-rule scenario at $T=0$. At very short (spacelike) distances (or M^2 large), the coordinate-space correlator is described by the free propagation of quarks; specifically, the dominant contribution to the correlator comes from the propagation of a free $q\bar{q}$ pair intermediate state. As we move to larger distances, power corrections start to become important relative to the free propagation of quarks, which shows up in the sum rule as $1/M^2$ corrections. These corrections (and perturbative corrections), when Borel transformed, will include the $e^{-4m_c^2/M^2}$ propagation factor, along with additional M^2 dependence [see Eq. (24)]. Higher-order power corrections should not change the exponential M^2 dependence if we restrict M^2 so that the lowest-order corrections are 30% or less.

For charmonium at finite temperature, these contributions to the sum rule are only slightly modified at temperatures below 200 MeV (except for the temperature dependence of the condensates themselves, which we are not considering here). This is because the temperature *corrections* involve factors of $e^{-m_c/T}$ in addition to the $e^{-4m_c^2/M^2}$ factors, and so are essentially negligible. However, as discussed above, the new contribution from the scattering term goes like $e^{-m_c/T}$ but is independent of M^2.

At low temperatures, the scattering term is comparable to other contributions only at larger times (or smaller M^2), where its effects are unobservable because higher-order power and perturbative corrections dominate and sum-rule methods cannot be applied. However, at a sufficiently high temperature, the scattering term is important at values of M^2 where the power corrections are still under control, so that it will significantly affect the correlator at larger separations. Since, as noted above, the appropriate scale of M^2 for charmonium sum rules is 1 GeV^2, the scattering term becomes comparable to the other sum-rule contributions at temperatures around 100 MeV; this is where the sum rule predicts a sudden drop of the threshold and resonance strength. (Note: A simple comparison of exponential factors is not sufficient; other numerical factors are important in determining the precise temperature.) One can argue that the sum-rule approach breaks down when this happens (e.g., perturbation theory for ρ_s might be inappropriate) and one should therefore be cautious about drawing strong conclusions. However, it does suggest that the nonperturbative physics (manifested as nonvanishing gluon condensate) is overwhelmed by the influence of the thermal heat bath and hence a perturbative treatment of the charmonium system could be a valid approximation.

The argument given above cannot be applied to the light meson system, because the current quark mass is effectively zero and only the high-momentum expansion is possible for the correlator:

$$\langle\langle JJ \rangle\rangle \sim \ln(Q^2/\mu^2) + d_1 \frac{\langle\langle G^2 \rangle\rangle}{Q^4} + d_2 \frac{\langle\langle \bar{q}q \rangle\rangle^2}{Q^6} + d_3 \frac{T^2}{Q^2} + d_4 \frac{T^4}{Q^4} \ . \tag{30}$$

Our numerical results show that in the small-Q^2 region, the power correction provides a dominant contribution until T gets large. In the large-Q^2 region, the perturbative contribution always dominates. Thus, at moderate temperatures the scattering term does not change the qualitative nature of the $T=0$ sum rule.

We have repeated the analysis of charmonium using moment sum rules, generalizing to finite temperature the sum rules of Ref. 21. In the deep Euclidean region $Q^2=-\omega^2$, the theoretical side of the sum rule reads

$$\begin{aligned} M_n(Q^2) &= \frac{1}{n!}\left[-\frac{d}{dQ^2}\right]^n \Pi_l \\ &= \int_0^\infty d\omega^2 \frac{1}{(\omega^2+Q^2)^{n+1}}[\theta(\omega^2-4m_c^2)\rho_g + \delta(\omega^2)\rho_s] \\ &\quad + \tfrac{1}{2}A(n)[a_n(\xi)\alpha_s + b_n(\xi)\phi] \ , \end{aligned} \tag{31}$$

where

$$\xi \equiv \frac{Q^2}{4m_c^2} \tag{32}$$

and the functions A, a_n, and b_n are tabulated in Ref. 21. (The overall normalization factor of $\frac{1}{2}$ follows the convention of Ref. 13.) The phenomenological side is modeled with the same spectral functions as before:

$$\int_0^\infty d\omega^2 \frac{1}{(\omega^2+Q^2)^{n+1}}[\rho_{\text{pole}} + \theta(\omega^2-S_0)\rho_g' + \delta(\omega^2)(\rho_D' + \rho_{D_s}')] \ . \tag{33}$$

