Long-range effects in K^0-$\bar{K}^0$ mixing calculated in the potential model

Ho Tso-hsiu and Ching Cheng-rui
Center of Theoretical Physics, Chinese Center of Advanced Science and Technology (World Laboratory), and Institute of Theoretical Physics, Academia Sinica, Beijing, China

Li Xue-qian
Center of Theoretical Physics, Chinese Center of Advanced Science and Technology (World Laboratory), and Theoretical Physics Division, Nankai University, Tianjin, China

Tao Zhi-jian
Institute of Theoretical Physics, Academia Sinica, Beijing, China
(Received 21 June 1989; revised manuscript received 23 January 1990)

We calculate the long-range effects in K^0-$\bar{K}^0$ mixing in terms of the potential model. In contrast with the conventional methods for evaluating these effects, in the nonrelativistic approximation all possible one-pseudoscalar-meson intermediate states with light-quark quantum numbers are taken into account altogether. The numerical result shows that the long-range effects may be 53% to 62% of the total $\mathrm{Re}(M_{12})$.

I. INTRODUCTION

The K_L-K_S mass difference ($\Delta_m \equiv m_l - m_s$) has been a long-standing and challenging problem for several decades.[1] It is believed that solving this problem can provide us a good test for the standard model and enrich our knowledge on many aspects of particle physics. The paper by Gaillard and Lee[2] on K^0-$\bar{K}^0$ mixing led to a correct prediction of the c-quark mass. In some following works,[3] the box diagrams were calculated in detail. However, the matrix-element calculations are related to hadronization, about which we still lack enough knowledge; therefore mostly vacuum saturation is applied to this estimation,[1,2,4] as well as to some decay matrix-element calculations.[5,6] In addition the MIT bag model[7] and QCD sum rules[8] are employed to include soft QCD, i.e., some nonperturbative effects in hadronization. Meanwhile there are many other models to approach the problem of the K_L-K_S mass difference, such as the nonminimal left-right-symmetric model,[9] Higgs-boson-exchange box diagrams,[10] and technicolor[11] to test how the mechanisms beyond the standard model can contribute to K^0-$\bar{K}^0$ mixing. Each of them comes in from a different angle, and most of them are based on short-range behavior, i.e., box diagrams. Instead, this paper will concentrate on the long-range effects in K^0-$\bar{K}^0$ mixing which are calculated in the potential model.

Wolfenstein[12] pointed out that there are not only short-distance effects described by box diagrams, but also long-distance effects in K^0-$\bar{K}^0$ mixing which cannot be ignored. The expression can be written as

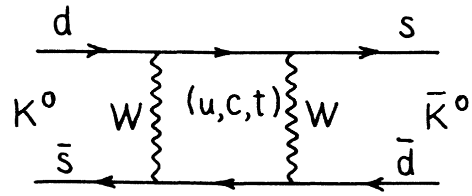
$$\Delta m = \Delta m|_{\mathrm{box}} + D\Delta m \ , \tag{1}$$

where $D = 1 - B\lambda$. Parameter D denotes the contribution from low-mass intermediate states. B is the famous parameter depending on the models for calculating matrix elements and λ corresponds to replacing m_u in the box diagrams by a characteristic hadronic mass, as Wolfenstein proposed; currently we know that almost all box contributions are from heavier c and t quarks in K^0-$\bar{K}^0$ or B^0-$\bar{B}^0$ mixing.

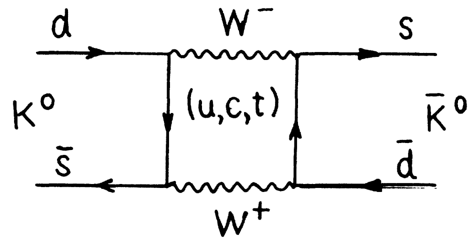
Many authors have estimated long-distance effects.[13,14] The significance of these effects is emphasized from both numerical results and physics intuition.

In this work, we introduce a new method which was developed in the double-β-decay calculations by Ho and Ching[15] to estimate long-distance effects in K^0-$\bar{K}^0$. This is based on the idea that there are not only degrees of freedom of real physical particles at long distances, but also free quark degrees of freedom at short distances. Therefore their contributions must be counted together. There may be a problem of overcounting because it is hard to draw a line separating long- and short-distance effects. Cea *et al.*[4] suggested a cutoff of 0.5–1 GeV which is about from the mass of the K meson to the chiral-symmetry-breaking scale Λ. In our mechanism overcounting is avoided when we count only the intermediate states with light-quark flavors.

The short-distance effects are obtained from the box diagrams depicted in Fig. 1. Looking at the two diagrams in Fig. 1, one can be convinced that only Fig. 1(a) can correspond to long-distance effects, while the intermediate states are not free-quark states, but physical particle states at long distances, propagating as $u\bar{u}$ pairs. This is because the W boson is very heavy in this energy scale. Furthermore, despite $c\bar{u}$, $u\bar{c}$ and $c\bar{c}$ might be D^0, $\bar{D}^0$, η_c and higher-excited states as intermediate states; however, since m_c is heavy, we can *a priori* ignore their contributions to long-distance effects.[16] Therefore, for evaluating long-distance effects in K^0-$\bar{K}^0$ mixing, only those states



(a)



(b)

FIG. 1. The box diagrams for K^0-$\bar{K}^0$

with quantum numbers of $u\bar{u}$ are taken into account. Generally speaking, as intermediate states there are not only one-particle contributions, but also two-, three-, and multiparticle contributions. In this paper, we only consider the one-particle contribution, because as is usually accepted, the multiparticle intermediate states are neglected, adopting the argument suggested by Regge phenomenology,[14] and we will give some further explanations by means of our numerical results later in this paper. It seems to contrast with the statement of Donoghue, Golowich, and Holstein[4] that there were no overall contributions from the single-octet particle states due to the Gell-Mann–Okubo formula; however, as a matter of fact, here our sum involves all possible excited states with the same $u\bar{u}$ flavors, and there is no strange-quark contribution, so it does not upset the Gell-Mann–Okubo formula.

