Production of Higgs particles through $e^+e^- \to Hgg$

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(Received 16 February 1990; revised manuscript received 4 June 1990)

We computed the differential cross section for $e^+e^- \to Hgg$ away from the Z^0 pole and for polarized beams. Form factors for $Z^0 \to Hgg$ are given. The four-point scalar integral needed for the present case is expressed in terms of Spence functions.

I. INTRODUCTION

Finding the Higgs boson is undoubtedly one of the greatest challenges facing the experimentalists now. If the Higgs-boson mass is below 50 GeV or so, the Z factories currently under operation should find them through the Bjorken process $Z^0 \to H\mu^+\mu^-$.[1-3] As the Higgs-boson mass increases, the branching ratio for $Z^0 \to H\mu^+\mu^-$ drops quickly and the search for the Higgs boson becomes difficult. It is then necessary to explore all the possible channels with reasonable branching ratios and controllable backgrounds. One possible process is $Z^0 \to H\gamma$, which has been proposed by Chan *et al.*[4] and investigated in detail by Baroso *et al.*[5] In a previous note,[6] we suggested that the process $Z^0 \to Hgg \to \tau^+\tau^- gg$ may be useful for Higgs-boson mass $m_H \gtrsim 50$ GeV. We intend to provide the details of the calculation in this work. In the next section, we shall derive the helicity amplitudes for $Z^0 \to Hgg$. The form factors are expressed in terms of scalar integrals. Although the calculation is straightforward in principle, subtleties arise as all the relevant momenta involved in the box diagram are either timelike or lightlike in most kinematical regions of interest. Using the results of 't Hooft and Veltman,[7] we derive in Appendix C an expression for the four-point scalar integrals in terms of sixteen Spence functions which is valid when at least one of the relevant momenta is lightlike. This formula is useful beyond the present context since evaluating the higher-point scalar integrals can usually be reduced to evaluating four-point scalar integrals with one lightlike momentum.[7]

To make our work useful away from the Z^0 pole and for polarized beams, we compute the differential cross section for $e^+e^- \to Hgg$ in Sec. III. The electron mass is neglected and the diagrams in which the Higgs boson couples directly to the electron or the positron are ignored. Some details concerning the scalar integrals and certain kinematical factors are given in the Appendixes.

II. HELICITY AMPLITUDES FOR $Z^0 \to Hgg$

To lowest order in electroweak and strong couplings, the Feynman diagrams contributing to the process $Z^0 \to Hgg$ are sketched in Figs. 1 and 2. We have to permute the external momenta so that there are two triangle diagrams and six box diagrams for each species of quark running in the loop. The helicity amplitude $\mathcal{M}_{\lambda\lambda_1\lambda_2}$, where λ and λ_1, λ_2 denote the polarization of Z^0 and the two gluons, respectively, can be written as

$$\mathcal{M}_{\lambda\lambda_1\lambda_2} = \epsilon^\mu(k_3,\lambda)\epsilon_1^\nu(k_1,\lambda_1)\epsilon_2^\sigma(k_2,\lambda_2)\mathcal{R}_{\mu\nu\sigma}(k_1,k_2,k_3) \quad (1)$$

and

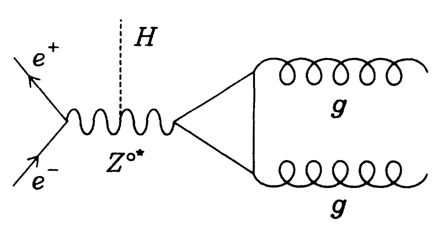

FIG. 1. Triangle diagram contributing to $e^+e^- \to Hgg$.

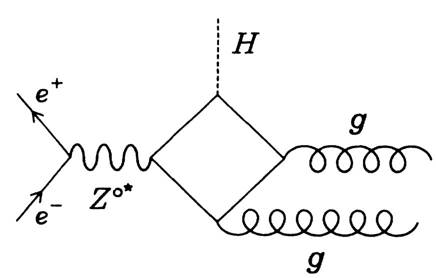

FIG. 2. Box diagram contributing to $e^+e^- \to Hgg$.

$$\mathcal{R}_{\mu\nu\sigma}=\frac{g^2 g_s^2 Q^j}{32\pi^2 m_Z \cos^2\theta_W}\left[\frac{m_Z^2 g_\mu^\alpha-(k_1+k_2)_\mu (k_1+k_2)^\alpha}{(k_1+k_2)^2-m_Z^2}\mathcal{R}^T_{\alpha\nu\sigma}+\tfrac{1}{2}\mathcal{R}^B_{\alpha\nu\sigma}\right] . \tag{2}$$

In the above formulas, color indices are suppressed, k_1 and k_2 are the momenta of the two gluons, k_3 is the momenta of Z^0, g and g_s denote the electroweak and the strong coupling constants respectively, $Q^j=1$ for $j=u,c,t$ quarks, and $Q^j=-1$ for $j=d,s,b$ quarks. $\mathcal{R}^T$ and $\mathcal{R}^B$ denote respectively the contributions from the triangle and the box diagrams. We have

$$\begin{aligned}\mathcal{R}^T_{\mu\nu\sigma}(k_1,k_2)= {} & A_1\Delta(k_1 e_\mu e_\nu e_\sigma)+A_2\Delta(k_2 e_\mu e_\nu e_\sigma)+A_3 k_{1\nu}\Delta(k_1 k_2 e_\mu e_\sigma)\\ & +A_4 k_{2\nu}\Delta(k_1 k_2 e_\mu e_\sigma)+A_5 k_{1\sigma}\Delta(k_1 k_2 e_\mu e_\nu)+A_6 k_{2\sigma}\Delta(k_1 k_2 e_\mu e_\nu) ,\end{aligned} \tag{3}$$

