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Constraints on new chiral fermions in the standard model

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In the light of the new experimental data on standard-model parameters, we investigate the implications of new types of chiral fermions with masses above present experimental detection capabilities. We show that new chiral fermions may exist, but only under very special circumstances; one is R-parity-violating supersymmetry. The analysis is based on the present bounds on the number of light neutrals and on the lack of flavor-changing neutral currents. No such limits exist for vectorlike fermions of the type anticipated in supersymmetric and in some grand unified extensions.

Latest measurements of the Z^0 width at CERN LEP and the SLAC Linear Collider (SLC) have confirmed that there are only three light neutrinos in nature.[1,2] Any other such neutral fermion that might appear must have a mass of 46 GeV or greater. These results rule out a variety of theoretical extensions of the standard model; for example, they are contrary to what one might expect if there were mirror fermions, since mirror families each have a massless neutrino. They rule out the simplest of the Majoron models,[3] and they of course rule out a fourth chiral family of fermions with the usual quantum numbers.

In this paper we set out to analyze in a systematic way the constraints imposed by the new data on the presence of chiral fermions beyond the three chiral families in the standard model. Fermions which can be added to the standard model are either vectorlike or chiral with respect to the standard-model quantum numbers. Typically, vectorlike fermions have masses which break no electroweak quantum numbers; we refer to these as $\Delta I_W=0$ masses. On the other hand, chiral fermions have masses which break weak isospin. For charged chiral fermions, consistency with present data demands that they break it by half a unit, since we have no evidence for substantial electroweak breaking of other types. We thus refer to these as $\Delta I_W=\frac{1}{2}$ masses. Since an upper limit of 246 GeV is set for this breaking by the Fermi constant, masses of chiral fermions are not expected to be much larger, unless perturbation theory breaks down—a possibility we do not entertain here. No such limits exist for vectorlike fermions as their masses are unconstrained by low-energy considerations. Thus the presence of chiral fermions beyond the three families is expected to be an experimentally answerable question in this century.

Any fermionic additions to the standard model, be they chiral or vectorlike, are expected to mix with the usual families, quantum numbers permitting. For this reason we will include in our discussion self-conjugate chiral sets which, although vectorlike as far as their masses are concerned, can alter the character of the neutrinos by mixing with the normal families, and thus allow for more standard families.

Some time ago all possible chiral sets of fermions which could be added to the standard model were cataloged.[4] These *addons* were built using rather general theoretical guidelines. (1) Fermions transform under representations of $SU(3)_c\times SU(2)_W\times U(1)_Y$. Here, each representation is given in two-component Weyl form, labeled by $(\mathbf{r}_c,2\mathbf{I}_W+1)^Y_{R(L)}$, with a subscript R (L) indicating that the fermions are right (left) handed, and $\mathbf{r}_c$ describing the color representation. (2) All color singlets have integer electric charge. (3) All nonsinglet complex color states and all electrically charged states are vectorial. (4) Charged fermions have $\Delta I_W=\frac{1}{2}$ masses. (5) All anomalies are absent.

Such chiral sets can be divided into three classes. (i) All fermions are color singlets (leptons). (ii) Some fermions are color triplets and some are singlets. (iii) All fermions are color triplets. For the purpose of this paper we do not entertain the possibility of exotic colors. (Fermions of color other than triplet could not mix with ordinary matter, and although allowed by present data, will not be discussed in the present context.)

As we alluded to, we consider self-conjugate chiral sets separately. These are described by left-handed fermions with self-conjugate quantum numbers; they are color representations with zero triality such as the singlet and the octet, and real representations of weak isospin such as the singlet, the triplet, the quintet, etc. The further requirement of anomaly cancellation demands that they appear either with zero hypercharge or in pairs of opposite hypercharge. Some of these sets contain unaccompanied neutrals which can serve as the Dirac partners of the left-handed neutrals in the standard chiral families.

The standard model contains three chiral sets of type (ii). Other chiral sets contain exotic charges for leptons and quarks, such as charge two leptons, and charge $-\frac{4}{3}$ and $\frac{5}{3}$ quarks. As we shall see, chiral sets other than these contain too many massless neutrals to describe the real world. Hence we must consider an amalgam of different chiral sets so as to allow mass terms for these exotic neutrinos.