We use the parameter set given in Ref. 21 (note that the parameters are ξ dependent):

$$m_c = 1.21 \text{ GeV} \ , \quad \alpha_s = 0.21, \quad \phi = 1.80\times 10^{-3} \ . \tag{34}$$

These parameters can be varied somewhat and still provide a reasonable description of the charmonium states at $T=0$.[21]

The analogous results for the moment sum rule to the Borel sum-rule results of Figs. 6 and 7 are shown in Figs. 8 and 9, respectively. The spectral parameters are deter-

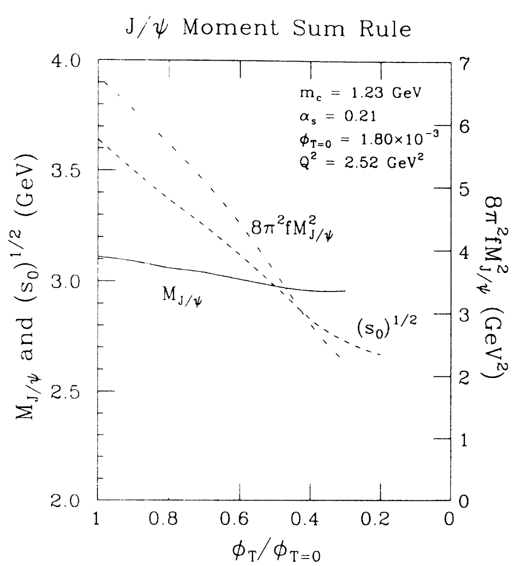

FIG. 8. Resonance parameters of the J/ψ as functions of the magnitude of the gluon condensate, by optimizing the moment sum rule.

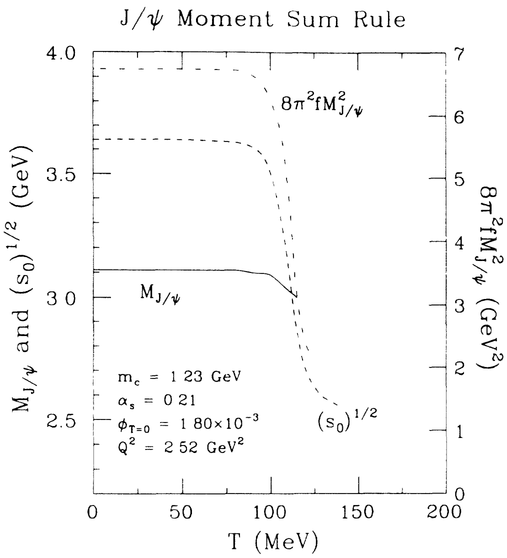

FIG. 9. Resonance parameters of the J/ψ as functions of temperature, by optimizing the moment sum rule.

mined by minimizing the relative difference of the theoretical and phenomenological sides, as summed over the moments from $n=5$ to $n=9$. This range in n corresponds to the stability region at $T=0$.[21] As can be seen from Fig. 8 and Fig. 9, similar qualitative results are obtained in the moment method as were found using the Borel-transform rule, although the predictions differ quantitatively in some details such as the absolute strength of the J/ψ resonance. (The latter discrepancy can be attributed, at least in part, to the differences in the parameters sets used for the two sum rules.) The systematic dependence of the spectral parameters on the magnitude of the gluon condensate at $T=0$ and on the temperature is quite similar, including the temperature range in which the continuum threshold and the resonance strength decrease rapidly. Thus, this behavior does not depend on details of the sum rules.

VI. SUMMARY AND DISCUSSION

Let us summarize our results. For the ρ meson, which we believe typifies the behavior of light-quark systems, the chiral condensate $\langle\langle \bar{q}q \rangle\rangle$ is the most important factor in determining bound-state properties. The changes in these properties as a function of temperature and density have interesting implications for hot quark-gluon plasmas, high density matter, and even for finite nuclei.[2,30,31]