$$\begin{aligned}\mathcal{R}^B_{\mu\nu\sigma}(k_1,k_2,k_3)= {} & A_7\Delta(k_1 e_\mu e_\nu e_\sigma)+A_8\Delta(k_2 e_\mu e_\nu e_\sigma)+A_9\Delta(k_3 e_\mu e_\nu e_\sigma)\\ & +(A_{10}k_1+A_{13}k_2+A_{16}k_3)_\nu\Delta(k_1 k_2 e_\mu e_\sigma)+(A_{11}k_1+A_{14}k_2+A_{17}k_3)_\nu\Delta(k_1 k_3 e_\mu e_\sigma)\\ & +(A_{12}k_1+A_{15}k_2+A_{18}k_3)_\nu\Delta(k_2 k_3 e_\mu e_\sigma)+(A_{19}k_1+A_{22}k_2+A_{25}k_3)_\sigma\Delta(k_1 k_2 e_\mu e_\nu)\\ & +(A_{20}k_1+A_{23}k_2+A_{26}k_3)_\sigma\Delta(k_1 k_3 e_\mu e_\nu)+(A_{21}k_1+A_{24}k_2+A_{27}k_3)_\sigma\Delta(k_2 k_3 e_\mu e_\nu)\\ & +(A_{28}k_{1\nu}k_{1\sigma}+A_{29}k_{1\nu}k_{2\sigma}+A_{30}k_{1\nu}k_{3\sigma}+A_{31}k_{2\nu}k_{1\sigma}+A_{32}k_{2\nu}k_{2\sigma}+A_{33}k_{2\nu}k_{3\sigma}\\ & \quad +A_{34}k_{3\nu}k_{1\sigma}+A_{35}k_{3\nu}k_{2\sigma}+A_{36}k_{3\nu}k_{3\sigma}+A_{37}g_{\nu\sigma})\Delta(k_1 k_2 k_3 e_\mu) ,\end{aligned} \tag{4}$$

where e_μ is an orthonormal basis and $\Delta(abcd)$ is the determinant of the matrix formed by the four four-vectors a,b,c,d so that

$$\Delta(k_1 e_\mu e_\nu e_\sigma)=k_1^\alpha \epsilon_{\alpha\mu\nu\sigma}, \quad \text{etc. ;}$$

A_i's are functions of Lorentz invariants formed from the relevant momenta. In writing down the general form of $\mathcal{R}^T$ and $\mathcal{R}^B$, we used the identities

$$k_\mu\epsilon_{\alpha\beta\gamma\delta}+k_\alpha\epsilon_{\beta\gamma\delta\mu}+k_\beta\epsilon_{\gamma\delta\mu\alpha}+k_\gamma\epsilon_{\delta\mu\alpha\beta}+k_\delta\epsilon_{\mu\alpha\beta\gamma}=0 , \tag{5}$$

$$g_{\sigma\mu}\epsilon_{\alpha\beta\gamma\delta}+g_{\sigma\alpha}\epsilon_{\beta\gamma\delta\mu}+g_{\sigma\beta}\epsilon_{\gamma\delta\mu\alpha}+g_{\sigma\gamma}\epsilon_{\delta\mu\alpha\beta}+g_{\sigma\delta}\epsilon_{\mu\alpha\beta\gamma}=0 \tag{6}$$

to suppress unnecessary terms.

Color gauge invariance implies the relations

$$A_1+k_1\cdot k_2 A_5+k_2^2 A_6=0 , \tag{7}$$

$$A_2+k_1^2 A_3+k_1\cdot k_2 A_4=0 , \tag{8}$$

$$k_1^2 A_{10}+k_1\cdot k_2 A_{13}+k_1\cdot k_3 A_{16}=-A_8 , \tag{9}$$

$$k_1^2 A_{11}+k_1\cdot k_2 A_{14}+k_1\cdot k_3 A_{17}=-A_9 , \tag{10}$$

$$k_1^2 A_{12}+k_1\cdot k_2 A_{15}+k_1\cdot k_3 A_{18}=0 , \tag{11}$$

$$k_1^2 A_{28}+k_1\cdot k_2 A_{31}+k_1\cdot k_3 A_{34}=A_{21}-A_{37} , \tag{12}$$

$$k_1^2 A_{29}+k_1\cdot k_2 A_{32}+k_1\cdot k_3 A_{35}=A_{24} , \tag{13}$$

$$k_1^2 A_{30}+k_1\cdot k_2 A_{33}+k_1\cdot k_3 A_{36}=A_{27}-A_{37} , \tag{14}$$

$$k_1\cdot k_2 A_{19}+k_2^2 A_{22}+k_2\cdot k_3 A_{25}=-A_7 , \tag{15}$$

$$k_1\cdot k_2 A_{20}+k_2^2 A_{23}+k_2\cdot k_3 A_{26}=0 , \tag{16}$$

$$k_1\cdot k_2 A_{21}+k_2^2 A_{24}+k_2\cdot k_3 A_{27}=A_9 , \tag{17}$$

$$k_1\cdot k_2 A_{28}+k_2^2 A_{29}+k_2\cdot k_3 A_{30}=-A_{11} , \tag{18}$$

$$k_1\cdot k_2 A_{31}+k_2^2 A_{32}+k_2\cdot k_3 A_{33}=-A_{14}-A_{37} , \tag{19}$$

$$k_1\cdot k_2 A_{34}+k_2^2 A_{35}+k_2\cdot k_3 A_{36}=-A_{17} . \tag{20}$$

Using Bose symmetry, we get relations such as

$$\begin{aligned}&A_3(k_1,k_2)=-A_6(k_2,k_1) ,\\ &A_8(k_1,k_2,k_3)=-A_7(k_2,k_1,k_3) .\end{aligned} \tag{21}$$

We shall not write down all of these relations explicitly. In our lowest-order calculation, we have

$$A_i=0, \quad i=28,29,\ldots,36 . \tag{22}$$

Together with Eqs. (7)–(20), we find

$$A_{11}=A_{17}=A_{24}=A_{27}=0 \tag{23}$$

and that we need only compute the form factors A_5, A_6, A_{10}, A_{12}, A_{13}, A_{15}, A_{16}, A_{18}, and A_{37}. The rest are given by Eqs. (7)–(9), (21), and