In the case of self-conjugate extensions, we will see that there is ample room for extra standard families of chiral fermions whenever the addons contain unaccompanied neutrals which serve as Dirac partners to the neutrals present in the standard chiral families. Interestingly, extra families would be hard to detect indirectly since they do not violate the custodial SU(2) appreciably enough to alter the value of the ρ parameter. Such self-conjugate sets appear in the supersymmetric extension of the standard model. If there is mixing between leptons and the superpartners of the gauge and Higgs bosons, then R parity is violated. Other than present bounds on the τ neutrino mass we find no additional bounds on this mixing.[5]

All purely leptonic truly chiral extensions suffer from having either too many massless neutrals, or charge 0 and -1 states with the wrong coupling to the Z^0. They are excluded by experiment.

For the other two types of chiral extensions, we shall see that it is rather simple to give masses to the neutrals, but at a large cost. With modest assumptions concerning the hierarchy of scales, we find that there will be an extra charged $\frac{2}{3}$ quark which is light and has the wrong coupling to the Z^0 and/or tree-level flavor-changing neutral currents. In the first case, this light quark is the byproduct of a seesaw mechanism in the quark sector. Obviously, this case is experimentally ruled out. As to the latter case, this too is almost ruled out by the latest available data on the decay:

$$K^+ \to \pi^+ \nu \bar{\nu} \ .$$

In order to get around this bound, one must have unnaturally small mixing angles.

In most cases, a crucial ingredient to ruling out chiral sets will be the absence of a $\Delta I_W = 0$ scale in the electroweak range. While it is unlikely that such a new *scale* occurs in the 100–300-GeV range, it should be noted that it is not unreasonable to expect $\Delta I_W = 0$ *masses* in that range since masses appear naturally depressed by small Yukawa couplings. In view of the large value of the top-quark mass, appreciable mixing in the charge $\frac{2}{3}$ sector between the top and vectorlike quarks may be expected.[6]

We now present a case-by-case analysis of the various chiral sets (addons) mentioned above. We shall only treat the simplest examples in each of the aforementioned cases.

Case (I): Self-conjugate chiral leptons

(A) The simplest way to chirally extend the standard model is to add a single left-handed neutral with zero hypercharge:

$$(1^c, 1)_L^0 \ . \tag{1}$$

With such a term it is possible to add a fourth family of fermions, because the left-handed neutrino can pair off with the extra left-handed neutrino in the fourth family to form a massive $\Delta I_W = \frac{1}{2}$ Dirac neutrino. If its mass is greater than 46 GeV, then there is no contradiction with experiment. Thus a fourth chiral family could very well exist provided that its neutral is coupled to this extra singlet. Lepton number is conserved, and in the case of one extra family with one such addon, three neutrinos will still be massless.

Should the massive neutrino be lighter than half the Z^0 mass, it would have to be identified with one of the existing neutrinos, presumably the τ neutrino which is known to be lighter than 35 MeV. Since lepton number is conserved, it would have to be an additional 10^7 times lighter to satisfy cosmological constraints.

It is possible to add this neutral without a fourth family if we allow lepton number to be violated.[7] In this case it is possible to have the electroweak invariant ($\Delta I_W = 0$) mass term $M \nu_0^T \sigma_2 \nu_0$. If M is much larger than the electroweak scale, then two of the three neutrinos with $\Delta I_W = \frac{1}{2}$ couplings to ν_0 will be massless and the third will have its mass seesawed to a small value.[8] However, if M is of the order of the electroweak symmetry-breaking scale, then it is still possible to have a fourth family. This chiral extension has been recently explored in Ref. 9.

We can give an upper bound on τ-neutrino mixing with this chiral addon based on latest Z^0 width data. Let the physical τ neutrino ν_τ' be given by the linear combination

$$\nu_\tau' = \nu_\tau \cos\theta + \nu_0 \sin\theta \ .$$

Each massless neutrino contributes 160 MeV to the Z^0 width, while the error on the width measurement is about 30 MeV.[1] Since Z^0 does not couple to ν_0, $\cos\theta$ satisfies the lower bound $\cos^4\theta > \frac{130}{160}$, and thus $\theta < 0.3$.

It could also be that an extra neutrino is hiding in the experimental error of the Z^0 width. But this can be ruled out. Assuming that the lightest neutrino combination, of mass m, is of the form

$$\nu = \nu_{1/2} \cos\theta + \nu_0 \sin\theta \ ,$$

we find that $\tan^2\theta > 1.23$. This bound, using the vanishing of the $\Delta I_W = 1$ entry in the mass matrix implies that the other linear combination is actually lighter than m, and this is ruled out by data as both states would contribute the full 160 MeV to the width.

