Coulomb interactions and fermion condensation

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The influence of the Coulomb interaction in states containing massless and flavorless fermion-antifermion pairs is studied, using a continuum formulation within the finite volume S^3. Several different forms for the Coulomb interaction are examined, including confining potentials as well as nonconfining potentials. The calculations show that if the interaction is strong enough, the Coulomb interaction leads to condensation of pairs, and that this condensation has a chiral character. The condensation does not depend on whether the interaction is confining. It is found that simplified variational approximations are not accurate enough for an adequate description of the states.

I. INTRODUCTION

Studies of QCD have suggested that light quarks may become condensed in pairs as a consequence of the Coulomb interaction.[1,2] The condensation of fermions has also been considered in strong-coupling QED, using both continuum[3,4] and lattice methods.[5] However, the association between this condensation and the phenomenon of confinement is not clear from the previous work. To examine further the role of the Coulomb interaction in the condensation of light quarks and the chiral properties of the states, along with the relation of these phenomena to confinement, we have carried out numerical calculations based on the continuum theory on a hypersphere.

The hypersphere S^3 provides a quantization volume for the fields which has good symmetry and topology. Momenta are given an upper limit Λ leading to a system with a finite number of degrees of freedom. We use a Hamiltonian formulation, together with variational methods, to investigate directly the low-lying states of the system in the strong-coupling region. Since we use continuum rather than lattice variables, there are no fundamental difficulties associated with fermions. It is possible to treat fermions that have a vanishing mass, so that chiral symmetry is explicitly preserved. We consider models in which the fermions have only one flavor. For our variational calculations, we shall use a convenient set of states introduced by Wang and Cutkosky in a previous paper, referred to here as WC.[6] States with vacuum quantum numbers are constructed by using operators formed from pairs of fermion operators, and the matrix elements of the Coulomb energy operator are evaluated between such states.

We shall study here the low-lying eigenstates of the WC effective Hamiltonian. In this model Hamiltonian, the bosonic degrees of freedom are not treated explicitly. Instead, we consider an effective Coulomb interaction, and investigate whether such an interaction can lead to a condensation of pairs and whether such a condensation affects the chiral properties of the ground state. To examine how the confining or nonconfining nature of the interaction influences the results, we shall consider a Coulomb interaction that has the general form

$$G_K = \left(\frac{3}{K(K+2)}\right)^{\mu}, \tag{1}$$

where K is the hyperspherical momentum-transfer variable. Note that on S^3, the Laplacian operator takes the values $\nabla^2 = -K(K+2)$, where K is a positive integer. The value $K = 0$ does not appear, and we impose the constraint $K \leq \Lambda$. We consider here the values $\mu = 1$ (ordinary QED), $\mu = 2$ (corresponding to a linear potential), and also $\mu = \infty$, a "superconfining" interaction previously discussed by WC, in which only the lowest value $K = 1$ is relevant. The number of colors is taken as n, where $n = 1$ corresponds to the case of ordinary QED. However, in these calculations, n only enters kinematically, in the degeneracy of fermion energy levels. We find that the value of μ does not significantly affect the condensation phenomena. However, the chiral properties are somewhat modified in the superconfining limit.

II. FERMION PAIRS ON S^3 AND THE COULOMB HAMILTONIAN

On the hypersphere S^3, bosonic modes are conveniently classified by the (l_1, l_2) representation of the SU(2)×SU(2) symmetry group.[7] The momentum index is $K = l_1 + l_2$, and the helicity is $h = l_1 - l_2$. To use the same classification scheme for fermionic modes, following Sen,[8] we resolve two-component spinors along a triad system formed from orthonormal vectors of the

representation $(1,0)$. This spin is then associated with the angular momenta $(\frac{1}{2},0)$, and combined with the orbital angular momenta (l,l) of a scalar mode. We write $\kappa = l_1 + l_2 + \frac{1}{2}$, where κ is a positive integer. Construction of four-component free Dirac spinors is done just as in flat space,[6,9] and gives a free-particle energy $E = \pm[m^2 + (\kappa + \frac{1}{2})^2]^{1/2}$, where m is the mass. The degeneracy, including the color factor n, is

$$\nu_\kappa = n\kappa(\kappa+1) \tag{2}$$

for each helicity $h = \pm\frac{1}{2}$. There is never an infrared problem, even if $m = 0$, because all modes have a finite energy and degeneracy.