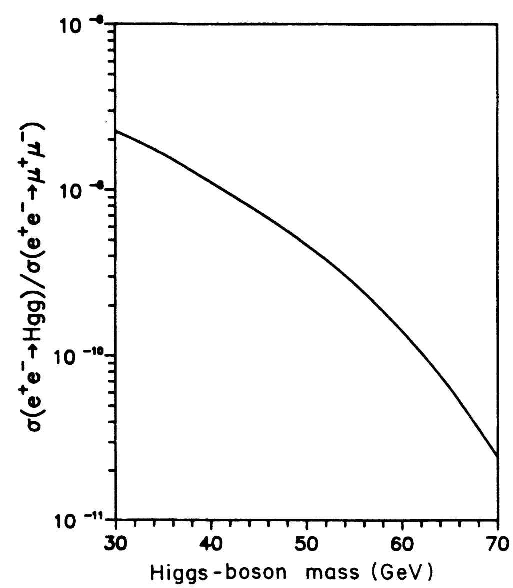

FIG. 3. The ratio for the integrated cross section $\sigma(e^+e^-\to Hgg)/\sigma(e^+e^-\to\mu^+\mu^-)$ near the Z^0 pole as a function of the Higgs-boson mass.

$$
\begin{aligned}
&A_9 = k_1 \cdot k_2 A_{37}, \quad A_{14} = -A_{37}, \quad A_{21} = A_{37} , \\
&A_{19}(k_1,k_2,k_3) = -A_{13}(k_2,k_1,k_3) , \\
&A_{20}(k_1,k_2,k_3) = A_{15}(k_2,k_1,k_3) , \\
&A_{22}(k_1,k_2,k_3) = -A_{10}(k_2,k_1,k_3) , \\
&A_{23}(k_1,k_2,k_3) = A_{12}(k_2,k_1,k_3) , \\
&A_{25}(k_1,k_2,k_3) = -A_{16}(k_2,k_1,k_3) , \\
&A_{26}(k_1,k_2,k_3) = A_{18}(k_2,k_1,k_3) .
\end{aligned} \tag{24}
$$

Moreover, we still have Eq. (11) relating A_{12}, A_{15}, and A_{18} and have also the relation

$$A_{37}(k_1,k_2,k_3) = -A_{37}(k_2,k_1,k_3) . \tag{25}$$

We note that gauge invariance and Bose symmetry allow us to express the naively divergent form factors A_1, A_2, A_7, A_8, and A_9 in terms of convergent ones. The needed form factors can be read off from the Feynman amplitudes using the method of Passarino and Veltman.[8] Using their notation, we have, for loop-quark mass m,

$$A_5(k_1k_2;m) = 2(C^1_{12} + C^2_{12} + C^1_{23} + C^2_{23}) , \tag{26}$$

$$A_6(k_1k_2;m) = 2(C^1_{12} + C^2_{11} + C^1_{23} + C^2_{21}) , \tag{27}$$

$$
\begin{aligned}
A_{10}(k_1,k_2,k_3;m) = 2m^2(&D^1_0 + 3D^1_{11} + 2D^1_{21} + D^2_0 + 3D^2_{12} + 2D^2_{22} + D^3_{12} + 2D^3_{22} \\
&+ D^4_{13} + 2D^4_{23} + D^5_0 + 3D^5_{11} + 2D^5_{21} + D^6_{13} + 2D^6_{23}) ,
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
A_{12}(k_1,k_2,k_3;m) = m^2(&D^1_0 + 2D^1_{11} + 2D^1_{13} + 4D^1_{25} + D^2_0 + 2D^2_{12} + 2D^2_{13} + 4D^2_{26} \\
&+ D^3_0 + 2D^3_{11} + 2D^3_{12} + 4D^3_{24} + D^4_0 + 2D^4_{11} + 2D^4_{13} + 4D^4_{25} \\
&+ D^5_0 + 2D^5_{11} + 2D^5_{12} + 4D^5_{24} + D^6_0 + 2D^6_{12} + 2D^6_{13} + 4D^6_{26}) ,
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
A_{13}(k_1,k_2,k_3;m) = 2m^2(&D^1_0 + 3D^1_{12} + 2D^1_{24} + D^2_0 + D^2_{11} + 2D^2_{12} + 2D^2_{24} + D^3_{13} + 2D^3_{26} \\
&- D^4_{12} + 2D^4_{13} + 2D^4_{26} + D^5_{13} + 2D^5_{25} + D^6_0 + D^6_{11} + 2D^6_{13} + 2D^6_{25}) ,
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
A_{15}(k_1,k_2,k_3;m) = 2m^2(&D^1_{12} + 2D^1_{26} + D^2_0 + D^2_{11} + 2D^2_{13} + 2D^2_{25} + D^3_{13} + 2D^3_{25} \\
&+ D^4_0 + 2D^4_{11} + D^4_{12} + 2D^4_{24} + D^5_{13} + 2D^5_{26} + D^6_0 + D^6_{11} + 2D^6_{12} + 2D^6_{24}) ,
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
A_{16}(k_1,k_2,k_3;m) = -m^2(&D^1_0 + 4D^1_{13} + 4D^1_{25} + D^2_0 + 4D^2_{13} + 4D^2_{26} + D^3_0 + 4D^3_{12} + 4D^3_{24} \\
&+ D^4_0 + 4D^4_{13} + 4D^4_{25} + D^5_0 + 4D^5_{12} + 4D^5_{24} + D^6_0 + 4D^6_{13} + 4D^6_{26}) ,
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
A_{18}(k_1,k_2,k_3;m) = -2m^2(&D^1_{13} + 2D^1_{23} + D^2_{13} + 2D^2_{23} + D^3_0 + 3D^3_{11} + 2D^3_{21} \\
&+ D^4_0 + 3D^4_{11} + 2D^4_{21} + D^5_{12} + 2D^5_{22} + D^6_0 + 3D^6_{12} + 2D^6_{22}) ,
\end{aligned} \tag{33}
$$

$$A_{37}(k_1,k_2,k_3;m) = m^2(D^1_0 + 2D^1_{13} - D^2_0 - 2D^2_{13} + D^3_0 + 2D^3_{11} - D^4_0 - 2D^4_{11} - D^5_0 - 2D^5_{12} + D^6_0 + 2D^6_{12}) , \tag{34}$$

where

$$
\begin{aligned}
&C^1_{ij} = C_{ij}(-k_1, -k_2; m) , \\
&C^2_{ij} = C_{ij}(-k_2, -k_1; m) , \\
&D^1_{ij} = D_{ij}(-k_1, -k_2, k_3; m) , \\
&D^2_{ij} = D_{ij}(-k_2, -k_1, k_3; m) , \\
&D^3_{ij} = D_{ij}(k_3, -k_1, -k_2; m) , \\
&D^4_{ij} = D_{ij}(k_3, -k_2, -k_1; m) , \\
&D^5_{ij} = D_{ij}(-k_1, k_3, -k_2; m) , \\
&D^6_{ij} = D_{ij}(-k_2, k_3, -k_1; m) .
\end{aligned} \tag{35}
$$

