# $CP$ violation in $\eta$, $K_L \to \mu\bar{\mu}$ decays and electric dipole moments of electron and muon

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The $CP$-violating longitudinal-polarization asymmetry $P_L$ of the outgoing muon in $\eta \to \mu\bar{\mu}$ and $K_L \to \mu\bar{\mu}$ decays and the electric dipole moments of an electron and muon ($d_l$, $l = e, \mu$) are studied in various extensions of the standard $CP$-violation model. The possibility of having large $P_L$ in both decays and $d_l$ is explored.

## I. INTRODUCTION

The standard model of electroweak interactions is a very successful theory in that presently no experimental result contradicts its predictions. The origin of $CP$ violation in the model comes from the complex Kobayashi-Maskawa (KM) matrix[1] in the quark sector where only one physical phase exists for three generations of quarks. The $CP$ violation observed in the neutral-kaon system[2] can be easily incorporated within this framework.[3] The origin of this $CP$ nonconservation lies in complex quark matrices which in turn can be traced back to complex Yukawa couplings. Spontaneous symmetry breaking (SSB) is then responsible for this phase in a roundabout way. None the less, it would be "unnatural" if the KM phase is not there. However, many extensions of the standard model could also give rise to $CP$ violation in a different manner.[3] In order to determine the mechanism of $CP$ violation and hence distinguish between different theoretical models, it is important to look for new $CP$-violating effects which are not within the standard model. It will be particularly important if these are within reach of the current round of experiments. Two examples of the most interesting such effects are $CP$ violation in the lepton sector such as the electric dipole moments[4] (EDM's) of charged leptons ($d_l$), and the muon longitudinal-polarization asymmetry[5] ($P_L$) in $\eta \to \mu\bar{\mu}$ and $K_L \to \mu\bar{\mu}$ decays.

In the standard model, there is no $CP$ violation in the lepton sector because of the absence of right-handed neutrinos. One expects $d_l$ to be zero at the two-loop level like the neutron EDM ($d_n$).[6,7] Recently, Hoogeveen[8] has calculated the contributions to the electron EDM ($d_e$) beyond two loops in the standard model and found $d_e$ to be $\leq 10^{-38}$ $e$ cm. The possibility of testing $CP$ violation in $P^0 \to l\bar{l}$ decays was first pointed out by Pais and Treimann[9] and Sehgal[10] where $P^0$ is a pseudoscalar meson. For $K_L \to \mu\bar{\mu}$ decay, the nonzero muon polarization can come from (1) indirect $CP$ nonconservation induced by the mixing[11] of $K^0$, $\bar{K}^0$ states which is characterized by $\epsilon$ and (2) direct $CP$-violating decay amplitude via the standard neutral-Higgs-boson exchange.[12,13] Because of the smallness of $|\epsilon| \simeq 2\times 10^{-3}$ and $K_L \simeq K_2 + \epsilon K_1$, where $K_1$ and $K_2$ are $CP$-even and -odd states, respectively, $|P_L|$ from (1) is expected to be quite small[11] $\sim 7\times 10^{-4}$. The contribution to $|P_L|$ from (2) has been studied recently by Botella and Lim[12] and the present authors.[13] It has been shown that in order to have large muon polarization a relatively light Higgs boson with the mass of order of 1 GeV/$c^2$ is required. However, the recent experimental searches at the CERN $e^+e^-$ collider LEP[14] have ruled out a Higgs-boson mass below 24 GeV/$c^2$. Accepting this limit we find that the muon polarization in the standard model is bounded by

$$|P_L(K_L \to \mu\bar{\mu})| \leq 10^{-3} \tag{1.1}$$

(see Ref. 13). Unlike the neutral-kaon system, the $\eta$ meson is a $CP$ eigenstate and thus the indirect type of $CP$-violating contributions does not exist. Furthermore no direct $CP$-violating contributions to $P_L(\eta \to \mu\bar{\mu})$ via the standard neutral Higgs boson can be induced up to two-loop level. Therefore, $P_L$ in the $\eta \to \mu\bar{\mu}$ decay practically vanishes in the standard model.

We now briefly summarize the experimental situations involving the above $CP$-violating effects. Beginning with the charged-lepton EDM's, the current bound on $d_e$ is given by[15]

$$|d_e| < 1.3\times 10^{-25}\ e\ \text{cm}\ , \tag{1.2}$$

which is extracted from the atomic experiments. An experiment[16] which will improve the bound in (1.2) by several orders of magnitude is ongoing. For the EDM of muon, the $(g_\mu - 2)$ experiment in CERN gives a bound[17]

$$|d_\mu| < 7.3\times 10^{-19}\ e\ \text{cm}\ . \tag{1.3}$$

This bound will be improved by a factor of 20 in a future BNL experiment.[18]

There have been two new measurements[19] of $K_L \to \mu\bar{\mu}$ decay at KEK and BNL giving the branching ratios $(8.4\pm1.1)\times 10^{-9}$ and $(5.8\pm1)\times 10^{-9}$, respectively. Both results are lower than the previous value[20] of $(9.5^{+2.4}_{-1.5})\times 10^{-9}$. Taking an average of all these measurements we obtain[21]

$$B(K_L \to \mu\bar{\mu}) \equiv \frac{\Gamma(K_L \to \mu\bar{\mu})}{\Gamma(K_L \to \text{all})} = (7.34^{+0.71}_{-0.66})\times 10^{-9}\ , \tag{1.4}$$

which is close to the unitarity bound of

$$B(K_L \to \mu\mu)_{2\gamma} = (6.83\pm0.28)\times 10^{9} \tag{1.5}$$

 

arising from the two-photon intermediate-state contribution calculated from the measured branching ratio[20] of $B(K_L \to \gamma\gamma) = (5.70 \pm 0.23) \times 10^{-4}$. Using the data in (1.4) and (1.5), one finds[22] that the experimental limit of $P_L$ in $K_L \to \mu\bar{\mu}$ decay is[23]

$$|P_L(K_L \to \mu\bar{\mu})| \leq 0.50 \; . \tag{1.6}$$

The experimental value for $\eta \to \mu\bar{\mu}$ decay is[20]

$$B(\eta \to \mu\bar{\mu}) = (6.5 \pm 2.1) \times 10^{-6} \; , \tag{1.7}$$

which is close to the unitarity bound $B(\eta \to \mu\bar{\mu})_{2\gamma} \geq 4.3 \times 10^{-6}$. This gives a limit of $P_L(\eta \to \mu\bar{\mu}) \lesssim 1$ which is not very useful. However, a strong constraint[24] of $P_L(\eta \to \mu\bar{\mu}) \lesssim 0.1$ could come from the current limit on the EDM of the neutron[25]

$$|d_n| < 1.2 \times 10^{-25} \; e\,\text{cm} \tag{1.8}$$

albeit with some additional theoretical assumptions. This will be discussed in more detail in Sec. III. Future measurements of muon polarizations in both $K_L \to \mu\bar{\mu}$ and $\eta \to \mu\bar{\mu}$ decays will be quite interesting. At KEK, there are plans[26] to measure $P_L$ in $K_L \to \mu\bar{\mu}$ decay with an accuracy of about 20% which is larger than the standard-model prediction given in (1.1). An experiment[27] with $\eta$ flux several orders of magnitude higher than previously available experiments, which can in principle measure $P_L(\eta \to \mu\bar{\mu})$ to $10^{-2}$ or better, is underway at Saclay. These recent developments have motivated us to study systematically the possible signature of the *CP*-violating effects in various existing extensions of the standard *CP*-violation model and examine how they are related to each other.

Recently, we have constructed extended Higgs-boson models[5,28,29] with *CP* violation arising from the scalar-pseudoscalar mixing mechanism. We showed that this *CP*-violating source would lead to sizable muon polarizations in $\eta, K_L \to \mu\bar{\mu}$ decays and charged-lepton EDM's. In this report, we will examine these leptonic *CP*-violating effects in this class of multi-Higgs-boson models as well as other *CP*-violation theories beyond the standard model, emphasizing the connections between the lepton EDM's and the muon polarization effects. Especially, we will explore the possibility of having large $P_L$ in both decays and $d_l$ $(l = e, \mu)$.

