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On-shell renormalization of the $\langle \bar{q}q \rangle$ contribution to the $\Delta I = \frac{1}{2}$ s-d self-energy transition

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The dimension-3 quark condensate contribution to the renormalized s-d self-energy transition is evaluated within the framework of on-shell renormalization with respect to the (Lagrangian) quark mass matrix. This transition is then related to $\Delta I = \frac{1}{2}$ matrix elements for nonleptonic kaon decay rates. The contributions to such matrix elements from the $\langle \bar{q}q \rangle$ component of the s-d self-energy transition, when considered in isolation, are seen to generate rates of the same order of magnitude as the experimental values for $K_L \to \gamma\gamma$ and $K_S \to \pi\pi$ decays.

I. INTRODUCTION

In a recent Letter, Guberina, Peccei, and Picek note that[1] "since the $\bar{d}s$ self-energy $[\Sigma_{sd}]$ is a purely $\Delta I = \frac{1}{2}$ contribution, the suggestion [Refs. 2–5] that it has something to do with the $\Delta I = \frac{1}{2}$ rule is very natural. . . Unfortunately, when one tries to translate this qualitative idea into practice one finds that, after properly renormalizing the self-energy, the GIM mechanism [Ref. 6] makes the magnitude of Σ_{sd} uninterestingly small [Ref. 7]."

Specifically, if one uses the on-mass-shell (OMS) renormalization prescription of Refs. 7 and 8 on the d generated by perturbative electroweak interactions (Fig. 1), one finds that the constant $O(M_W^{-2})$ leading contribution to the self-energy[4] is subtracted away, leaving only $O(M_W^{-4})$ contributions off shell.[7]

In the present paper, we demonstrate that if nonperturbative quark condensate ($\langle \bar{q}q \rangle$) QCD contributions to Σ_{sd} are evaluated using the same OMS prescription discussed above, then the OMS-renormalized s-d two-point function can generate a $\Delta I = \frac{1}{2}$ matrix element $\langle \pi^0 | H_W | K^0 \rangle$ that remains of order M_W^{-2} in magnitude. Indeed, for generally accepted estimates of the Lagrangian (current quark) s- and d-quark masses, we find $\langle \bar{q}q \rangle$ contributions to this matrix element (when considered in isolation) to be somewhat larger than empirical $K_L \to \gamma\gamma$ and $K_S \to \pi\pi$ decay rates would suggest.

This strong enhancement of the OMS-renormalized $\langle \bar{q}q \rangle$ component of Σ_{sd} relative to the purely perturbative component of Σ_{sd} results from the strong momentum dependence present only in the $\langle \bar{q}q \rangle$ contribution. Thus, for the former case only do $O(M_W^{-2})$ terms survive the OMS subtractions imposed by the renormalization conditions.[8]

$$[\Sigma_{sd}(p)|s(p)\rangle]|_{\not{p}=m_s} = 0 = [\langle d(p)|\Sigma_{sd}(p)]|_{\not{p}=m_d} \ . \tag{1.1}$$

The values of m_s and m_d in the above conditions are assumed to correspond to "current" quark masses, i.e., the explicit masses generated through Yukawa couplings to the electroweak vacuum expectation value $\langle \phi \rangle$. Upon introducing a scale parameter b to characterize the K-π transition matrix element,

$$\langle \pi^0 | H_W | \bar{K}^0 \rangle|_{q_\pi^2=0} \equiv \sqrt{2} b m_K^2 f_K / f_\pi \ , \tag{1.2}$$

we utilize a one-loop Landau-gauge calculation in order to obtain the following lower-bound estimate for b:

$$\begin{aligned}|b| &\gtrsim |G_F \cos\theta_C \sin\theta_C \\ &\quad \times (m_c\langle \bar{c}c \rangle - m_u \langle \bar{u}u \rangle)/(6\sqrt{2} m_d m_s)| \\ &\approx 1.5 \times 10^{-7} \ ,\end{aligned} \tag{1.3}$$

where we have assumed "standard" values $m_d \approx 10$ MeV, $m_s \approx 200$ MeV, and where we have employed the "$m_c \to \infty$" approximation to estimate $m_c \langle \bar{c}c \rangle = -(\alpha_s/12\pi)\langle GG \rangle = -0.001$ GeV^4 (Refs. 9 and 10). The contribution of $m_u \langle \bar{u}u \rangle$ in (1.3) is down from that of $m_c \langle \bar{c}c \rangle$ by a factor ~ 10, owing to the smallness of m_u; the comparable contribution of $m_t \langle \bar{t}t \rangle$ is ignored because of the smallness of the product of t-d and t-s mixing angles. The motivation for our use of the Landau gauge is to avoid Ward-identity difficulties discussed in the final section of Ref. 11.

The above value for b is at least a factor of 2 larger than the value corresponding to observed $K_L \to \gamma\gamma$ and $K_S \to \pi\pi$ decay rates. However, given the uncertainties intrinsic to our one-loop estimate (which neglects completely interference effects with other $|\Delta S| = 1$ processes), as well as the uncertainties in the parameters m_d, m_s, and $m_c \langle \bar{c}c \rangle$, we consider our result meaningful as an order-of-magnitude estimate. Our result indicates that the off-diagonal self-energy $\Sigma_{sd}(p)$, upon inclusion of nonperturbative QCD order-parameter insertions, may indeed prove important in explaining the $\Delta I = \frac{1}{2}$ enhancement observed in nonleptonic kaon decays. We note that other $\Delta I = \frac{1}{2}$ enhancing mechanisms, such as the recent work on diquark effects[12] or the apparently large QCD corrections to "penguin"-type processes,[13] may be operating in addition to the mechanism studied in the present paper;[14]

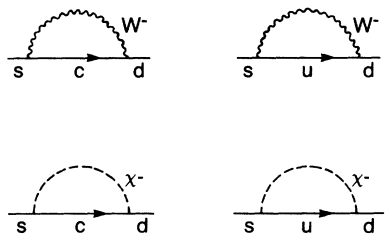

FIG. 1. Purely perturbative electroweak contributions to the off-diagonal self-energy Σ_{sd}. χ is the scalar partner of the intermediate vector boson W.

a reliable quantitative understanding of the $\Delta I = \frac{1}{2}$ enhancement may involve the careful amalgamation of several complementary effects.