In WC, a set of operators $M(\kappa,h)^\dagger$ was defined which contain products of two creation operators: one for a fermion and one for an antifermion. Applied to the perturbative vacuum, these operators create a pair in which two particles having a definite magnitude of momentum κ and helicity h have been combined to give a total momentum of zero, as well as no net color. The fermions are taken to have only a single flavor. A state with k such pairs is given by

$$|k\rangle = C(k,\kappa)\left[M(\kappa,h)^\dagger\right]^k|0\rangle. \tag{3}$$

We define

$$\langle k|M(\kappa,h)^\dagger|k-1\rangle = F(k,\kappa), \tag{4}$$

which gives

$$C(k,\kappa) = \prod_{l=1}^{k} 1/F(l,\kappa). \tag{5}$$

By using commutation properties of the $M^\dagger$ and M, it can be shown that

$$F(k,\kappa) = \sqrt{k[1-(k-1)/\nu_\kappa]}. \tag{6}$$

Apart from a normalization factor and a shift of the origin, the matrix elements of $M^\dagger$ equal those of an angular-momentum-raising operator for spin $\nu_\kappa/2$.

In general, we consider linear combinations of the states

$$|k_1,k_2,\ldots,k_{2\Lambda}\rangle = \prod_i C(k_i,\kappa_i)[M(\kappa_i,h_i)^\dagger]^{k_i}|0\rangle, \tag{7}$$

where the product extends over the 2Λ values of κ and h with $\kappa \le \Lambda$. This defines a variational approximation, because there are also other states which are colorless and have a total momentum of zero. The WC effective Coulomb Hamiltonian, evaluated between states of the form given by (7), can be written in terms of the number operators for pairs, $N_{\kappa h}$, and the operators M and $M^\dagger$. By use of Eqs. (4) and (6) this provides a convenient way to express all matrix elements. For massless fermions, helicity conservation holds at each vertex, and there are only two basic kinds of terms. We denote terms that involve fermion scattering at both vertices by $s = +$, and terms that involve pair creation or annihilation at both vertices by $s = -$. The effective Hamiltonian has only 6 types of operators:

$$H = \sum_\kappa (N_{\kappa+} + N_{\kappa-})(2\kappa+1) + \gamma \sum_{i=1}^{3} \sum_s \sum_{\kappa,\lambda} \frac{\mathcal{M}_{is}(\kappa,\lambda)}{\Delta_i(\kappa,\lambda)} \sum_K G_K \mathcal{R}_s(\lambda,K,\kappa), \tag{8}$$

where, with $D(\kappa)^2 = \kappa(\kappa+1)$,

$$\Delta_1(\kappa,\lambda) = D(\kappa)^2,$$

$$\Delta_2(\kappa,\lambda) = nD(\kappa)^2 D(\lambda)^2, \tag{9}$$

$$\Delta_3(\kappa,\lambda) = D(\kappa)D(\lambda),$$

$$\mathcal{R}_+(\lambda,K,\kappa) = \tfrac{1}{4}(K+\kappa+\lambda+2)(\kappa+\lambda-K)(K+1), \tag{10}$$

$$\mathcal{R}_-(\lambda,K,\kappa) = \tfrac{1}{4}(K+\kappa-\lambda+1)\times(K-\kappa+\lambda+1)(K+1),$$

and

$$\begin{aligned}
\mathcal{M}_{1+} &= N_{\kappa+} + N_{\kappa-}, \\
\mathcal{M}_{2+} &= -:(N_{\kappa+}N_{\lambda+} + N_{\kappa-}N_{\lambda-}):, \\
\mathcal{M}_{3+} &= -[M(\lambda,+)^\dagger M(\kappa,+) + M(\lambda,-)^\dagger M(\kappa,-)],
\end{aligned} \tag{11}$$

$$\begin{aligned}
\mathcal{M}_{1-} &= -(N_{\kappa+} + N_{\kappa-}), \\
\mathcal{M}_{2-} &= 2N_{\kappa+}N_{\lambda-}, \\
\mathcal{M}_{3-} &= M(\kappa,+)^\dagger M(\lambda,-)^\dagger + M(\kappa,+)M(\lambda,-).
\end{aligned}$$

The coefficient γ in (8) is

$$\gamma = \frac{g^2 n_g}{4\pi^2 n}, \tag{12}$$

where n_g is the number of exchanged vector particles; $n_g = n^2-1$ for SU(n) and $n_g = 1$ for U(1). In $\mathcal{M}_{3+}(\kappa,\kappa)$, $M(\kappa,h)^\dagger M(\kappa,h)$ as given by (11) is to be replaced by $N_{\kappa h}$.

