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Two-loop chiral anomaly as an infrared phenomenon

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We consider a two-loop chiral anomaly within the dispersion approach to matrix elements of the axial-vector current. The anomaly is known to be ultraviolet ambiguous to this order and our goal is to trace the ambiguity in terms of imaginary parts of the matrix elements which are "observable." We find that the matrix elements depend crucially on details of infrared regularization which is needed to specify the imaginary part. In particular, if one sets the masses of the particles inside the loop equal to zero first while letting the virtuality of external particles go to zero next then the anomaly does not receive a two-loop contribution. If the limiting procedure is reversed then one reconstructs the so-called supersymmetric current which acts as a partner of the energy-momentum tensor in the supermultiplet of currents. The results are applied to the problem of renormalization of topological charges.

I. INTRODUCTION

There exist two alternative approaches to the anomaly issue. The most common one[1] assumes the following sequence of steps: (a) first, identify the chiral-symmetry transformations of the Lagrangian, say, chiral rotations, $\Psi_{L,R} \Longrightarrow \Psi_{L,R}\exp(\pm i\alpha)$; (b) construct then the corresponding axial-vector current

$$a_\mu = \bar{\Psi}\gamma_\mu\gamma_5\Psi, \quad \partial_\mu a_\mu = 0 \text{ (classically)} \; ; \tag{1}$$

(c) add a heavy regulator field with mass M_H ($M_H \Longrightarrow \infty$):

$$\delta L = M_H \bar{\Psi}_H \Psi_H + \cdots$$

or use some other ultraviolet regularization; (d) finally, calculate the triangle graph of Fig. 1(a) both with physical and regulator fields inside and establish the anomaly

$$\partial_\mu a_\mu = \frac{\alpha}{4\pi} F_{\mu\nu}\tilde{F}_{\mu\nu} \; , \tag{2}$$

where $F_{\mu\nu}$ is electromagnetic field-strength tensor and $\tilde{F}_{\mu\nu}$ is its dual, $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}$, and α is the electromagnetic coupling constant.

Clearly enough, this picture blames regulators, or short-distance physics, for the anomaly. The other approach avoids mentioning regulator fields at all.[2] Namely, let us consider the same triangle graph but for the current, not its divergence:

$$\langle 0 | a_\mu | 2\gamma \rangle = f(q^2) q_\mu F_{\rho\sigma}\tilde{F}_{\rho\sigma} \; . \tag{3}$$

Here q_μ is the four-momentum carried by the current, $f(q^2)$ is the form factor to be calculated, and the photons are considered to be on mass shell, i.e., $k_1^2 = k_2^2 = 0$. The next step is to evaluate the matrix element associated with the triangle graph via dispersion relations.

The corresponding imaginary part is determined by the Born graphs and respects for this reason all the symmetries of the classical Lagrangian. In particular, as a manifestation of the chiral symmetry the imaginary part is proportional to the fermion mass squared m_f^2, so that it does vanish as m_f tends to zero. From dimensional considerations alone one would conclude then

$$\mathrm{Im} f(q^2) \propto m_f^2/q^4$$

while the explicit calculation brings some extra log factor:[2]

$$\mathrm{Im} f(q^2) = -\frac{\alpha}{2}\frac{m_f^2}{q^4}\ln\frac{1+v}{1-v} \; , \tag{4}$$

where v is the fermion velocity in the c.m. system.

Finally, evaluation of the real part of $f(q^2)$ reveals the anomaly

$$\mathrm{Re} f(q^2) = \frac{1}{\pi}\int_{4m_f^2}^{\infty}\frac{\mathrm{Im} f(s)ds}{s-q^2}$$
$$= \frac{\alpha}{2\pi}\frac{m_f^2}{q^2}\int_{4m_f^2}^{\infty}\frac{ds}{s^2}\ln\frac{1+v}{1-v} = \frac{\alpha}{4\pi q^2} \; . \tag{5}$$

Indeed, unlike the imaginary part the real part of $f(q^2)$ is not proportional to m_f^2. The reason is that the dispersion

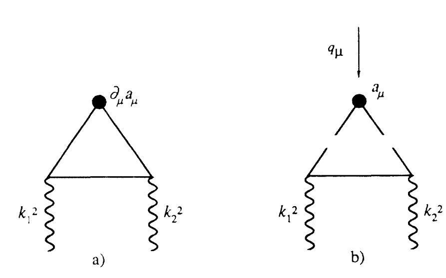

FIG. 1. (a) Anomalous triangle graph for the divergence of the axial-vector current. Solid line represents fermions, wavy lines stand for photons or gluons. (b) Unitarity cut of a triangle graph associated with the matrix element of the axial-vector current a_μ carrying momentum q_μ. Breaking of a line means that the corresponding particle is on mass shell.

integral is saturated by $s \propto m^2$ and produces the m_f^{-2} factor.