(B) The second simplest self-conjugate extension involves a weak triplet with zero hypercharge

$$(1^c, 3)_L^0 \ ,$$

with charges

$$\begin{pmatrix} +1 \\ 0 \\ -1 \end{pmatrix}_L^0 \ . \tag{2}$$

In this case the lepton-number-violating Majorana $\Delta I_W = 0$ mass is needed in order to give the charged lepton a mass; the central mass, not bounded by standard-model considerations, can be arbitrarily large. The triplet can, however, have a Yukawa coupling to the usual lepton and Higgs doublets. Its effect is to produce mixing among the charged leptons, the amount determined by the strength of the $\Delta I_W = 0$ scale relative to the electroweak scale, but producing no seesaw mechanism for the light mass. On the other hand, in the neutral sector the effect of this coupling is to provide a seesaw mechanism, depressing a normal neutrino's mass below electroweak scale, should the singlet scale be large compared to the electroweak one. The same caveats apply as in the previous case.

(C) In the third case we have the self-conjugate set

$$(1^c,2)_L^{-1}+(1^c,2)_L^{1} ,$$

with charges

$$\begin{bmatrix} 0 \\ -1 \end{bmatrix}_L^{-1} + \begin{bmatrix} +1 \\ 0 \end{bmatrix}_L^{+1} . \tag{3}$$

These can clearly acquire a $\Delta I_W=0$ mass by pairing. However, the doublet with negative hypercharge can, in principle, mix with its counterparts in the standard families. This results in mixings with the normal leptons, but does not affect their masses significantly. Lepton number is conserved so that the neutrinos stay massless.

The singlet extension occurs naturally in left-right-symmetric gauge extensions, and in their grand unified origins, SO(10) and E_6. The doublet extension (3) also appears in E_6.

All three types of self-conjugate chiral fermions appear in the minimal supersymmetric extension of the standard model, with the first two sets acting as the superpartners of the U(1) and SU(2) gauge bosons and the last set acting as the superpartners of the Higgs bosons. (Recall that the minimal extension requires that there be two Higgs bosons, H_1 and H_2, so that the theory is anomaly-free and the charge $\frac{2}{3}$ quarks can get masses.)

If we consider such an extension, then it is possible to give masses to the neutrinos, should R parity be violated. In this case one could easily have one or two more families of chiral fermions, with their neutrinos pairing with the b-ino and W-ino neutrals. In this context, the states of the standard model (including the second Higgs boson) are even under R parity, and their superpartners are odd under the transformation. Stated differently, the R-parity operator is $(-1)^{2J+L+3B}$, where L and B are, respectively, the lepton- and baryon-number operators and J is the spin. Many authors have pointed out that there is no reason why R parity should be a symmetry of the supersymmetric Lagrangian and even if it were, why it should survive supersymmetry breaking.

If R parity is violated, then a possible mass matrix for the neutral sector is

$$(\tilde{B}\tilde{W}^3\tilde{H}_1^0\tilde{H}_2^0\nu_\tau)_L \begin{bmatrix} M_1 & 0 & g'v_1/2 & -g'v_2/2 & 0 \\ 0 & M_2 & -gv_1/2 & gv_2/2 & 0 \\ g'v_1/2 & -gv_1/2 & 0 & \mu\cos\beta & \mu\sin\beta \\ -g'v_2/2 & gv_2/2 & \mu\cos\beta & 0 & 0 \\ 0 & 0 & \mu\sin\beta & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{B} \\ \tilde{W}^3 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \\ \nu_\tau \end{bmatrix}_L , \tag{4}$$

where $\tilde{B}$ is the b-ino, $\tilde{W}^3$ is the neutral W-ino, $\tilde{H}_{1,2}^0$ are Higgsinos, and ν_τ is the τ neutrino. The v_i are the vacuum expectation values for the H_i, and g and g' are the SU(2)$\times$U(1) coupling constants. The R-parity violation appears here as a mixing term between $\tilde{H}_1^0$ and ν_τ, hence violating lepton-number conservation.

The mixing gives an approximate mass to this neutrino

$$m \approx \frac{[M_1(gv_2)^2+M_2(g'v_1)^2]\tan^2\beta}{M_1M_2} . \tag{5}$$

Since lepton number is no longer conserved, it is possible for this neutrino to decay; hence, the cosmological bound to a neutrino mass need not apply if they decay fast enough. On the other hand, the upper bound on the τ-neutrino mass of 35 MeV sets a bound on $\tan^2\beta$. Assuming $M_i \sim 1$ TeV, we find that $\tan\beta < 0.2$.