In Sec. II, we calculate explicitly the quark condensate contribution to electroweak off-diagonal self-energies, following the detailed treatment of electroweak diagonal self-energies in Ref. 11. Renormalization conditions corresponding to (1.1) are then employed to determine the OMS-renormalized off-diagonal self-energy. In Sec. III, we relate this renormalized self-energy to the K-π transition matrix element by direct comparison of the one-loop diagrams corresponding to weak and strong kaon axial-vector currents; a cutoff-insensitive estimate for the parameter b in (1.2) is then obtained from the ratio of such currents. In particular, we demonstrate that the $1/m_d$ dependence of b (1.3) is explicitly a consequence of the OMS renormalization procedure and is not a reflection of infrared sensitivity within the quark-loop graph containing the Σ_{sd} transition amplitude. Finally, in Sec. IV we relate the $\langle \bar{q}q \rangle$-generated K-π transition matrix element to $K_L \rightarrow \gamma\gamma$ and $K_S \rightarrow \pi\pi$ decay rates, and we briefly discuss our results.

II. THE $\langle \bar{q}q \rangle$ CONTRIBUTION TO THE *sd* SELF-ENERGY TRANSITION

True standard-model physics is the physics of an $SU(3)_c \times SU(2)_L \times U(1)$ gauge theory, in which the unbroken-$SU(3)_c$ sector is sufficiently strong to permit the formation of vacuum condensates. Such condensates, whose phenomenological utility for QCD sum rules is well established,[10] characterize normal-ordered nonlocal vacuum expectation values of quark and gluon fields. For example, a nonvanishing value for the quark condensate $\langle \bar{q}q \rangle$ necessarily implies a nonvanishing value for the nonlocal vacuum expectation value of normal-ordered quark fields:[15]

$$-\langle 0|:q_n(y)\bar{q}_r(z):|0\rangle = \langle \bar{q}q \rangle \left[\sum_{j=0}^{\infty} C_j(-im)^j[\gamma\cdot(y-z)]^j \right]_{nr} + O(\langle \bar{q}G\cdot\sigma q \rangle) \, . \tag{2.1}$$

In this expression, n and r are Dirac-spinorial indices, q and $\bar{q}$ are quark fields of the same flavor and color, and m is the mass characterizing the operator-product expansion of the nonperturbative vacuum expectation value (NPVEV) on the left-hand side (LHS) of (2.1). The coefficients C_j are given by[15]

$$C_j = \begin{cases} [12(j/2)!(1+j/2)!]^{-1}4^{-j/2}, & j \text{ even,} \\ \{6[(j-1)/2]![(j+3)/2]!\}^{-1}4^{-(j+1)/2}, & j \text{ odd.} \end{cases} \tag{2.2}$$

Since the SU(3) QCD Lagrangian is merely a projection of the full $SU(3)\times SU(2)\times U(1)$ standard-model Lagrangian, there is no field-theoretical reason why NPVEV's [such as (2.1)] known to enter the Wick expansions of QCD amplitudes (as in QCD sum-rule applications pedagogically developed in Ref. 16) should not also contribute to the electroweak amplitudes. In particular, computation of s-d self-energies (Fig. 2) in which the NPVEV (2.1) is coupled to the exchanges of electroweak bosons[11] should proceed analogously with self-energy calculations in which the same NPVEV is coupled to the exchange of a gluon.[15–17] The contributions of these Fig. 2 amplitudes are then, respectively, found to be

$$\Sigma_{W,c}(p) = (G_F M_W^2 \sin\theta_C \cos\theta_C/\sqrt{2}) \times \{-\gamma^\mu(1-\gamma_5)[\mathcal{P}_{W,c}(p)]_{\mu\nu}\gamma^\nu(1-\gamma_5)\} \, , \tag{2.3a}$$

$$\Sigma_{W,u}(p) = (G_F M_W^2 \sin\theta_C \cos\theta_C/\sqrt{2}) \times \{\gamma^\mu(1-\gamma_5)[\mathcal{P}_{W,u}(p)]_{\mu\nu}\gamma^\nu(1-\gamma_5)\} \, , \tag{2.3b}$$

$$\Sigma_{\chi,c}(p) = (G_F \sin\theta_C \cos\theta_C/\sqrt{2}) \times \{[(m_c-m_d)+\gamma_5(m_c+m_d)] \times \mathcal{P}_{\chi,c}(p)[(m_s-m_c)+\gamma_5(m_s+m_c)]\} \, , \tag{2.3c}$$

$$\Sigma_{\chi,u}(p) = (G_F \sin\theta_C \cos\theta_C/\sqrt{2}) \times \{-[(m_u-m_d)+\gamma_5(m_u+m_d)] \times \mathcal{P}_{\chi,u}(p)[(m_s-m_u)+\gamma_5(m_s+m_u)]\} \, , \tag{2.3d}$$

where

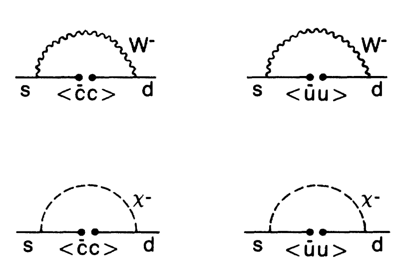

FIG. 2. Contribution to the off-diagonal self-energy Σ_{sd} generated through the coupling of QCD-generated nonperturbative vacuum expection values proportional to $\langle \bar{u}u \rangle$ and $\langle \bar{c}c \rangle$ condensates.

