Learning about the Cabibbo-Kobayashi-Maskawa matrix from *CP* asymmetries in B^0 decays

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We show how relations among various classes of *CP* asymmetries in B^0 decays can be used to test the unitarity of the three-generation Cabibbo-Kobayashi-Maskawa matrix, independently of the mechanism of mixing in the B^0 and K^0 systems. We suggest various ways to determine the sign of $\sin\delta$, independently of the sign of the B_K parameter.

I. INTRODUCTION

The standard model (SM) with three quark generations has so far provided a sufficient explanation of all electroweak phenomena. However, in the area of *CP* violation we have as yet not enough evidence to ascertain whether the SM explanation is the only source of such effects or whether new physics beyond the three-generation SM is needed. The neutral *B* mesons provide a sensitive laboratory in which to study this question.[1] The beauty of this system is that it allows several independent measurements. The SM predicts specific relations among the results, and thus these measurements probe physics beyond the SM which may cause the relations to be violated. It is interesting to analyze how one can separately test specific features of the SM, by careful choice of the quantities to be compared. This paper extends the previous analysis of this subject given in Ref. 2. We analyze two specific features: first we consider tests for the unitarity of the three-generation Cabibbo-Kobayashi-Maskawa (CKM) matrix that are independent of any assumptions about the mechanism of mixing in either the *B* or the *K* systems; second we address the information from the signs of the various asymmetries.

II. TESTING UNITARITY OF THE CKM MATRIX

Our study involves those classes of asymmetries for which, within the SM, the direct decay is expected to be dominated by a single combination of CKM parameters. The asymmetries are denoted by $\mathrm{Im}\lambda_{iq}$. The subscript $i=1,\ldots,5$ denotes the quark subprocess. The subscript $q=d,s$ denotes the type of decaying meson, B_q. In Tables I and II we list *CP* asymmetries in B_d and B_s decays, respectively. The list of hadronic final states gives examples only. Other states may be more favorable experimentally. We always quote the *CP* asymmetry for *CP*-even states, regardless of the specific hadronic state listed. The angles α, β, and γ are the three angles of the unitarity triangle; a recent analysis of the standard-model predictions for this triangle was given by Dib, Dunietz, Gilman, and Nir.[3]

A clean theoretical interpretation of the experimental measurement of *CP* asymmetries is possible only if the two following conditions are met.[4]

(a) $\Gamma_{12}(B_q) \ll M_{12}(B_q)$.

(b) The *CP* asymmetries arise dominantly from interference of amplitudes corresponding to two paths to the same final state, one of which involves B-$\bar{B}$ mixing. This means that the direct decay is dominated by a single combination of CKM parameters, or by a single strong-interaction phase.[5]

Under these two assumptions the *CP* asymmetries are given by

$$A_{iq}(t) = \mathrm{Im}\lambda_{iq}\sin(\Delta M_q t) , \tag{1}$$

where $\Delta M_q \equiv M\,(\text{heavy}) - M\,(\text{light})$ is the mass difference in the B_q system and λ_{iq} is of the form ($|\lambda_{iq}|=1$)

$$\lambda_{iq} = \left[\frac{X_i}{X_i^*}\right]\left[\frac{Y_q}{Y_q^*}\right]\left[\frac{Z_{iq}}{Z_{iq}^*}\right] . \tag{2}$$

The X_i factor depends on the quark subprocess amplitude. The Y_q factor depends on the mixing amplitude of the decaying meson. The Z_{iq} factor depends on the K-$\bar{K}$ mixing amplitude. Z_{iq} can be different from one only for those asymmetries where there is a single unpaired neutral kaon in the final state, and depends on whether this comes from a K^0 or a $\bar{K}^0$. Thus, independent of any model for K-$\bar{K}$ mixing,

$$\begin{aligned} &Z_{2d} = Z_{3d} = Z_{5d} = Z_{1s} = Z_{4s} = 1 , \\ &Z_{1d} = Z_{4d} = Z_{2s}^* = Z_{3s}^* = Z_{5s}^* . \end{aligned} \tag{3}$$

Equations (2) and (3) imply relations among the various λ_{iq} such as

TABLE I. *CP* asymmetries in B_d decays.