We see that within the latter approach when one examines first the matrix elements of the current the anomaly emerges as a pure infrared effect. Since the imaginary parts are associated, generally speaking, with some observable cross sections one might think that the dispersive approach is more directly related to physics. The 't Hooft consistency condition[3] which relates the quark graphs to physical coupling constants of pseudoscalars may be considered as a realization of this idea (in cases when the condition applies).

The final result for the classic one-loop anomaly is the same, no matter what technique is used. However, the two-loop anomaly is known to be ambiguous (see, e.g., Ref. 4) and this knowledge seems to conflict with the general belief that imaginary parts of matrix elements are uniquely determined. Motivated by this seeming contradiction we will address ourselves to considering the two-loop anomaly by means of dispersion relations.

Our main conclusion is that the result depends crucially on the way the infrared regularization is performed although finally we set both m_f and $k_{1,2}^2$ equal to zero. The results allow for a fresh approach to the problem of topological charge renormalization. In particular, if one chooses the regularization in a physically motivated way the topological charge is not renormalized.

The organization of the paper is as follows. In the next section, we review the results on the two-loop chiral anomaly. In Sec. III we introduce an alternative way to regularize the imaginary part of the triangle graph by means of a nonvanishing virtuality of external particles. In Sec. IV we demonstrate the infrared instability of the imaginary part considered. In Sec. V compare ambiguities inherent to ultraviolet and infrared regularizations and claim to establish a one-to-one correspondence between the two. In Sec. VI we apply the results obtained to the problem of renormalization of topological charges.

II. TWO-LOOP PUZZLE

Since a lot of work on the anomaly has already been done by considering ultraviolet regularization we review briefly the results obtained, emphasizing possible implications for the infrared treatment we keep in mind.

It is convenient sometimes to invoke supersymmetry (SUSY) and consider SUSY Yang-Mills (YM) theory. It is worth emphasizing however that supersymmetry is actually not essential and the conclusions we are going to reach are of a general nature. The function of supersymmetry is to fix some matrix elements and provide us with insight as to which structures may occur in the matrix elements in general. Once supersymmetry is abandoned these structures can appear with an arbitrary weight.

Historically, the so-called Adler-Bardeen current was introduced first into the theory.[5] It is specified by the condition that the anomaly equation has no higher-order contributions:

$$(\partial_\mu a_\mu)_{\text{Adler-Bardeen}} = \frac{\alpha_s N}{4\pi} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} + (\text{no } \alpha_s^2 \text{ corrections}) , \tag{6}$$

where $G^a_{\mu\nu}$ is the gluon field-strength tensor, a_μ is the gluino axial-vector current, the gauge group is SU(N) and a is the color index while α_s is the coupling constant. Note also that we have changed the chiral anomaly for electromagnetic background into a very similar one, for an external gluonic field.

On the other hand, supersymmetry allows us to unify various anomalies.[6] In particular, the conformal anomaly is proportional to the whole β function:

$$\vartheta_{\mu\mu} = \frac{\beta(\alpha_s)}{4\alpha_s} G^a_{\mu\nu} G^a_{\mu\nu} , \tag{7}$$

where $\vartheta_{\mu\mu}$ is the trace of the energy-momentum tensor. The two anomalies naturally fall into the same supermultiplet so that there exists the so-called supersymmetric axial-vector current whose divergence is given by

$$(\partial_\mu a_\mu)_{\text{SUSY}} = -\frac{\beta(\alpha_s)}{3\alpha_s} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} . \tag{8}$$

It is worth emphasizing that classically the axial-vector current is uniquely determined:

$$(a_\mu)_{\text{classical}} = \tfrac{1}{2} \bar{\lambda}^a \gamma_\mu \gamma_5 \lambda^a , \tag{9}$$

where λ^a is the gluino field and a is the color index. Thus the difference in ultraviolet regularization prescriptions has been blamed for the difference in the resulting anomaly equations.[7] However, as we have already learned the anomaly condition can be converted into an equation for the matrix element of the current with a nontrivial imaginary part. Therefore, having both Eq. (7) and Eq. (8) implies varying the imaginary part although we start from one and the same classical current (9). Thus one may wonder what happens to the unitarity condition.