Although the Dirac equation can be constructed to be invariant under a parity transformation $\mathcal{P}$, the effect on spinor modes is quite complicated, because the two SU(2) subgroups of O(4) are interchanged. In the operators $M(\kappa,h)^\dagger$, however, these complications cancel out, and we have, as required,

$$\mathcal{P}M(\kappa,h)^\dagger\mathcal{P} = -M(\kappa,-h)^\dagger. \tag{13}$$

The effective Hamiltonian H of Eq. (8) therefore conserves parity, although the states (7) do not separately have definite parities. Four of the operators listed in Eq. (11) are diagonal in the occupation numbers $N_{\kappa h}$. The other two have the property that the difference $Z = \sum_\kappa (N_{\kappa+} - N_{\kappa-})$ remains constant, so Z is a good quantum number. It follows from (13) that the parity operation gives $Z \to -Z$. However, the energy of eigenstates is independent of the sign of Z, so for $Z \neq 0$ there are pairs of degenerate states, which form parity doublets. For $Z = 0$, there are both scalar and pseudoscalar

states. These are not degenerate, and there are also more scalar states. If the mass of the fermions were not zero, the helicity selection rules would no longer hold. Then Z would no longer be a good quantum number, and there would be, in general, no parity doublets.

III. THE LARGE-N LIMIT

In the limit $n \to \infty$, the terms $\mathcal{M}_{2s}$ can be dropped, and the Hamiltonian (8) takes the form

$$H_\infty = \sum_\kappa (N_{\kappa+} + N_{\kappa-})[2\kappa + 1 + \gamma A_+(\kappa) - \gamma A_-(\kappa)] + \gamma \sum_{\kappa\lambda s} \mathcal{M}_{3s}(\kappa,\lambda) B_s(\kappa,\lambda), \tag{14}$$

where

$$B_s(\kappa,\lambda) = \sum_K \frac{G_K \mathcal{R}_s(\kappa,K,\lambda)}{D(\kappa)D(\lambda)} \tag{15}$$

and

$$A_s(\kappa) = \sum_\lambda B_s(\kappa,\lambda)\frac{D(\lambda)}{D(\kappa)}. \tag{16}$$

(In this limit, γ is kept fixed.) Furthermore, as seen from Eq. (6), the operators $M^\dagger_{\kappa h}$ become ordinary boson creation operators, and we can write

$$M(\kappa,h) = \sqrt{\tfrac{1}{2}}(x_{\kappa h} + ip_{\kappa h}), \tag{17}$$

using conjugate variables $x_{\kappa h}$ and $p_{\kappa h}$. This gives

$$H_\infty = \tfrac{1}{2}\sum_{\kappa h}[2\kappa + 1 + \gamma A_+(\kappa) - \gamma A_-(\kappa)](x^2_{\kappa h} + p^2_{\kappa h}) - \tfrac{1}{2}\gamma \sum_{\kappa\lambda h} B_+(\kappa,\lambda)(x_{\kappa h}x_{\lambda h} + p_{\kappa h}p_{\lambda h}) + \gamma \sum_{\kappa\lambda} B_-(\kappa,\lambda)(x_{\kappa+}x_{\lambda-} - p_{\kappa+}p_{\lambda-}) \tag{18}$$

(which is normal ordered).