Our motivation is to investigate *CP* violation beyond the standard model, so we assume that a nonvanishing KM phase exists and one can account for *CP* violation in $K \to \pi\pi$ decay via this phase and a heavy $t$ quark through the Glashow-Iliopoulos-Maiani mechanism. The physics that we are trying to probe is additional to this source of *CP* violation.

The paper is organized as follows. In Sec. II we briefly review various *CP*-violation models which involve leptons. We then study the muon polarization asymmetry in $\eta, K_L \to \mu\bar{\mu}$ decays and the EDM of an electron and muon in Sec. III. Our conclusions are summarized in Sec. IV.

## II. CP-VIOLATION MODELS INVOLVING LEPTONS

Although there is no experimental evidence so far to indicate that *CP* is not conserved in the lepton sector, it is widely speculated that the leptonic *CP* violation would exist if there is new physics beyond the standard model. Models with such *CP* violation have been constructed by introducing more fermions and/or scalar bosons, such as the multi-Higgs-boson, leptoquark, supersymmetry (SUSY), etc., models, or by enlarging the gauge group of the standard model such as the left-right-symmetric models, the horizontal-symmetry models, etc. In this section, we will review four classes of models which are relevant to our discussions on the EDM's and $P_L$. Since many models can be constructed within each class, we shall focus on the simplest one in each category. The emphasis is to bring out the physics involved without being overwhelmed by details of the models.

### A. Multi-Higgs-boson models

In the standard model with three generations, observable *CP*-violating phenomena come from the *W*-boson–fermion coupling because of the existence of a physical phase in the KM matrix. Another source of *CP* violation can occur when *CP* symmetry is violated spontaneously with multi-Higgs bosons, which was first pointed out by Lee.[30] In purely spontaneous *CP* violation (SCPV) models in which the Yukawa couplings are real, *CP* is assumed to be good prior to symmetry breakdown and *CP* violation is due to different relative phases of the vacuum expectation values (VEV's) of Higgs fields. It was shown that in the context of the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ model, two Higgs doublet are the minimal number required for SCPV[30] to take place. This minimal two-Higgs-doublets model has flavor-changing neutral currents (FCNC) due to neutral-Higgs-boson exchange, which result in *CP*-violating $\Delta S = 2$ superweak interaction at the tree level through scalar-pseudoscalar ($R$-$I$) mixing to the two doublets,[30,31] where $R$ and $I$ represent the real and imaginary parts of the neutral scalars in the weak-eigenstate basis. The branching ratio of $K_L \to \mu\bar{\mu}$ dictates that the spin-0 boson mediating the FCNC must be heavier than several TeV. It is doubtful that such a heavy particle makes theoretical sense in the theory. An alternative will be to suppress FCNC's by symmetry consideration. To achieve this naturally, one imposes the principle of natural flavor conservation[32] (NFC) in the Higgs sector. Unfortunately, NFC will automatically keep *CP* invariance after SSB because $R$-$I$ mixing has been eliminated. With Higgs-doublet fields alone, it has been shown that the minimal model with SCPV and NFC is the Weinberg three-Higgs-doublets model[33,34] in which flavor-changing neutral-Higgs-boson couplings are forbidden by a $Z_2 \times Z_2'$ discrete symmetry. With the additional doublets *CP* violation can then come from charged-Higgs-boson exchanges[33] and/or neutral scalars of $R$-$I$ mixings.[35] If $SU(3)_C \times SU(2)_L \times U(1)_Y$ Higgs-singlet fields are introduced, we have shown recently that the two Higgs doublets ($\phi_i$, $i = 1,2$) and one Higgs singlet ($\chi$) with a discrete symmetry[28] or two $\chi$'s with a Peccei-Quinn[36] (PQ) U(1) global symmetry can achieve SCPV and NFC simultaneously. This model has the virtue of reducing the number of Higgs fields. Furthermore, *CP* nonconservation is purely due to the *R-I* mixings between the components of the Higgs-doublet and Higgs-singlet fields.

In order to isolate *CP* violation arising from neutral-Higgs-boson exchanges, i.e., *R-I* mixings, it is sufficient to study the simplest multi-Higgs-boson model. This is the one that contains two $\phi$'s and one $\chi$. We will assume that the Yukawa couplings are complex and as a result the dominant observed *CP* violation in the kaon system is given by the phase in the KM matrix. In this model, there are a pair of physical charged Higgs bosons ($H^{\pm}$) and five physical neutral spin-0 fields $H_k$ ($k=1,2,\ldots,5$) after spontaneous symmetry breaking. The $H^{\pm}$ fields carry the same KM phase as that of the *W* bosons in their couplings to the fermions[37] and play a negligible role in the kaon system.[38] The neutral scalar fields $H_k$ will mix and the *R-I* mixing is the only new *CP*-violation source in this multi-Higgs-boson model. The weak eigenstates $R_i$ and $I_j$ and the mass eigenstates $H_k$ are related by a real orthogonal transformation. This transformation mixes different real and imaginary components of the Higgs-doublet and -singlet fields, and their coupling to fermions will then contain both scalar (1) and pseudoscalar ($i\gamma_5$) terms. The Lagrangian density of the Yukawa terms involving $R_i, I_j$ fields has the expression

$$\mathcal{L}_Y = \frac{R_1}{v_1}\bar{u}M_u u - \frac{I_1}{v_1}\bar{u}M_u i\gamma_5 u + \frac{R_2}{v_2}(\bar{d}M_d d + \bar{e}M_e e) + \frac{I_2}{v_2}(\bar{d}M_d i\gamma_5 d + \bar{e}M_e i\gamma_5 e) \tag{2.1}$$

which, in terms of the mass eigenstates $H_k$, can be rewritten as

$$\mathcal{L}_Y = (2\sqrt{2}G_F)^{1/2}\sum_k [\alpha_1^k \bar{u}M_u u + \beta_1^k \bar{u}M_u i\gamma_5 u + \alpha_2^k(\bar{d}M_d d + \bar{e}M_e e) + \beta_2^k(\bar{d}M_d i\gamma_5 d + \bar{e}M_e i\gamma_5 e)]H_k \ , \tag{2.2}$$

where $M_d$, $M_u$, $M_e$ are the fermion mass matrices for *d*- and *u*-type quarks and charged leptons, respectively, and $v_i$ ($i=1,2$) are the VEV's of the Higgs doublets $\phi_i$. The mixing parameters $\alpha_i^k$ and $\beta_i^k$ depend on the strength of *R-I* mixing and ratio of VEV's. We assume that all these free parameters are of the same order of magnitude.

### B. Leptoquark

We concentrate on a class of leptoquark models proposed by Hall and Randall.[39] The models have been studied by Barr and Masiero[40] (BM) and reexamined by one of us[41] (C.Q.G.) recently. The quantum numbers of the scalar leptoquark $\phi$ are $(3,2,\frac{7}{3})$ under the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ group. The general $\phi$-fermion-fermion couplings are given by

$$L = \sum_{i,j} (\lambda_{ij}\bar{u}^i_R e^j_L + \lambda'_{ij}\bar{u}^i_L e^j_R)\phi \ , \tag{2.3}$$

where $e^i_L(u^i_L)$ and $e^i_R(u^i_R)$ are left- and right-handed charged leptons (up-type quarks), respectively, and $i,j$ are family indices. *CP* violation in these models comes from the complex coupling constants $\lambda_{ij}$ and $\lambda'_{ij}$. The couplings in (2.3) do not involve the down-type quarks and thus it has no impact on neutral-kaon decays at the tree level. It has been pointed out by BM[40] that the experimental limit on $\mu \to e\gamma$ decay could give the strongest bound on $d_e$. If we take the couplings as $|\lambda_{ij}| \sim |\lambda'_{ij}| \sim (m_{e^i} m_{u^j})^{1/2}/M_\phi$, we find[41]

$$M_\phi > 300 \text{ GeV}/c^2 \tag{2.4}$$

from the current limit[20] of

$$B(\mu \to e\gamma) < 5 \times 10^{-11} \tag{2.5}$$

on the $\mu \to e\gamma$ decay.