C_{ij} and D_{ij} are scalar integrals defined in Ref. 8. We give the relevant formulas in Appendix A. To get the helicity amplitude $\mathcal{M}_{\lambda\lambda_1\lambda_2}$, we still need to compute determinants such as $\Delta(k_1\epsilon\epsilon_1\epsilon_2)$, $\Delta(k_1k_2\epsilon\epsilon_1)$, $\Delta(k_1k_2k_3,\epsilon)$, etc. The convention for the polarization vectors and the formulas for the determinants are given in Appendix B. We define the polarization density matrix for $Z^0 \to Hgg$ by

$$\mathcal{P}_{\lambda\lambda'}=(N^2-1)\sum_{\lambda_1\lambda_2}\mathcal{M}_{\lambda\lambda_1\lambda_2}\mathcal{M}^*_{\lambda'\lambda_1\lambda_2}\ , \tag{36}$$

where summing over SU(N) color produces the color factor N^2-1.

III. $e^+e^-\to Hgg$ FOR POLARIZED BEAMS

Since only the axial-vector current contributes to the triangle and the box diagrams in the present case, $e^+e^-\to Hgg$ cannot go through the virtual photon. It can go through the Higgs particle but this is suppressed by the small electron mass. Similarly, the processes in which the final-state Higgs boson is emitted from the incoming beam can be ignored. As a result, the longitudinal part of the propagator of the virtual Z^0 coming from e^+e^- annihilation does not contribute to the Feynman amplitude. It is then needed only to compute the polarization density matrix $E_b^2\mathcal{D}_{\lambda\lambda'}$ for the Z^0 production by $e^+e^-\to Z^0$ with a beam energy E_b and to put in the Breit-Wigner propagator for the virtual Z^0 to obtain the complete scattering cross section for $e^+e^-\to Hgg$. We let θ be the angle between the Higgs boson and the incoming beam and ϕ be the azimuthal angle measuring from the Higgs-boson–gluon–gluon plane to the Higgs-boson–beam plane. Choosing the polarization vector $\epsilon(\lambda)$ of Z^0 to be

$$\begin{aligned}&\epsilon(\lambda)=(0,0,0,1),\quad \lambda=0\ ,\\ &\epsilon(\lambda)=\frac{1}{\sqrt{2}}(0,\mp 1,-i,0),\quad \lambda=\pm\ ,\end{aligned} \tag{37}$$

which has to be consistent with the choice made in computing $\mathcal{P}_{\lambda\lambda'}$, we have

$$\mathcal{D}_{\lambda\lambda'}=\frac{g^2}{2\cos^2\theta_W}\{\mathcal{A}(\delta_{\lambda\lambda'}-l_\lambda l^*_{\lambda'})+(a^2-b^2)(l_{1\lambda}l^*_{2\lambda'}+l_{2\lambda}l^*_{1\lambda'})-[(a^2+b^2)(P^3-\bar{P}^3)+2ab(1-P^3\bar{P}^3)]\mathcal{C}_{\lambda\lambda'}\} \tag{38}$$

where

$$\mathcal{A}=(a^2+b^2)(1-P^3\bar{P}^3)+2ab(P^3-\bar{P}^3)-(a^2-b^2)[(P^1\bar{P}^1+P^2\bar{P}^2)\cos2\phi+(P^1\bar{P}^2-P^2\bar{P}^1)\sin2\phi]\ , \tag{39}$$

$$l_\lambda=-\left[\cos\theta,-\frac{1}{\sqrt{2}}\sin\theta\, e^{i\phi},\frac{1}{\sqrt{2}}\sin\theta\, e^{-i\phi}\right]_{\lambda=0,+,-}\ , \tag{40}$$

$$\begin{aligned}l_{1\lambda}=&\,(P^1\cos\phi+P^2\sin\phi)\left[\sin\theta,\frac{1}{\sqrt{2}}\cos\theta\, e^{i\phi},-\frac{1}{\sqrt{2}}\cos\theta\, e^{-i\phi}\right]_{\lambda=0,+,-}\\ &+i(-P^1\sin\phi+P^2\cos\phi)\left[0,\frac{1}{\sqrt{2}}e^{i\phi},-\frac{1}{\sqrt{2}}e^{-i\phi}\right]_{\lambda=0,+,-}\ ,\end{aligned} \tag{41}$$

$$\begin{aligned}l_{2\lambda}=&-(\bar{P}^1\cos\phi+\bar{P}^2\sin\phi)\left[\sin\theta,\frac{1}{\sqrt{2}}\cos\theta\, e^{i\phi},-\frac{1}{\sqrt{2}}\cos\theta\, e^{-i\phi}\right]_{\lambda=0,+,-}\\ &+i(-\bar{P}^1\sin\phi+\bar{P}^2\cos\phi)\left[0,\frac{1}{\sqrt{2}}e^{i\phi},-\frac{1}{\sqrt{2}}e^{-i\phi}\right]_{\lambda=0,+,-}\ ,\end{aligned} \tag{42}$$

$$\begin{aligned}&\mathcal{C}_{00}=0,\quad \mathcal{C}_{+-}=C_{-+}=0,\quad \mathcal{C}_{++}=-\mathcal{C}_{--}=\cos\theta\ ,\\ &\mathcal{C}_{0+}=\mathcal{C}_{-0}=\frac{1}{\sqrt{2}}\sin\theta\, e^{-i\phi},\quad \mathcal{C}_{0-}=\mathcal{C}_{+0}=\frac{1}{\sqrt{2}}\sin\theta\, e^{i\phi}\ .\end{aligned} \tag{43}$$