We express the helicity-labeled variables in terms of new variables as

$$x_{\kappa\pm} = \sqrt{\tfrac{1}{2}}(\pm P_{\kappa+} + X_{\kappa-}), \qquad p_{\kappa\pm} = \sqrt{\tfrac{1}{2}}(+P_{\kappa-} \mp X_{\kappa+}). \tag{19}$$

Here, the label $p = \pm$ refers to parity:

$$\mathcal{P}X_{\kappa p}\mathcal{P} = pX_{\kappa p}, \qquad \mathcal{P}P_{\kappa p}\mathcal{P} = pP_{\kappa p}. \tag{20}$$

The variables in (19) have been arranged so that the parity structure of the Hamiltonian is exhibited clearly:

$$H_\infty = \tfrac{1}{2}\sum_{\kappa\lambda p}(U_{\kappa\lambda}P_{\kappa p}P_{\lambda p} + V_{\kappa\lambda}X_{\kappa p}X_{\lambda p}), \tag{21}$$

where

$$V_{\kappa\lambda} = \delta_{\kappa\lambda}(2\kappa+1) + \gamma[\,\delta_{\kappa\lambda}(A_+(\kappa) - A_-(\kappa)) - B_+(\kappa,\lambda) + B_-(\kappa,\lambda)], \qquad U_{\kappa\lambda} = V_{\kappa\lambda} - 2\gamma B_-(\kappa,\lambda). \tag{22}$$

The Hamiltonian (21) corresponds to a set of coupled harmonic oscillators. If γ is small enough, the matrices U and V will be positive definite. The ground state is nondegenerate and scalar. The excited states are given by different excitation levels of the normal modes. These excitation levels correspond to different numbers of noninteracting particles. The normal modes all occur in degenerate pairs, giving scalar and pseudoscalar excitations of equal energy.

Equation (16) implies that $D(\lambda)$ is an eigenvector with eigenvalue zero of the Coulomb term V_C in the matrix V (the part proportional to γ). Therefore, since B_- is positive definite, we must have $\sum_{\kappa\lambda} D(\kappa)U_{\kappa\lambda}D(\lambda) < 0$ if γ is large enough. This implies that the matrix U must develop a negative eigenvalue if γ is larger than some critical value γ_c. Numerical diagonalization (for $\Lambda \lesssim 45$) of the matrices V and U confirms that there is such a $\gamma_c(\mu)$, for various values of μ, whether or not μ corresponds to a confining potential. If $\gamma > \gamma_c$, the spectrum does not have a lower bound; the energy can be made arbitrarily small by increasing the number of pairs. This condensation, when it occurs, involves pairs of both parities.

For finite n, the signal for condensation would be somewhat different and less pathological, because the spectrum is always bounded for any finite Λ. The reason is that the number of pairs has an upper limit given by the degeneracy (2).

In Fig. 1, the value of γ_c is plotted against Λ for various

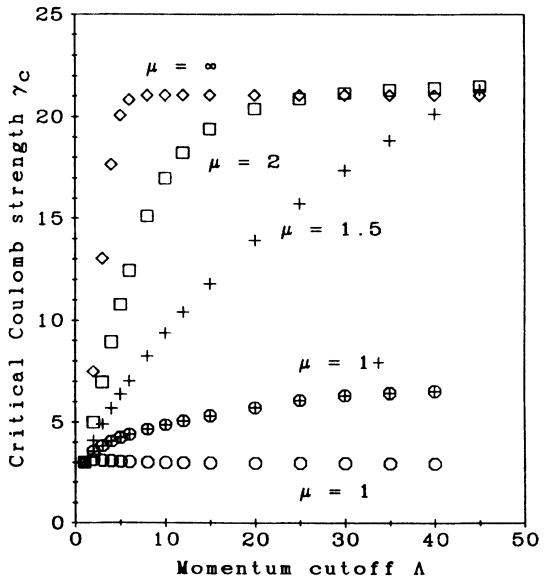

FIG. 1. The critical Coulomb strength parameter γ_c at $n = \infty$ as a function of the cutoff Λ, for various forms of the Coulomb potential.

values of μ. In addition to the values mentioned above, we made calculations for $\mu = \frac{3}{2}$, which marks the transition between confining and nonconfining potentials, and for $\mu = 1+$. By $\mu = 1+$ we mean that the potential for $\mu = 1$ has been multiplied by the factor L_1/L_K, where $L_K = 1 + \ln(K+1)$. A logarithmic modification can be motivated by considerations of asymptotic freedom.