First, one turns to the question of which counterterms can arise in the process of evaluating the first loop. If there existed some freedom in choosing one-loop subtraction constants these counterterms could produce a change in the two-loop imaginary part and distinguish between various currents. However the only counterterm possible is proportional to the classical current itself,

$$\delta a_\mu \propto \alpha_s \bar{\lambda} \gamma_\mu \gamma_5 \lambda , \tag{10}$$

and introduction of such a counterterm reduces to a mere redefinition of the classical current which is of little interest. So we can forget about this possibility (for a thorough discussion see Refs. 7 and 8).

An ingenious solution to the problem has been found in Ref. 9. In this paper the ultraviolet dimensional regularization is used. If one changes the dimension some components of a $d=4$ vector field, gluon in our case, are to be treated as scalar particles since the Lorentz vectors incorporate different number of fields in different dimensions. Thus there emerges a new freedom as to which axial charge to be ascribed to these scalar particles. In somewhat symbolic way the new counterterm can be written as

$$\delta a_\mu \propto (d-4) \cdots , \tag{11}$$

where d is the number of dimensions and the factor $d-4$

indicates that the current is constructed from the fields disappearing at $d=4$. Although the counterterm vanishes once d tends to $d=4$ it produces a nonvanishing effect to next order because of ultraviolet divergencies. In the same symbolic way the extra piece in the divergence of the current can be written as

$$\delta(\partial_\mu a_\mu) \propto \alpha_s^2(d-4)/(d-4) \propto \alpha_s^2 \text{ (finite number)} . \quad (12)$$

This solution of the anomaly puzzle makes our problem even more acute since if one translates the result, so to say literally, into the language of the infrared regularization one would expect the effect of the unphysical particles to show up in the unitarity condition. Although this is not exactly what actually happens there is some truth in the hint. Namely, if we stick just to the physical dimension $d=4$ and rely on the unitarity condition to evaluate the anomaly the result still turns out to be uncertain because the imaginary part appears to be dependent on the way one performs the infrared regularization.

III. MORE ON INFRARED REGULARIZATION

Now we are coming closer to the subject of our immediate interest and consider the infrared regularization in more detail. For simplicity we shall consider in this section photons as external particles in Fig. 1.

The first question is why we need any regularization at all to calculate the imaginary part. The reason is that for all the particles being massless and on mass shell the fermionic pole in the graph describing the transition of two photons into two fermions [see Fig. 1(b)] falls into the boundary of the physical region and the imaginary part is not formally determined. Thus there arises the necessity for an infrared regularization.

Once one starts to regularize the imaginary part there appear two alternative possibilities both of which deserve discussion. First, one can introduce an infinitesimal fermionic mass and let this mass go to zero at the end of the calculation. Such a procedure was implied in fact in our discussion above. The advantage of regularizing by means of the fermionic mass is its simplicity. In particular the matrix element of the current is described by a single form factor [see Eq. (3)] and for this reason calculating the matrix element of the current and of its divergence is actually one and the same thing. The disadvantage is that this regularization does not respect chiral symmetry and it is just this violation of the symmetry which goes into the final answer and manifests itself as the anomaly.

Thus one might be encouraged to look for another regularization which does observe chiral symmetry. Indeed such a regularization can be constructed. Namely, let us consider the external photons to be off mass shell:

$$k_1^2 = k_2^2 \equiv k^2 \neq 0 ,$$

where k_1 and k_2 are the four-momenta of the vector particles.

Then both the imaginary part is well defined and chiral symmetry is observed. Therefore at least the imaginary part of the matrix element of the divergence of the current is identical to zero. Indeed, we have already mentioned that the imaginary part is determined by the Born graphs and all the symmetries of the classical Lagrangian become manifest. What is going wrong however is that the matrix element of the current is no longer described by a single form factor and for this reason the knowledge of the divergence of the current does not imply the matrix element of the current to be fixed.