$$[\mathcal{P}_{W,c}(p)]^{\mu\nu}_{nr} \equiv \int d^4(y-z)e^{ip\cdot(y-z)}\langle 0|:c_n(y)\bar{c}_r(z):|0\rangle D^{\mu\nu}_{(W)}(y-z)$$
$$= \langle \bar{c}c \rangle \sum_{j=0}^{\infty} C_j(m_c)^j \left[\left[-\gamma\cdot\frac{\partial}{\partial p} \right]^j D^{\mu\nu}_{(W)}(p) \right]_{nr} , \quad (2.4a)$$

$$[\mathcal{P}_{\chi,c}(p)]_{nr} = \langle \bar{c}c \rangle \sum_{j=0}^{\infty} C_j(m_c)^j \left[\left[-\gamma\cdot\frac{\partial}{\partial p} \right]^j D_{(\chi)}(p) \right]_{nr} . \quad (2.4b)$$

In (2.3b) [(2.3d)], the corresponding function $\mathcal{P}_{W,u}$ $[\mathcal{P}_{\chi,u}]$ is obtained trivially from (2.4a) [(2.4b)] by replacing $\langle \bar{c}c \rangle$ and m_c with $\langle \bar{u}u \rangle$ and m_u.

The fermion masses appearing in (2.3c) and (2.3d) are the current masses generated via Yukawa couplings to the electroweak VEV $\langle \phi \rangle$; we will assume here that the same current masses characterize (2.4) [and the operator-product expansion (2.1)]. The momentum-space W and χ propagators $D_{(W)}(p), D_{(\chi)}(p)$ are evaluated in the Landau gauge to avoid explicit violation of electroweak Ward identities at the W vertices [$\langle \bar{q}q \rangle$ necessarily breaks SU(2)$\times$U(1) gauge symmetry].[11] Away from the Landau gauge, $O(M_W^{-2})$ contributions from Fig. 2 will arise explicitly from scalar and pseudoscalar Ward-identity corrections to the W vertices upon inclusion of $O(\langle \bar{q}q \rangle)$ dynamical components[17] of quark masses. Substitution of the Landau-gauge propagators[18]

$$D^{\mu\nu}_{(W)}(p) = \frac{g^{\mu\nu}}{p^2-M_W^2} + \frac{p^\mu p^\nu}{p^2 M_W^2} - \frac{p^\mu p^\nu}{(p^2-M_W^2)M_W^2}, \quad D_{(\chi)}(p) = -1/p^2 , \quad (2.5)$$

into (2.4) and, subsequently, into (2.3) yields the following $\langle \bar{q}q \rangle$ contribution to the s-d self-energy:

$$\bar{d}(p)\Sigma_{sd}(p)s(p) = (G_F/\sqrt{2})\cos\theta_C \sin\theta_C [m_c\langle \bar{c}c \rangle - m_u \langle \bar{u}u \rangle]$$
$$\times \bar{d}(p) \left[\left[\frac{1}{4p^2} + O(M_W^{-2}) \right] \not{p}(1-\gamma_5) + \left[\frac{m_d m_s}{12p^4} \right] \not{p}(1+\gamma_5) - \frac{m_d}{6p^2}(1-\gamma_5) - \frac{m_s}{6p^2}(1+\gamma_5) \right] s(p) . \quad (2.6)$$

In obtaining (2.6) from (2.4), we have made use of the identities

$$\left[\gamma\cdot\frac{\partial}{\partial p} \right]^j (p^{-2}) = \begin{cases} 1/p^2, & j=0, \\ -2\not{p}(p^2)^{-2}, & j=1, \\ 0, & j\geq 2, \end{cases} \quad (2.7a)$$

$$\gamma^\mu \left[\gamma\cdot\frac{\partial}{\partial p} \right]^j \left[\frac{p_\mu p_\nu}{p^2} \right] \gamma^\nu = \begin{cases} 1, & j=0, \\ 6\not{p}/p^2, & j=1, \\ 0, & j=2 \text{ or } j\geq 4, \\ -24\not{p}(p^2)^{-2}, & j=3, \end{cases} \quad (2.7b)$$

which serve to truncate the series in j for the contributions of $D_{(\chi)}$ and the (p^{-2}) portions of $D_{(W)}$. In (2.6) it is interesting to note that only the $V-A$ coefficient has $O(M_W^{-2})$ corrections, which arise from successive differentiations of $(p^2-M_W^2)^{-1}$ in (2.4a). The leading p^{-2} dependence of all coefficients in (2.6) is to be contrasted with the much weaker momentum dependence of corresponding coefficients within the usual perturbative s-d self-energy transition of Fig. 1. We further note that if m_d, m_s, and m_u current masses are all small, corresponding to a flavor-SU(3) chiral limit, the transition (2.6) is effectively a $V-A$ transition [i.e., proportional to $\not{p}(1-\gamma_5)$] for hadronic momentum scales $(p^2 \sim m_K^2)$.

