Null-plane Bethe-Salpeter dynamics: Mass spectra, decay constants of pseudoscalar mesons, and the pion form factor

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A new relativistic definition of the reduced mass (μ_{12}) of a $q\bar{q}$ pair, so as to be in conformity with the standard Wightman-Garding definition of its relative four-momenta q_μ, is introduced into the kernel of an ongoing Bethe-Salpeter (BS) program on a two-tier basis. The new definition of μ_{12} (involving the hadron mass M) is found to produce a natural Regge asymptotic behavior ($M^2 \sim N$) in the hadron mass spectra, while retaining the property of an asymptotically linear ($\sim r$) confinement in the three-dimensional structure of the BS kernel. The relativistic structure of μ_{12} is responsible for a significant improvement in the fits to the ground-state masses of $q\bar{q}$ and $Q\bar{q}$ mesons as compared to its nonrelativistic definition $m_1 m_2/(m_1+m_2)$. The leptonic decay constants f_p and the charge radii thus calculated are also in excellent agreement with data (π, k) where available, while f_p predictions for $Q\bar{q}$ mesons have good overlap with recent lattice predictions. Further, the scaling property ($\sim k_\mu^{-2}$) of the hadron's electromagnetic form factor at large k^2 is a consequence of the "on-shell" form of its null-plane wave function. All these results (which are indicated in the barest outline) are preceded by a perspective summary of the theoretical premises and practical working of the BS equation with a four-fermion interaction kernel as a necessary background on a two-tier basis.

I. INTRODUCTION

In some recent publications,[1-6] we have attempted to give an exposition of the contact with data of a null-plane formulation of hadron dynamics based on the Bethe-Salpeter equation (BSE) with a Lorentz-invariant vector-type kernel.[4] The model which works on a two-tier basis is characterized on the one hand by a covariant three-dimensional formulation of the BSE, using the null-plane ansatz (NPA) and, on the other, by a reconstructed four-dimensional BS wave function so as to restore the (virtual) $q\bar{q}$ degrees of freedom [suppressed during the three-dimensional (3D) reduction], and is directly adapted to the language of standard 4D quark-loop diagrams. The first stage is appropriate for "on-shell" manifestations, viz., the mass spectra of $q\bar{q}$[1] and qqq[2] hadrons, while the second stage concerns various "off-shell" applications typified by different types of hadronic transition amplitudes.[3] Nevertheless the seemingly unconventional nature of the two-tier BS approach vis-à-vis more orthodox perception does call for some elucidation as regards to observance of the basic rules, so as to avoid possible misconceptions on its raison d'etre. To this end, we offer a fresh resume of the theoretical premises and practical working of the BSE together with the logic of its adaptation to the present two-tier form.

Another aspect concerns the need for a more conscious incorporation of *internal* relativistic kinematics in the spatial structure of the four-fermion kernel which involves the *reduced mass* μ_{12} of the constituent quarks as a proportionality factor of the spring constant in accordance with a standard theoretical [albeit nonrelativistic (NR)] picture.[6] In view of its pivotal role in determining the anatomy of a general-relativistic interaction, a *relativistic* definition of μ_{12}, which must be in consonance with the Wightman-Garding definition of the internal four-momentum q_μ of the system, seems to be extremely desirable[6] on grounds more general than the immediate (BS) model under study. In this paper we offer a theoretical motivation for this quantity in the context of the BS model, together with the structural changes in the kernel necessitated by its introduction which ensures among other things an explicit Regge asymptotic behavior ($M^2 \sim N$) in the mass spectrum. The bulk of this investigation constitutes an exposé of the above two items in the rest of this section. The immediate numerical consequences of the new definition of the reduced mass, vis-à-vis its traditional NR definition, are merely collected (without derivation) in the form of a few tables (Sec. II) in respect to mass spectra, f_p values, and charge radii (small-k^2 behavior) of P mesons. For large k^2, on the other hand, it is noted that the scaling behavior ($\sim k_\mu^{-2}$) in the hadron's electromagnetic (e.m.) factor is a consequence of the on-shell form of the null-plane wave function in this formalism.

A. Perspectives on BSE

The conventional BSE which is rooted in field theory is a ladder approximation to the detailed Schwinger-Dyson equation involving irreducible vertex functions of arbitrarily high order. A ladder approximation may be perceived in concrete terms in a QED-like theory which allows a systematic classification of higher-order kernels, the consistency of which can be monitored (in a perturbative manner) by a coupling constant (α) to any desired order of accuracy beyond the lowest order (one-boson-exchange) kernel. Such methods are useful for providing consistency checks on, e.g., Lorentz and gauge invariance, especially if a 3D reduction of the BSE (in the instantaneous or null-plane approximation) has to be performed, as has been the case in most practical applications. For a systematic discussion of such problems in the QED context, see a recent review.[7] [In principle it would also be natural to take a similar view in a QCD-like treatment,[8] were it not for the uncertainties which the much larger value of the corresponding coupling constant (α_s) would entail].