### C. SUSY models

*CP* violation in SUSY theory has been studied as a test of effects beyond the standard model.[42,43] There are many new *CP*-violating sources in addition to the standard KM phase. For example, in an $N=1$ supergravity model, in which the local SUSY is broken by a "hidden sector," the *CP*-violating phases can arise from the gaugino masses, the $\mu HH'$ terms in the superpotential, and soft SUSY-breaking terms. Although the new phases may not be all independent, there are many free parameters which are model dependent. As an example, we will concentrate our discussion on a special model inspired by superstring theory.[44] The model is based on $E_8 \otimes E_8'$ heterotic string in ten dimensions leading, upon compactification, to an observable four-dimensional $E_6$ grand unified theory coupled to $N=1$ supergravity.[44] In order to establish our notation, we give the SO(10) content of the $E_6$ fundamental representation

$$\begin{aligned} \mathbf{16} &= \left[ Q \equiv \begin{bmatrix} u \\ d \end{bmatrix}, u^c, e^c, L \equiv \begin{bmatrix} \nu \\ e \end{bmatrix}, d^c, \nu^c \right] , \\ \mathbf{10} &= \left[ D, H \equiv \begin{bmatrix} H^+ \\ H^0 \end{bmatrix}, \bar{D}, \bar{H} \equiv \begin{bmatrix} \bar{H}^+ \\ \bar{H}^0 \end{bmatrix} \right] , \\ \mathbf{1} &= N \ . \end{aligned} \tag{2.6}$$

The matter fields transforms as the fundamental representation of $E_6$ and the most general cubic superpotential arising from the coupling of three 27-plets of $E_6$ can be written as

$$\begin{aligned} \mathcal{L} &= \sum_{i=1}^{4} \mathcal{L}_i \ , \\ \mathcal{L}_1 &= \lambda^1 HQu^c + \lambda^2 Q\, d^c\bar{H} + \lambda^3 L\bar{H}e^c + \lambda^4 H\bar{H}N + \lambda^5 DD^cN \ , \\ \mathcal{L}_2 &= \lambda^6 De^cu^c + \lambda^7 D^cLQ + \lambda^8 D\, d^c\nu^c \ , \\ \mathcal{L}_3 &= \lambda^9 DQQ + \lambda^{10} D^cu^cd^c, \quad \mathcal{L}_4 = \lambda^{11} HL\nu^c \ , \end{aligned} \tag{2.7}$$

where the Yukawa couplings $\lambda^k$ are tensors in generation space. To avoid rapid proton decay and have naturally small neutrino masses generated through radiative corrections, one introduces a $Z_2 \times Z_3$ discrete symmetry[45] to forbid $\mathcal{L}_3$ and $\mathcal{L}_4$ terms in (2.7).

### D. Left-right-symmetric models

Left-right-symmetric models[46,47] are based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with the quantum numbers of quarks and leptons assigned as follows:

$$q_L = \begin{bmatrix} u \\ d \end{bmatrix}_L = (2,1,\tfrac{1}{3}), \quad q_R = \begin{bmatrix} u \\ d \end{bmatrix}_R = (1,2,\tfrac{1}{3}),$$
$$l_L = \begin{bmatrix} \nu \\ e \end{bmatrix}_L = (2,1,-1), \tag{2.8}$$
$$l_R = \begin{bmatrix} \nu \\ e \end{bmatrix}_R = (1,2,-1).$$

$CP$ violation in left-right models has been studied extensively in the literature.[48] In contrast to the standard model, physical $CP$-violating phases can be introduced even for two generations of quarks because of the existence of right-handed currents. A two-generation version of such models can be regarded as an effective model in which $CP$ violation from the third generation is negligible. The minimal left-right model contains one Higgs field $\Phi$ which transforms as a (2,2,0) under the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. To achieve the correct symmetry-breaking pattern, other representations of Higgs fields such as triplets $\Delta_{L,R}$ are also required. These field usually acquire large masses when $SU(2)_R$ is broken and as a result their contributions to $CP$-violation effects are negligible. Also, one can show that $CP$ violations associated with the $\Phi(2,2,0)$ field are very small.

The charged-current interaction of quarks is given by

$$\mathcal{L}_{CC}^{q} = -\sum_{ij} \left[ \frac{g_L}{\sqrt{2}} W_L^{\mu} \bar{u}_L^{\,i} \gamma_\mu U_L^{ij} d_L^j + \frac{g_R}{\sqrt{2}} W_R^{\mu} \bar{u}_R^{\,i} \gamma_\mu U_R^{ij} d_R^j \right] + \text{H.c.}, \tag{2.9}$$

where

$$U_L = \begin{bmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{bmatrix},$$
$$U_R = e^{i\gamma} \begin{bmatrix} e^{-i\delta_2}\cos\theta_C & e^{-i\delta_1}\sin\theta_C \\ -e^{i\delta_1}\sin\theta_C & e^{i\delta_2}\cos\theta_C \end{bmatrix}, \tag{2.10}$$

with $\theta_C$ being the Cabibbo angle. We recall that the left-right mixing of the model is given by

$$\xi = \eta \left[ \frac{M_{W_L}}{M_{W_R}} \right]^2 \le \left[ \frac{M_{W_L}}{M_{W_R}} \right]^2 \le 2 \times 10^{-3}, \tag{2.11}$$

where $\eta = 2kk'/(k^2 + k'^2)$ and $kk'$ are the VEV's of $\Phi$. The bound on $\xi$ in (2.11) comes from the constraint imposed by $\Delta M_K$. For the lepton sector, the charged current can be written as

$$\mathcal{L}_{CC}^{l} = -\sum_{i} \left[ \frac{g_L}{\sqrt{2}} W_L^{\mu} \bar{e}_L^{\,i} \gamma_\mu \nu_L^i + \frac{g_R}{\sqrt{2}} W_R^{\mu} \bar{e}_R^{\,i} \gamma_\mu \nu_R^i \right] + \text{H.c.} \tag{2.12}$$

where, for simplicity, the generation mixing of leptons are assumed to be zero. For each generation, there is a $CP$-violation phase because of the Majorana neutrino mass matrix

$$M_\nu = \begin{bmatrix} m & D \\ D & M \end{bmatrix}, \tag{2.13}$$

where the mass $m$ ($M$) and $D$ are the left- (right)- handed Majorana and the Dirac terms, respectively, and $m \ll D \ll M$. Taking $m=0$, one can diagonalize $M_\nu$ by the unitary matrix

$$U = \begin{bmatrix} e^{-i\alpha'}\cos\theta & -\sin\theta \\ e^{-\alpha'}\sin\theta & \cos\theta \end{bmatrix}, \tag{2.14}$$

for each generation with $\tan 2\theta = 2D/M \ll 1$ and the light-neutrino mass is

$$m_{\nu_1} \simeq \frac{D^2}{M} \tag{2.15}$$

as given by the "seesaw" mechanism. The phase convention reveals that all $CP$-violating processes in this model are associated with a right-handed gauge-boson $W_R$ exchange. Thus, in the limit $M_{W_R} \to \infty$ the model is $CP$ conserving as in the case of a two generation $SU(2)_L \times U(1)_Y$ model. However, it has been shown that the EDM's of leptons[49,50] and muon polarization[51,52] in $K_L \to \mu\bar{\mu}$ are proportional to the mass term $D$ which cannot be large in the simplest version of the models. This is because of the experimental constraints on $m_{\nu_1}$ and $M$ ($\sim M_{W_R}$) in (2.15). Especially, $d_l$ and $P_L$ vanish in the limit $m_{\nu_1} \to 0$. Recently, Frère and Liu[53] (FL) have proposed an extension of the minimal left-right model by introducing one more neutrino $S_L$ which is a singlet under the gauge group in each family with the neutrino mass matrix as

$$M_\nu = \begin{bmatrix} 0 & D & 0 \\ D & 0 & M \\ 0 & M & m' \end{bmatrix} \tag{2.16}$$

in the basis of $(\nu_L, \nu_R^c, S_L)$. The light neutrino is then given by

$$m_{\nu_1} \simeq \frac{m' D^2}{M^2} \tag{2.17}$$

for $m'$, $D \ll M$. Thus the constraint on $D$ is released when one chooses a small $m'$.