No such bound on β can be derived by examining the charged sector.[5] Still, it is instructive to examine the charged sector in some detail. There it is possible to have mixing of $\tilde{H}_2^+$ with τ^+, where the second term is an SU(2) singlet. This mixing is described by the mass matrix

$$(\tilde{W}^-\tilde{H}_1^-\tau^-)_L \begin{bmatrix} M_2 & gv_1 & 0 \\ gv_2 & \mu\cos\beta & 0 \\ 0 & -\mu\sin\beta & m_\tau \end{bmatrix} \begin{bmatrix} \tilde{W}^+ \\ \tilde{H}_2^+ \\ \tau^+ \end{bmatrix}_L . \tag{6}$$

What is pertinent here is the mixing of the lightest eigenvalue into the Higgsino. We assume that β is small, $M_2 \approx \mu$, and that these two values are large compared to gv_i and m_τ. The lowest mass eigenstates are given by

$$\tau_L^{-\prime} \simeq \tau_L^- + \beta\tilde{H}_{1L}^- , \quad \tau_L^{+\prime} \simeq \tau_L^+ + \frac{m_\tau\beta}{\mu}\tilde{H}_{2L}^+ . \tag{7}$$

Since $\tilde{H}_2^+$ and τ^+ have different values of I_W^3, one expects their mixing to affect the Z^0 decay rate into the τ mass eigenstate. However, the appearance of the extra suppression factor m_τ/μ does not yield a bound on β comparable to that found in the neutral sector. Experimental signatures of R-parity violation at LEP have recently been explored in Ref. 10. There they find limits on branching ratios of Z^0 decays $Z^0 \to \tau^+\chi^-$ and $Z^0 \to \nu_\tau\chi^0$, where χ^- and χ^0 are respectively a chargino and a neutralino.

Case (II): Leptons only

The simplest chiral extension with leptons only is actually quite a bit more complicated than our previous examples. This is the set

$$(1^c,3)_L^{-2}+(1^c,2)_R^{-3}+5(1^c,2)_R^{-1}+5(1^c,1)_L^{-2} , \tag{8}$$

with charges

$$\begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix}_L^{-2} + \begin{pmatrix} -1 \\ -2 \end{pmatrix}_R^{-3} + 5\begin{pmatrix} 0 \\ -1 \end{pmatrix}_R^{-1} + 5(-1)_L^{-2} . \qquad (9)$$

Obviously the charge 2 state must have a large mass in order to escape detection. In this set we also have four massless neutrals transforming as $(1^c,2)_R^{-1}$, as well as one neutral with a $\Delta I_W = \frac{1}{2}$ Dirac mass. Now the $(1^c,2)_R^{-1}$ states can combine with the $(1^c,2)_L^{-1}$ neutrinos of the standard model to form $\Delta I_W = 0$ Dirac neutrals. Hence if we consider a model with the states in (8) and seven families of fermions with standard-model quantum numbers, there will be only three massless neutrinos. However, such a scenario is suspect. If the $\Delta I_W = 0$ mass is larger than the electroweak scale, and there is no reason why it should not be, then one of these light neutrinos is the $(1^c,3)_L^{-2}$ state. For this state $I_W^3 = 1$, hence its coupling to the Z^0 is twice as big as that of a regular neutrino and its branching ratio in Z^0 decay is four times that of the electron neutrino. Thus there would appear to be effectively six light neutrinos. Obviously this violates experiment.

Furthermore, if we look at the charge -1 leptons, we see that five of the left-handed charge -1 states of the seven families will have large masses by combining with the five $(1^c,2)_R^{-1}$ states of (9). Assuming that this mass scale is larger than the electroweak scale, we conclude that only two of the lighter charge -1 states will have the same quantum numbers as the electron. This is inconsistent with experiment as it leaves no room for the τ lepton's measured coupling to the Z^0, and adding an eighth family would only give too many light neutrals. Therefore, the set (8) is possible only if the $\Delta I_W = 0$ mass is smaller than 250 GeV, an eventuality that can be ruled out with the next generation of experiments.

If we flip the chirality of set (9), then clearly we will have too many neutrals since four of the $(1^c,2)_L^{-1}$ leptons cannot pair off with a right-handed neutral to form a Dirac mass. This eliminates such a possibility.

All other purely leptonic chiral addons suffer from the same problem as this last set. Basically there will either be too many neutrals, or light neutrals and charge -1 states with the wrong value of I_W^3. The only way out is to have $\Delta I_W = 0$ masses lighter than the electroweak scale.