An alternative point of view for the formulation of the BSE would be in terms of a single effective kernel arising out of a corresponding four-fermion Lagrangian in the Nambu–Jona-Lasinio (NJL) sense[9] (including its ramifications on dynamical symmetry breaking). In this kind of treatment the entire information on the dynamics is in principle contained in the effective kernel which makes it logically rather inconsistent to ask for its higher-order corrections. It is quite another matter if the theoretical value of such effective kernels in a BSE is far below the standard of a full-fledged QED-like field theory. Nevertheless this is the kind of price one would expect to pay for insisting on a BS-type language for the nonperturbative sector of QCD at today's state of its art. And indeed, several BSE formulations in the NJL[10] spirit of chiral-symmetry breaking (CSB) have been in evidence in the QCD context,[11] as an alternative to the one-pion exchange (OPE) and allied languages.[12] Another advantage of the BSE formalism is that its effective 4D kernel can be attuned to the successful Coulomb plus linear potentials[13] by first giving them a suitable Lorentz-invariant generalization.

B. 3D and 4D aspects of BSE

The present approach to the BSE which was motivated by similar considerations was designed to make *simultaneous* use of both its 3D and 4D aspects. The 3D form which has had a long history by itself,[14] provides the basic dynamical equation for the lowest Fock component in the quark structure of hadrons, and is particularly relevant to their observed mass spectra which seem to resist a 4D (Wick-rotated) description.[15] On the other hand, the 4D form formally incorporates another important feature, viz., the higher Fock components which would be suppressed during the 3D reduction.[3,14] This aspect can be seen more clearly by analyzing the full content of the 4D BSE as a chain of 3D equations[8,16] connecting successively higher Fock components, much like the Tamm-Dancoff equation of the 1950s. And it is these higher Fock components which provide a field-theoretic orientation to the BSE in its full 4D form, thus liberating it from the possible stigma of a "fixed-particle basis" which the BSE (without the higher Fock components) would otherwise connotate in a 3D form. Both are, of course, fully relativistic in content, the 4D form being explicitly Lorentz invariant, while the appearance of the corresponding property in the 3D form depends on its actual formulation.[17]

In the BS program under study we have employed an alternative strategy of reconstructing the 4D wave function[18,4] (as a means of identifying the hadron-quark vertex function) in terms of its 3D structure. This is a rather crucial step which restores, albeit in a perturbative fashion, the effect of higher Fock states to provide access to different types of transition amplitudes through appropriate 4D quark-loop diagrams. Such a point of view is perhaps not entirely new, for the possibility of the 4D BS amplitude containing higher Fock components was at least indicated by Karmanov[19] within a simple-minded Wick-Cutkosky model. On the other hand, the more general nature of the connection between the 3D and 4D amplitudes (especially the reconstruction of the latter in terms of the former),[4,18] in the context of an arbitrary kernel, does not seem to have been recognized[6] in contemporary literature. It may be asked whether such a 4D wave function which merely incorporates the higher Fock components on a rather selective basis would be physically more meaningful than its 3D form with a fixed-particle basis. A possible answer in the first place lies in the recognition that a perturbative connection between the 3D and 4D forms is in keeping with the picture of a gradual unfolding of the higher Fock components since a suppression of their effects seems to be strongly indicated by the O(3)-like structure of the observed mass spectra. Second, a 4D wave function which is reconstructed[6] from its 3D form so as to satisfy the 4D BSE thus incorporates the higher Fock components on a definite field-theoretic basis (albeit perturbatively), and is not merely an *ad hoc* construct. This feature seems to be particularly relevant for providing a nontrivial connection between spectroscopy and transition amplitudes and hence a rich observational basis for a closer look at the detailed anatomy of the confining potential. Indeed the only firm knowledge available on the latter is an asymptotically linear ($\sim r$) flavor-independent behavior,[20] which by itself says very little about its detailed structure at nonasymptotic distances, which may well be strongly influenced by other available length scales (e.g., quark and hadron masses).

C. Two-tier BS program

Before addressing this question of r dependence in greater detail further below, we first recapitulate the main points of our two-tier BS model.[3,6] The QCD motivation of the otherwise empirical confining kernel is limited to a color dependence ($\sim \frac{1}{2}\lambda_1 \cdot \frac{1}{2}\lambda_2$) and a vector-vector ($VV$) -type ($\sim \gamma_\mu^{(1)}\gamma_\mu^{(2)}$) structure,[21,22] analogous to one-gluon exchange (OGE). For a ($q\bar{q}$) system involving spin-$\frac{1}{2}$ constituents, the vector-vector form of confinement, unlike a scalar-scalar form simulates to a significant extent the operative aspects of gauge invariance[23] that are usually sought to be incorporated through standard phase integral structures involving gluon fields.[24] (Gauge invariance for a pair of spin-one constituents again favors the vector-vector form except for a simple modification arising from the effect of the contact g^4 interaction.[25]) A second advantage of the vector-vector form is that the 3D content of the corresponding