Of course, Eq. (2.6) leads to an $O(G_F)$ mixing between d- and s-quark states, a consequence of the fact that $\Sigma_{sd}(p)s(p)$ and $\bar{d}(p)\Sigma_{sd}(p)$ are nonvanishing, respectively, on the s- and d-quark mass shells. This mixing may be eliminated entirely by constructing an "on-shell-renormalized" (OSR) Σ_{sd}, in which an appropriate set of $V-A$, $V+A$, scalar, and pseudoscalar counterterms is added to (2.6). Specifically, the OSR requirements that[8]

$$\Sigma_{sd}^{\mathrm{OSR}}(p)s(p) \xrightarrow[\not{p}\to m_s]{} 0 , \quad (2.8a)$$

$$\bar{d}(p)\Sigma_{sd}^{\mathrm{OSR}}(p) \xrightarrow[\not{p}\to m_d]{} 0 \quad (2.8b)$$

dictate a unique choice of counterterms that alters three of the four coefficients in (2.6):

$$\bar{d}(p)\Sigma_{sd}^{\mathrm{OSR}}(p)s(p) = (G_F/\sqrt{2})\cos\theta_C \sin\theta_C (m_c\langle \bar{c}c \rangle - m_u \langle \bar{u}u \rangle)$$
$$\times \bar{d}(p) \left[\left[\frac{1}{4p^2} - \frac{1}{12m_s^2} - \frac{1}{12m_d^2} + O(M_W^{-2}) \right] \not{p}(1-\gamma_5) \right.$$
$$\left. + \left[\frac{m_d m_s}{12p^4} \right] \not{p}(1+\gamma_5) + \left[\frac{-m_d}{6p^2} + \frac{m_d}{12m_s^2} \right] (1-\gamma_5) + \left[\frac{-m_s}{6p^2} + \frac{m_s}{12m_d^2} \right] (1+\gamma_5) \right] s(p) . \quad (2.9)$$

The p-dependent terms of (2.9) are identical to those of (2.6), but are now supplemented by additional constant counterterms to ensure the on-shell conditions (2.8).

It is worth recalling that an identical on-shell renormalization drastically diminishes the contribution of (Fig. 1) the purely perturbative sd transition to $\Delta S=1$ processes.[7] We shall show in the sections following that such is *not* the case for the condensate-driven contribution of (2.9).

III. EMBEDDING Σ_{sd} INTO THE WEAK KAON AXIAL-VECTOR CURRENT

The weak-interaction s-d transition obtained in the preceding section can be used to generate a portion of the weak-interaction $\Delta S=1$ matrix element $\langle \pi | H_W | K \rangle$ through the divergence of the weak kaon axial-vector current,[19]

$$M_\mu \equiv -i \int d^4x\, e^{iq\cdot x} \langle 0 | T[A^3_\mu(x) H_W^{\Delta S=1}] | \bar{K}^0 \rangle \ . \quad (3.1)$$

We can model a direct comparison to the strong-interaction axial-vector current $\langle 0 | A^{6+i7}_\mu | \bar{K}^0 \rangle$ as

$$i\sqrt{2} f_K q_\mu = -i \int d^4x\, e^{iq\cdot x} \langle 0 | A^{6+i7}_\mu(x) | \bar{K}^0 \rangle \quad (3.2)$$

by, respectively, identifying (3.2) with Fig. 3 and (3.1) with Fig. 4. We then find that the weak axial-vector current is approximately

$$M_\mu \cong i\sqrt{2} f_K q_\mu b \ , \quad (3.3)$$

where b is the ratio of the Fig. 4 amplitude to the Fig. 3 amplitude in the soft-q^2 limit. The shaded boxes in both figures factor out all the strong interactions into a net pseudoscalar coupling $g_{Kqq}\gamma_5$; the corresponding fermion propagators are thus regarded to have poles at their Lagrangian (as opposed to constituent) mass values.

In this section we estimate the parameter b in (3.3) that would be obtained by assuming that $H_W^{\Delta S=1}$ in (3.1) is *entirely* due to the OSR s-d transition amplitude (2.9). Specifically, we find that

$$b = \lim_{q^2\to 0} \frac{\int \mathrm{Tr}[\gamma_5(\not{q}+\not{k}-m_d)^{-1}\gamma_\mu\gamma_5(\not{k}-m_d)^{-1}\Sigma_{sd}(k)(\not{k}-m_s)^{-1}]d^4k}{\int \mathrm{Tr}[\gamma_5(\not{q}+\not{k}-m_d)^{-1}\gamma_\mu\gamma_5(\not{k}-m_s)^{-1}]d^4k} \ . \quad (3.4)$$

Our use of the soft-q^2 limit, as opposed to the kaon mass shell, is intended (in part) to probe any infrared problems that may arise in our model; the $q^2=m_K^2$ kaon mass-shell alternative is briefly discussed at the end of this section. Although the logarithmic divergences of both integrals in (3.4) divide out, an approximate evaluation of the right-hand side (RHS) of (3.4) can proceed through explicit use of an ultraviolet cutoff Λ. This cutoff can then be eliminated entirely by relating the Λ-dependent evaluation of Fig. 3 directly to f_K via (3.2):

$$\begin{aligned} -i\sqrt{2} f_K q_\mu &= \lim_{q^2\to 0} \left[\frac{3\sqrt{2} g_{Kqq}}{(2\pi)^4} \right] \int^\Lambda d^4k\, \mathrm{Tr}[\gamma_5(\not{q}+\not{k}-m_d)^{-1}\gamma_\mu\gamma_5(\not{k}-m_s)^{-1}] \\ &= -\left[\frac{3\sqrt{2} g_{Kqq}}{(2\pi)^4} \right] (i\pi^2 q_\mu)(4m_s) \left[\frac{1}{2} \ln \left[\frac{\Lambda^2}{m_s^2} \right] + \frac{m_s^2}{2\Lambda^2} + O\left[\frac{m_d^2}{m_s^2} \right] \right] . \end{aligned} \quad (3.5)$$

In obtaining (3.5) we have dropped all Λ^{-4} contributions. The absence of nonlogarithmic contributions and the explicit value of the Λ^{-2} contribution follows from careful attention to the shift-of-integration-variable surface term[20]

$$\int^\Lambda \frac{d^4k\, k_\mu}{[(k+qz)^2-\mu^2]^2} - \int^\Lambda \frac{d^4k'(k'-qz)_\mu}{(k'^2-\mu^2)^2} = (i\pi^2 q_\mu z/2)[1-(6\mu^2-q^2z^2)/3\Lambda^2] \ . \quad (3.6)$$