Class (iq)	Quark subprocess	Final state (example)	SM prediction
$1d$	$\bar{b}\to\bar{c}c\bar{s}$	ψK_S	$-\sin2\beta$
$2d$	$\bar{b}\to\bar{c}c\bar{d}$	D^+D^-	$-\sin2\beta$
$3d$	$\bar{b}\to\bar{u}u\bar{d}$	$\pi^+\pi^-$	$\sin2\alpha$
$4d$	$\bar{b}\to\bar{s}s\bar{s}$	ϕK_S	$-\sin2\beta$
$5d$	$\bar{b}\to\bar{s}s\bar{d}$	$K_S K_S$	0

TABLE II. CP asymmetries in B_s decays.

Class (iq)	Quark subprocess	Final state (example)	SM prediction
$1s$	$\bar b \to \bar c c \bar s$	$D_s^+ D_s^-$	0
$2s$	$\bar b \to \bar c c \bar d$	ψK_S	0
$3s$	$\bar b \to \bar u u \bar d$	ρK_S	$-\sin 2\gamma$
$4s$	$\bar b \to \bar s s \bar s$	$\eta' \eta'^c$ (Ref. 6)	0
$5s$	$\bar b \to \bar s s \bar d$	ϕK_S	$\sin 2\beta$

$$\begin{aligned}&\arg\lambda_{1d}-\arg\lambda_{4d}-\arg\lambda_{1s}+\arg\lambda_{4s}=0 ,\\&\arg\lambda_{1d}-\arg\lambda_{5d}-\arg\lambda_{1s}+\arg\lambda_{5s}=0 .\end{aligned}\tag{4}$$

These relations can be experimentally tested. As assumption (a) is very mild and holds on rather general grounds, and as assumption (b) is rather safe for $\bar b \to \bar c c \bar s$,[7,8] what we really test with Eq. (4) is whether $\bar b \to \bar s s \bar s$ and $\bar b \to \bar s s \bar d$ processes satisfy condition (b). Similarly, one can test assumption (b) for the other classes of asymmetries, as explained in Ref. 2. There are additional ways in which violations of (b) can be discovered: first,[9] the time dependence of the asymmetry is different from Eq. (1); second, various hadronic states corresponding to the same quark subprocess are likely to exhibit different asymmetries.

Let us now consider the implications of one further assumption.

(c) The single channel that dominates a direct decay is given by the relevant SM diagram. In other words, we assume that the direct decays are not dominated by processes from new physics beyond the SM.

For $i=1,2,3$ this dominant contribution comes from tree-level W-mediated diagrams, and thus

$$\begin{aligned}&X_1\equiv X(\bar b\to\bar c c\bar s)=V_{cb}V_{cs}^* ,\\&X_2\equiv X(\bar b\to\bar c c\bar d)=V_{cb}V_{cd}^* ,\\&X_3\equiv X(\bar b\to\bar u u\bar d)=V_{ub}V_{ud}^* ,\end{aligned}\tag{5}$$

while for $i=4,5$ the dominant contribution comes from the real part of penguin diagrams, which give[7]

$$\begin{aligned}X_4\equiv X(\bar b\to\bar s s\bar s)&=(V_{ub}V_{us}^*+V_{cb}V_{cs}^*)\ln(m_b^2/M_W^2)\\&\quad+(V_{tb}V_{ts}^*)\ln(m_t^2/M_W^2) ,\\X_5\equiv X(\bar b\to\bar s s\bar d)&=(V_{ub}V_{ud}^*+V_{cb}V_{cd}^*)\ln(m_b^2/M_W^2)\\&\quad+(V_{tb}V_{td}^*)\ln(m_t^2/M_W^2) .\end{aligned}\tag{6}$$

Note that under this assumption each of the processes in Eq. (6) depends on several CKM combinations, but on a common strong-interaction phase.

We make no assumptions about the mechanism of mixing in the B_q-$\bar B_q$ and K-$\bar K$ systems. Consequently, Y_d and Y_s remain unspecified, while the various Z_{iq}'s are not specified beyond the model-independent relations of Eq. (3). Note, however, that combinations such as $(\arg\lambda_{1q}-\arg\lambda_{4q})$, $(\arg\lambda_{2q}-\arg\lambda_{5q})$, and $(\arg\lambda_{3q}-\arg\lambda_{5q})$ are independent of the Y and Z factors. This fact will allow us to test the unitarity constraints:

$$\begin{aligned}&\mathcal{U}_{db}\equiv V_{ub}V_{ud}^*+V_{cb}V_{cd}^*+V_{tb}V_{td}^*=0 ,\\&\mathcal{U}_{sb}\equiv V_{ub}V_{us}^*+V_{cb}V_{cs}^*+V_{tb}V_{ts}^*=0 ,\end{aligned}\tag{7}$$

in a way which is independent of the mechanism of mixing in the K and B_q systems.