BSE would remain essentially unaltered by the replacement[6] $VV \to \frac{1}{2}(VV - AA)$ which makes for approximate chiral invariance, thus making the VV form more analogous to the NJL spirit.[9]

A third advantage of the VV form is that it ensures a *common sign* for the $q\bar{q}$ (color-singlet) and qq (color-antitriplet) interactions, unlike the scalar form which gives opposite signs for them.[2] In this way the VV form is compatible with the possibility of a common origin for $q\bar{q}$ and qqq confinement, while a scalar form is not.[26]

An additional point about the QCD motivation in our BS kernel is an empirical ansatz of its proportionality[1] to α_s among other things. This does not preclude a more complicated dependence on α_s, but is merely intended to suggest that any additional dependence is probably weaker (say, $\ln\alpha_s$). This ansatz has also received strong observational support from the data on all quarkonia (from light to heavy) to provide an integrated view of the spectroscopy.[1,27] We should like to emphasize in this connection that the formal analogy of the confining term with the OGE with respect to color and spin does not connote a lowest-order ladder approximation in the usual OGE sense, since the *spatial* structure given to the former confining term abundantly simulates the features of higher-order ladders.

D. Spatial structure of BS kernel

While the requirement of Lorentz invariance for the BS kernel is trivially satisfied for the OGE part, it proved less trivial for the confining part and took quite some time to implement since this program was begun[21] with a 3D harmonic kernel as an instantaneous approximation to its (yet to be discovered) 4D form, since the obvious 4D generalization $\Box^2\delta^4(q-q')$ did not suffice. Eventually a Lorentz-invariant generalization of a rather interesting representation[28] for r^2 was found in momentum space as[4]

$$\lim_{n\to 0}\frac{\delta^3}{\delta m^3}\frac{4\pi}{m^2+(\mathbf{q}-\mathbf{q}')^2} \Longrightarrow \lim_{m\to 0}\frac{\delta^3}{\delta m^3}\frac{4\pi}{m^2+(q_\mu-q'_\mu)^2} \tag{1.1}$$

which turned out to be ideally suited to a 3D reduction through the null-plane ansatz (NPA), in the following sense: In the NP scheme that has been employed,[4] the third component of the internal three-momentum $\mathbf{q}$ is defined as $q_3 = Mq_+/P_+$, where M is the invariant mass of the composite and P_μ its four-momentum.[29] This NPA argument eventually proved to be more generally applicable to the 3D reduction of any Lorentz-invariant kernel of the form $K(q-q')^2$ as $K(\mathbf{q}-\mathbf{q}')^2$ with the third component defined as above.[14] This permits a simple NPA generalization of any NR potential $V(r)$ to qualify for insertion as part of a BSE kernel, merely through a reinterpretation of the third components of $\mathbf{r}$ and ∇_r as[4]

$$z = x_+M/P_+ , \quad \partial_z = \frac{P_+}{M}\partial_+ . \tag{1.2}$$

As regards the actual parametrization for $V(r)$ the only tangible theoretical constraint[20] is an asymptotically linear ($\sim r$), flavor-independent, behavior, but does not by itself tell much more about its structure at nonasymptotic distances, which is presumably a matter of deeper theory, such as the role of background fields, together with appropriate feedback from data. In this regard we are able to provide a formal defense for the assumption of a harmonic kernel,[4] since it has been known[30] to arise from background gluon fields in the vacuum as the *lowest-order term* in a Fock-Schwinger gauge expansion of the latter.[31] The same result, viz., an r^2 behavior for small r has been shown to emerge from the Bethe-Salpeter equation for a $q\bar{q}$ system wherein the confinement is sought to be simulated[32] (instead of being put in by hand) through the replacement[33] of the quark four-momenta p_μ by $p_\mu - gA_\mu^\alpha \frac{1}{2}\lambda_\alpha$ (A_μ^α=background gluon field) in the inverse quark propagators appearing on the left-hand side of the BSE.[32] A very similar conclusion has also been reached through path-integral techniques with background fields.[34] These two theoretical constraints, viz., linear ($\sim r$) behavior for large r, while remaining harmonic ($\sim r^2$) for small r, the kernel in this model, had led us to suggest the following interpolating formula which explicitly satisfies both these constraints:

$$(2\pi)^{-3}V(r) = \tfrac{3}{4}\omega_{q\bar{q}}^2 \times \left[r^2(1 + A_0\hat{m}_1\hat{m}_2\hat{m}_{12}^2 r^2)^{-1/2} - \frac{C_0}{\omega_0^2} \right] \tag{1.3}$$

with the spring constant defined as[1]

$$\omega_{q\bar{q}}^2 = 4\mu_{12}\omega_0^2\alpha_s(m_{12}^2), \quad \mu_{12} = m_{12}\hat{m}_1\hat{m}_2 , \tag{1.4}$$