For heavy-quark bound states such as J/ψ, thermal fluctuations (calculated in lowest-order perturbation theory) overwhelm the gluon condensate above a certain temperature, at which point a perturbative description of the bound state would seem to be justified. (We caution the reader that this temperature is rather low, ~ 100 MeV, while the finite-temperature sum rule should be most reliable at higher temperatures, so its quantitative value might not be well determined.) This behavior of the spectrum is similar to that obtained by a potential model with a temperature-dependent string tension,[3] although the exact connection is not so clear. The mechanism for the phase transition is difficult to explain in the sum-rule approach; however, our results seem to be consistent with the description of J/ψ at high temperature advocated by Matsui and Satz.[32]

It would be interesting to study the behavior of D mesons with the QCD sum-rule approach. The first reason is to include the temperature dependence of the D-meson mass in the hadronic scattering term of Eq. (24). Second, one can study potential level crossings of the J/ψ spectrum and D mesons,[33] which would have important implications for signatures of the quark-gluon plasma. In addition, one must investigate corrections to the present calculations, such as contributions to the thermal spectral density from multiparticle intermediate states, to test the robustness of our results. Work in these areas is in progress.

Let us turn our discussion to the nonperturbative physics of QCD at high temperature. As we have shown, when considering hadronic parameters the influence of a persisting nonvanishing gluon condensate is overwhelmed by other contributions so that the naive picture of the QGP as a gas of weakly interacting quarks and gluons seems to be justified. However, the surviving gluon condensate might induce non-negligible corrections to other plasma properties such as the electric or magnetic masses. For example, the temperature dependence of the screening length in lattice gauge calculations disagrees with the perturbative calculation at finite temperature.[34] The screening length given in a perturbative calculation goes like

$$r_D T = \frac{\sqrt{3}}{g\sqrt{N+N_f/2}} \ . \tag{35}$$

Here $N=3$ and N_f is the number of flavors.

The product $r_D T$ increases with temperature according to Eq. (30), since g decreases with temperature. However, lattice calculations show that $r_D T$ decreases with temperature above the phase transition. Based on the assumption that the infrared behavior of QCD at high temperature is the same as three-dimensional QCD,[6] Nadkarni calculated the screening length in three-dimensional QCD with the further assumption that the vacuum expectation value of the adjoint Higgs field $\langle A_0 \rangle = v$ is not equal to zero.[35] In this approach,

$$r_D T = \frac{1}{g} \frac{1-11/12\pi c}{[(N+N_f/2)/3 - 2cg^2/\pi]^{1/2}} \ , \tag{36}$$

where

$$v = cg(T)\sqrt{T} \ , \tag{37}$$

g is the running coupling constant, and c is a nonperturbatively determined dimensionless constant.

As can be seen in the equation, this expression modifies the original value in the direction that decreases with temperature. In Nadkarni's approach, the nonperturbative nature of QCD at high temperature is expressed in terms of the nonvanishing vacuum expectation value (VEV) of the A_0 field and including this effect seems to give a temperature dependence of the screening length more consistent with the lattice result. At this stage, it would be interesting to see how the gluon condensate compares with the VEV of A_0 in modifying the screening length in a manageable way to further study other properties of the plasma that could be modified by a nonvanishing gluon condensate. Some of these issues are presently under investigation.

ACKNOWLEDGMENTS

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APPENDIX A

In this appendix we give a brief account of the derivation of the spectral function $\rho(\omega^2)$.[13] At zero temperature, it is finite only above the two-particle threshold $\omega^2 > 4m^2 + \mathbf{p}^2$. On the other hand, at finite temperature the spacelike region $\omega^2 < \mathbf{p}^2$ is also allowed since the ground state contains thermally excited particles that can absorb the external current. As shown in the text, this contribution becomes quite important in the case of charmonium. To lowest order in the strong-coupling constant $O(\alpha_s^0)$, the longitudinal component of the spectral density reads

$$\begin{aligned}
\rho_l^g(\omega,\mathbf{p}) &= \frac{3}{32\pi^2}\int_{-v}^{v} dx(1-x^2)\tanh\left[\frac{px+\omega}{4T}\right] \quad \text{for } \omega^2 - p^2 > 4m^2 , \\
\rho_l^s(\omega,\mathbf{p}) &= \frac{3}{32\pi^2}\int_{v}^{\infty} dx(1-x^2)\left[2n_F\left[\frac{px+\omega}{2T}\right] - 2n_F\left[\frac{px-\omega}{2T}\right]\right] \quad \text{for } p^2 > \omega^2 ,
\end{aligned} \tag{A1}$$

with $p = |\mathbf{p}|$, $v = [1 - 4m^2/(\omega^2 - \mathbf{p}^2)]^{1/2}$, and $n_F(x) = [\exp(x)+1]^{-1}$, which are obtained from the time-time component of the time-ordered correlation function.