Upon assuming that $g_{Kqq} \approx g_{\pi qq} \approx 2\pi/\sqrt{N_c} \approx 3.6$,[17] we find that

$$\Lambda^2 \simeq m_s^2[\exp(843\ \mathrm{MeV}/m_s)-1] \ . \quad (3.7)$$

The trace in the numerator of (3.4) projects out only the parity-conserving (PC) pieces of (2.9):

$$\Sigma_{sd}^{\mathrm{PC}}(k) \equiv (G_F/\sqrt{2})\cos\theta_C \sin\theta_C (m_c\langle \bar{c}c \rangle - m_u \langle \bar{u}u \rangle)[\mathcal{A}(k^2)\not{k} + \mathcal{C}(k^2)] \ , \quad (3.8a)$$

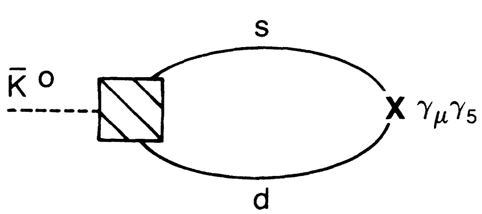

FIG. 3. The kaon-to-vacuum transition of Eq. (3.2).

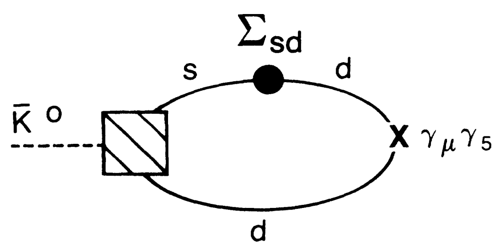

FIG. 4. The kaon-to-vacuum transition of Eq. (3.1).

$$\mathcal{A}(k^2)=\left[-\frac{1}{12m_s^2}-\frac{1}{12m_d^2}+\frac{1}{4k^2}+\frac{m_d m_s}{12k^4}\right], \tag{3.8b}$$

$$\mathcal{C}(k^2)=\left[\frac{m_d}{12m_s^2}+\frac{m_s}{12m_d^2}-\frac{m_d}{6k^2}-\frac{m_s}{6k^2}\right]. \tag{3.8c}$$

We find after some algebra that this numerator trace is given by

$$\begin{aligned}&\mathrm{Tr}[\gamma_5(\not{q}+\not{k}-m_d)^{-1}\gamma_\mu\gamma_5(\not{k}-m_d)^{-1}\Sigma_{sd}(k)(\not{k}-m_s)^{-1}]\\&\quad=\frac{4(G_F/\sqrt{2})\cos\theta_C\sin\theta_C(m_c\langle\bar{c}c\rangle-m_u\langle\bar{u}u\rangle)}{(k^2-m_s^2)(k^2-m_d^2)[(q+k)^2-m_d^2]}\\&\qquad\times\{k_\mu[m_s\mathcal{A}(k^2)+\mathcal{C}(k^2)](k^2-m_d^2)+q_\mu[(m_s+m_d)k^2\mathcal{A}(k^2)+(k^2+m_sm_d)\mathcal{C}(k^2)]\}\\&\quad=-\frac{G_F\cos\theta_C\sin\theta_C[m_c\langle\bar{c}c\rangle-m_u\langle\bar{u}u\rangle]}{3\sqrt{2}k^2[(q+k)^2-m_d^2]}\left[k_\mu\left[\frac{1}{m_s}-\frac{m_d}{m_s^2}+\frac{m_d}{k^2}\right]+q_\mu\left[\frac{1}{m_d}+\frac{1}{m_s}\right]\right].\end{aligned} \tag{3.9}$$

The cancellation of $(k^2-m_s^2)^{-1}$ and $(k^2-m_d^2)^{-1}$ propagator denominators in (3.9) follows from the on-shell character of $\Sigma_{sd}^{\mathrm{OSR}}(k)$, as reflected in (2.8).

We then see from (3.9) that the numerator of (3.4) is given by

$$\begin{aligned}&\mathrm{Tr}\int^\Lambda d^4k[\gamma_5(\not{q}+\not{k}-m_d)^{-1}\gamma_\mu\gamma_5(\not{k}-m_d)^{-1}\Sigma_{sd}(k)(\not{k}-m_s)^{-1}]\\&\quad=-[G_F\cos\theta_C\sin\theta_C(m_c\langle\bar{c}c\rangle-m_u\langle\bar{u}u\rangle)/3\sqrt{2}]\\&\qquad\times\left[\left[\frac{1}{m_d}+\frac{1}{m_s}\right]\int^\Lambda\frac{d^4k\,q_\mu}{k^2[(q+k)^2-m_d^2]}+\left[\frac{1}{m_s}-\frac{m_d}{m_s^2}\right]\int^\Lambda\frac{d^4k\,k_\mu}{k^2[(q+k)^2-m_d^2]}\right.\\&\qquad\left.+m_d\int^\Lambda\frac{d^4k\,k_\mu}{k^4[(q+k)^2-m_d^2]}\right].\end{aligned} \tag{3.10}$$