IV. NUMERICAL CALCULATIONS FOR FINITE N

In this section we present the results of some numerical calculations for several finite values of n and various values of Λ. We use trial wave functions which are general linear combinations of the states (7):

$$\Psi = \sum B(k_1, k_2, \ldots, k_{2\Lambda})|k_1, k_2, \ldots, k_{2\Lambda}\rangle . \qquad (23)$$

We have been restricted to consideration of quite small values of n and Λ, because we have not yet been able to find a sufficiently accurate variational approximation for the coefficients B which appear in (23), and the total number of terms included in (23) increases very rapidly with n and Λ. We use the notation $\Omega = \sum_{\kappa=1}^{\Lambda} \nu_\kappa$ for the maximum number of pairs of each helicity, and write $Z = \zeta\Omega$.

Our method of calculation was first to find a sufficient number of the lowest-energy states, for the simplified problem in which the total number of pairs N is kept fixed at various values, and then to find the lowest energies when these states are coupled through the remaining terms of the Hamiltonian. This procedure allows an interpretation in which N is considered as a collective variable with an effective potential $V(N)$. The interpretation would be precise if only one eigenstate of lowest energy needed to be considered, for each N. We found that this approximation gives a qualitative understanding of the lowest states of the system, even in the strong-coupling regime. For accurate results, however, we needed to include as many as 5–9 states for some N, and the required number increased with n and Λ.

For displaying our results, we express the Coulomb strength coefficient γ in terms of the infinite-n critical strength γ_c by the parametrization

$$\gamma = \frac{7\gamma_c\Gamma}{3(1-\Gamma)} . \qquad (24)$$

In other words, $\Gamma = 0.3$ corresponds to γ_c, the value of γ shown in Fig. 1, at which, for given values of Λ and μ, the system becomes unstable when $n \to \infty$. We found that the energies and the properties of the states depend strongly on Γ, but depend relatively weakly on Λ, μ, and n when γ is expressed this way. For example, when $\mu = 1$ or $\mu = 2$, there is a chiral transition when $\Gamma \approx 0.5$.

Figure 2 shows the mean occupancy for several values of μ and ζ. The mean occupancy is defined to be the average number of positive-helicity fermion pairs divided by the maximum possible number of such pairs for the given values of Λ and n. The curves shown are for $n = 2$ and $\Lambda = 3$, but other values of n and Λ give almost identical results. We see that if $\Gamma \lesssim 0.3$, the number of pairs remains close to the minimum needed to provide a given ζ. If the critical value is exceeded, however, the number of pairs increases steadily with Γ.

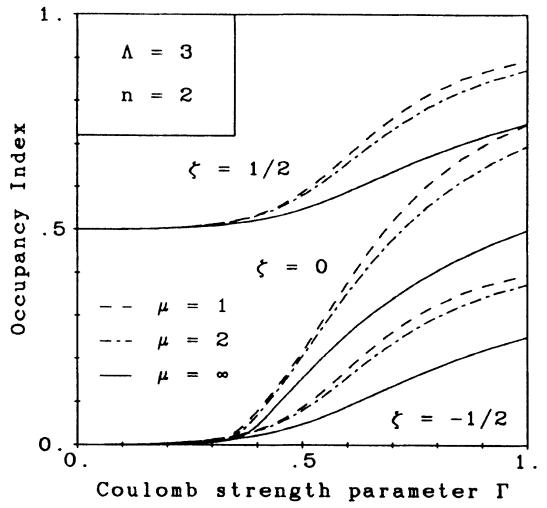

FIG. 2. The mean number of positive-helicity pairs, divided by the maximum possible number, as a function of the Coulomb interaction strength for several values of ζ and μ.

Figure 3 shows how the lowest energy depends on Λ and ζ, as well as on Γ, for $\mu = 1, 2$ and $n = 2$. The reference energy is the ground-state energy for $\zeta = 0$, and the energy unit is taken to be the calculated first excitation energy for $\zeta = 0$ multiplied by Ω, as this removes most of the dependence on n, Λ and μ. Figure 4 shows the same quantities for $\Lambda = 2$ and various values of n. Figure 5 gives similar information for $\mu = \infty$. From the curves shown in these figures we can draw several general conclusions. First, we note that if $\gamma \lesssim 2\gamma_c$, the energies increase with ζ and are roughly proportional to ζ. In this region, the energies are still dominated by the free-particle energy term of the Hamiltonian. There are peaks in the curves at $\gamma \approx \gamma_c$, which are more pronounced for the larger values of n. These peaks arise from the fact that the energy unit ΔE_0 is relatively small for this value of γ, and are presumably associated with the large-n instability.