In more detail, the matrix element of the current now reads

$$\langle 0|a_\mu|2\gamma\rangle = f_1(q^2,k^2)q_\mu F\tilde{F} + f_2(q^2,k^2)(k^2\epsilon_{\mu\nu\rho\sigma}A_\nu F_{\rho\sigma} + \cdots) , \quad (13)$$

where A_μ is a vector potential and the ellipsis denote terms proportional to $\partial_\mu A_\mu$. Note the appearance of the vector potential A_μ, not just of the field-strength tensor $F_{\mu\nu}$, which became possible since the whole form factor f_2 is proportional to k^2 and vanishes on mass shell.

The vanishing of the imaginary part of the divergence now implies

$$q^2 \mathrm{Im} f_1 = k^2 \mathrm{Im} f_2 . \quad (14)$$

As for the $\mathrm{Im} f_{1,2}$ themselves they do not vanish. Moreover applying the same kind of dimensional analysis as in Sec. I we estimate $\mathrm{Im} f_1$ as

$$\mathrm{Im} f_1 \propto k^2/q^4$$

and the corresponding dispersion integral remains finite once k^2 tends to zero:

$$\mathrm{Re} f_1(q^2,k^2) \propto k^2 \int_{4k^2}^{\infty} \frac{ds}{s^2(s-q^2)} \underset{k^2 \to 0}{\Longrightarrow} \frac{\text{const}}{q^2} . \quad (15)$$

As for the contribution of f_2, it disappears in the limit considered and we see that the anomaly reemerges in new terms.

To summarize, already at the one-loop level one can use various infrared regularizations. Moreover, within the approach developed it is not *a priori* clear that different regularizations would lead to the same numerical answer (although the very existence of the anomaly can readily be substantiated).

IV. TWO-LOOP INFRARED REGULARIZATION

We are in a position now to consider infrared regularization of two-loop graphs. Actually the procedure is no more complicated than that for one loop. The point is that the intermediate three-particle cut (see Fig. 2) which might cause the most trouble is actually very simple to handle. Indeed, consider the regularization

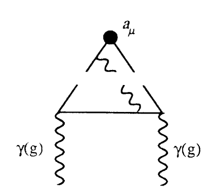

FIG. 2. A three-particle unitarity cut. Only one graph of the whole set is depicted.

$$m_f=0,\quad k^2=0,\quad m_\gamma\neq 0\ ,$$

where k^2 is the external particle four-momentum squared while m_γ is the infinitesimal mass of the photon in the intermediate state.

As we have already mentioned, in the case of external momenta $k^2=0$ there exists a single form factor [see Eq. (3)]. Moreover the imaginary part is regularized now in a chiral-invariant way. Therefore $\mathrm{Im}f(q^2)$ vanishes identically implying vanishing of the whole contribution. Thus, there is no anomaly associated with the three-particle intermediate state. It might be worth noting that our derivation echoes at this point the original arguments of Ref. 5, where the ultraviolet regularization was considered. Namely, vanishing of the three-particle contribution corresponds to the observation[5] that higher loops are not divergent and do not contribute to the anomaly.

Thus one has to worry only about the two-particle intermediate state (see Fig. 3) since this contribution cannot be regularized in a chiral-invariant fashion by ascribing mass to the photon inside the loop.

Moreover let us imagine that the first loop results in the contribution

$$\delta a_\mu \propto \alpha_s \frac{q_\mu q_\nu}{q^2}\bar{\lambda}^a\gamma_\nu\gamma_5\lambda^a \tag{16}$$

which can be called a "nonlocal zero." Indeed, the nonlocality is evident since we have the q^{-2} factor. As for the "zero," the extra piece (16) vanishes by virtue of the Dirac equation for massless gluinos which can be applied since we are considering imaginary part. Because it is equal to zero the term (16) is beyond control within the dispersion approach we are pursuing. However, in view of the experience of the previous sections we should not be surprised if the contribution of this term at the next-loop level is not vanishing.