## III. MUON POLARIZATION ASYMMETRY AND EDM'S

In $P^0 \to \mu\bar{\mu}$ decays, where $P^0$ denotes either the meson $\eta$ or $K_L$, the muon polarization asymmetry is defined by

$$P_L = \left[ \frac{N_R - N_L}{N_R + N_L} \right] , \tag{3.1}$$

where $N_L$ and $N_R$ are the numbers of left-handed and right-handed outgoing muons, respectively. A nonvanishing $P_L$ will be a clear indication of $CP$ violation. The most general matrix element of the decays is given by

$$M = a\bar{u}i\gamma_5 v + b\bar{u}v , \tag{3.2}$$

where $u$ and $v$ denote spinor and antispinors for the outgoing muons. The width $\Gamma$ of the decays is calculated to be

$$\Gamma = \frac{M_{P^0} r}{8\pi}(|a|^2 + r^2|b|^2) , \tag{3.3}$$

where $r^2 = (1 - 4m_\mu^2/M_{P^0}^2)$. In terms of $a$ and $b$ polarization asymmetry $P_L$ is given by

$$P_L = \frac{2r\,\mathrm{Im}(ba^*)}{|a|^2 + r^2|b|^2} = \frac{M_{P^0} r^2 \mathrm{Im}(ba^*)}{4\pi\Gamma} . \tag{3.4}$$

As is well known, $\Gamma$ is dominated by the two-photon intermediate state via the chain $P^0 \to \gamma\gamma \to \mu\bar{\mu}$. This two-photon amplitude is the main contribution to the imaginary part of $a$. If we write

$$a = a_{\gamma\gamma} + a_n \tag{3.5a}$$

and

$$b = b_{\gamma\gamma} + b_n , \tag{3.5b}$$

where the subscripts $\gamma\gamma$ and $n$ denote the two-photon and nonelectromagnetic contributions, respectively, one can argue that[54]

$$|\mathrm{Im} b_{\gamma\gamma}| \ll |\mathrm{Im} a_{\gamma\gamma}| . \tag{3.6}$$

Thus, Eq. (3.4) can be rewritten as

$$P_L \simeq -\frac{M_{P^0} r^2}{4\pi\Gamma} b_n \mathrm{Im} a_{\gamma\gamma} . \tag{3.7}$$

The most general effective Lagrangian contributing to $P^0 \to \mu\bar{\mu}$ is

$$\mathcal{L}^{\mathrm{eff}} = \sum_{i \le j} \frac{G_F}{\sqrt{2}} (g_{AA}^{ij} J_{A\mu}^{ij} \bar{\mu}\gamma^\mu\gamma_5\mu + g_{PP}^{ij} J_P^{ij} \bar{\mu}\gamma_5\mu + g_{SP}^{ij} J_P^{ij} \bar{\mu}\mu) + \mathrm{H.c.} , \tag{3.8}$$

where $i,j = 1,2$ and

$$\begin{aligned} J_{A\mu}^{11} &= \tfrac{1}{2}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d) , \\ J_P^{11} &= \tfrac{1}{2}(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d) , \\ J_{A\mu}^{22} &= \bar{s}\gamma_\mu\gamma_5 s, \quad J_P^{22} = \bar{s}i\gamma_5 s , \\ J_{A\mu}^{21} &= \bar{s}\gamma_\mu\gamma_5 d, \quad J_P^{21} = \bar{s}i\gamma_5 d . \end{aligned} \tag{3.9}$$

Following the discussions in Refs. 11 and 24 and using the experimental values of $\Gamma(\eta, K_L \to \mu\bar{\mu})$ and $\Gamma(\eta, K_L \to 2\gamma)$, from Eqs. (3.7)–(3.9) we find that

$$|P_L(\eta \to \mu\bar{\mu})| \le 2.34 |\mathrm{Re} g_{SP}^{11} - 4.3 \times 10^{-2} \mathrm{Re} g_{SP}^{22}| \tag{3.10}$$

and

$$|P_L(K_L \to \mu\bar{\mu}| \simeq 2.36 \times 10^6 |\mathrm{Re} g_{SP}^{21}| , \tag{3.11}$$

respectively. The ratio of (3.10) and (3.11) is

$$R \equiv \frac{|P_L(\eta \to \mu\bar{\mu})|}{|P_L(K_L \to \mu\bar{\mu})|} \simeq 10^{-6} \left| \frac{\mathrm{Re} g_{SP}^{11} - 4.3 \times 10^{-2} \mathrm{Re} g_{SP}^{22}}{\mathrm{Re} g_{SP}^{21}} \right| . \tag{3.12}$$

In general $P_L(\eta \to \mu\bar{\mu})$ and $P_L(K_L \to \mu\bar{\mu})$ measure different $g_{SP}$ that are associated with different quark currents.

Next we discuss the EDM of a fermion. A nonvanishing EDM signals a $P$- and $T$-violating interaction

$$if_D(k^2)\bar{\psi}\left[p + \frac{k}{2}\right]\sigma_{\mu\nu}k_\nu\gamma_5\psi\left[p - \frac{k}{2}\right]A_\mu(k) , \tag{3.13}$$

where $A_\mu$ is the electromagnetic field potential. The magnitude of the EDM is

$$d_f = f_D(0) . \tag{3.14}$$

In gauge field theory, the EDM also implies $CP$ violation by virtue of the $CPT$ theorem. In a given model of $CP$ violation one can calculate $P_L$ and $f_D$ in terms of the parameters of the model. In most cases the experimental limit achieved on neutron EDM's gives a stringent bound on the $CP$-violating parameters of a model. We can use this to set upper bounds on $P_L$ as well as EDM's of leptons. Now we present the analysis for the models listed in Sec. II.

### A. Multi-Higgs-boson models

In a recent Letter,[5] we have emphasized some interesting physical implications of the neutral-Higgs-scalar exchanges in the multi-Higgs-boson models by assuming that spin-0 boson exchanges have little contribution to $K_L \to 2\pi$. For example, if the charged Higgs bosons are heavier than $M_W$, then their contributions to $\epsilon$ and $\epsilon'$ will be negligible. The standard-model results for the above $CP$-violation parameters will hold. Furthermore, the $CP$ violation from neutral-Higgs-boson exchanges takes place at the two-loop level[28,29] and thus $K_L \to 2\pi$ decays will not be a good probe of this $CP$-violation mechanism. On the other hand, a stringent constraint on the parameters of the model comes from the EDM of the neutron $d_n$, which would be induced from the following three dominant contributions:[55] (a) the quark electric dipole moments arising from the one-loop graph shown in Fig. 1; (b) three- and four-gluon operators[56,57] generated from the neutral-Higgs-boson exchange, i.e., $R$-$I$ mixing in Figs. 2(a) and 2(b), respectively; (c) the quark electric dipole moments coming from the typical two-loop diagram[58] of Fig. 3.

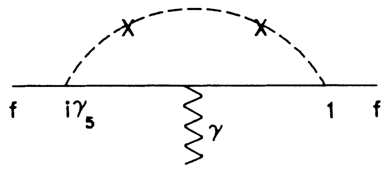


FIG. 1. Feynman diagram for the fermion EDM due to scalar-pseudoscalar mixing at the one-loop level.