The $q^2\to 0$ limits of the three Feynman integrals in (3.10) are, respectively, given by

$$\int^\Lambda\frac{d^4k\,q_\mu}{k^2[(q+k)^2-m_d^2]}\underset{q^2\to 0}{\sim} i\pi^2 q_\mu\left[\ln\left[\frac{\Lambda^2}{m_d^2}\right]+\frac{m_d^2}{\Lambda^2}\right], \tag{3.11}$$

$$\int^\Lambda\frac{d^4k\,k_\mu}{k^2[(q+k)^2-m_d^2]}\underset{q^2\to 0}{\sim} i\pi^2 q_\mu\left[-\tfrac{1}{2}\ln\left[\frac{\Lambda^2}{m_d^2}\right]+1-\frac{m_d^2}{\Lambda^2}\right], \tag{3.12}$$

$$\int^\Lambda\frac{d^4k\,k_\mu}{k^4[(q+k)^2-m_d^2]}\underset{q^2\to 0}{\sim} i\pi^2 q^\mu\left[\frac{1}{2m_d^2}-\frac{1}{3\Lambda^2}\right]. \tag{3.13}$$

An estimate of the numerator of b [Eq. (3.4)] is obtained first from substitution of (3.11), (3.12), and (3.13) into (3.10), and subsequently from use of (3.7) to eliminate the ultraviolet cutoff Λ. The denominator of (3.4) is expressed in terms of f_K and g_{Kqq} via the first equality in (3.5). Assuming $m_d \ll m_s \ll \Lambda$, we find that

$$\begin{aligned}b\simeq&\left\{\frac{G_F\cos\theta_C\sin\theta_C(m_c\langle\bar{c}c\rangle-m_u\langle\bar{u}u\rangle)}{6\sqrt{2}m_dm_s}\right\}\\&\times\left\{1+\left[\frac{m_s}{843\ \mathrm{MeV}}\right]\left[\ln\left[\frac{m_s^2}{m_d^2}\right]+\tfrac{1}{2}\right.\right.\\&\left.\left.-e^{-843\ \mathrm{MeV}/m_s}\right]\right\}.\end{aligned} \tag{3.14}$$

The first factor in the curly brackets on the RHS of (3.14) is the answer for b one would obtain from the ratio of just the divergent parts (i.e., the ratio of coefficients of $\ln\Lambda$) occurring in the numerator and denominator of (3.4). The remaining factor in curly brackets is substantially more model dependent. Modification of this factor, which for $m_s\simeq 200$ MeV, $m_d\simeq 10$ MeV is about 2.5, occurs if we modify (3.11), (3.12), and (3.13) to include an infrared (IR) cutoff μ. Although all three integrals are IR finite for nonzero m_d, one could certainly argue for the nonpropagation of quarks beyond a length scale μ^{-1} of order m_s^{-1}. Upon incorporation of such an IR cutoff μ within the range $m_d<\mu<m_s$, we find that (3.11)–(3.13) should be replaced by

$$\int_{\mu}^{\Lambda} \frac{d^4k\, q_\mu}{k^2[(q+k)^2-m_d^2]} \underset{q^2\to 0}{\sim} i\pi^2 q_\mu \left[\ln\left[\frac{\Lambda^2}{\mu^2}\right]+\frac{m_d^2}{\Lambda^2}-\frac{m_d^2}{\mu^2}\right], \tag{3.15}$$

$$\int_{\mu}^{\Lambda} \frac{d^4k\, k_\mu}{k^2[(q+k)^2-m_d^2]} \underset{q^2\to 0}{\sim} -i\pi^2 q_\mu \left[\frac{1}{2}\ln\left[\frac{\Lambda^2}{\mu^2}\right]+\frac{m_d^2}{\Lambda^2}-\frac{m_d^2}{\mu^2}\right], \tag{3.16}$$

$$\int_{\mu}^{\Lambda} \frac{d^4k\, k_\mu}{k^4[(q+k)^2-m_d^2]} \underset{q^2\to 0}{\sim} i\pi^2 q_\mu \left[\frac{1}{3\mu^2}-\frac{1}{3\Lambda^2}\right]. \tag{3.17}$$

In (3.15) and (3.16), we have dropped terms of order m_d^4/μ^4 as well as m_d^4/Λ^4; similarly, terms proportional to m_d^2/μ^4 have been dropped from the right-hand side of (3.17).

Upon substitution of (3.15)–(3.17) into (3.10), one finds that the net effect of including an IR cutoff μ between m_d and m_s is to replace the second factor in curly brackets in (3.14) with a factor somewhat closer to unity:

$$b \simeq \left\{\frac{G_F\cos\theta_C\sin\theta_C(m_c\langle\bar{c}c\rangle-m_u\langle\bar{u}u\rangle)}{6\sqrt{2}m_d m_s}\right\} \times\left\{1+\left[\frac{m_s}{843\ \text{MeV}}\right]\left[\ln\left[\frac{m_s^2}{\mu^2}\right]-\frac{2m_d^2}{3\mu^2}\right]\right\}. \tag{3.18}$$

For values of μ between 25 and 200 MeV, we see that the second term in curly brackets in (3.18) runs from 2.0 to approximately 1.0 (we retain our current-mass assignments of $m_s \simeq 200$ MeV, $m_d \simeq 10$ MeV). Moreover, if μ is larger than m_s, then both the numerator and denominator of (3.4) will be dominated by terms proportional to $\ln(\Lambda^2/\mu^2)$, in which case the second factor in curly brackets in (3.18) is effectively replaced by unity.