Construction (16) may look artificial and one could feel inclined to disregard such terms altogether. It is supersymmetry which provides us here with a guide, implying that such terms do appear. The trick is that SUSY relates the divergence term $\partial_\mu a_\mu$ which vanishes on mass shell to the $G\tilde{G}$ term which is generated by the first loop with a well-defined coefficient. Indeed, it is well known (see, e.g., Ref. 10) that only the combination

$$G\tilde{G}-2(\partial_\mu a_\mu)$$

appears in superfield language. Therefore the one-loop matrix element of $(a_\mu)_{\mathrm{SUSY}}$ actually looks like

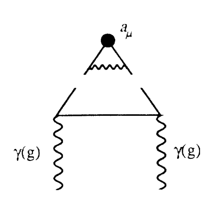

FIG. 3. Two-loop, two-fermion unitarity cut.

$$(a_\mu)_{\mathrm{SUSY}}=\frac{\alpha_s N}{4\pi}q_\mu\frac{G\tilde{G}-q_\nu\bar{\lambda}^a\gamma_\nu\gamma_5\lambda^a}{q^2}\ . \tag{17}$$

Thus we are led to consider the effect of the term (16) carefully in the general case as well.

The crucial point is that different infrared regularizations produce different results for the two-loop anomaly induced by (16). Consider first the case of a nonvanishing fermionic mass:

$$\text{(a)}\ m_f\neq 0,\quad m_f\Longrightarrow 0,\quad k^2\equiv 0\ .$$

Then one can readily check that the first-loop anomaly reiterates itself.[10,11] Indeed, in this case we just have

$$\alpha_s\frac{q_\mu(q_\nu a_\nu)}{q^2}\Longrightarrow\alpha_s\frac{q_\mu q_\nu}{q^2}a_\nu\Longrightarrow\alpha_s^2\frac{q_\mu q_\nu}{q^2}\frac{q_\nu G\tilde{G}}{q^2}$$
$$\Longrightarrow\alpha_s^2\frac{q_\mu G\tilde{G}}{q^2}\ . \tag{18}$$

Or, in more detail, the two-loop contribution into the imaginary part $\mathrm{Im}f(q^2)$ associated with the term (17) is equal to

$$[\mathrm{Im}f(q^2)]_{\text{two loop}}=+\frac{\alpha_s N}{4\pi}\frac{\alpha_s N}{\pi}\frac{m_f^2}{q^4}\ln\frac{1+v}{1-v} \tag{19}$$

so that the contribution to the two-loop anomaly is nonvanishing and well defined:

$$\langle 0|(\partial_\mu a_\mu)|2g\rangle_{\text{two loop}}=\left[\frac{\alpha_s N}{4\pi}\right]\left[-\frac{\alpha_s N}{2\pi}\right]G\tilde{G}\ . \tag{20}$$

Since in the supersymmetric case the axial-vector current does receive higher-order contributions such a reiteration of anomalies suits $(a_\mu)_{\mathrm{SUSY}}$.

Let us try, however, another regularization:

$$\text{(b)}\ m_f\equiv 0,\quad k^2\neq 0,\quad k^2\Longrightarrow 0\ .$$

Then the effect of our "nonlocal zero" term is identical to zero. Indeed, since the gluinos are on mass shell we can rely on the Dirac equation for massless particles so that

$$[\mathrm{Im}f(q^2,k^2)]_{\text{two loop}}=\mathrm{const}\times\mathrm{Im}\langle 0|\partial_\mu a_\mu|2g\rangle_{k^2\neq 0}\equiv 0\ . \tag{21}$$

and we come to the realization of the Adler-Bardeen current.

Thus, in the both cases (a) and (b) we end up at the same physical point, i.e., $m_f,k^2=0$. However, the limiting procedures are different. What we find is that, unlike the first-loop result, the two-loop matrix element of the current does depend on the details of the infrared regularization procedure.

V. INTERPRETATION

To be sure that we identified correctly the origin of the ambiguity it is desirable to establish links to the results obtained within other approaches and to access the physical meaning of the "nonlocal zero" discussed above. In this section we will address ourselves to both issues.

From supersymmetry we learn that once the axial-vector current is postulated to be in the same multiplet as the energy-momentum tensor its matrix elements get uniquely determined.[9] What is the analog of this observation in our language?