For $\gamma > \gamma_c$, the dependence on ζ changes character, first becoming weaker, and then changing sign at $\Gamma \approx 0.5$ ($\gamma \approx 2.3\gamma_c$) for $\mu = 1$ and $\mu = 2$. In this regime, the lowest-energy state is a parity doublet. Furthermore, the energy for $\zeta = \pm 1$ is the lowest, whenever $E_{\pm 1} < E_0$. The condensation of pairs thus occurs with a maximal concentration in a single helicity, and all possible states with $\kappa \leq \Lambda$ and that helicity are filled. This phenomenon occurs with both a nonconfining interaction ($\mu = 1$) and a confining interaction ($\mu = 2$). The "superconfining" limit $\mu = \infty$ is somewhat different, however. There is no chiral transition for finite γ, but there is a "superdegeneracy" when $\gamma = \infty$.

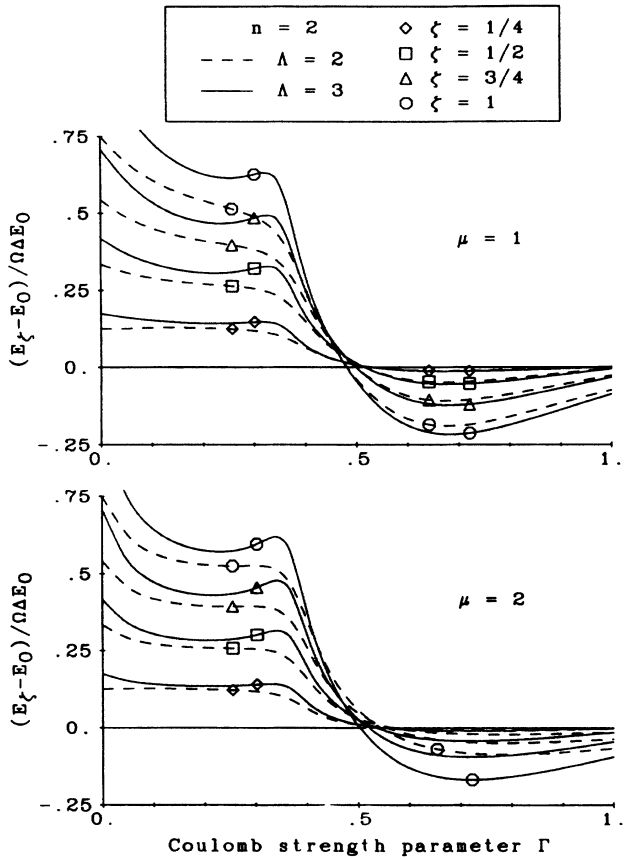

FIG. 3. Normalized energy differences as a function of the Coulomb interaction strength for $\mu = 1$ (the ordinary Coulomb potential) and $\mu = 2$ (a string-like potential), with $n = 2$ and $\Lambda = 2, 3$.