$$\alpha_s(Q^2) = \frac{12\pi}{33-2f}(\ln Q^2/\Lambda^2)^{-1} , \tag{1.5}$$

$$\hat{m}_{1,2} = m_{1,2}/m_{12}, \quad m_{12} = m_1 + m_2 . \tag{1.6}$$

Operationally the small value[1] of A_0 (=0.0283) produces an effectively harmonic kernel for light quark (u,d,s) spectroscopy (characterized by small m_1m_2) in conformity with the earliest findings,[21] while the large value of m_1m_2 (despite the smallness of A_0) ensures an almost linear confinement for $b\bar{b}$ quarkonia. Note also the proportionality of the r^2 terms to μ_{12} in both the numerator and denominator of (1.3), in conformity with more general considerations usually employed for r^2 potentials in the NR limit.[35] Note further that (1.3) is linear and flavor independent in the $r\to\infty$ limit, in conformity with lattice QCD theory.[20]

E. Scope of this investigation: relativistic reduced mass

Further sharpening of this structure (which is the main theme of the present investigation) requires an interplay of additional theoretical principles and data on mass spectra and possibly beyond. An important theoretical principle in this regard is the Regge asymptotic behavior $M^2 \sim N$, a requirement which has so far been only approximately satisfied by (1.3) since it gives merely $M^2 \sim N^{4/3}$ for both meson[1] and baryon[2] spectra. This has necessitated a reexamination of the parametric structure of (1.3) so as to conform more closely to the accepted norms of Lorentz invariance. Thus, e.g., a natural Lorentz-invariant generalization of m_{12}, the sum of two quark masses, is the invariant hadron mass M since in the NR limit

$$M \approx m_{12} = m_1 + m_2 \ . \tag{1.7}$$

In a similar way, the fractional momenta $\hat{m}_{1,2}$ of the two quarks $p_{1\mu}$, $p_{2\mu}$ in a hadron which are defined in terms of the total (P_μ) and relative (q_μ) four-momenta as

$$p_{1\mu}, p_{2\mu} = (\hat{m}_1, \hat{m}_2) P_\mu \pm 1 q_\mu \tag{1.8}$$

have the values (1.6) only in the NR limit. These definitions are not satisfactory in the relativistic regime since these do not permit the condition $P \cdot q = 0$ to be satisfied for on-shell quarks ($m_i^2 + p_i^2 = 0$), except for equal-mass kinematics ($m_1 = m_2$). The correct values of $\hat{m}_{1,2}$ come from the Wightman-Garding (WG) definition[36] of the relative four-momentum q_μ:

$$q_\mu = \hat{m}_2 p_{1\mu} - \hat{m}_1 p_{2\mu} \ , \tag{1.9}$$

$$\hat{m}_{1,2} = \tfrac{1}{2}[1 \pm (m_1^2 - m_2^2)/M^2] \ , \tag{1.10}$$

which not only satisfies the condition $P \cdot q = 0$ exactly but also agrees with (1.6) in the NR limit (1.7). Nevertheless, the NR spirit has continued in (1.3)–(1.6) through the appearance of the factor m_{12} whose natural Lorentz-invariant extension appropriate to a $q\bar{q}$ composite is $m_{12} \to M$. In the initial stages we had refrained from this generalization so as to avoid invoking the hadron mass M (to be eventually determined) at the input level. However, the empirical success of the model already achieved[1–3] warrants a closer look at the observational effect of this nontrivial theoretical refinement which comes more under the head of relativistic kinematics than of real dynamics. Thus the WG-inspired refinement (1.10) in the definition of $\hat{m}_{1,2}$ calls for a matching replacement

$$m_{12} = m_1 + m_2 \Longrightarrow M \tag{1.11}$$

in (1.3) and (1.4) to yield a WG-motivated definition for the spring constant, viz.,

$$\omega_{q\bar{q}}^2 = 4 \hat{m}_1 \hat{m}_2 M \omega_0^2 \alpha_s(M^2) \tag{1.12}$$

with the hadron mass M replacing m_{12} consistently in (1.3). This also implies the following definition for the *relativistic* reduced mass:

$$\mu_{12} = \hat{m}_1 \hat{m}_2 M \ . \tag{1.13}$$

Slight precaution is needed before this definition can be applied to all situations, including states (mostly pseudoscalar mesons), where $M < m_1 + m_2$. This is achieved through the simple expedient of $M \to M_>$, where $M_>$ is the larger of M and $m_1 + m_2$ in the relevant equations so that, e.g.,

$$\omega_{q\bar{q}}^2 = 4 \hat{m}_1 \hat{m}_2 M_> \omega_0^2 \alpha_s(M_>^2) \ , \text{ etc.} \tag{1.14}$$