Our use of the soft-q^2 limit in (3.4), (3.5), (3.11)–(3.13), and (3.15)–(3.17), motivated by soft-pion reductions discussed in the next section, leads to a somewhat larger estimate of b than would be obtained using the kaon-shell $q^2=m_K^2$ limit. For this latter case, Feynman integrals analogous to (3.5) and (3.11)–(3.13) develop real parts corresponding to the breakup of an on-shell kaon into "physical" on-shell s and d quarks. Confinement disallows such contributions; the remaining imaginary parts are dominated by Λ-sensitive logs regulated by the physical kaon mass, rather than by m_s (3.4), m_d (3.11)–(3.13), or μ (3.15)–(3.17). The net effect of evaluating the ratio (3.4) on the kaon mass shell (as opposed to the $q^2=0$ limit) is to equilibrate all such logs appearing in the numerator and denominator of (3.4), thereby eliminating sensitivity of their ratio to the actual value of Λ, and thus yielding the estimate

$$b \simeq \left\{\frac{G_F\cos\theta_C\sin\theta_C(m_c\langle\bar{c}c\rangle-m_u\langle\bar{u}u\rangle)}{6\sqrt{2}m_d m_s}\right\}. \tag{3.19}$$

We thus conclude that the magnitude of the parameter b characterizing the ratio of weak-to-strong kaon axial-vector currents (3.1)–(3.3) is approximately given by (3.19), corresponding to the first term in curly brackets on the right-hand sides of (3.14) and (3.18). We reiterate that (3.19) corresponds to the ratio of coefficients of $\ln(\Lambda)$ in (3.4), and, as such, is *not* contingent on any further estimate of the ultraviolet cutoff (3.7). Moreover, the Feynman integrals characterizing (3.10) are all IR-finite for nonzero m_d; the inclusion of a phenomenological IR cutoff (or the abandonment of the soft-q^2 limit) serves to reduce a multiplicative factor of order 2.5 [i.e., the second term in curly brackets in (3.14)] to unity.

It is particularly worth noting that the factor of m_d in the denominator of (3.19) survives infrared regulation of the integrals in (3.10). This factor may be traced ultimately to the terms proportional to $1/m_d^2$ in the on-shell renormalized self-energy (3.8). Indeed, such $1/m_d^2$ terms are *absent* prior to mass-shell subtractions utilized to ensure the on-shell conditions (2.8), as is evident from examination of the unrenormalized self-energy (2.6). Thus, mass-shell subtractions, which are known to suppress the contribution of the Fig. 1 off-diagonal self-energy to $\Delta S=1$ transitions, serve to *enhance* the $\langle\bar{q}q\rangle$-mediated contribution of the Fig. 2 off-diagonal self-energy.

IV. APPLICATIONS AND DISCUSSION

The scale of $\langle\bar{q}q\rangle$-generated off-diagonal self-energy contributions to kaon-decay matrix elements is somewhat larger than (and certainly of comparable magnitude to) the scale appropriate for experimental kaon decay rates. To see this, we first utilize the lower bound (3.19) to estimate the scale parameter b appearing in (3.3). The lead term in the heavy-quark expansion[9] allows the following estimate of the charmed condensate term (which dominates $m_u\langle\bar{u}u\rangle$ if $m_u \lesssim 10$ MeV):

$$m_c\langle\bar{c}c\rangle \approx -(\alpha_s/12\pi)\langle GG\rangle \approx -0.001\ \text{GeV}^4 \tag{4.1}$$

for the standard-gluon condensate[10] $(\alpha_s/\pi)\langle GG\rangle \approx 0.012$ GeV^4. This estimate may also be regarded as a lower bound, in that recent work has argued for estimates for $\langle GG\rangle$ two to five times larger than that utilized in (4.1).[21]

The fact that the charm-quark term (driven by the gluon condensate) gives the dominant contribution in (3.19) may appear surprising. Nevertheless, the usual intuitive biases (often based on decoupling-theorem considerations) against having a heavy flavor dominate a low-momentum-transfer process have been shown to be wrong in a number of applications. In b-s penguins and in B^0-$\bar{B}^0$ mixing the top quark dominates, although we would naively expect it to be decoupled because of its large mass. In these cases, as in our own calculation, the breaking of the Glashow-Iliopoulos-Maiani (GIM) mechanism by a relatively large disparity in fermion masses provides the driving mechanism for the process in question to proceed. As mentioned earlier, the top-quark term $m_t\langle \bar{t}t \rangle$ ($\approx m_c\langle \bar{c}c \rangle$) is unimportant for our effect because of the smallness of the relevant t-quark mixing angles. Consequently, the $m_c\langle \bar{c}c \rangle$ contribution dominates the process under consideration. If one further utilizes the current-mass values $m_d \approx 10$ MeV, $m_s \approx 200$ MeV, one then finds from (3.19) that[22]

$$b \approx 1.5\times 10^{-7} . \quad (4.2)$$

This parameter can then be utilized to generate a portion of the matrix element $\langle \pi | H_W^{\rm PC} | K \rangle$, as remarked at the beginning of the preceding section. We employ the soft-pion theorem to relative the $\langle \pi H | K \rangle$ matrix element to the divergence of the weak kaon axial-vector current (3.1):[19]

$$q^\mu M_\mu \approx i f_\pi \langle \pi^0 | H_W^{\rm PC} | \bar{K}^0 \rangle \quad (4.3)$$

in which case we find from (3.3) that, for kaon decays ($q^2 = m_K^2$),

$$\langle \pi^0 | H_W^{\rm PC} | \bar{K}^0 \rangle \approx \sqrt{2} b m_K^2 f_K / f_\pi . \quad (4.4)$$

The matrix element in (4.4) is directly tested by the decay of a neutral kaon into two photons. Assuming this decay proceeds through an intermediate neutral pion, we find that

$$\langle \gamma\gamma | H_{\rm W}^{\rm PC} | \bar{K}^0 \rangle = \frac{-\langle \gamma\gamma | \pi^0 \rangle \langle \pi^0 | H_W | \bar{K}^0 \rangle}{m_K^2 - m_\pi^2} = \frac{-\sqrt{2} b m_K^2 f_K}{(m_K^2 - m_\pi^2) f_\pi} \langle \gamma\gamma | \pi^0 \rangle . \quad (4.5)$$

The proportionality between measured $K_L \to \gamma\gamma$ and $\pi^0 \to \gamma\gamma$ amplitudes suggests that

$$|b| \approx (0.52)[(0.23\times 10^{-8}\ {\rm GeV}^{-1})/(0.025\ {\rm GeV}^{-1})] \approx 4.8\times 10^{-8} , \quad (4.6)$$

which is one-third of our estimate (4.2).