Case (III): Quarks and leptons

This case was of greatest interest to us as the standard-model fermions also fall in this category. Aside from adding a fourth normal chiral family—a case previously studied at great length—there are a few fairly simple true chiral addons possible (see Ref. 4).

(A) The most economical set of this type is

$$(3^c,2)_L^{-5/3} + (3^c,1)_R^{-2/3} + (3^c,1)_R^{-8/3} + (1^c,3)_L^2 + (1^c,2)_R^3 , \qquad (10)$$

with charge assignments

$$\begin{pmatrix} -\frac{1}{3} \\ -\frac{4}{3} \end{pmatrix}_L^{-5/3} + (-\tfrac{1}{3})_R^{-2/3} + (-\tfrac{4}{3})_R^{-8/3} + \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}_L^2 + \begin{pmatrix} 2 \\ 1 \end{pmatrix}_R^3 . \qquad (11)$$

Note that the chiral reverse ($L \leftrightarrow R$) set includes a right-handed isotriplet neutrino with the wrong hypercharge assignment, preventing it from obtaining a mass by combining with normal neutrinos. This exotic neutrino would contribute twice as much as a normal neutrino to the Z^0 decay amplitude. This possibility corresponds to an experimental value of $N_\nu = 7$ when three normal neutrinos are included, and is ruled out by data.

For the chirality indicated in Eq. (11), the neutral state N_L Lorentz transforms like $\sigma_2 \nu_R^*$ and can form a $\Delta I_W = \frac{1}{2}$ Dirac mass with a standard neutrino ν_L. Its mass could be either unnaturally small or greater than 46 GeV. The first possibility can be ruled out by the fact that N_L contributes four times as much to the Z^0 width as normal neutrinos. For the second possibility one must add an additional massless neutrino species to this set to be in agreement with Z^0 width data. It is simplest to add another normal chiral family, but problems arise when we consider mixing in the charge $-\frac{1}{3}$ sector.

Consider the mixing of the $I_W^3 = \frac{1}{2}$ charge $-\frac{1}{3}$ exotic quark, which we call B, with the light quarks $d_i = (d,s,b)$. By the theorems of Glashow and Weinberg[11] we anticipate tree-level flavor-changing interactions. The mass term with mass matrix $\mathbf{M}^{(-1/3)}$ for this case is written as

$$(d_i B)_R^\dagger \begin{pmatrix} m_{ij} & m_i \\ m_j & M \end{pmatrix} \begin{pmatrix} d_j \\ B \end{pmatrix}_L , \qquad (12)$$

where B_R has the same quantum numbers as d_R and B_L is the aforementioned exotic. The lower-case masses are known to be small compared to the upper-case one, although all masses are $\Delta I_W = \frac{1}{2}$. For simplicity, let us ignore the b quark, as there is better data for the d and s quarks, and assume that the mass matrix is approximately symmetric. This does not alter the following qualitative analysis. The mixing matrix, $\mathbf{U}(\mathbf{U}^T \simeq \mathbf{U}^{-1})$, is reduced to a 3-by-3 matrix, and is given approximately by

$$\begin{pmatrix} 1 - \left[\dfrac{\alpha^2+\beta^2}{2}\right] & \alpha & \beta \\ -\alpha & 1 - \left[\dfrac{\alpha^2+\gamma^2}{2}\right] & \gamma \\ -\beta & -\gamma & 1 - \left[\dfrac{\beta^2+\gamma^2}{2}\right] \end{pmatrix} , \qquad (13)$$

where $\mathbf{U}\mathbf{M}^{(-1/3)}\mathbf{U}^{-1} = \mathrm{diag}(m_d, m_s, M_B)$. The mixing angles are assumed to obey the typical Oakes relations

$$\begin{aligned} \tan\alpha &\approx \sqrt{m_d/m_s} \approx \sin\theta_C = 0.22 \ , \\ \tan\beta &\approx \sqrt{m_d/M_B} \ , \\ \tan\gamma &\approx \sqrt{m_s/M_B} \ . \end{aligned} \tag{14}$$

The neutral current for the left-handed charge $-\frac{1}{3}$ quarks is

$$J^\mu = \bar{D}_L \gamma^\mu Y_L D_L \ , \tag{15}$$

where $Y_L = I_L^3 - \sin^2\theta_W Q$ and $D_L = (d,s,B)_L$. In terms of mass eigenstates, $D_L' = \mathbf{U} D_L$,

$$J^\mu = \bar{D}'_L \gamma^\mu \mathbf{U} Y_L \mathbf{U}^{-1} D_L' \ . \tag{16}$$