In Ref. 5, we have concentrated only on contribution (a). It has been argued by Anselm, Bunakov, Godkov, and Uraltsev[57] that for the Weinberg three-Higgs-doublets model,[33] in which $d_n$ arises also from the charged Higgs scalars, $d_n$ from the four-gluon operator

$$O_1 = C_1 G_{\alpha\mu\nu} G^{\mu\nu}_{\alpha} G_{\beta\rho\sigma} G_{\beta\kappa\lambda} \epsilon^{\rho\sigma\kappa\lambda} \tag{3.15}$$

generated from Fig. 2(b) is estimated to be $10^{-22}$ $e$ cm by assuming the $R$-$I$ mixing parameters are the same order of magnitude as the charged one. However, the prediction depends on the Higgs-pseudoscalar coupling with

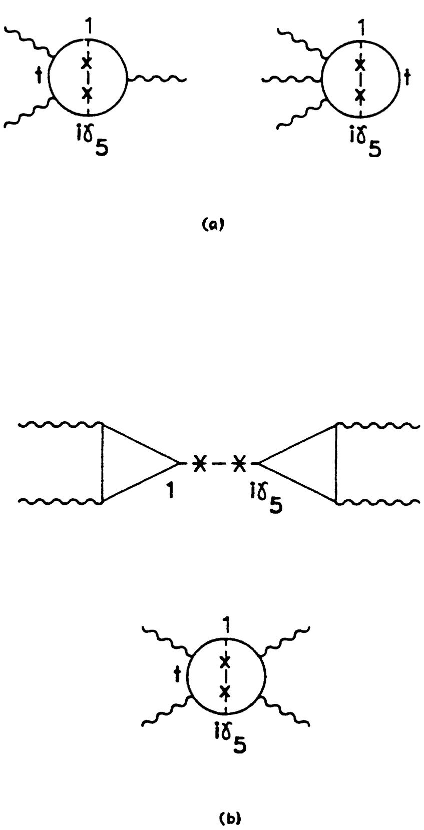


FIG. 2. Graphs contributing to (a) the three-gluon operator and (b) the four-gluon operator due to scalar-pseudoscalar mixing where wavy lines represent gluons.

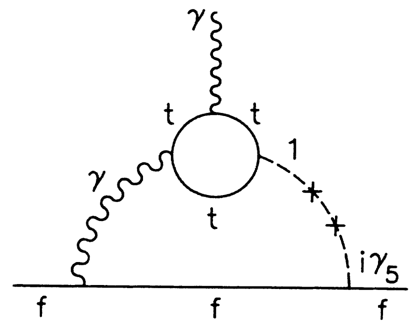


FIG. 3. A representative diagram for the fermion EDM due to scalar-pseudoscalar mixing at the two-loop level.

the neutron which is proportional to[59]

$$\langle n | -\mathrm{Tr}(\epsilon^{\mu\nu\lambda\rho} G_{\mu\nu} G_{\lambda\rho} | n \rangle \tag{3.16}$$

by considering the recent European Muon Collaboration effect.[60,61] The value of (3.16) can be consistent with zero[62] although it is hard to do an exact calculation of this quantity. Thus, the four-gluon effect may be vanishing small.[59] Recently, Weinberg[56] has shown that a large contribution to $d_n$ can arise from the three-gluon operator

$$O_2 = -\tfrac{1}{6} C_2 f_{abc} G^{\rho}_{a\mu} G_{b\rho\sigma} G_{c\lambda\tau} \epsilon^{\mu\sigma\lambda\tau} \tag{3.17}$$

generated from Fig. 2(a), where $f_{abc}$ is the totally antisymmetric SU(3) Gell-Mann structure constant. Again, the contribution from $O_2$ is uncertain to the extent that we cannot give a precise calculation of $\langle n | O_2 | n \rangle$. We can only use naive dimensional analysis (NDA) as a guide. The factor $C_2$ in (3.17) can be calculated in QCD and the renormalization-group technique. In a recent report, Barr and Zee[58] (BZ) have pointed out that a class of two-loop graphs may also give a significant contribution to $d_n$. These graphs have been neglected previously. Unlike Weinberg's three-gluon operator, BZ's graphs could also give a large contribution to $d_e$. In the following, we first obtain constraints on $R$-$I$ mixing based on the contributions to $d_n$ from (a) the dimension-5 quark operator, (b) the three-gluon operator $O_2$, and (c) the typical two-loop graph involving a top-quark loop shown in Fig. 3 and then estimate the $CP$-violating effects.

The contribution to $d_f$ ($f = q, l$) coming from $R$-$I$ mixing at one loop depicted in Fig. 1 is given by[5]

$$d_f = Q_i \frac{e X_i m_f^3}{4\pi^2 M_0^2 v^2} I(m_f / M_0) \, , \tag{3.18}$$

where $Q_f$ is the fermion charge and $v = (\sqrt{2} G_F)^{-1/2} \sim (2v_1^2)^{1/2} \sim (2v_2^2)^{1/2}$. The parameter $X_i$ is the product of $\alpha_i^k$ and $\beta_i^k$ for the lightest $H_k$, denoted by $H_0$, whose mass is $M_0$, and

$$I(Y)=\left[\frac{1}{Y^2}+\frac{1}{2Y^4}\ln Y^2+\frac{1-2Y^2}{2Y^4(1-4Y^2)^{1/2}}\times\ln\frac{2Y^2}{1-2Y^2-(1-4Y^2)^{1/2}}\right] . \quad (3.19)$$

The neutron-EDM contribution from $d_q$ in Eq. (3.17) is denoted by $d_n^a$ and is explicitly given by

$$d_n^a=\tfrac{4}{3}d_d-\tfrac{1}{3}d_u\sim-(10^{-21}\ \mathrm{GeV}^4)\left[\frac{X}{M_0^2v^2}\right] e\ \mathrm{cm} , \quad (3.20)$$

where we have used $m_u \sim 4.2$ MeV, $m_d \sim 7.5$ MeV, and the assumption that all the $X_i$'s are of the same order, i.e., $X \sim X_i$ and $M_0$ is given in units of GeV. Applying the current limit on $d_n$ in (1.8), we find

$$\frac{X}{M_0^2v^2}<1.2\times10^{-4}\mathrm{GeV}^{-4} . \quad (3.21)$$

For the three-gluon operator effect, the value of $C_2$ in (3 17) calculated from the graphs in Fig. 2(a) has the form[56,63]

$$C_2=\frac{\zeta X}{4\pi v^2}h(m_t^2/M_0^2) , \quad (3.22)$$

where

$$\zeta=[g_s(\mu)/g_s(\lambda)]^{-108/23}[g_s(\mu)/4\pi]^3\simeq9.2\times10^{-5}$$

and $h(Y)\sim\frac{1}{8}$ and $h(Y)\sim-\frac{1}{2}Y\ln Y$ for $Y \gg 1$ and $Y \ll 1$, respectively. The EDM of the neutron induced by $O_2$ denoted by $d_n^b$ can be expressed as

$$d_n^b\simeq\frac{e}{4\pi}MC_2=\frac{e\zeta MX}{16\pi^2v^2}h(m_t^2/M_0^2) \quad (3.23)$$

with the use of NDA, where $M=2\pi F_\pi\simeq1190$ MeV is the chiral-symmetry-breaking scale. Assuming $m_t \gg M_0$ and using Eqs. (1.8) and (3.23) and the value of $\zeta$ we find that

$$\frac{X}{M_0^2v^2}<7\times10^{-5}\ \mathrm{GeV}^{-4} \quad (3.24)$$

for $M_0 \sim 1$ GeV. It is clear that these limits should be taken as a guide only. There are too many theoretical uncertainties involving QCD as well as hadronic matrix elements to warrant taking them seriously. We now consider the two-loop graph in Fig. 3. For $m_t \gg M_0$, the EDM of the neutron arising from this two-loop diagram is given by[58]

$$d_n^c\simeq\tfrac{4}{3}d_d\sim3\times10^{-22}\left[\ln\frac{m_t^2}{M_0^2}+2\right]\left[\frac{X}{v^2}\right]\ \mathrm{GeV}^2\ e\ \mathrm{cm} \quad (3.25)$$

which leads to

$$\frac{X}{M_0^2v^2}<4\times10^{-5}\ \mathrm{GeV}^{-4} \quad (3.26)$$

for $M_0 \sim 1$ GeV. The bounds in Eqs. (3.21), (3.24), and (3.26) are of the same order of magnitude. Henceforth, we shall use the stronger bound given by (3.26) for estimating other $CP$-violating effects.