Assuming $\mathcal{U}_{db}=0$ gives

$$X(\bar b\to\bar s s\bar d)=V_{tb}V_{td}^*\ln(m_t^2/m_b^2) ,\tag{8}$$

and consequently

$$\begin{aligned}&\arg\lambda_{2q}-\arg\lambda_{5q}=2[\arg(V_{cb}V_{cd}^*)-\arg(V_{tb}V_{td}^*)] ,\\&\arg\lambda_{3q}-\arg\lambda_{5q}=2[\arg(V_{ub}V_{ud}^*)-\arg(V_{tb}V_{td}^*)] .\end{aligned}\tag{9}$$

The same assumption further leads to

$$\begin{aligned}&\arg\lambda_{2q}-\arg\lambda_{5q}=-2\beta ,\\&\arg\lambda_{3q}-\arg\lambda_{5q}=2\alpha .\end{aligned}\tag{10}$$

The following relation is predicted:[10]

$$\frac{\sin[(\arg\lambda_{5q}-\arg\lambda_{2q})/2]}{\sin[(\arg\lambda_{3q}-\arg\lambda_{5q})/2]}=\left|\frac{V_{ub}}{V_{cb}}\right|\frac{1}{\sin\theta_c} .\tag{11}$$

If it fails, it will be a strong indication that $\mathcal{U}_{db}\neq 0$.

Similarly, assuming $\mathcal{U}_{sb}=0$ gives

$$X(\bar b\to\bar s s\bar s)=V_{tb}V_{ts}^*\ln(m_t^2/m_b^2) ,\tag{12}$$

and consequently

$$\arg\lambda_{1q}-\arg\lambda_{4q}=2[\arg(V_{cb}V_{cs}^*)-\arg(V_{tb}V_{ts}^*)] .\tag{13}$$

The same assumption, together with the experimental information

$$|V_{ub}V_{us}^*| \ll |V_{cb}V_{cs}^*|$$

further leads to

$$\arg\lambda_{1q}-\arg\lambda_{4q}\approx 0 .\tag{14}$$

(The exact constraint that follows from $|V_{ub}/V_{cb}|\le 0.16$ is $|\arg\lambda_{1q}-\arg\lambda_{4q}|\le 0.07$.) Equation (14) can be rewritten as

$$\mathrm{Im}\lambda_{1q}\approx \mathrm{Im}\lambda_{4q} .\tag{15}$$

If this prediction fails, it will be a strong indication that $\mathcal{U}_{sb}\neq 0$.

Both predictions above depend on the fact that in Eq. (6) the terms arising from up and charm quarks contribute equally. This is true for the dominant real term from the lowest-order penguin graph.[7] The absorptive part of the penguin graph represents a contribution to a final-state rescattering from states involving u or c quarks to those with strange quarks. Here there is no reason to expect that the lowest-order penguin graph gives a correct estimate, and the differences in kinematics may lead to differing u- and c-quark contributions. As discussed by Wolfenstein for the case of charged-B decays,[11] there are competing processes with multiple mesons in the final state that can be expected to deplete the amplitude for the exclusive processes of interest here. Hence we have assumed that the contribution to the asymmetry from the phase introduced by this absorptive part is small, at most of the order of a few percent. A discrepancy of this order in Eq. (11) or Eq. (15) cannot be taken as evidence of physics beyond the SM. However, if the relations are untrue by significantly more than 10%, this would strongly suggest a nonstandard source. A test for this conclusion can be made by measuring *CP* asymmetries in *charged-B* decays which proceed via the quark subprocesses of interest, namely, $b \to \bar{s}s\bar{s}$ (e.g., $B^{\pm} \to \phi K^{\pm}$) and $\bar{b} \to \bar{s}s\bar{d}$ (e.g., $B^{\pm} \to K_S K^{\pm}$). Asymmetries in such charged-*B* decays arise solely from interference between absorptive parts and real parts of penguin diagrams.[12] Therefore, the magnitude of these asymmetries should be comparable to the modification of Eqs. (11) and (15) due to the absorptive part. If the charged-*B* asymmetries are much smaller than the discrepancies in Eqs. (11) or (15), this would be a strong indicator of physics beyond the standard three generations.[13]

In the relations given in Eqs. (11) and (15), we used the asymmetries of classes $2q$ and $1q$, respectively. In this way, these predictions are independent of any assumption on the mechanism for K-$\bar{K}$ mixing. However, as explained in Ref. 2, the measurement of the ϵ parameter determines the phase of the Z factor. Consequently, it is rather safe to assume that $\mathrm{Im}\lambda_{1d} = \mathrm{Im}\lambda_{2d}$ and $\mathrm{Im}\lambda_{1s} = \mathrm{Im}\lambda_{2s}$. From the experimental point of view, it would be advantageous then to combine measurements within both classes $1q$ and $2q$ in each of Eqs. (11) and (15), to achieve a test of three-generation unitarity that is independent of the nature of mixing in the B system but not of mixing in the K system.