In the above formulas, P^i and $\bar{P}^i$ are the components of the polarization vectors of the electron and the positron respectively, measuring in the frame in which the direction of motion of the Higgs boson is taken to be the z axis and the Higgs-boson–gluon–gluon plane is taken to be the xz plane. a and b are the coupling of Z^0 to the fermions so that, for electrons,

$$a=-\tfrac{1}{2}+2\sin^2\theta_W,\quad b=\tfrac{1}{2}\ . \tag{44}$$

The differential cross section is then given by

$$d\sigma=\frac{1}{8}\frac{1}{(4E_b^2-m_Z^2)^2+m_Z^2\Gamma_z^2}\left[\sum_{\lambda\lambda'}\mathcal{D}_{\lambda\lambda'}\mathcal{P}_{\lambda'\lambda}\right]d\Phi_3\ , \tag{45}$$

where $d\Phi_3$ is the three-particle phase-space element for the Higgs-boson–gluon–gluon final state. Let ψ be the angle between the Higgs-boson–beam plane and the plane containing the beam and the polarization vector of the electron, measuring from the latter plane. We have, explicitly,

$$d\sigma=\frac{1}{2(4\pi)^5}J(z,\omega)\left[\sum_{\lambda\lambda'}D_{\lambda\lambda'}(\theta,\phi,\psi)\mathcal{P}_{\lambda'\lambda}(z,\omega)\right]dz\,d\cos\omega\,d\cos\theta\,d\phi\,d\psi\;, \tag{46}$$

where $0\le\omega\le\pi$ is the angle between the two gluons and

$$J(z,\omega)=\frac{E_b^2}{(4E_b^2-m_Z^2)^2+m_Z^2\Gamma_Z^2}\frac{1-z+l}{(1-\cos\omega)^2}\left[(2-z)^2-\frac{8(1-z+l)}{1-\cos\omega}\right]^{-1/2}, \tag{47}$$

$$z=\frac{E_H}{E_b},\quad l=\frac{m_H^2}{4E_b^2} \tag{48}$$

with E_H being the energy of the Higgs boson in the center-of-mass frame. Note that when the electron is longitudinally polarized we have

$$P^i=(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta)_{i=1,2,3} \tag{49}$$

and when it is transversely polarized we have

$$P^i=(-\sin\phi\sin\psi-\cos\theta\cos\phi\cos\psi,\cos\phi\sin\psi-\cos\theta\sin\phi\cos\psi,\sin\theta\cos\psi)\;. \tag{50}$$

For the unpolarized beam, the formula is simplified if we carry out the integration over θ, ϕ, and ψ. We have, in this case,

$$d\sigma=\frac{a^2+b^2}{6(4\pi)^3}\frac{g^2}{2\cos^2\theta_W}J(z,\omega)\left[\sum_\lambda\mathcal{P}_{\lambda\lambda}\right]dz\,d\cos\omega\;. \tag{51}$$

IV. DISCUSSION

As the possibility of having polarized beams at the SLAC Linear Collider and the CERN collider LEP is being seriously considered, it is useful to consider an example of how to apply our formulas to this case. Assume that the positron is unpolarized. Equation (38) for $\mathcal{D}_{\lambda\lambda'}$ is greatly simplified. For longitudinally polarized electrons, we have, from Eq. (49), $P^3=\cos\theta$. Substituting $\mathcal{D}_{\lambda\lambda'}$ into Eq. (46) and carrying out the integrations over θ, ϕ, and ψ, we get

$$d\sigma=\frac{a^2+b^2}{6(4\pi)^3}\frac{g^2}{2\cos^2\theta_W}J(z,\omega)\left[\sum_\lambda d_\lambda^l\mathcal{P}_{\lambda\lambda}\right]dz\,d\cos\omega,\quad d_\lambda^l=\langle 1,\tfrac{1}{2},\tfrac{3}{2}\rangle_{\lambda=0,+,-}\;. \tag{52}$$

Similarly for transversely polarized electron, we have $P^3=\sin\theta\cos\psi$ so that

$$d\sigma=\frac{a^2+b^2}{6(4\pi)^3}\frac{g^2}{2\cos^2\theta_W}J(z,\omega)\left[\sum_\lambda d_\lambda^t\mathcal{P}_{\lambda\lambda}\right]\frac{1}{2\pi}dz\,d\cos\omega\,d\psi\;,$$
$$d_\lambda^t=\langle 1,1,1\rangle-\frac{3\pi ab}{32(a^2+b^2)}\cos\psi\langle 6,5,5\rangle\;. \tag{53}$$

To summarize, we have obtained a formula for the differential cross section of $e^+e^-\to Hgg$ for polarized beams. To calculate $\mathcal{P}_{\lambda\lambda'}$, we have to calculate the form factors for the helicity amplitudes and using the results in the Appendix B for the kinematical factors. The calculation of the form factors is reduced to solving the linear equations given in Appendix A and evaluating Spence functions for the C_0 and D_0 functions using the formula in Ref. 7 for C_0 and that in Appendix C for D_0.

In Fig. 3, we plotted the ratio $\sigma(e^+e^-\to Hgg)/\sigma(e^+e^-\to\mu^+\mu^-)$ near the Z^0 pole as a function of the Higgs-boson mass. The branching ratio turns out to be too small to have practical significance. Nevertheless, some of the formulas we derived will be useful in computing the helicity amplitudes of other processes.

ACKNOWLEDGMENTS

This work was supported in part by a grant from the National Science Council, Taiwan, Republic of China, under Contract No. NSC78-0208-M001-68.