The new WG-inspired definitions of $\hat{m}_{1,2}$ also induce a corresponding change in the structure of the one-gluon-exchange interaction ($V_{\rm OGE}$) for which we advocate the *full package* of short-range ($\sim r^{-3}$) corrections to the Coulomb interaction and not merely the spin-dependent Fermi-Breit term considered earlier.[1] The complete structure of this operator (cf. Ref. 21) on a given state ϕ is now

$$V_{\rm OGE}(r)\phi = \tfrac{4}{3}\alpha_s \left[4M^2 \hat{m}_1 \hat{m}_2 r^{-1} + 4\left\{ \tfrac{1}{4}\nabla^2 - \frac{1}{r^2}\frac{\partial}{\partial r} \right\} - 4 L \cdot S \left\{ \frac{1}{r^3} \right\} + 4\pi(1 - \tfrac{2}{3}\sigma_1 \cdot \sigma_2)\delta^3(\mathbf{r}) - \frac{1}{r^3}(3\sigma_1 \cdot \hat{\mathbf{r}} \sigma_2 \cdot \hat{\mathbf{r}} - \sigma_1 \cdot \sigma_2) \right] \phi \ . \tag{1.15}$$

A major consequence of these WG-inspired replacements is found to been explicit Regge-like behavior $M^2 \sim N$ for large excitation quantum numbers N, compared to the $M^2 \sim N^{4/3}$ trend[1,2] found with the formulas (1.3)–(1.6). With these WG-oriented structures, the following items have been specially investigated in this paper.

(a) A recalculation of the mass spectra of $q\bar{q}$ and $Q\bar{q}$ mesons which are expected to be sensitive to these WG modifications compared to the (equal-mass) heavy ($Q\bar{Q}$) quarkonia which, being more nearly NR systems, should be much less sensitive to these changes.

(b) A complete (new) calculation of the $l\bar{l}$ decay amplitudes (f_p) of P mesons, especially of the $Q\bar{q}$ type, which are particularly sensitive to the NR (1.6) versus WG (1.10) definitions for the fractional momenta $\hat{m}_{1,2}$.

(c) A fresh evaluation of the pion form factor $F_\pi(k_\mu^2)$ at high energies to bring out the feature of a basically k_μ^{-2} behavior[37] at large k_μ^2.

Since the calculational techniques of the model in respect to $q\bar{q}$ states have been provided in earlier publications[1–3] and partly summarized in a recent review,[6] we shall omit all these details and merely draw attention to the effect of the WG-inspired definitions of the $\hat{m}_{1,2}$ and the relativistic reduced mass μ_{12} on the basic structure of the model, as well as the numerical results thereof in respect to the above items in the form of a few tables. The 3D form of the BSE now reads

$$D_+ \phi(\mathbf{q}) = \frac{P_+}{M} \omega_{q\bar{q}}^2 \tilde{D}(\mathbf{q}) \phi(\mathbf{q}) \ , \tag{1.16}$$

$$D_+ = 2P_+[\mathbf{q}^2 - \lambda(M^2, m_1^2, m_2^2)/4M^2] \ . \tag{1.17}$$

The expression for $\tilde{D}$ is omitted for brevity[6] except to note that all *internal* three-vectors $\mathbf{A}$ ($=\mathbf{q}, \sigma, \mathbf{L}, \ldots$) have their third components defined as $A_3 = A_+ M/P_+$, which is consistent with the Lorentz-invariant on-shell

TABLE I. Predictions of mass spectra (in MeV) of $q\bar{q}$ $(1=u,d)$ and $S\bar{S}$ states and their comparison with earlier BS predictions (Ref. 1) as well as data (Ref. 38).

Meson	N	J	L	S	M (Ref. 1)	M [BS (WG)]	β^2 (WG)
$\Pi(140)$	0	0	0	0	163	140	0.0310
$\rho(775)$	0	1	0	1	915	776	0.0692
$b_1(1235)$	1	1	1	0	1182	1142	0.0899
$f_1(1284)$	1	1	1	1		1190	0.0922
$a_2(1320)$	1	2	1	1	1352	1310	0.0939
$\rho^1(1600)$	2	1	0	1	1590	1544	0.1058
$\Pi_2(1680)$	2	2	2	0	1532	1516	0.1095
$\rho_3(1690)$	2	3	2	1	1682	1662	0.1100
$f_4(2030)$	3	4	3	1	2009	2038	0.1239
$\rho_5(2350)$	4	5	4	1	2288	2365	0.1364
$a_6(\cdots)$	4	6	5	1	2571	2710	0.1389
$\phi(1020)$	0	1	0	1	1051	990	0.0840
$f'_0(1300)$	1	0	1	1		1245	0.0962
$f_1(1420)$	1	1	1	1	1364	1353	0.1020
$f_2(1525)$	1	2	1	1	1460	1468	0.1070
$\phi'(1680)$	2	1	0	1	1644	1696	0.1124
$\phi_J(1850)$	2	3	2	1	1776	1802	0.1203
$f_4(2230)$	3	4	3	1		2157	0.1352

TABLE II. Predictions of mass spectra (in MeV) of strange, charm, and beauty states and their comparison with earlier BS predictions (Ref. 1) as well as data (Ref. 38).