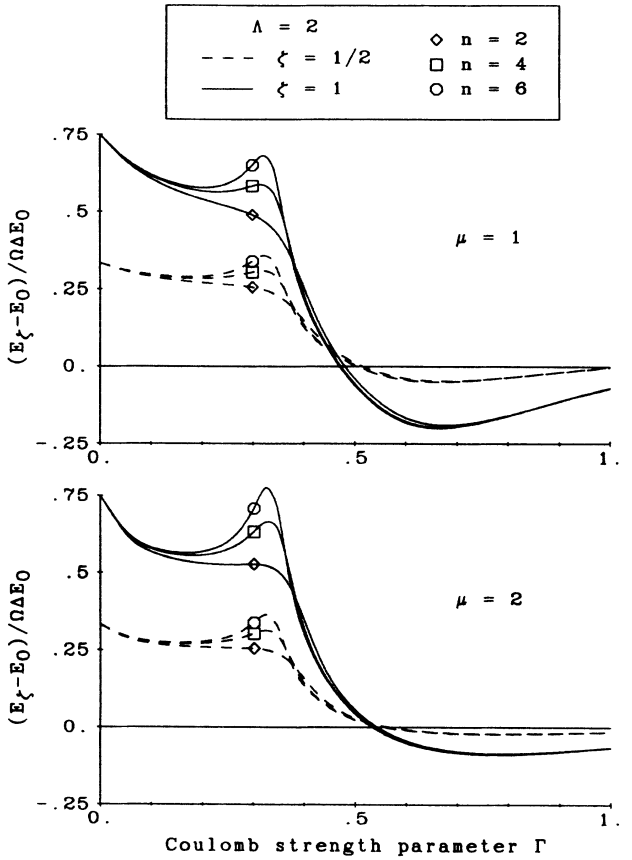

FIG. 4. Normalized energy differences as a function of the Coulomb interaction strength for several values of n, with $\Lambda = 2$ and $\mu = 1, 2$.

The wave function (23) has defined a variational approximation that may itself be too restrictive in some respects. Unfortunately, it also has so many adjustable parameters (the coefficients B) that it can only be used if Λ and n are very small. In this range, it provides a basis for judging the accuracy of further approximations which might allow simpler calculations for larger Λ and n, as well as calculations with more than one flavor. One possibility is to limit the total number of pairs that are included. This is satisfactory if $\gamma \lesssim \gamma_c$. However, when condensation of pairs occurs, we found that a limitation that reduces the total number of states included in the sum (23) by only a few percent can introduce a sizable error in the energy differences.

A trial wave function similar to that used in previous work[1,2] is

$$\Psi_a = \exp\left(\sum_{\kappa h} a_{\kappa h} M(\kappa, h)^\dagger\right)|0\rangle\,, \tag{25}$$

where the $a_{\kappa h}$ are the adjustable parameters. This wave function does not respect the conservation of Z; it contains a mixture of states with different values of Z. The minimum energy calculated using (25) would approximate an average of the energies E_ζ, for ζ near the value ζ_0 giving the smallest energy. [From our calculations using Ψ, see Eq. (23), we already know that $\zeta_0 = 0$ for $\Gamma \lesssim 0.5$, and $\zeta_0 = 1$ for $\Gamma \gtrsim 0.5$.] Lower energies can be obtained by introducing a projection operator P_Z onto the Z subspace:

$$\Psi_{Za} = P_Z \Psi_a\,. \tag{26}$$

This will give a set of energies $\tilde{E}_\zeta$ from which we can choose the smallest. Use of Ψ_{Za} instead of Ψ_a will always give a lower energy.

In Fig. 6 we compare the "exact" energies (obtained from Ψ) with the approximation Ψ_{Za}. The unit is the energy difference ΔE_0 obtained using Ψ. For $\zeta = 1$ there is only one state in the sum for Ψ, in which there are Ω pairs with positive helicity and none with negative helicity. Therefore Ψ_{Za} is independent of the $a_{\kappa h}$, and necessarily gives the same energy as Ψ for $\zeta = 1$. For small values of $|\zeta|$, however, the energy errors in the condensed regime are much larger than ΔE_0, showing that conventional formulations related to Ψ_a can give misleading results.