Since $\mathbf{U}$ commutes with Q, the flavor-changing neutral currents are produced by the matrix $\mathbf{U} I_L^3 \mathbf{U}^{-1}$ given approximately by

$$\begin{bmatrix} -\frac{1}{2} & \beta\gamma & \beta \\ \beta\gamma & -\frac{1}{2} & \gamma \\ \beta & \gamma & \frac{1}{2} \end{bmatrix} , \tag{17}$$

which will induce flavor-changing processes such as $K^+ \to \pi^+ \nu\bar{\nu}$. Comparing this rate to that for $K^+ \to \pi^0 e^+ \nu_e$ we obtain[12]

$$\begin{aligned} \frac{\Gamma(K^+ \to \pi^+ \nu\bar{\nu})}{\Gamma(K^+ \to \pi^0 e^+ \nu_e)} &= \frac{3(g/2)^4 (2\beta\gamma)^2}{\frac{1}{2}(g/\sqrt{2})^4 \sin^2\theta_C} \\ &\le \frac{3.4\times 10^{-8}}{5.0\times 10^{-2}} \ , \end{aligned} \tag{18}$$

where the 3 in the numerator is the number of neutrino species and the $\frac{1}{2}$ in the denominator comes from the pion wave function. Using $\beta\gamma \simeq \sqrt{m_s m_d}/M_B$ we obtain the limit

$$M_B \gtrsim 500 \text{ GeV} \ . \tag{19}$$

The exotic B quark should not have a mass larger than the $\Delta I_W = \frac{1}{2}$ scale ($\sim$250 GeV). Therefore, this set is close to being ruled out by recent data on flavor-changing neutral transitions. In fact, it is expected that the upper limit on the branching ratio in (18) will soon be improved to 10^{-9},[13] in which case the lower limit on the B-quark mass will be about 3 TeV. This will then effectively eliminate this theory.

(B) The next most complicated example in this category is a combination of a set previously obtained in Ref. 4 with a normal chiral set. Omitting self-conjugate states, one obtains the set

$$\begin{aligned} &(3^c,3)_L^{-2/3} + (3^c,2)_R^{1/3} + (3^c,2)_R^{-5/3} + (3^c,2)_L^{1/3} \\ &\quad + (3^c,1)_L^{-2/3} + (3^c,1)_R^{-2/3} + (3^c,1)_R^{4/3} \\ &\qquad + (1^c,2)_L^3 + (1^c,1)_R^2 + (1^c,1)_R^4 \ , \end{aligned} \tag{20}$$

with charge assignments

$$\begin{aligned} &\begin{bmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ -\frac{4}{3} \end{bmatrix}_L^{-2/3} + \begin{bmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{bmatrix}_R^{1/3} + \begin{bmatrix} -\frac{1}{3} \\ -\frac{4}{3} \end{bmatrix}_R^{-5/3} \\ &\quad + \begin{bmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{bmatrix}_L^{1/3} + (-\tfrac{1}{3})_L^{-2/3} + (-\tfrac{1}{3})_R^{-2/3} \\ &\qquad + (\tfrac{2}{3})_R^{4/3} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}_L^3 + (1)_R^2 + (2)_R^4 \ . \end{aligned} \tag{21}$$

This set has no neutral states and has $\Delta I_W = 0$ masses in the charge $\frac{2}{3}$ and $-\frac{1}{3}$ sectors. When set (21) is combined with the three normal families we must diagonalize nontrivial mass matrices in these sectors. We use the notation

$$\begin{aligned} &\tilde{Q}_L^T = (\tilde{U}, \tilde{D}, A)_L^{-2/3}, \quad Q_R^T = (\tilde{U}, \tilde{D})_R^{1/3} \ , \\ &Q_R'^T = (D, A)_R^{-5/3}, \quad D_L = (-\tfrac{1}{3})_L \ , \\ &q_L^T = (u', d')_L^{1/3}, \quad d_R' = (-\tfrac{1}{3})_R, \quad u_R' = (\tfrac{2}{3})_R \ , \end{aligned}$$

for the extra quarks. With upper-case letters indicating exotics, the mass term in the charge $-\frac{1}{3}$ sector is

$$(\tilde{D} D d' d_i)_R^\dagger \begin{bmatrix} m_1 & m & M & M & M & M \\ m & m_2 & 0 & 0 & 0 & 0 \\ 0 & M & m_3 & m & m & m \\ 0 & M & m & & & \\ 0 & M & m & & m_{ij}^{(-1/3)} & \\ 0 & M & m & & & \end{bmatrix} \begin{bmatrix} \tilde{D} \\ D \\ d' \\ d_j \end{bmatrix}_L . \tag{22}$$