With the limit on $CP$-violation parameters established we can calculate the upper bound on the lepton EDM's. The contributions to $d_l$ from Figs. 1 and 3 give[5]

$$d_l^a=\frac{eXm_l^3}{4\pi^2M_0^2v^2}I(m_l/M_0) , \quad (3.27)$$

where $I(Y)$ is defined by Eq. (3.19) and[58]

$$d_l^c\simeq\frac{5eXm_l\alpha}{72\pi^3v^2}\left[\ln\frac{m_t^2}{M_0^2}+2\right] , \quad (3.28)$$

respectively. Thus we obtain

$$d_e=d_e^a+d_e^c\simeq d_e^c<7\times10^{-26}\ e\ \mathrm{cm} , \quad (3.29a)$$

and

$$d_\mu=d_\mu^a+d_\mu^c<10^{-22}\ e\ \mathrm{cm} . \quad (3.29b)$$

It is interesting to note that the ratio of the EDM of the electron and muon in Eq. (29), i.e., $d_e/d_\mu\simeq7\times10^{-4}$, is much larger than the value of $d_e^a/d_\mu^a\simeq(m_e/m_\mu)^3\sim10^{-7}$ predicted[5] by the one-loop graph in Fig. 1.

We now study the muon polarization in $\eta, K_L \to \mu\bar{\mu}$ decays. The contributions to $P_L(\eta\to\mu\bar{\mu})$ and $P_L(K_L\to\mu\bar{\mu})$ due to the neutral-scalar exchanges arise from the tree-level and one-loop graphs in Fig. 4, respectively. We estimate that

$$g_{SP}^{11}\simeq\frac{4m_dm_\mu X}{M_0^2} \quad (3.30a)$$

and

$$g_{SP}^{22}\simeq\frac{4m_sm_\mu X}{M_0^2} \quad (3.30b)$$

from Fig. 4(a) and

$$g_{SP}^{21}\simeq\frac{Xm_sm_\mu m_c^2}{4\pi^2M_0^2v^2}f(m_c^2/M_W^2)\sin\theta_C \quad (3.30c)$$

from Fig. 4(b), where $f(m_c^2/M_W^2)\sim1$ and we have neglected the contribution from the KM phase/ From

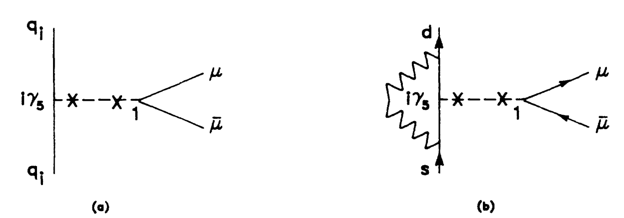


FIG. 4. Contribution to $P_L$ in (a) $\eta\to\mu\bar{\mu}$ and (b) $K_L\to\mu\bar{\mu}$ decays due to scalar-pseudoscalar mixing.

Eqs. (3.12) and (3.30), we find

$$R \simeq 0.2 . \quad (3.31)$$

With the bound in (3.26), from Eqs. (3.10), (3.11), and (3.30) we get

$$|P_L(\eta \to \mu\bar{\mu})| < 0.3 \times 10^{-2} \quad (3.32a)$$

and

$$|P_L(K_L \to \mu\bar{\mu})| < 1.4 \times 10^{-2} . \quad (3.32b)$$

We emphasize here that the polarization effects in (3.32) depend on the bound in (3.26) which holds only for $M_0 \sim 1$ GeV. Obviously, for a larger $M_0$, $P_L$'s in (3.32) will be smaller but $d_n$ and $d_{e,\mu}$ can still be large.[58]

### B. Leptoquark model

In this model, there is no tree-level contribution to $K_L \to \mu\bar{\mu}$ decay because of no down quarks and leptoquark couplings in (2.3). The one-loop box diagram shown in Fig. 5 that contributes to the decay amplitude does not induce a $g_{SP}$ term. Thus, $P_L$ in $K_L \to \mu\bar{\mu}$ decay cannot arise from the leptoquark interaction in (2.3). On the other hand, the contribution to $\eta \to \mu\bar{\mu}$ decay can proceed at the tree level and the Feynman diagram depicting this is shown in Fig. 6. The effective interaction that contributes to $g_{SP}$ is given by

$$L_{\rm eff} \sim \frac{G_F}{\sqrt{2}} g_{SP} \bar{u} i \gamma_5 u \bar{\mu} \mu , \quad (3.33)$$

where

$$g_{SP} = \frac{v^2 {\rm Im}(\lambda_{12}\lambda'^*_{12})}{M_\phi^2} . \quad (3.34)$$

This is straightforwardly derived from the leptoquark couplings in (2.3). From (3.10) we find that the muon polarization is

$$|P_L(\eta \to \mu^+\mu^-)| \le 1.5 \times 10^5 \frac{{\rm Im}(\lambda_{12}\lambda'^*_{12})}{M_\phi^2} \ {\rm GeV}^2 . \quad (3.35)$$

If we take

$${\rm Im}(\lambda_{ij}\lambda'^*_{ij}) \sim \frac{m_{\mu^i} m_{e^j}}{M_\phi^2} , \quad (3.36)$$

one obtains

$$|P_L(\eta \to \mu^+\mu^-)| \le 10^{-8} , \quad (3.37)$$

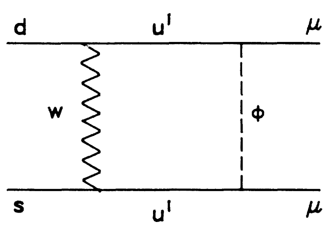


FIG. 5. One-loop contribution to $K_L \to \mu\bar{\mu}$ decay due to the leptoquark interaction.

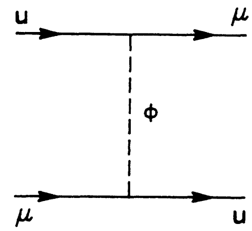


FIG. 6. Contribution to $P_L(\eta \to \mu\bar{\mu})$ due to the leptoquark interaction.

by using (3.35) and the constraint on $M_\phi$ in (2.4). Obviously, such a muon polarization is far below an experimentally detectable level.

The EDM of the electron has been studied by BM.[40] We will summarize their result and evaluate $d_\mu$ which they did not do. The relevant Feynman diagrams are depicted in Fig. 7. These one-loop diagrams lead to[40]

$$d_l \simeq \frac{e}{24\pi^2} \sum_i {\rm Im}(\lambda_{ij}\lambda^*_{ij}) \frac{m_{u^i}}{M_\phi^2} \left[ \frac{11}{4} + \ln \frac{m_{u^i}^2}{M_\phi^2} \right] , \quad (3.38)$$

where $l = e$ and $\mu$ for $j = 1$ and 2, respectively. With the couplings in (3.36), the dominant contribution to $d_l$ comes from the $t$ quark which gives

$$d_l \simeq \frac{e}{24\pi^2} \frac{m_l m_t^2}{M_\phi^4} \left[ \frac{11}{4} + \ln \frac{m_t^2}{M_\phi^2} \right] . \quad (3.39)$$

Taking $m_t \sim 100$ GeV/$c^2$, one finds

$$d_e < 3 \times 10^{-26} \ e \ {\rm cm} \quad (3.40a)$$

and

$$d_\mu = \frac{m_\mu}{m_e} d_e < 6 \times 10^{-24} \ e \ {\rm cm} . \quad (3.40b)$$

Notice that this type of model gives the scaling

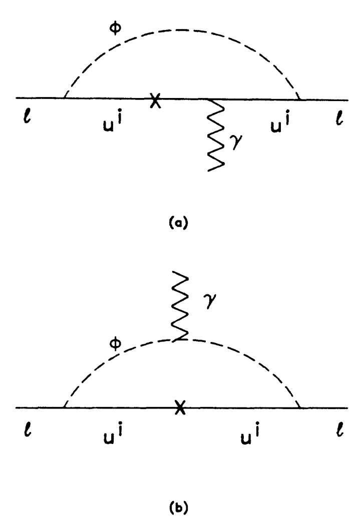


FIG. 7. Feynman diagrams for lepton electric dipole moment, where $l = e$ and $\mu$ due to the leptoquark interaction.