We want to take the limit $p \to 0$, but is not trivial for $\rho_l^s(\omega,\mathbf{p})$ because of the constraint $0 < \omega < p$. However, we note that $\rho_l^s(\omega,\mathbf{p})$, considered as a function of ω, is nonzero only for $0 < \omega < p$ but becomes increasingly large in this region as $p \to 0$. By integrating over this region with p finite and then taking the limit, we find

$$\lim_{p\to 0}\int_0^{p^2} d\omega^2 \rho_l^s(\omega,\mathbf{p}) = \int_{4m^2}^{\infty} du^2 2n_F\left[\frac{u}{2T}\right]\rho^0(u^2;m) , \tag{A2}$$

so that $\rho_l^s(\omega,\mathbf{p})$ effectively becomes a δ function in ω. Thus, the final result is that the spectral functions reduce to

$$\begin{aligned}
\lim_{p\to 0}\rho_l^g(\omega,\mathbf{p}) &= \tanh\left[\frac{\omega}{4T}\right]\rho^0(\omega^2;m) \equiv \rho_l^g(\omega) , \\
\lim_{p\to 0}\rho_l^s(\omega,\mathbf{p}) &= \delta(\omega^2)\int_{4m^2}^{\infty} du^2 2n_F\left[\frac{u}{2T}\right]\rho^0(u^2;m) \equiv \delta(\omega^2)\rho_l^s ,
\end{aligned} \tag{A3}$$

where $\rho^0(\omega^2;m) \equiv v(3-v^2)/16\pi^2$ and $v = (1 - 4m^2/\omega^2)^{1/2}$, which were first given by Bochkarev and Shaposhinkov.[13]

On the phenomenological side of the sum rule, we should take into account the scattering terms due to color-singlet hadrons. For the ρ-meson sum rule, the dominant term arises from the thermal pions

$$\rho_\pi = \tfrac{1}{3}\int_{4m_\pi^2}^{\infty} d\omega^2 2n_B\left[\frac{\omega}{2T}\right]\rho^0(\omega^2;m_\pi) , \tag{A4}$$

where $n_B(x) = [\exp(x)-1]^{-1}$. Note that the last factor $\rho^0(\omega^2;m_\pi)$, which is numerically important for $m_\pi \simeq 140$ MeV, is neglected in Ref. 13. For charmonium, the dominant contributions come from the hybrid pseudoscalar mesons $D^0\,\bar{D}^0$, $D^\pm$, and $D_s^\pm$. The bosonized form of the current $J_\mu = \bar{c}\gamma_\mu c$ written in terms of these fields is

$$J_\mu \sim -i(D^+\overleftrightarrow{\partial}_\mu D^- + \bar{D}^0\overleftrightarrow{\partial}_\mu D^0 + D_s^+\overleftrightarrow{\partial}_\mu D_s^-) . \tag{A5}$$

The spectral function using this current is easily obtained by replacing the pion mass in (A4) with the D-meson masses:

$$\begin{aligned}
\rho_D &= 2\rho_\pi(m_\pi \to m_D) , \\
\rho_{D_s} &= \rho_\pi(m_\pi \to m_{D_s}) .
\end{aligned} \tag{A6}$$

APPENDIX B

Here we will estimate the temperature-dependent α_s contribution to the charmonium sum rule. This contribution will come from thermal gluons, which are characterized by the Boltzmann factor n_B. Compared to the zero-temperature α_s correction, these temperature-dependent contributions will have n_B cutting off the gluon phase-space integral. The ratio of these two is then of the order of $(T/m_c)^2$. Hence it is clear that this effect can be safely neglected, at around $T=200$ MeV, compared to the α_s correction at zero temperature.