Here i and j range over the three normal families, upper-case M's indicate $\Delta I_W = 0$ masses, and lower-case m's indicate $\Delta I_W = \frac{1}{2}$ masses. It is not difficult to determine that for $M \gg m$, there will be two eigenvalues of (22) of $O(M)$ and four of $O(m)$. Here one must take care that the extra nonexotic quark d' has small mixing with the usual charge $-\frac{1}{3}$ quarks as an extra quark has not yet been seen. For the typical hierarchy of scales ($\Delta I_W = 0 \gg \Delta I_W = \frac{1}{2}$) flavor-changing neutral currents are not a problem here.

Now consider the mass term in the charge $\frac{2}{3}$ sector

$$(\tilde{U}u'u_i)^\dagger_R \begin{bmatrix} m_1 & M & M & M & M \\ 0 & m_2 & m & m & m \\ 0 & m & & & \\ 0 & m & & m_{ij}^{(2/3)} & \\ 0 & m & & & \end{bmatrix} \begin{bmatrix} \tilde{U} \\ u' \\ u_j \end{bmatrix}_L , \tag{23}$$

where we use the same conventions as above. Diagonalization of (23) yields one eigenvalue of $O(M)$, three of $O(m)$, and one of $O(m^2/M)$. Hence there is a seesaw mechanism for a very light charge $\frac{2}{3}$ quark. However it is the exotic quark that becomes light. Consider the case of one normal family. Then the mass matrix, $\mathbf{M}^{(2/3)}$, is

$$\begin{bmatrix} m_1 & M \\ 0 & m_2 \end{bmatrix} . \tag{24}$$

Diagonalizing $\mathbf{M}^{(2/3)}\mathbf{M}^{(2/3)T}$, we find mass-squared eigenvalues

$$m_\pm^2 = \frac{M^2+m_1^2+m_2^2}{2}\left[1 \pm \left[1-\frac{4m_1^2m_2^2}{(M^2+m_1^2+m_2^2)^2}\right]^{1/2}\right] \tag{25}$$

and eigenvectors

$$u_\pm = \frac{1}{\left[1+\left[\dfrac{m_\mp^2}{Mm_2}\right]^2\right]^{1/2}} \left[1, \mp\frac{m_\mp^2}{Mm_2}\right] . \tag{26}$$

For $M \gg m$,

$$m_+ \simeq M, \quad m_- \simeq \frac{m_1 m_2}{M} , \tag{27}$$

and the mass eigenstates are

$$\begin{aligned} \tilde{U}'_L &= u_+ = \tilde{U}_L \sin\theta - u_L \cos\theta , \\ u'_L &= u_- = \tilde{U}_L \cos\theta + u_L \sin\theta , \end{aligned} \tag{28}$$

where $\theta \simeq m_2/M$. Thus the exotic quark obtains the suppressed mass.

On the other hand, the mass M could easily be of the same magnitude as the electroweak scale. The u and c quarks would have to have small mixing with $\tilde{U}$ in order to account for their small masses. However, the t quark, whose mass is now known to lie above 89 GeV, could have appreciable mixing with $\tilde{U}$. In fact, this mixing could account for its large mass compared to the other quarks. For instance, if the terms m_{ij} are suppressed by small Yukawa couplings, but the values m, m_1, and m_2 are unsuppressed, then two of the three usual quarks will have small masses but the third will have a large mass. Of course, it will have nonstandard coupling to Z^0 because it is a mixture of a normal and an exotic quark, which could possibly act as an experimental signature for such a configuration. We make this point because the top-quark mass is not very different from the electroweak scale, which makes it a prime laboratory for studying mixing with $\Delta I_W=0$ fermions, should they be generated by new physics at the TeV scale. However, there is one problem with this scheme. By making the same sort of assumptions about the masses in charge $-\frac{1}{3}$ sector as in the charge $\frac{2}{3}$ sector, we would find only one light charge $-\frac{1}{3}$ quark.