$d_\mu : d_e \sim m_\mu : m_e$. One notes that the model predicts a very small $d_n$ ($\sim 10^{-28}$ e cm) since it is proportional to the mass of the $u$ quark.

### C. SUSY models

For the minimal standard *CP*-violation SUSY model in which only the complex KM phase is considered, it has been shown that the muon polarization in $K_L \to \mu\bar{\mu}$ decay can arise from one-loop diagrams (cf. Fig. 8) involving superparticles exchanges[64] and this gives $P_L \leq 10^{-3}$, i.e., at best it is of the same order as that given by the standard model. Furthermore, $d_l$ vanishes at the two-loop level[65] and as a result, it is expected to be very small and takes place only at three loops.

In the superstring-inspired model described in Sec. II, apart from the ordinary and superparticles, there are leptoquark-like exotic heavy particles $D$ and $D^c$ which couple to both up and down quarks differentially from the previously considered non-SUSY leptoquark model. The *CP* violation involving these exotics has been examined by several authors.[66,67] The muon polarization in $K_L \to \mu\bar{\mu}$ decay can arise from one-loop diagrams[67] that are not contained in the set dictated by the minimal standard *CP*-violation SUSY model. A typical graph that contributes to $P_L$ through intermediate charged gaugino, squark, and $D$ exchanges is shown[67] in Fig. 9. One estimates that

$$g_{SP}^{21} \sim \frac{1}{12\pi^2} |\lambda_{22}^6 \lambda_{22}^7| \frac{m_s m_c M_{\tilde{W}}^2}{M^4} \sin\theta_C \sin\delta , \qquad (3.41)$$

where the lower indices of $\lambda$ stand for generations, $\theta_C$ is the Cabibbo angle arising from the $\tilde{W}$-$d_L$-$\tilde{c}_L$ vertex, $\delta$ is a combination of *CP*-violating phases in the diagram, and $M$ is the mass parameter for all the exotic particles. (Here all unknown masses are assumed to be the same which is sufficient for an estimate.) Using Eqs. (3.11), we get

$$|P_L(K_L \to \mu\bar{\mu})| \sim 7 \times 10^{-2} |\lambda_{22}|^2 \sin\delta , \qquad (3.42)$$

where we explicitly take the scale $M \sim 100$ GeV/$c^2$ and $\lambda^6 \sim \lambda^7 \sim \lambda$. Similar to the discussions in the leptoquark model, the upper limit of the couplings $\lambda$ can be extract-

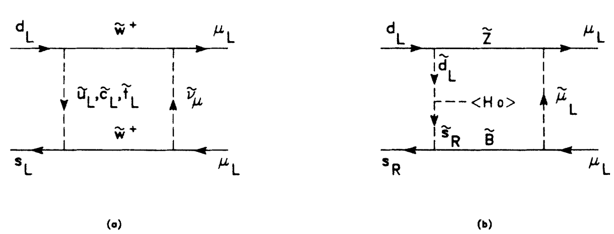


FIG. 8. The dominant contributions to $P_L(\eta, K_L \to \mu\bar{\mu})$ due to the superparticle exchanges.

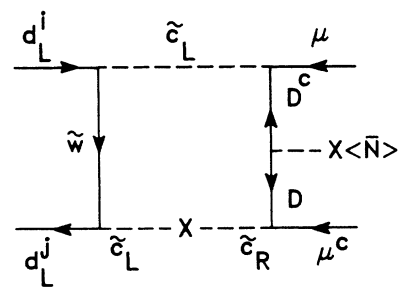


FIG. 9. Diagram that contributes to $P_L(\eta, K_L \to \mu\bar{\mu})$ through intermediate *W*-ino, squark, and *D*-leptoquark exchange with $d^{i,j} = d$ and $s$.

ed from $\mu \sim e\gamma$ decay and one finds from Eq. (2.5) that

$$|\lambda_{12}\lambda_{22}| < 2 \times 10^{-7} . \qquad (3.43)$$

If one takes $\lambda_{12} \sim \lambda_{22}$ or $|\lambda_{12}|^2 \sim |\lambda_{22}|^2 < 2 \times 10^{-7}$, one finds

$$|P_L(K_L \to \mu\bar{\mu})| < 1.4 \times 10^{-8} \sin\delta \qquad (3.44)$$

which is vanishingly small. The diagram in Fig. 9 also gives the contribution to $P_L$ in the $\eta \to \mu\bar{\mu}$ decay if we replace $d\,(s)$ by $s\,(d)$. We expect that

$$g_{SP}^{11} \sim g_{SP}^{21} \sin\theta_C , \quad g_{SP}^{22} \sim g_{SP}^{21} \sin\theta_C , \qquad (3.45)$$

and thus

$$|P_L(\eta \to \mu\bar{\mu})|_{\square} \leq 2.5 \times 10^{-8} |P_L(K_L \to \mu\bar{\mu}) \lesssim 10^{-8} , \qquad (3.46)$$

where $\square$ stands for the loop contributions. On the other hand, the tree contribution similar to (3.35) is given by

$$|P_L(\eta \to \mu\bar{\mu})|_{\text{tree}} \leq 0.4 [B(\mu \to e\gamma)]^{1/2} < 3 \times 10^{-6} \qquad (3.47)$$

with $\lambda_{12} \sim \lambda_{22}$. Here, the bound in (3.47) is more general than that in (3.37) where special couplings have been assumed in (3.36). We thus see that it is impossible to have large muon polarization in $\eta \to \mu\bar{\mu}$ decay because of the bound on the $\mu \to e\gamma$ decay of $P_L(K_L \to \mu\bar{\mu})$.

However, large muon polarization in $K_L \to \mu\bar{\mu}$ decay is possible if the constraint from the $\mu \to e\gamma$ decay can be evaded. This can be achieved if the coupling $\lambda_{12}$ is small. In fact, $\lambda_{12}$ can naturally be zero if some discrete or global family-type symmetries are imposed on the model. If $\lambda_{12} \sim 0$ because of such symmetries then the constraint from (3.43) disappears and $\lambda_{22}$ can be $\sim 1$. This in turn gives

$$|P_L(K_L \to \mu\bar{\mu})| \leq 7\% \qquad (3.48)$$

from (3.42).

As for the lepton EDM's, we need only study the vanishing $\lambda_{12}$ case and we find

$$d_{e^i} \simeq \frac{e}{24\pi^2} |\lambda_{ii}|^2 \frac{m_{u^i}}{M^2} \left[ \frac{11}{4} + \ln \frac{m_{u^i}^2}{M^2} \right] . \qquad (3.49)$$

Since there is no constraint on $\lambda_{11}$ we expect $d_e$ can be as large as $10^{-25}$ e cm which requires $|\lambda_{11}|^2 \sim 1.4\times 10^{-4}$. The muon EDM is similarly estimated to be

$$d_\mu = c\frac{m_c}{m_u} d_e \ , \tag{3.50a}$$

where

$$c = |\lambda_{22}/\lambda_{11}|^2 \left[\frac{11}{4} + \ln\frac{m_c^2}{M^2}\right] \Big/ \left[\frac{11}{4} + \ln\frac{m_u^2}{M^2}\right] .$$

It is easy to see that

$$d_\mu \sim 6\times 10^{-20}\ e\,\mathrm{cm} \tag{3.50b}$$

for $\lambda_{22} \sim 1$.