The scale factor b can also be related to $K_{2\pi}^0$ decays, an area of obvious interest insofar as the $\langle \bar{q}q \rangle$-mediated s-d self-energy is a purely $\Delta I = \frac{1}{2}$ transition amplitude. If one accounts for the rapidly varying K^0 pole in $K_{2\pi}^0$, then pion PCAC (partial conservation of axial-vector current) and current algebra lead to[4,23,24]

$$|\langle 2\pi | H_W^{\rm PV} | K^0 \rangle| \approx (1/f_\pi) |\langle \pi^0 | H_W^{\rm PC} | K^0 \rangle| (1 - m_\pi^2/m_K^2) , \quad (4.7)$$

for $f_\pi \approx 93$ MeV. Upon further application of (4.4), one obtains

$$|\langle 2\pi | H_W^{\rm PV} | \bar{K}^0 \rangle| \approx \sqrt{2} b f_K (m_K^2 - m_\pi^2)/f_\pi^2 . \quad (4.8)$$

The experimental $\Delta I = \frac{1}{2}$ magnitude of (4.8), 26×10^{-8} GeV,[23] suggests a value for $b \approx 6.0\times 10^{-8}$, a value consistent with (4.6) and less than one-half of our estimate (4.2).

Consequently, quark-condensate mediated self-energy transitions appear to give an overly *large* contribution to the $\Delta I = \frac{1}{2}$ component of nonleptonic kaon decays. We emphasize that the results discussed above are obtained using the same on-shell-renormalization recipe used in Ref. 7 that is seen to *suppress* the contribution of the conventional off-diagonal self-energy of Fig. 1. Such suppression follows from the absence of momentum dependence in the lead terms of the unrenormalized Fig. 1 amplitudes, which are entirely eliminated by the double-subtraction procedure needed to ensure the renormalization conditions (2.8). By contrast, the unrenormalized $\langle \bar{q}q \rangle$-mediated off-diagonal self-energy (2.6) of Fig. 2 has a steep $1/p^2$ momentum dependence, which generates the $O(1/m_{d,s}^2)$ counterterms responsible for the $1/m_d$ dependence in the scale factor b of (3.19). Moreover, this dependence is insensitive to infrared regulation, as has been noted in the previous section.

The $1/p^2$ momentum dependence of the $\langle \bar{q}q \rangle$-mediated off-diagonal self-energy follows from the $1/p^2$ dependence within the χ and W propagators occurring in the Landau gauge [Eqs. (2.5)]. As remarked earlier, our use of the Landau gauge was to avoid problematical $\langle \bar{q}q \rangle$ corrections to the $W\bar{q}q$ vertex, corresponding to alteration of standard electroweak Feynman rules as a result of the SU(2)×U(1) noninvariance of the additional $\langle \bar{q}q \rangle$ order parameter now present in the theory.[11] Landau gauge transversality of the W propagator ensures that the Fig. 2 Feynman amplitudes are insensitive to any Ward-identity driven corrections to the $W\bar{q}q$ vertex:

$$\Delta\Gamma^\mu_{W\bar{q}q}(p, k_{\rm in}, k_{\rm out}) \sim \frac{p^\mu}{p^2} \left[\sum_{\langle \bar{q}q \rangle} (k_{\rm in}) - \sum_{\langle \bar{q}q \rangle} (k_{\rm out}) + \gamma_5 \left[\sum_{\langle \bar{q}q \rangle} (k_{\rm in}) + \sum_{\langle \bar{q}q \rangle} (k_{\rm out}) \right] \right] . \quad (4.9)$$

Such corrections (considered *outside* the Landau gauge) provide "constituent-mass" corrections to $W\bar{q}q$ vertices that are currently under investigation.[25] The non-Landau-gauge contribution of such corrections applied to Fig. 2 processes are comparable in magnitude to the explicit Landau-gauge amplitude we have calculated above. Specifically, the W-exchange diagrams of Fig. 2 evaluated outside the Landau gauge would acquire contributions of order $g^2\langle \bar{q}q \rangle m_q^2/p^2 M_W^2$ from the vertex corrections (4.9), a result comparable in magnitude and $1/p^2$ kinematic structure to the amplitude (2.6) we obtain in a gauge (Landau) for which the vertex corrections (4.9) are irrelevant.

We further note that the s-d self-energy transition amplitude we have considered would not occur if the W and χ fields were integrated out in order to generate an effective electroweak Hamiltonian with local four-fermion operators, as is the case in the calculation of Ref. 13. Of course, such an approach essentially corresponds to letting $\xi M_W^2 - p^2 \to \xi M_W^2$ within covariant-(ξ) gauge χ and W propagator denominators, a simplification that is inappropriate for the Fig. 2 process when using the M_W^2-independent propagator denominators specific to the Landau gauge (2.5).[26] Thus, one cannot integrate out W or its scalar partner in the Landau-gauge version of Fig. 2 (which is sensitive to the nonperturbative content of the

QCD vacuum). Nevertheless we reiterate that our result would be expected to arise from Ward-identity-driven corrections to the vertices in Fig. 2 that occur outside the Landau gauge.

Finally, we note that our overly large estimate for the scale factor b [and matrix elements (4.4) and (4.8)] can be reduced to phenomenological acceptable values through use of a larger magnitude for the current quark-mass values.[27] Indeed, the use of larger quark masses is suggested by earlier work linking the quark condensate to the dynamical component of quark masses.[17]

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