III. DETERMINING THE SIGN OF (sinδ)

It is interesting to remark that the sign of $\sin\delta$, the phase parameter of the standard (Particle Data Group) parametrization[14] of the CKM matrix, is not unambiguously known. The ϵ-parameter measurement, which is often quoted as fixing this sign, depends in fact only on the combination $B_K \sin\delta$, and fixes this combination to be positive.[15] Theoretical methods to determine B_K have historically given either sign[16] or often simply determined $|B_K|$.[17] Statements in the literature that the sign of the K_L-K_S mass difference fixes the sign of B_K should be taken with care. The fact that the long-lived kaon is heavier implies that the relative phase between M_{12} and Γ_{12} is π but does not give the overall phase. Only the fact that the *CP*-odd kaon is heavier gives the sign of M_{12}. The sign of B_K is still not cleanly predicted, but if short-distance contributions dominate then it is positive.

Recent calculations on the lattice[18] all give positive values of B_K, but they are subject to the uncertainty of the uncontrolled "quenched" approximation[19] (namely, the suppression of disconnected quark loops). The situation is similar for calculations based on the $1/N$ expansion.[20] Even though a positive value of B_K is indeed favored, it would still be informative to have a measurement of sgn(sinδ) that does not depend on knowledge of sgn(B_K). Measurement of *CP* asymmetries in B^0 decays offer this opportunity.

Two versions of the unitarity triangle are shown in Fig. 1. *A priori* either orientation is possible. The two possible orientations correspond to the two possible signs of $\sin\delta$. We will now show how measuring *CP* asymmetries in B^0 decays will decide between the two. We emphasize that this part of our analysis is carried out within the three-generation SM: in models of extended quark sector there is no "unitarity triangle" (and the phase δ has to be redefined); in models with new sources of *CP* violation, ϵ and $\mathrm{Im}\lambda_{iq}$ may not give information on $\sin\delta$.

The angles in the unitarity triangle are related to the asymmetries by

$$\sin 2\alpha = \mathrm{Im}\lambda_{3d} \; ,$$
$$\sin 2\beta = -\mathrm{Im}\lambda_{1d} \; , \qquad (16)$$
$$\sin 2\gamma = -\mathrm{Im}\lambda_{3s} = \sin 2\delta \; .$$

All angles are defined by convention to lie between 0 and 2π. Measurement of any one asymmetry, $\sin 2\phi$, thus determines the corresponding angle only up to a fourfold ambiguity: ϕ, $\pi/2-\phi$, $\phi+\pi$, and $3\pi/2-\phi$ [mod (2π)]. The four solutions lie within two separate quadrants, corresponding to two different signs of $\sin\delta$. When all three asymmetries are measured, the fact that α, β, and γ define a triangle resolves the ambiguity between the quadrants for them all, since not more than one of the internal angles of the triangle can be greater than or equal to $\pi/2$. The sign of $\sin\delta$ is thus determined; it is the sign of at least two of the three asymmetries: $\mathrm{Im}\lambda_{3d}$, $-\mathrm{Im}\lambda_{1d}$, and $-\mathrm{Im}\lambda_{3s}$. (The remaining twofold ambiguity for each of the angles is also resolved, unless one angle is $\pi/2$ or $3\pi/2$.) We emphasize that this method of determining sgn(sinδ) is independent of knowledge of any additional SM parameters, or of hadronic matrix elements.

An experimentally simpler test relies on the fact that $|V_{ub}/V_{cb}| < \sin\theta_C$. As a result of this relation, the angle β is constrained to lie within the range $\{0,\pi/2\}$ or

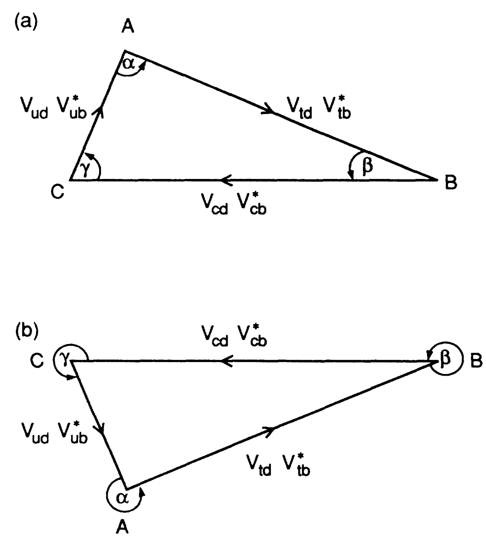

FIG 1. The unitarity triangle. The two orientations of the triangle correspond to (a) $\sin\delta > 0$ and (b) $\sin\delta < 0$.