APPENDIX A

Using the same conventions as in Ref. 8, we may rearrange the results there to have the linear equations relating the form factors C_{ij} to B_0, B_1, and C_0,

$$\begin{vmatrix} 2p_1^2 & 2p_1\cdot p_2 & 0 & 0 & 0 & 0 \\ 2p_1\cdot p_2 & 2p_2^2 & 0 & 0 & 0 & 0 \\ \frac{3}{2}p_1^2 & \frac{1}{2}(p_5^2-p_1^2) & 2p_1^2 & 2p_1\cdot p_2 & 0 & 0 \\ p_5^2-p_1^2 & 0 & 2p_1\cdot p_2 & 2p_2^2 & 0 & 0 \\ 0 & p_1^2 & 0 & 0 & 2p_1^2 & 2p_1\cdot p_2 \\ \frac{1}{2}p_1^2 & \frac{3}{2}(p_5^2-p_1^2) & 0 & 0 & 2p_1\cdot p_2 & 2p_2^2 \end{vmatrix} \begin{bmatrix} C_{11} \\ C_{12} \\ C_{21} \\ C_{23} \\ C_{23} \\ C_{22} \end{bmatrix} = \begin{bmatrix} -p_1^2C_0+B_0(2,3)-B_0(1,3) \\ (p_1^2-p_5^2)C_0+B_0(1,3)-B_0(1,2) \\ \frac{1}{2}[1-2m^2C_0-B_0(2,3)-2B_1(1,3)] \\ B_1(1,3)-B_1(1,2) \\ B_1(2,3)-B_1(1,3) \\ \frac{1}{2}[1-2m^2C_0+B_0(2,3)+2B_1(1,3)] \end{bmatrix} , \quad \text{(A1)}$$

and the linear equations relating D_{ij} to C_0, C_{ij}, and D_0:

$$\begin{bmatrix} & X_{3\times 3} & & 0 & 0 & 0 \\ 2p_1^2 & p_5^2-p_1^2 & p_4^2-p_5^2 & & & \\ p_5^2-p_1^2 & 0 & 0 & X_{3\times 3} & 0 & 0 \\ p_4^2-p_5^2 & 0 & 0 & & & \\ 0 & p_1^2 & 0 & & & \\ p_1^2 & 2(p_5^2-p_1^2) & p_4^2-p_5^2 & 0 & X_{3\times 3} & 0 \\ 0 & p_4^2-p_5^2 & 0 & & & \\ 0 & 0 & p_1^2 & & & \\ 0 & 0 & p_5^2-p_1^2 & 0 & 0 & X_{3\times 3} \\ p_1^2 & p_5^2-p_1^2 & 2(p_4^2-p_5^2) & & & \end{bmatrix} \begin{bmatrix} D_{11} \\ D_{12} \\ D_{13} \\ D_{21} \\ D_{24} \\ D_{25} \\ D_{24} \\ D_{22} \\ D_{26} \\ D_{25} \\ D_{26} \\ D_{23} \end{bmatrix} = \begin{bmatrix} -p_1^2D_0+C_0(2,3,4)-C_0(1,3,4) \\ (p_1^2-p_5^2)D_0+C_0(1,3,4)-C_0(1,2,4) \\ (p_5^2-p_4^2)D_0+C_0(1,2,4)-C_0(1,2,3) \\ -C_{11}(1,3,4)-2m^2D_0 \\ C_{11}(1,3,4)-C_{11}(1,2,4) \\ C_{11}(1,2,4)-C_{11}(1,2,3) \\ C_{11}(2,3,4)-C_{11}(1,3,4) \\ C_{11}(1,3,4)-C_{12}(1,2,4)+C_0(2,3,4)-2m^2D_0 \\ C_{12}(1,2,4)-C_{12}(1,2,3) \\ C_{12}(2,3,4)-C_{12}(1,3,4) \\ C_{12}(1,3,4)-C_{12}(1,2,4) \\ C_{12}(1,2,4)+C_0(2,3,4)-2m^2D_0 \end{bmatrix} , \quad \text{(A2)}$$

where

$$X_{3\times 3} = \begin{bmatrix} 2p_1^2 & 2p_1\cdot p_2 & 2p_1\cdot p_3 \\ 2p_1\cdot p_2 & 2p_2^2 & 2p_2\cdot p_3 \\ 2p_1\cdot p_3 & 2p_2\cdot p_3 & 2p_3^2 \end{bmatrix} ,$$

$p_1+p_2+p_3+p_4=0$, and $p_5=p_1+p_2$. B_0, B_1 are elementary functions while C_0 and D_0 are scalar one-loop integrals. They are given in Ref. 7. For the D_0 function, the result there does not apply immediately to our case (see Appendix C).

APPENDIX B

For the polarization vector $e_\lambda(k)$ of a vector boson with the four-momentum $k=(k^0,k_x,k_y,k_z)$, we use the following convention:

$$e_0(k)=\frac{1}{m}k, \quad m^2=k^2 ,$$

$$e_1(k)=\frac{1}{|\mathbf{k}(|\mathbf{k}|+k_z)} \times(0,-k_z(|\mathbf{k}|+k_z)-k_y^2,k_xk_y,k_x(|\mathbf{k}|+k_z)) ,$$

$$e_2(k)=\frac{1}{|\mathbf{k}(|\mathbf{k}|+k_z)} \times(0,-k_xk_y,k_z(|\mathbf{k}|+k_z)+k_x^2,-k_y(|\mathbf{k}|+k_z)) , \quad \text{(B1)}$$

$$e_3(k)=-\frac{1}{m}\left\langle|\mathbf{k}|,\frac{k^0}{|k|}\mathbf{k}\right\rangle ,$$

where $|\mathbf{k}|=(k_x^2+k_y^2+k_z^2)^{1/2}$. If the particle moves in the negative z direction, we choose to let k_y approach zero first in the above formulas. The helicity eigenstates of the vector boson are given by

$$\epsilon(k,\lambda=0)=e_3(k) ,$$
$$\epsilon(k,\lambda=\pm)=\frac{1}{\sqrt{2}}[\pm e_1(k)-ie_2(k)] \quad \text{(B2)}$$

for zero, positive, and negative helicity, respectively.