One can consider the conformal anomaly starting from the matrix elements of the energy-momentum tensor itself $\vartheta_{\mu\nu}$, not just of its trace $\vartheta_{\mu\mu}$. What happens then is that we get a similar pole structure. Indeed Eq. (17) can be readily generalized to the case of matrix elements of the supercurrent $J_{\alpha\dot{\alpha}}$ (Ref. 6) which unifies the axial-vector current and the energy-momentum tensor:

$$(J_{\alpha\dot{\alpha}})_{\text{pole}}=\frac{\beta(\alpha_s)}{3\alpha_s}\frac{i\partial_{\alpha\dot{\alpha}}}{\partial^2}(W^2+\overline{W}^2) , \tag{22}$$

where α and $\dot{\alpha}$ are Lorentz indices in chiral notations, W^a_α is the superfield describing gluino and gluon, $\overline{W}^a_{\dot{\alpha}}$ is its conjugate, and $\partial_{\alpha\dot{\alpha}}$ is a derivative in chiral notation.

In particular, Eq. (22) implies the following matrix element of the energy-momentum tensor:

$$\langle 0|\vartheta_{\mu\nu}|2g\rangle=\frac{\beta(\alpha_s)}{3\alpha_s}\left[g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right]G^2 , \tag{23}$$

where $|2g\rangle$ is a two-gluon state. Note that Eq. (23) actually incorporates both the pole term and the local term needed to uphold the energy-momentum conservation. Taking the trace of Eq. (23) we come back to the conformal anomaly (7).

Now, in case of the energy-momentum tensor there exists one extra requirement. Namely, the energy, or the matrix element of ϑ_{00}, should not get renormalized by the loop corrections (23). Therefore, we must treat the pole term in Eq. (23) in such a way that it does not vanish and cancels the contribution of the local term proportional to g_{00}. Since the supersymmetry relates all the pole terms it means that the pole term in the axial-vector current [see Eq. (17)] is not vanishing either. In Sec. IV we came just to the same conclusion starting from very different considerations.

As for the physical meaning of the "nonlocal zero" (16) it can be clarified by considering the definition of the axial charge upon accounting for one-loop effect. Let us remind the reader that there exist in fact two different definitions of the charge.

(a) One can define the charge Q_A by considering the integral over the zeroth component of the axial-vector current:

$$Q_A=\lim_{q_0\to 0}\int \exp(iq_0 t)a_0(\mathbf{x},t)d^3x . \tag{24}$$

In momentum space such a procedure corresponds to the space components of q_μ being identical to zero while the time component q_0 tends to zero if we are interested in large times.

(b) One can introduce some auxiliary field coupled to the current a_μ and consider the Coulomb-like scattering of gluinos on an external field. Then at small momentum transfer the cross section is determined by the charge providing an alternative way to measure the charge.

Now, the point is that these two definitions result in different answers once one accounts for the term (16)—a phenomenon which has no parallel in the case of an electrical charge. Moreover our definitions (a) and (b) of the charge turn out to be in one-to-one correspondence with the (a) and (b) infrared regularizations described in Sec. IV.

Indeed definition (a) above implies a nonvanishing change in the axial charge:

$$(\delta a_0)_{\text{SUSY}}=\frac{\alpha_s N}{4\pi}(-2)\frac{q_0 q_0}{q_0^2}\frac{1}{2}\bar{\lambda}^a\gamma_0\gamma_5\lambda^a$$

$$=-\frac{\alpha_s N}{2\pi}(a_0)_{\text{classical}} \tag{25}$$

which means that the charge gets renormalized as an effect of the first loop. Moreover, the coefficient turns out to be exactly the same so as to reproduce correctly the two-loop anomaly (20). Note also that taking $\mathbf{q}=0$ while $q_0\neq 0$ ensures a vanishing loop correction to the matrix element of ϑ_{00} [see Eq. (23)].

If one relies on definition (b), however, then one considers the physical values of the momentum transfer—while the point $q_0\neq 0, \mathbf{q}\equiv 0$ is unphysical for a massless particle scattering—and $\partial_\mu a_\mu=0$ since the gluino is on mass shell. Thus, in this case there is no renormalization of the axial charge,

$$(\delta a_0)_{\text{Adler-Bardeen}}=0 , \tag{26}$$

and the current constructed in this way is just the Adler-Bardeen current.

Thus the axial charge is not uniquely determined and depends on the details of infrared regularization. The two-loop calculation of the anomaly transforms this ambiguity in defining the charge into the ambiguity in the anomaly equation.