This contribution is also small compared to the power correction, as can be seen as follows. If the operator-product expansion is exact, this term would come from nonvanishing of the condensate term at finite temperature. Namely,

$$\left\langle\!\!\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle\!\!\right\rangle_{\text{pert}} = \frac{\alpha_s}{\pi}\left(\tfrac{8}{15}\pi^2 T^4\right) .$$

The ratio of this at $T=200$ MeV to the nonperturbative condensate contribution

$$\left\langle\!\!\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle\!\!\right\rangle_{\text{nonpert}} ,$$

using the parameters from Sec. V, is less than 5%. This is not an exact estimate because the operator-product expansion is valid only at large four-momentum, but it gives an estimate of how large the relevant scales are between the temperature-dependent α_s correction and the gluon condensate.

APPENDIX C

Here we summarize the expression for Eq. (24) in the text, which has been derived in Ref. 28:

$$G(b,c,\omega)=\frac{1}{\Gamma(c)}\int_0^\infty e^{-x}x^{c-1}(\omega+x)^{-b}dx \; , \qquad \omega=4m_c^2/M^2 \; . \tag{C1}$$

Using these formulas, the moments are given by functions

$$\pi A(M^2)=\frac{3}{16\sqrt{\pi}}\frac{4m_c^2}{\omega}G(\tfrac{1}{2},\tfrac{5}{2},\omega) \; , \tag{C2}$$

$$a(M^2)=\frac{4}{3\sqrt{\pi}}G^{-1}(\tfrac{1}{2},\tfrac{5}{2},\omega)\times[\pi-c_1G(1,2,\omega)+\tfrac{1}{3}c_2G(2,3,\omega)] - c_2-\frac{4\ln 2}{\pi}h(\omega) \; , \tag{C3}$$

where

$$c_2=\tfrac{1}{2}\pi-\frac{3}{4\pi} \; , \quad c_1=\tfrac{1}{3}\pi+\tfrac{1}{2}c_2 \; , \quad h(\omega)=\omega G(\tfrac{1}{2},\tfrac{3}{2},\omega)G^{-1}(\tfrac{1}{2},\tfrac{5}{2},\omega) \; , \tag{C4}$$

and

$$b(M^2)=-\tfrac{1}{2}\omega^2 G(-\tfrac{1}{2},\tfrac{3}{2},\omega)G^{-1}(\tfrac{1}{2},\tfrac{5}{2},\omega) \; , \tag{C5}$$

$$\frac{-A'}{A}=\frac{1}{\omega}+\tfrac{1}{2}G(\tfrac{3}{2},\tfrac{5}{2},\omega)G^{-1}(\tfrac{1}{2},\tfrac{5}{2},\omega) \; , \tag{C6}$$

$$-a'(M^2)=\frac{4}{3\sqrt{\pi}}G^{-1}(\tfrac{1}{2},\tfrac{5}{2},\omega)\{-c_1G(2,2,\omega)+\tfrac{2}{3}c_2G(3,3,\omega) -\tfrac{1}{2}G(\tfrac{3}{2},\tfrac{5}{2},\omega)G^{-1}(\tfrac{1}{2},\tfrac{5}{2},\omega)[\pi-c_1G(1,2,\omega)+\tfrac{1}{3}c_2G(2,3,\omega)]\}+\frac{4\ln 2}{\pi}f(\omega) \; , \tag{C7}$$

where

$$f(\omega)=G^{-1}(\tfrac{1}{2},\tfrac{5}{2},\omega)[G(\tfrac{1}{2},\tfrac{3}{2},\omega)-\tfrac{1}{2}\omega G(\tfrac{3}{2},\tfrac{3}{2},\omega)+\tfrac{1}{2}\omega G(\tfrac{1}{2},\tfrac{3}{2},\omega)G(\tfrac{3}{2},\tfrac{5}{2},\omega)G^{-1}(\tfrac{1}{2},\tfrac{5}{2},\omega)] \tag{C8}$$

and

$$-b'(M^2)=\omega G^{-1}(\tfrac{1}{2},\tfrac{5}{2},\omega)[G(-\tfrac{1}{2},\tfrac{3}{2},\omega)+\tfrac{1}{4}\omega G(\tfrac{1}{2},\tfrac{3}{2},\omega)+\tfrac{1}{4}\omega G(-\tfrac{1}{2},\tfrac{3}{2},\omega)G(\tfrac{3}{2},\tfrac{5}{2},\omega)G^{-1}(\tfrac{1}{2},\tfrac{5}{2},\omega)] \; . \tag{C9}$$

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