Case (IV): Quarks only

The simplest example involves exotic charge $\frac{5}{3}$ quarks

$$\begin{bmatrix} \frac{5}{3} \\ \frac{2}{3} \\ -\frac{1}{3} \end{bmatrix}_R^{4/3} +2\begin{bmatrix} \frac{5}{3} \\ \frac{2}{3} \end{bmatrix}_L^{7/3} +2\begin{bmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{bmatrix}_L^{1/3} +(\tfrac{5}{3})_R^{10/3}$$
$$+3(\tfrac{2}{3})_R^{4/3}+(-\tfrac{1}{3})_R^{-2/3} . \tag{29}$$

Note that one cannot change the chirality of this set as it would produce large $\Delta I_W=0$ masses for two $(3^c,2)_L^{1/3}$ states, three $(3^c,1)_R^{4/3}$ states, and one $(3^c,1)_R^{-2/3}$ state, all within the normal families of quarks.

Since all masses for set (29) and three normal families are $\Delta I_W=\frac{1}{2}$, i.e., $\lesssim 250$ GeV, there is no natural mechanism to ensure that the additional quarks obtain large masses and have negligible mixing with normal quarks. Because of the presence of two exotic $I_W^3=-\frac{1}{2}$ left-handed charge $\frac{2}{3}$ quarks and an $I_W^3=-1$ right-handed charge $-\frac{1}{3}$ quark B_R, there are flavor-changing neutral currents produced by this set. The mass matrix in the charge $\frac{2}{3}$ sector is similar to the matrix in (12). One obtains a simplified mixing matrix of the form (13) but since the neutral-current data from D^+ decays is not as restrictive as that of K^+ decays, it is not as difficult to accommodate this mixing.

The presence of the exotic charge $-\frac{1}{3}$ quark, however, contradicts experimental observations. The mixing of this quark with normal right-handed quarks (d_R and s_R, say) leads again to a mass matrix (12) and a mixing matrix $\mathbf{U}$ of Eq. (13). The neutral current for the right-handed charge $-\frac{1}{3}$ quarks above is

$$J^\mu = \bar{D}'_R \gamma^\mu \mathbf{U} Y_R \mathbf{U}^{-1} D'_R , \tag{30}$$

where $D_R=(d_i,B)_R$ and D'_R denotes the mass eigenstates. Flavor-changing neutral currents are produced by the matrix $\mathbf{U}I_R^3\mathbf{U}^{-1}$ given approximately by (again neglecting the b quark)

$$\begin{bmatrix} \beta^2 & 2\beta\gamma & \beta \\ 2\beta\gamma & \gamma^2 & \gamma \\ \beta & \gamma & 1 \end{bmatrix} . \tag{31}$$

Following our previous analysis, replacing $\beta\gamma$ by $2\beta\gamma$, in (18), we find that this mixing produces observable tree-level flavor-changing neutral currents unless $M_B \gtrsim 1$ TeV. Perhaps a worse result of this mixing is that it induces couplings (quadratic in mixing angle) of normal right-handed quarks to the Z^0, thereby modifying the $V-A$ low-energy structure of the theory. Hence we can rule out (29). Further examples are far more complicated, having greater numbers of quark states with unusual charges, and display similar problems.

Further chiral addons can be constructed using the results of Ref. 4. The leptonic addons involve increasing numbers of neutral particles obtaining $\Delta I_W = \frac{1}{2}$ masses. In the absence of sophisticated seesaw mechanisms these addons can be ruled out by current Z^0 width data. There is an increasing menagerie of exotic quarks and leptons in these addons whose mixing with known fermions becomes more complicated and more difficult to suppress.

We could also consider combining the sets (10) and (20). In this case the neutrino acquires a $\Delta I_W = \frac{1}{2}$ mass and there is no seesaw mechanism in the quark sector. However, this set suffers from the same neutral-current difficulties as the combination of (10) and four families of standard-model fermions.

SUMMARY AND OUTLOOK

We have shown that generic chiral extensions of the standard model are ruled out by present experimental data, assuming that mixing angles satisfy the usual Oakes relations. The exceptions to this are extensions by self-conjugate fields, that is, those fermions that can form Majorana masses.

The few windows that remain open are likely to be closed in the near future. For instance, $\Delta I_W = 0$ masses smaller than 250 GeV are not at present ruled out for our second quark-lepton extended model. Hopefully this possibility will be tested in the Superconducting Super Collider era. Furthermore, as the number of Z^0 events increases, the statistics on its width should improve enough to decrease the upper bound on R-parity-violating mixing angles.

Perhaps the most striking aspect evident throughout our analysis is the success of the standard model with no chiral extensions whatsoever. As other potential theories fall victim to experiment, one cannot help but be enchanted by this remarkable theory of nature.

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