In the rest frame of Z^0, we introduce four dimensionless parameters $x=2k_1\cdot k_3/m_Z^2$, $y=2k_2\cdot k_3/m_Z^2$, $z=2k_4\cdot k_3/m_Z^2$, and $l=m_H^2/m_Z^2$ so that the four-momenta of each particle in the $Z^0\to Hgg$ process are given as

$$k_1=\frac{m_Z}{2}x(1,\sin\theta_1,0,\cos\theta_1) ,$$
$$k_2=\frac{m_Z}{2}y(1,-\sin\theta_2,0,\cos\theta_2) ,$$
$$k_3=m_Z(1,0,0,0) , \quad \text{(B3)}$$
$$k_4=\frac{m_Z}{2}z(1,0,0,\beta) ,$$

where $l=\frac{1}{4}(1-\beta^2)z^2$. k_1, k_2, k_3, and k_4 are the momenta of the two gluons, Z^0, and Higgs boson, respectively. The trigonometric functions of the angles can be expressed in terms of the Lorentz invariants. We have

$$\cos\theta_1=\frac{1}{2\beta zx}(y^2-x^2-\beta^2z^2)\ ,$$
$$\cos\theta_2=\frac{1}{2\beta zy}(x^2-y^2-\beta^2z^2)\ ,$$
$$\sin\theta_1=\frac{1}{2\beta zx}\Theta^{1/2}\ , \tag{B4}$$
$$\sin\theta_2=\frac{1}{2\beta zy}\Theta^{1/2}\ ,$$

where $\Theta=[(x+y)^2-\beta^2z^2][\beta^2z^2-(x-y)^2]$. The kinematically allowed regions are determined by requiring the trigonometric functions to be well defined and are given by

$$2\sqrt{l}\le z\le 1+l,\quad -\beta z\le x-y\le\beta z\quad (x+y+z=2). \tag{B5}$$

Computing the determinants, we have the following.

(1) For Z^0 with helicity $\lambda=0$,

$$\Delta(k_1\epsilon\epsilon_1\epsilon_2)=\frac{im_Z}{8\beta yz}\delta_1(\lambda,\lambda_1,\lambda_2)\ ,$$
$$\Delta(k_2\epsilon\epsilon_1\epsilon_2)=\frac{im_Z}{8\beta xz}\delta_1(\lambda,\lambda_1,\lambda_2)\ ,$$
$$\Delta(k_3\epsilon\epsilon_1\epsilon_2)=\frac{im_Z}{4\beta xyz}\delta_1(\lambda,\lambda_1,\lambda_2)\ ,$$
$$\Delta(k_1k_2\epsilon\epsilon_1)=\Delta(k_1k_2\epsilon\epsilon_2)=-\frac{im_Z^2}{8\sqrt{2}\beta z}(x+y)\Theta^{1/2}\ , \tag{B6}$$
$$\Delta(k_1k_3\epsilon\epsilon_1)=\Delta(k_1k_3\epsilon\epsilon_2)=-\Delta(k_2k_3\epsilon\epsilon_1)=-\Delta(k_2k_3\epsilon\epsilon_2)=-\frac{im_Z^2}{4\sqrt{2}\beta z}\Theta^{1/2}\ ,$$
$$\Delta(k_1k_2k_3\epsilon)=0\ ,$$

where

$$\delta_1(\lambda,\lambda_1,\lambda_2)\equiv-(\lambda_2x-\lambda_1y)[(\lambda_1x+\lambda_2y)^2-(\beta z)^2]\ , \tag{B7}$$

(2) For Z^0 with helicity $\lambda=\pm$,

$$\Delta(k_1\epsilon\epsilon_1\epsilon_2)=\frac{im_Z}{8\beta yz}\delta_2(\lambda,\lambda_1,\lambda_2)\ ,$$
$$\Delta(k_2\epsilon\epsilon_1\epsilon_2)=\frac{im_Z}{8\beta xz}\delta_2(\lambda,\lambda_1,\lambda_2)\ ,$$
$$\Delta(k_3\epsilon\epsilon_1\epsilon_2)=\frac{im_Z}{4\beta xyz}\delta_2(\lambda,\lambda_1,\lambda_2)\ ,$$
$$\Delta(k_1k_2\epsilon\epsilon_1)=\frac{im_z^2}{16\beta z}\lambda(x-y+\lambda\lambda_1\beta z)[(x+y)^2-(\beta z)^2]\ ,$$
$$\Delta(k_1k_2\epsilon\epsilon_2)=\frac{im_z^2}{16\beta z}\lambda(x-y-\lambda\lambda_2\beta z)[(x+y)^2-(\beta z)^2]\ , \tag{B8}$$
$$\Delta(k_1k_3\epsilon\epsilon_1)=\frac{im_Z^2}{8\beta z}\lambda[(x+\lambda\lambda_1\beta z)^2-y^2]\ ,$$
$$\Delta(k_1k_3\epsilon\epsilon_2)=\frac{im_Z^2}{8\beta yz}(\lambda x-\lambda_1\beta z)[y^2-(\lambda_1x+\lambda\beta z)^2]\ ,$$
$$\Delta(k_2k_3\epsilon\epsilon_1)=\frac{im_Z^2}{8\beta xz}(\lambda y-\lambda_2\beta z)[x^2-(\lambda_2y+\lambda\beta z)^2]\ ,$$
$$\Delta(k_2k_3\epsilon\epsilon_2)=\frac{im_Z^2}{8\beta z}\lambda[(y+\lambda\lambda_2\beta z)^2-x^2]\ ,$$
$$\Delta(k_1k_2k_3\epsilon)=\frac{im_Z^3}{8\sqrt{2}}\Theta^{1/2}\ ,$$

where

$$\delta_2(\lambda,\lambda_1,\lambda_2)\equiv-\frac{1}{\sqrt{2}}(\lambda\lambda_2x+\lambda\lambda_1y+\lambda_1\lambda_2\beta z)\Theta^{1/2}\ . \tag{B9}$$

For the $e^+e^-\to Hgg$ process away from the Z^0 pole, we have to replace m_Z by $2E_b$, the total beam energy, in the above formulas.

APPENDIX C

The scalar one-loop integral had been studied by 't Hooft and Veltman.[7] They showed that a given scalar four-point function can in most cases be expressed in terms of 24 Spence functions in addition to some logarithms. When one of the external momenta is lightlike as in our case, their formula cannot be applied directly, in general. However, one can derive from their result an alternate formula by taking the proper limit.