Meson	N	J	L	S	M (Ref. 1)	M [BS (WG)]	β^2 (WG)
$K(496)$	0	0	0	0	565	510	0.0467
$K^*(892)$	0	1	0	1	983.5	87	0.0765
$Q_1(1270)$	1	1	1	0	1312	1229	0.0948
$Q_2(1350)$	1	1	1	1		1272	0.0988
$K^{**}(1430)$	1	2	1	1	1412	1393	0.1027
$K'(1460)$	2	0	0	0		1406	0.1027
$L(1770)$	2	2	2	0	1595	1596	0.1176
$K^{***}(1780)$	2	3	2	1	1738	1741	0.1180
$K^{****}(2060)$	3	4	3	1	2064	2113	0.1292
$D(1869)$	0	0	0	0	2010	1868	0.0716
$D^*(2010)$	0	1	0	1	2098	1946	0.0905
$D_s(1971)$	0	0	0	0	2113	1966	0.0895
$D_s^*(\cdots)$	0	1	0	1	2198	2041	0.1082
$B(5271)$	0	0	0	0	5253	5272	0.0692
$B_s(\cdots)$	0	0	0	0		5373	0.0862

TABLE III. Predictions of f_p values (in MeV) of p mesons in the BS model together with the lattice QCD results for their comparison. The notation NR and WG refer to the nonrelativistic and relativistic definition for the reduced mass. All numbers are normalized to the value of f_π^+ (=133 MeV) for $\Pi^+ \rightarrow \mu^+ \nu$.

Model	Π	K	D	D_s	B	B_s
BS (NR)	157	164	110	161	47.7	78.0
BS (WG)	134	166	158	191	94.9	114
Lattice QCD (Ref. 40)	(Input)	161.04±0.05	174.26±46.00	234.9±2.14	105.17±30.00	155.31±48.00
Lattice QCD (Ref. 41)	141.42(21)	155.56(+21)	200(20)		130(20)	

TABLE IV. Predictions of charge radii of P mesons in the BS model, together with experimental data (in fm). The notations WG and NR refer to the relativistic and nonrelativistic definitions, respectively, for the reduced mass.

P meson	r_p (WG) (Present)	r_p (NR)	Expt. results (r_p)
π	0.695	0.458	0.663±0.006[a]
K	0.422	0.425	0.58 ±0.04[b]
D	0.295	0.267	
D_s	0.308	0.289	
B	0.185	0.156	
B_s	0.189	0.191	

condition $A_\mu P_\mu = 0$. Likewise, the hadron-quark vertex function $\Gamma(\mathbf{q})$ which can be deduced from the reconstructed 4D form of the BS wave function[4,6] is

$$\Gamma(q) = N_H^{(-)} \Gamma_i D_+(\bar{q}) \phi(\mathbf{q})/2\pi i$$
$$(\Gamma_i = \gamma_5 \text{ for} \pi,\ i\gamma\cdot\varepsilon \text{ for } \rho, \text{etc.}) \ . \quad (1.18)$$

For the structure of the BS normalizer $N^{(-)}$ which is proportional to P_+^{-1} (Ref. 4), see Refs. 3 and 6. Finally, two kinds of wave functions at the 3D level have been employed, viz., the on-shell form ϕ and half-off-shell form $\tilde{\tilde{\phi}}$, both of which are functions of q_μ^2, but defined strictly on the 3D surfaces $P\cdot q = 0$ and $p_2^2 + m_2^2 = 0$ (m_2=mass of the lighter quark), respectively.[3,6] (We have not so far succeeded in achieving a full 4D extension of ϕ so as to be valid over all space.)

II. RESULTS AND CONCLUSION

A. Mass spectra of $q\bar{q}$ and $Q\bar{q}$, mesons

These results are summarized in Tables I and II for $m_1 = m_2$ and $m_1 \neq m_2$, respectively, together with the data[38] and the old BS prediction employing the nonrelativistic definition of the reduced mass. The input parameters are

$$\omega_0 = 158 \text{ MeV}, \quad C_0 = 0.27, \quad A_0 = 0.0283 \ , \quad (2.1)$$

$$m_{ud}, m_s, m_c, m_b = (265, 415, 1530, 4900) \text{ MeV} \ . \quad (2.2)$$

The most striking feature seems to be an improvement in the ground-state masses which may be attributed partly to the relativistic (WG) definition of μ_{12} and partly to the full OGE package. For excited states, there is not much to comment upon, except to note that these are now in conformity with Regge asymptotic behavior ($M^2 \sim N$) unlike the older ones[1] ($M^2 \sim N^{4/3}$). Comparison with other contemporary models is omitted for brevity.