$\{3\pi/2, 2\pi\}$. Consequently, the $\pm\pi$ ambiguity is resolved for β. This can be easily seen in Fig. 2, which shows various constraints on the form of unitarity triangle. The constraints that follow from the measurement of ϵ are given with the hypothetical range: $-1 \leq B_K \leq +1$ (all other ranges of parameters are taken from Ref. 21). The conclusion is that the sign of $\sin\delta$ can be determined from $\text{Im}\lambda_{1d}$ alone. If this result gives a value for the angle β that lies in the region $\{3\pi/2, 2\pi\}$ ($\text{Im}\lambda_{1d} > 0$), then we are confronted with a choice: either we have evidence for physics beyond the SM or B_K is negative and so is $\text{sgn}(\sin\delta)$. The conclusion one draws from this measurement thus depends on the level at which the calculations of B_K convincingly rule out negative values.

A third method gives a direct measurement of $\sin\delta$ (rather than $\sin 2\delta$), but is expected to be experimentally difficult. The exact SM prediction for the asymmetry in classes 1s and 2s is not zero but rather

$$2|V_{ub}/V_{cb}|\sin\theta_C \sin\delta \ .$$

Thus, the sign of these asymmetries directly gives the sign of $\sin\delta$. However, even for $|\sin\delta| = 1$, the absolute value of this asymmetry is constrained to be smaller than 0.07, and consequently difficult to measure.

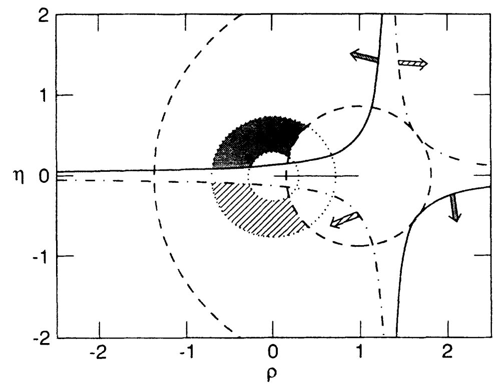

FIG. 2. Constraints on the vertex A of the rescaled unitarity triangle from the measurement of $|V_{ub}/V_{cb}|$ (dotted circles) and x_d (dashed circles). The ϵ constraint is given with the hypothetical range $-1 \leq B_K \leq +1$. The dotted area is the allowed region for positive B_K (solid curves), while the crosshatched area is the allowed region for negative B_K (dotted-dashed curves). The top-quark mass is provisionally fixed at 120 GeV.

IV. CONCLUSIONS

This paper describes two further tests of the standard model, or of the nature of its breakdown, that can be made using B^0 decay asymmetries. It is important to remark that the measurement of these asymmetries will provide important new information even if all the tests are passed by the standard model. The parameters of the CKM matrix are important physical quantities which merit careful measurement. The B^0 decays provide us with the opportunity to pin down some fundamental parameters of the standard model. Further they offer sufficient redundancy in this process that a number of tests can be devised, each of which probes a different set of the assumptions that comprise the standard model. Here we discussed two features: first how to test three-generation unitarity in a way that is independent of the mixing in either the B or the K systems, and second the information on the phase of the CKM matrix given by the signs of the asymmetries.

ACKNOWLEDGMENTS

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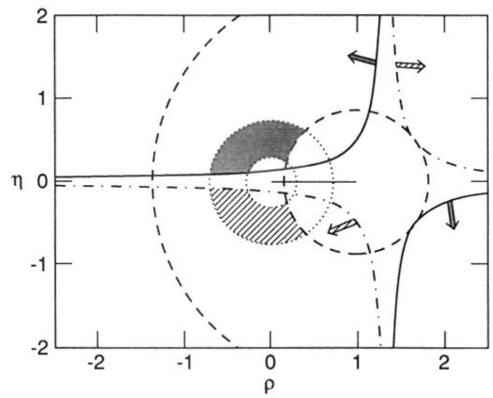

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