VI. GENERALIZATION TO BOSONIC AXIAL CHARGE

Inspection of numbers shows however that the nonlocal term (16) is not sufficient to account in full for the difference between $(a_\mu)_{\text{SUSY}}$ and $(a_\mu)_{\text{Adler-Bardeen}}$. To resolve the remaining discrepancy one has to consider the gluonic loops of Fig. 4. In terms used in the present paper the graphs introduce another two-particle intermediate state, namely, a two-gluon state. From the point of view of a supersymmetric theory it is only natural that

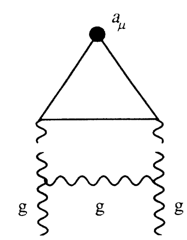

FIG. 4. Two-loop, two-gluon unitarity cut.

gluons and gluinos produce similar contributions, as we will demonstrate later. It is worth emphasizing, however, that the crucial role of these graphs to resolve the anomaly puzzles was established first earlier.[8,11] In particular in Ref. 11 all the numbers are traced up to the very end and Eq. (8) is checked explicitly to two-loop order. However, it is implicitly assumed in this paper that the two-loop anomaly is unique so that the correct equation is just Eq. (8), and this assumption is in variation with our conclusions.

To consider a reiteration of the gluonic piece of the anomaly one has to evaluate the matrix element of the $G\tilde{G}$ term associated with the first loop. All the calculations can be made parallel to those outlined above for the fermionic piece. Indeed, let us represent first $G\tilde{G}$ as a divergence:

$$G\tilde{G} = -2\partial_\mu K_\mu ,$$

where

$$K_\mu = \epsilon_{\mu\nu\rho\sigma}\left(A_\nu^a \partial_\rho A_\sigma^a + \tfrac{1}{3}\epsilon_{abc} A_\nu^a A_\rho^b A_\sigma^c \right) \tag{27}$$

and proceed then to evaluate the matrix elements of K_μ. It is worth mentioning at this point that although the current K_μ is not gauge invariant classically this noninvariance does not go onto the loop corrections. The reason is that by splitting A_μ^a into classical and quantum parts one actually confines all the gauge noninvariants into the purely classical part of K_μ while evaluation of a loop graph implies integration over the quantum part of A_μ^a.

Therefore for the external gluons being on mass shell the graph of Fig. 4 is described by a single form factor f:

$$\langle 0|K_\mu|2g\rangle = \alpha_s f_g(q^2) q_\mu G\tilde{G} . \tag{28}$$

Then one can proceed just in the same way as in the case of the matrix element associated with the graph of Fig. 1 (see Sec. I). The final result looks very similar to Eq. (5):

$$\langle 0|K_\mu|2g\rangle = \frac{\alpha_s N}{2\pi}\frac{q_\mu}{q^2} G\tilde{G} . \tag{29}$$

Moreover if one applies this treatment to evaluate the matrix element of the $G\tilde{G}$ term associated with the first loop the two-loop anomaly equation (8) for $(a_\mu)_{\rm SUSY}$ is reproduced.[11]

For the sake of completeness let us also mention that practically exactly the same technique can be applied[12] to evaluate the matrix element of the K_μ current associated with photonic field over a gravitational background:

$$\langle 0|F_{\mu\nu}\tilde{F}_{\mu\nu}|2\text{grav}\rangle \equiv -2\partial_\mu \langle 0|\epsilon_{\mu\nu\rho\sigma} A_\nu \partial_\rho A_\sigma|2\text{grav}\rangle$$
$$= q_\mu \frac{q_\mu}{96\pi^2 q^2} R_{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma} . \tag{30}$$

Here $|2\text{grav}\rangle$ stands for a two-graviton state, $R_{\mu\nu\rho\sigma}$ is the Riemann tensor and $\tilde{R}_{\mu\nu\rho\sigma}$ is its dual. The final result for the divergence of K_μ can be derived by means of ultraviolet regularization as well.[13]

Moreover Eq. (30) implies the following two-lop chiral anomaly in a gravitational background:[14]

$$(\partial_\mu a_\mu)_{\text{two loop}} = \frac{1}{192\pi^2}\left[1 - \frac{\alpha}{2\pi}\right] R\tilde{R} \tag{31}$$

which exhibits the same phenomenon of the reiteration of anomalies.