B. Leptonic decay constants (f_p)

These quantities are defined in terms of the BS vertex function $\Gamma(q)$ in Eq. (1.18) as[6]

$$f_p P_\mu = \sqrt{3} \int d^4q \, \text{Tr}[S_F(p_1)\Gamma(q) S_F(-p_2) i\gamma_\mu \gamma_5] \ . \quad (2.3)$$

The half-off-shell form ϕ of the 3D wave function, with the lighter quark (mass m_2) on shell, has been employed for reasons discussed elsewhere.[39] The results of the relativistic versus nonrelativistic definitions of μ_{12} are summarized in Table III, together with recent results[40,41] of lattice QCD for comparison. Comparison with other models may be found elsewhere.[39] The main conclusion is that the WG definition μ_{12} not only results in a marked improvement with respect to its NR definition, but also has a significant overlap with lattice QCD values.[40,41] An in-depth discussion of the significance of f_p values, especially of the $Q\bar{q}$ system for which direct data are not available, is given elsewhere.[39]

C. Electromagnetic form factors of pseudoscalar mesons

Calculation of e.m. form factors of pseudoscalar mesons for equal-mass kinematics are already given in Ref. 3. The corresponding details for unequal-mass kinematics may be found elsewhere.[42] For small k^2, the behavior is mainly controlled by the charge radii or the respective P mesons, the result for which are summarized in Table IV together with data.[43,44] It is again seen that the pion radius is significantly improved with the WG definition of μ_{12} thus ensuring a good fit to the data[13] up to $k^2 = 0.2$ GeV^2 (Ref. 42). Note that this is achieved without invoking vector-meson-dominance (VMD) contributions which seem to be needed[45] for a pointlike Nambu-Goldstone pion. The present model which is more in the Nambu–Jona-Lasinio spirit[9] and is consistent with a composite pion picture, has no provision for additional VMD contributions.

As a last point, we should like to comment on the predictions of this model for the large-k^2 behavior of the pion form factor (for details see Ref. 42). In this regard the half-off-shell form (ϕ) which is responsible for good fits at low k^2, predicts too rapid a fall with large k^2 as noticed by others.[46] On the other hand, the on-shell form (ϕ) given by[47]

$$\phi = \exp\{-[q_\perp^2 + (q_+ M/P_+)^2]/2\beta^2\} \quad (2.4)$$

has the right ingredients *in principle* for producing a scaling behavior, viz., $F(k^2) \sim k^{-2}$ at large k^2, for any hadron (pion included). Numerical fits to the pion data at large k^2 (which are possible), however, requires additional assumptions bearing on the variation of the mass of the pion at high k^2, for which arguments have been given in the literature[48,49] [for details see Ref. 42].

To summarize, the concept of a relativistic reduced mass within the BS framework has played a most helpful role, not only in providing a better analytical structure to the various predictions, but has also resulted in significantly improved fits to the data on mass spectra and electroweak transition amplitudes. The details may be found in Ref. 42.