### D. Left-right-symmetric models

In contrast with the standard KM model in which the lepton EMS's start appearing at the three-loop level, the left-right models allow one-loop contributions to $d_l$ if neutrino masses are nonzero.[49,50,68] The diagram depicting this is shown in Fig. 10. Most of the studies in the literature have concentrated on the EDM of the electron[49,50] which is estimated to lie in the range $10^{-24}$–$10^{-29}$ $e$ cm. This is because the mixings between the lepton generations are unknown. For vanishing mixings, the discussion on $d_\mu$ will follow as that of $d_e$ directly. Calculation of $P_L(K_L\to\mu\bar{\mu})$ in the simple left-right-symmetric models was first considered by Chang and Mohapatra[51] who estimate it to be in the range $10^{-3}$–$10^{-2}$. However, the predictions on $d_l$ and $P_L$ depend sensitively on the values of neutrino masses chosen. For example, by assuming no arbitrary fine-tuning among the parameters in the neutrino mass matrix, Liu[50,52] showed that $d_e$ and $P_L(K_L\to\mu\bar{\mu})$ are less than $10^{-26}$ $e$ cm and $10^{-3}$, respectively. The constraints from the neutrino masses become minimal in the extension of the simple left-right models by Ref. 53. They have shown that both $d_e$ and $P_L(K_L\to\mu\bar{\mu})$ can be large with small or even vanishing neutrino masses. Specifically, they find from Fig. 10 that

$$d_e \simeq \frac{G_F}{2\sqrt{2}\pi^2} e D_1 \xi \sin\alpha_1 f(M_1^2/M_{W_R}^2) \tag{3.51}$$

with $M_{W_R} \gg M_{W_L}$ and $g_L = g_R$, where $f(Y)$ is a smooth function varying from 1 to 0.25. For $D_1 \sim 10$ MeV, Eqs. (3.51) and (2.11) give $d_e \le 10^{-25}$ $e$ cm. For the calculation of $P_L(K_L\to\mu\bar{\mu})$, the dominant contribution arises from the left-right box diagram shown in Fig. 11 with $d^i$ and $d^j$ being $d$ and $s$ quarks, respectively, which leads to

$$g_{SP}^{21} \simeq \frac{G_F}{2\sqrt{2}\pi^2}\sin\theta_C \sin\alpha_2 m_c D_2 \left[\frac{M_{W_L}}{M_{W_R}}\right]^2 \times \left[\frac{M_{W_R}}{M_2}\right]^2 \ln\frac{M_2^2}{M_{W_R}^2} \tag{3.52}$$

by assuming $M_2 \ge M_{W_R}$ and $g_L = g_R$ and ignoring the phases in the quark sector. Taking $M_2 \sim M_{W_R} \sim 1$ TeV and $D_2 \sim 100$ GeV, one expects

$$P_L(K_L\to\mu\bar{\mu}) \le 0.02 \ . \tag{3.53}$$

We now extend the discussions to the muon EDM and polarization on $\eta\to\mu\bar{\mu}$ decay. The contributions to $P_L(\eta\to\mu\bar{\mu})$ can be obtained from the graph in Fig. 11 by substituting the $d$ or $s$ quark for $d^{i,j}$ which is the case shown in the SUSY models. It is straightforward to show that the relations in (3.45) hold with $g_{SP}^{21}$ now given by Eq. (3.52). Therefore, from (3.46) we find

$$P_L(\eta\to\mu\bar{\mu}) \lesssim 10^{-8} \ . \tag{3.54}$$

By analogy with the derivation of $d_e$ in Eq. (3.51), we obtain the EDM of muon as

$$d_\mu \simeq \frac{G_F}{2\sqrt{2}\pi^2} e D_2 \xi \sin\alpha_2 f(M_2^2/M_{W_R}^2) \simeq \frac{D_2 \sin\alpha_2}{D_1 \sin\alpha_1} d_e \le 10^{-21}\ e\,\mathrm{cm} \tag{3.55}$$

from Fig. 10 with $l=\mu$. Here we do not have a relation between $d_\mu/d_e$ and $m_\mu/m_e$ unless the masses of the Dirac neutrinos and charged leptons are related.

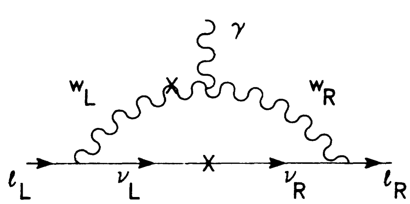


FIG. 10. One-loop contributions to electric dipole moment of lepton $d_l$. The crosses on the gauge-boson line represents the left-right mixing. The cross on the internal neutrino line represents a Dirac mass term insertion.

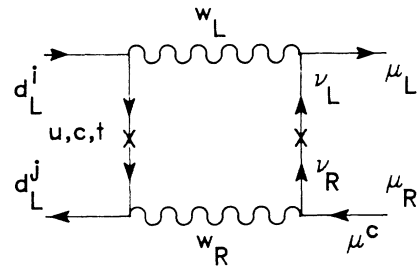


FIG. 11. Left-right box diagram that contributes to $P_L$ in $\eta, K_L\to\mu\bar{\mu}$ decays.

## IV. CONCLUSIONS

We have studied the longitudinal muon polarization in $\eta\to\mu\bar{\mu}$ and $K_L\to\mu\bar{\mu}$ decays and the electric dipole moment of an electron and muon in various extensions of the standard KM $CP$-violation model. In the multi-Higgs-boson models, the upper bounds in $P_L$ in $\eta\to\mu\bar{\mu}$ and $K_L\to\mu\bar{\mu}$ decays are estimated to be $<0.3\times 10^{-2}$ and

$1.4\times10^{-2}$, respectively, which are accessible to the planned experiments in Saclay and KEK, and the EDM's of the electron muon are found to be $<7\times10^{-26}$ and $10^{-22}$ $e$ cm, respectively. The leptoquark model gives vanishingly small $P_L$ in both decays and $d_l<3\times10^{-26}$ and $6\times10^{-24}$ $e$ cm for $l=e$ and $\mu$, respectively. In the SUSY model, $P_L(K_L\to\mu\bar{\mu})$ can be up to 0.07, whereas $P_L(\eta\to\mu\bar{\mu})$ is expected to be $<10^{-6}$. The main constraint here is provided by the experimental bounds of the value of $P_L(K_L\to\mu\bar{\mu})$ in (1.6) or the branching ratio of $\mu\to e\gamma$. The electron EDM can be as large as the present experimental limit while $d_\mu$ is estimated to be in the order of $10^{-20}$ $e$ cm which is within the measurable range of the approved BNL experiment. The results in the left-right-symmetric model are that $P_L(\eta,K_L\to\mu\bar{\mu})\lesssim10^{-8}$, 0.02, and $d_{e,\mu}<10^{-25}$, $10^{-21}$ $e$ cm, respectively, which are similar to those in the SUSY model.

In conclusion, the muon polarization asymmetry in $K_L\to\mu\bar{\mu}$ decay is accessible to the experiments in most of the *CP*-violating gauge theories beyond the standard model, whereas $P_L$ in $\eta\to\mu\bar{\mu}$ decay is a good probe of *CP* violation in the multi-Higgs-boson models. The electron and muon EDM's are within the ranges $10^{-25}-10^{-26}$ and $10^{-20}-10^{-22}$ $e$ cm, respectively, which are hard to get as may be within reach in the not too distant future. Measurement of the EMD of the electron or muon, especially the ratio of $d_\mu/d_e$, will be a powerful tool for distinguishing between the various *CP*-violating mechanisms. Furthermore, since $d_l$ are free of QCD uncertainties such as the strong $\theta$ parameter[69] and the gluon operators in (3.15) and (3.17), they will be of great importance for studying purely electroweak source of *CP* violation.

*Note added.* After the completion of this paper we received three papers: (1) by X. He and B. McKellar [Melbourne Report No. UM-90-09 (unpublished)]; (2) by J. Gunion and D. Wyler [Santa Barbara Report No. NSF-ITP-90-109 (unpublished)]; and (3) by D. Chang, W. Keung, and T. Yuan [Northwestern Report No. NUHEP-TH-90-22 (unpublished)], respectively, where (1) dealt with $\eta\to\mu\bar{\mu}$ and $d_n$ (2) and (3) studied the chromo-EDM of the light quarks via the BZ's two-loop mechanism. From (2) and (3), we find that $d_n$ is about four times larger than the value of $d_n^c$ in Eq. (3.25) for $m_t\gg M_0\sim1$ GeV. However, since the estimate of $d_n$ from the gluonic two-loop mechanism is more uncertain than the photonic one due to the low-energy hadronic physics, the orders of magnitude for the *CP* violating effects estimated in the text will not change. We thank the authors for sending their work to us before publication.

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