Note that the analogy between a_μ and K_μ extends in fact further. In particular the charge associated with the K_μ current measures the helicity of free photons (or gluons) (for more detail see Refs. 12 and 14). Therefore, if one defines the charge by taking $\mathbf{q}\equiv 0, q_0 \neq 0$ (see Sec. V) then photons acquire a nonvanishing axial charge as a result of the one-loop contribution to a_0:

$$\left\langle \gamma \left| \int a_0(\mathbf{x},x_0) d^3\mathbf{x} \right| \gamma \right\rangle = -\frac{\alpha_s}{2\pi}\left\langle \gamma \left| \int K_0 d^3x \right| \gamma \right\rangle$$
$$= \mp \frac{\alpha_s}{2\pi} , \tag{32}$$

where the plus and minus signs correspond to left-handed and right-handed photons, respectively. Thus we see that photons are no longer neutral with respect to the axial charge and this effect is to be taken into account once we decide to look for an interpretation of Eq. (8).

However, as is the case with the two-fermion intermediate state, the contribution of the two-gluon state is in fact ambiguous. The point is that a convenient way to derive, say, Eq. (30) is to introduce an infinitesimal photon mass. Then derivation of Eq. (30) runs parallel to that of Eq. (5).

Moreover, one can also argue that the result is infrared dependent. Indeed, the chirality of a photon propagating through a gravitational background is known to be conserved (for a thorough discussion of the corresponding chiral symmetry see Refs. 14 and 15). As a manifestation of this the matrix elements of $F\tilde{F}$ naively vanish which means that the imaginary part of these matrix elements vanishes for sure. The only problem is to provide an infrared regularization which observes the symmetry. This can be done again by ascribing virtuality to external gravitons. In that case the two-loop contribution to the chiral anomaly in the gravitational background vanishes. Seemingly, we may skip further discussion of the infrared regularization of the bosonic axial charge since it just reiterates the argument of Sec. V.

VII. APPLICATIONS

The infrared dependence of the chiral anomaly to two-loop order seems to resolve the apparent clash between the topological nature of some operators and the fact that these operators get renormalized by quantum correction. The point is that to establish the topological nature of, say, $G\tilde{G}$ it is convenient to represent $G\tilde{G}$ as a total divergence. The corresponding current is not conserved, however, and as a result the topological charge gets renormalized by quantum corrections. Now we have learned that even representing $G\tilde{G}$ as $\partial_\mu K_\mu$ does not fix its matrix elements. Concentrating on the gravitational anomaly for a moment we find, to two-loop order,

$$[\Delta Q_A]^{k^2_{\rm grav}=0}_{m_f,m_\gamma=0}=\frac{1}{192\pi^2}\int d^4x\, R\tilde{R}$$
$$=(\text{integer number}) \qquad (33)$$

but

$$[\Delta Q_A]^{k^2_{\rm grav}=0}_{m_f,m_\gamma=0}=\frac{1}{192\pi^2}\int d^4x\, R\tilde{R}\left[1-\frac{2\alpha}{\pi}\right]$$
$$=(\text{noninteger number})\ , \qquad (34)$$

where k^2 is the four-momentum squared of the external gravitons. While Eq. (33) is readily reconciled with the production of fermions in a nontrivial gravitational background, Eq. (34) disavows our physical intuition.

Now that we have learned about the infrared dependence of the results we can try to approach the problem anew and look for a proper infrared regularization. Then the regularization

$$m_f,m_\gamma\equiv 0,\quad k^2_{\rm grav}\neq 0\ , \qquad (35)$$

where $m_{f,\gamma}$ are the infinitesimal photon and fermion masses of the corresponding quantum fields and k^2 is the virtuality of the external field, seems to be the appropriate one. Indeed, the nontriviality of the background field implies that k^2 is nonvanishing. Moreover, since we are discussing actually the change of the axial charge, or particle production by this field, the masses of the particles are to be smaller than the virtuality of the field. The condition applies both to bare and physical fermions constraining both the fermion and photon masses.

Thus, we see that a physically motivated regularization (35) leads to the reasonable answer. The observation that it is just the infrared regularization which matters fits well the fact that establishing topological properties of any operator assumes integration over all the distances.

Therefore the topological charges—at least those which can be represented as full divergencies—are getting the status of nonrenormalizability provided that a motivated infrared regularization is picked up.

Infrared instability might have some implications for the theory itself. The point is that evaluation of the β function of, say, SUSY-YM theory can be reduced to an evaluation of the matrix elements of $G\tilde{G}$, as emphasized in Ref. 11. Then the β function becomes dependent on details of the infrared regularization starting from the third loop.

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