The inadequacy of Ψ_a does not arise as long as the

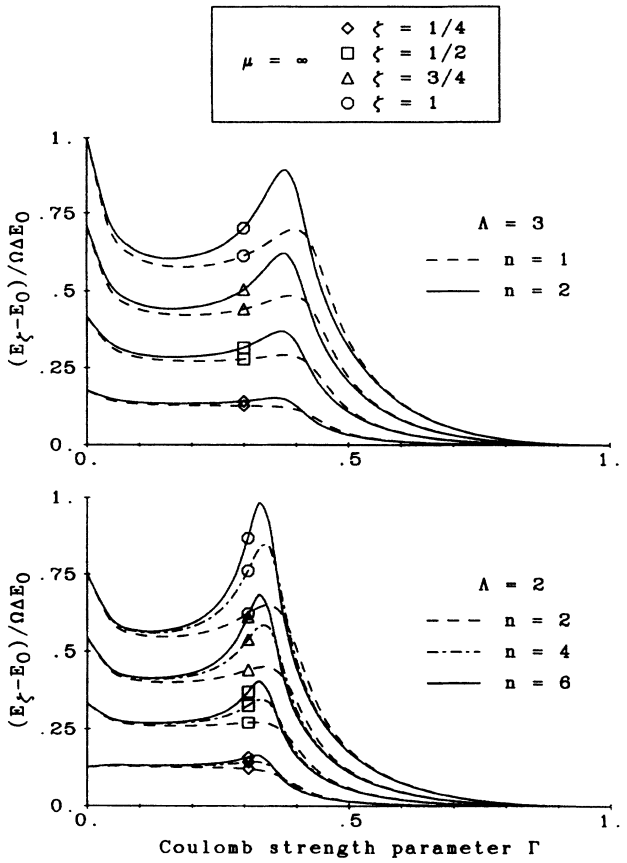

FIG. 5. Normalized energy differences as a function of the Coulomb interaction strength for the "superconfining" interaction $\mu = \infty$.

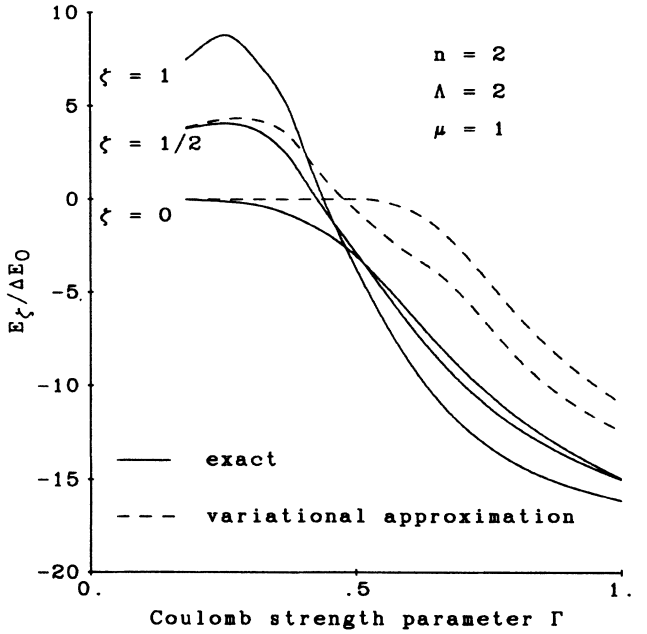

FIG. 6. Comparison of the normalized energies obtained using a simplified variational approximation with the more exact values, for $n = \Lambda = 2$ and $\mu = 1$.

occupation numbers remain close to the weak-interaction values. A chiral transition does occur when the crude approximation (26) to Ψ is used, but with incorrect characteristics, because fluctuations in the occupation numbers are not treated accurately enough. Our "effective potential" approximation is more accurate, but also gives only qualitatively reliable results. To go beyond a qualitative understanding of the properties of a model with strong coupling, very complicated calculations seem to be needed.

ACKNOWLEDGMENTS

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[1] J. T. Finger and J. E. Mandula, Nucl. Phys. **B199**, 168 (1982); J. Govaerts, J. E. Mandula, and J. Weyers, *ibid.* **B237**, 59 (1984).
[2] S. J. Adler and A. C. Davis, Nucl. Phys. **B244**, 469 (1984); A. C. Davis and A. M. Matheson, *ibid.* **B246**, 203 (1984); Phys. Lett. **165B**, 395 (1985); A. Matheson, Phys. Rev. D **38**, 3566 (1988).
[3] V. A. Miransky, Phys. Lett. **91B**, 421 (1980); Nuovo Cimento **90A**, 149 (1985); Zh. Eksp. Teor. Fiz. **88**, 1514 (1985) [Sov. Phys. JETP **61**, 905 (1985)].
[4] V. P. Gusynin, V. A. Kushnir, and V. A. Miransky, Phys. Lett. B **220**, 635 (1989).
[5] J. Kogut, E. Dagotto, and A. Kocić, Phys. Rev. Lett. **60**, 772 (1988); **61**, 2417 (1988); E. Dagotto, A. Kocić, and J. B. Kogut, Nucl. Phys. **B317**, 253 (1989).
[6] K. C. Wang and R. E. Cutkosky, preceding paper, Phys. Rev. D **42**, 1260 (1990).
[7] R. E. Cutkosky, J. Math. Phys. **25**, 939 (1984).
[8] D. Sen, J. Math. Phys. **27**, 427 (1986).
[9] R. E. Cutkosky and K. C. Wang, Czech. J. Phys. B **38**, 825 (1988).