[1]A. Mittal and A. N. Mitra, Phys. Rev. Lett. **57**, 290 (1986).
[2]D. S. Kulshreshtha and A. N. Mitra, Phys. Rev. D **37**, 1268 (1988).
[3]N. Nimai Singh and A. N. Mitra, Phys. Rev. D **38**, 1454 (1988).
[4]N. Nimai Singh, Y. K. Mathur, and A. N. Mitra, Few Body Syst. **1**, 47 (1986).
[5]A. N. Mitra, A. Pagnamenta, and N. N. Singh, Phys. Rev. Lett. **59**, 2408 (1987).
[6]S. Chakrabarty *et al.*, Prog Part. Nucl. Phys. **22**, 43-180 (1989); A. N. Mitra, in *Quarks 88*, proceedings of the International Seminar, Tbilisi, USSR, 1988, edited by A. N. Tavkhelidze, V. A. Matveev, S. A. Larin, and S. Yu. Khlebnikov (World Scientific, Singapore, 1989).
[7]G. T. Bodwin *et al.*, Rev. Mod. Phys. **57**, 723 (1985).
[8]E.g., S. Brodsky *et al.* in *Particle and Fields*, proceedings of the Banff Summer School, Banff, Canada, 1981, edited by A. Capri and A. N. Kamal (Plenum, New York, 1983), Vol. 2.
[9]Y. Nambu and L. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); referred to as NJL.
[10]K. Lane, Phys. Rev. D **10**, 2605 (1974).
[11]L. N. Chang and N. P. Chang, Phys. Rev. D **29**, 312 (1983); K. Higashijima, *ibid.* **29**, 1229 (1983); A. Le Yaouanc, L. Oliver, O. Pène, and J-C. Raynal, *ibid.* **29**, 1233 (1983).
[12]H. D. Politzer, Nucl. Phys. **B117**, 397 (1976); H. Pagels, Phys. Rev. D **19**, 3084 (1979); V. Elias and M. Scadron, *ibid.* **30**, 647 (1984).
[13]E. Eichten *et al.*, Phys. Rev. Lett. **34**, 369 (1975).
[14]For a discussion on these aspects, together with a fuller list of references, see Ref. 6, Secs. V and VI.
[15]M. Bohm, H. Joos, and M. Krammer, Nucl. Phys. **B50**, 397 (1973); Acta Phys. Austriaca **41**, 401 (1975).
[16]See, e.g., C. R. Ji, Phys. Lett. **167B**, 16 (1986).
[17]In this connection it is useful to recall that a 3D reduction in the null-plane language [J. Kogut and D. Soper, Phys. Rev. D **1**, 2901 (1970)] is more akin to the infinite-momentum frame [S. Weinberg, Phys. Rev. **150**, 1313 (1966)] in terms of its physical content than with the NR limit despite formal "looks." On the other hand, inasmuch as NPA also incorporates the finite-mass corrections to the infinite-momentum limit, it effectively enjoys the property of a closed-form interpolation between the infinite momentum and the NR limits.
[18]A. N. Mitra and D. S. Kulshreshtha, Phys. Rev. D **26**, 2123 (1982).
[19]V. A. Karmanov, Nucl. Phys. **B166**, 378 (1980).
[20]K. Wilson, Phys. Rev. D **10**, 2445 (1974).
[21]A. N. Mitra and I. Santhanam, Z. Phys. C **8**, 25 (1981); **8**, 33 (1981).
[22]It may be noted that in a BS context, involving a *pair* of particles, the interaction kernel is always a Lorentz scalar, though made up of $\gamma_\mu \cdot \gamma_\mu$ or 1×1, etc., combinations. This is unlike the case of a single particle in an external field where it is more meaningful to speak of a Lorentz-scalar or a Lorentz-vector interaction depending on the nature of the field it interacts with.
[23]Such a conclusion is reinforced by the observation that the 4D framework of the BSE does not suffer from the fixed-particle limitation.
[24]E. Eichten and F. Feinberg, Phys. Rev. D **23**, 2724 (1981); W. Buchmüller and S. H. H. Tye, *ibid.* **24**, 132 (1981).
[25]S. R. Choudhury and A. N. Mitra, Phys. Rev. D **28**, 2201 (1983).
[26]On the other hand, the scalar form has been claimed [e.g., D. Gromes, Phys. Lett. B **202**, 262 (1988)] to provide a more natural understanding of hyperfine splittings in $3\chi_J$ states of $C\overline{C}$, but such questions concern the details of dynamics.
[27]In this connection it is, of course, recognized that the OGE term is less important for light quarks than for the heavy ones, but this feature should get automatically reflected in the numerical results under the common ansatz of OGE and confinement for all sectors.
[28]J. R. Henley, Phys. Rev. D **20**, 2532 (1979).
[29]Despite its seemingly NR appearance, such a definition is actually Lorentz invariant. It permits an interpretation $\mathbf{q}^2 = \mathbf{q}_\perp^2 + q_3^2$ as the value of q_μ^2 on the 4D surface $P \cdot q = 0$.
[30]I. I. Balitsky, Nucl. Phys. **B254**, 166 (1985).
[31]M. A. Shifman, Nucl. Phys. **B173**, 13 (1980); M. B. Voloshin, *ibid.* **B154**, 365 (1979).
[32]Y. K. Mathur and A. N. Mitra, Yad. Fiz. **49**, 537 (1989) [Sov. J. Nucl. Phys. **49**, 336 (1989)].
[33]V. A. Novikov, Nucl. Phys. **B191**, 30 (1981).
[34]Yu. A. Simonov, Nucl. Phys. **B324**, 67 (1989).
[35]V. A. Novikov *et al.*, Phys. Rep. **41C**, 1 (1978).
[36]A. J. MacFarlane, Rev. Mod. Phys. **34**, 41 (1962).
[37]S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. **31**, 1153 (1973).
[38]Particle Data Group, G. P. Yost *et al.*, Phys. Lett. B **204**, 1 (1988).
[39]A. Pagnamenta *et al.*, J. Phys. G **16**, 677 (1990).
[40]C. Bernard, T. Draper, G. Hockney, and A. Soni, Phys. Rev. D **38**, 3540 (1988).
[41]Herbert W. Hamber, Phys. Rev. D **39**, 896 (1989).
[42]K. K. Gupta, Ph.D. thesis, 1990.
[43]S. R. Amendolia *et al.*, Nucl. Phys. **B277**, 168 (1986).
[44]E. B. Dally *et al.*, Phys. Rev. Lett. **45**, 232 (1980); **48**, 375 (1982).
[45]V. Bernard and Ulf. G. Meissner, Phys. Rev. Lett. **61**, 2296 (1988).
[46]Z. Dziembowski, Phys. Rev. D **37**, 778 (1988).
[47]A consistent use of the NPA condition $P \cdot q = 0$ has automatically dispensed with the (non-Hermitian) L_g operator which had characterized the IA treatment necessitating the introduction of the "true wave function" [A. N. Mitra and I. Santhanam, Z. Phys. C **8**, 33 (1981)]. There is now no need to define anew a "true wave function" since the wave function is already "true" as it stands.
[48]S. J. Brodsky and G. R. Farrar, Phys. Rev. D **11**, 1309 (1975).
[49]A. N. Mitra and I. Kumari, Phys. Rev. D **15**, 261 (1977); D. P. Stanley and D. Robson, *ibid.* **26**, 223 (1982).