We shall follow the conventions of Ref. 7. Assuming the external momentum p_1 is lightlike. In our case, $m_1=m_2$ and we have to be careful in taking the limit $m_1\to m_2$ in the formula for the D function in Ref. 7. After some algebra, we get

$$\frac{D}{i\pi^2}=-\frac{1}{k(k+h)\eta_0}\Bigg[-\mathrm{Sp}\left[\frac{y_1}{y_1-y_{1+}}\right]-\mathrm{Sp}\left[\frac{y_1}{y_1-y_{1-}}\right]$$
$$-\tfrac{1}{2}\ln^2(y_1-y_{1+})-\tfrac{1}{2}\ln^2(y_1-y_{1-})-i\pi\theta(k)[\ln(y_1-y_{1+})-\ln(y_1-y_{1-})]$$
$$+\mathrm{Sp}\left[\frac{y_2}{y_2-y_{2+}}\right]-\mathrm{Sp}\left[\frac{y_2-1}{y_2-y_{2+}}\right]+\mathrm{Sp}\left[\frac{y_2}{y_2-y_{2-}}\right]-\mathrm{Sp}\left[\frac{y_2-1}{y_2-y_{2-}}\right]$$
$$+i\pi\theta[-k(k+h)][\ln(y_2-y_{2+})-\ln(y_2-y_{2-})]$$
$$+\mathrm{Sp}\left[\frac{y_3}{y_3-y_{3+}}\right]+\mathrm{Sp}\left[\frac{y_3}{y_3-y_{3-}}\right]+\tfrac{1}{2}\ln^2(y_3-y_{3+})+\tfrac{1}{2}\ln^2(y_3-y_{3-})$$

$$+i\pi\theta(k+h)[\ln(y_3-y_{3+})-\ln(y_3-y_{3-})]$$
$$+\mathrm{Sp}\left[\frac{y_4}{y_4-y_{4+}}\right]+\mathrm{Sp}\left[\frac{y_4}{y_4-y_{4-}}\right]+\tfrac{1}{2}\ln^2(y_4-y_{4+})+\tfrac{1}{2}\ln^2(y_4-y_{4-})$$
$$+i\pi\theta(k)[\ln(y_4-y_{4+})-\ln(y_4-y_{4-})]$$
$$-\mathrm{Sp}\left[\frac{y_5}{y_5-y_{5+}}\right]+\mathrm{Sp}\left[\frac{y_5-1}{y_5-y_{5+}}\right]-\mathrm{Sp}\left[\frac{y_5}{y_5-y_{5-}}\right]+\mathrm{Sp}\left[\frac{y_5-1}{y_5-y_{5-}}\right]$$
$$-i\pi\theta[-k(k+h)][\ln(y_5-y_{5+})-\ln(y_5-y_{5-})]$$
$$-\mathrm{Sp}\left[\frac{y_6}{y_6-y_{6+}}\right]-\mathrm{Sp}\left[\frac{y_6}{y_6-y_{6-}}\right]-\tfrac{1}{2}\ln^2(y_6-y_{6+})-\tfrac{1}{2}\ln^2(y_6-y_{6-})$$
$$-i\pi\theta(k+h)[\ln(y_6-y_{6+})-\ln(y_6-y_{6-})]\Bigg] , \quad \text{(C1)}$$

where $k=p_{14}^2-p_{24}^2$, $k+h=p_{13}^2-p_{23}^2$,

$$y_1=\frac{1}{2b_2}[-e_1-(2d_0b_2-c_1e_1)\eta_0^{-1}] ,$$
$$y_3=\frac{1}{2b_2}[b_1-(2c_0b_2-c_1b_1)\eta_0^{-1}] ,$$
$$y_4=\frac{1}{2b_2}\{-(e_1-1)-[2d_0b_2-c_1(e_1-1)]\eta_0^{-1}\} ,$$
$$y_6=\frac{1}{2b_2}\{(b_1-1)-[2c_0b_2-c_1(b_1-1)]\eta_0^{-1}\} , \quad \text{(C2)}$$
$$y_2=\frac{1}{2a_0}[-d_0+(2a_0e_1-c_1d_0)\eta_0^{-1}] ,$$
$$y_5=\frac{1}{2a_0}[-d_0+(2a_0(e_1-1)-c_1d_0)\eta_0^{-1}] ,$$
$$\eta_0=\sqrt{c_1^2-4a_0b_2} ,$$

and

$$y_{1\pm}=\frac{1}{2b_2}\left[-e_1\pm\sqrt{e_1^2-4b_2(f_0-i\varepsilon)}\right] ,$$
$$y_{3\pm}=\frac{1}{2b_2}\left[b_1\pm\sqrt{b_1^2-4b_2(b_0-i\varepsilon)}\right] ,$$
$$y_{4\pm}=\frac{1}{2b_2}\left[-(e_1-1)\pm\sqrt{(e_1-1)^2-4b_2(f_0-i\varepsilon)}\right] ,$$
$$\text{(C3)}$$
$$y_{6\pm}=\frac{1}{2b_2}\left[(b_1-1)\pm\sqrt{(b_1-1)^2-4b_2(b_0-i\varepsilon)}\right] ,$$
$$y_{2\pm}=y_{5\pm}=\frac{1}{2a_0}\left[-d_0\pm\sqrt{d_0^2-4a_0(f_0-i\varepsilon)}\right] .$$

Our convention is such that the imaginary parts of y_{i+} and y_{i-} be chosen positive and negative respectively. In the above formulas, a_0, b_0, b_1, etc. are given as

$$a_0=-\frac{l_{34}}{k(k+h)}+\frac{m_4^2}{k^2}+\frac{m_3^2}{(k+h)^2} ,$$
$$b_0=\frac{m_3^2}{(k+h)^2},\quad b_1=\frac{l_{23}}{k+h},\quad b_2=m^2 ,$$
$$c_0=\frac{l_{34}}{k(k+h)}-\frac{2m_3^2}{(k+h)^2},\quad c_1=\frac{l_{24}}{k}-\frac{l_{23}}{k+h} , \quad \text{(C4)}$$
$$d_0=\frac{l_{34}}{k(k+h)}-\frac{2m_4^2}{k^2} ,$$
$$e_0=-\frac{l_{34}}{k(k+h)},\quad e_1=-\frac{l_{24}}{k} ,$$
$$f_0=\frac{m_4^2}{k^2} .$$

where $l_{ij}=p_{ij}^2+m_i^2+m_j^2$. The substitutions $p_{ij}^2=-p_{ij}^2$ have to be made in Minkowski space. We have assumed η_0 to be real which is true if either p_2 or p_3 is timelike or when both are lightlike. For complex η_0, the terms proportional to θ function in (C1) have to be modified in accordance with Eq. (6.14) of Ref. 7.

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