In the standard Kobayashi-Maskawa (KM) framework,[17] the short-distance box diagrams are calculated, and the results are[18–19]

$$M_{12}=\frac{G_F^2}{6\pi^2}F_K^2 m_K B M_W^2[\lambda_c^2\eta_1 S(x_c)+\lambda_t^2\eta_2 S(x_t)+2\lambda_c\lambda_t\eta_3 S(x_c,x_t)]\ , \tag{2}$$

where $\lambda_i=V_{id}^* V_{is}$ with V_{ij} being the entries in the KM matrix and B takes various values for different models,[7–11] and, for $B=1$, the vacuum saturation is recovered. η_1, η_2, and η_3 are the QCD corrections[19] with $\eta_1\simeq 0.85$, $\eta_2\simeq 0.6$, and $\eta_3\simeq 0.4$. The functions $S(x_i)$ and $S(x_i,x_j)$ are given by

$$S(x_i)=x_i\left[\frac{1}{4}+\frac{9}{4}\frac{1}{1-x_i}-\frac{3}{2}\frac{1}{(1-x_i)^2}\right]+\frac{3}{2}\left[\frac{x_i}{(x_i-1)^3}\right]\ln x_i \equiv x_i F(x_i)\ , \tag{3}$$

$$S(x_i,x_j)=x_i x_j\left[\frac{1}{4}+\frac{3}{2}\frac{1}{1-x_j}-\frac{3}{4}\frac{1}{(1-x_j)^2}\right]\frac{\ln x_j}{x_j-x_i}+(x_j\leftrightarrow x_i)-\frac{3}{4}\frac{1}{(1-x_i)(1-x_j)}\ , \tag{4}$$

where $x_i=m_i^2/M_W^2$.

These are the short-distance effects from the box diagrams. It is easy to see that since m_u is very small, it gives no substantial contribution to the box diagrams calculated in the quark picture. On the contrary, the physical-particle states with $u\bar{u}$ flavor dominate the long-distance effects, as discussed above; therefore, overcounting is minimized in our calculations.

In the next section, we introduce the Ho-Ching method and apply it to our calculations. Then we give numerical results and a discussion.

II. OUR MODEL

Generally, the mixing matrix elements can be expressed as[1]

$$M_{ij}=m_k\delta_{ij}+\langle i|H_{W,\Delta s=2}|j\rangle+P\sum_{\lambda}\frac{\langle i|H_{W,\Delta s=1}|\lambda\rangle\langle\lambda|H_{W,\Delta s=1}|j\rangle}{m_k-m_\lambda} \tag{5}$$

and

$$\Gamma_{ij}=2\pi\sum_{\lambda}\rho_\lambda\langle i|H_{W,\Delta s=1}|\lambda\rangle\langle\lambda|H_{W,\Delta s=1}|j\rangle\ , \tag{6}$$

where ρ_λ is the density of the λ state. Without $H_{W,\Delta s=2}$ from the standard model, M_{12} only obtains a contribution from the last term of Eq. (5). At short distances, the intermediate states $|\lambda\rangle$'s of a complete set are free-quark states such as $u\bar{u},u\bar{c},c\bar{u},\ldots,t\bar{t}$, etc., and calculated by the regular Feynman rules. The results are given in Eq. (2) by a calculation in the box diagrams. When we turn to the long-distance effects, there $|\lambda\rangle$'s are physical-particle states and include many one-particle states, both ground and excited, and multiparticle states as well. Our task is to find all their contributions and sum them with the correct relative phases. The strategy is to avoid calculating individual contributions and obtain an overall result instead, as Ho and Ching did for double-β decays.[15]

The long-distance effect is related to the physical-particle intermediate states, so

$$-M_{fi}=\sum_n{}' \frac{\langle f|H_W|n\rangle\langle n|H_W|i\rangle}{E_n-E_f}$$
$$=\sum_n{}'\left\langle f\left|H_W\frac{1}{E_n-E_f}\right|n\right\rangle\langle n|H_W|i\rangle$$
$$=\sum_n{}'\left\langle f\left|H_W\frac{1}{H_S-E_f}\right|n\right\rangle\langle n|H_W|i\rangle\ , \qquad (7)$$

where $|f\rangle=|K^0\rangle$, $|i\rangle=|\bar{K}^0\rangle$, and $E_f=E_i=m_k$. H_W is the effective $\Delta s=1$ weak Hamiltonian and $|n\rangle$'s are states which are not $|K^0\rangle$ and $|\bar{K}^0\rangle$. H_s is the strong-interaction Hamiltonian and $|n\rangle$'s are eigenstates of H_s. Principally, $|n\rangle$ includes any number of physical particles, so

$$-M_{fi}=\sum_l{}'\left\langle f\left|H_W\frac{1}{H_s^{(1)}-E_f}\right|1,l\right\rangle\langle 1,l|H_W|i\rangle$$
$$+\sum_l{}'\left\langle f\left|H_W\frac{1}{H_s^{(2)}-E_f}\right|2,l\right\rangle\langle 2,l|H_W|i\rangle$$
$$+\cdots\ , \qquad (8)$$

where the superscripts denote the number of intermediate particles,

$$H_S^{(2)}=H_{S,1}^{(2)}+H_{S,2}^{(2)}+H_{S,12}^{(2)}\ , \qquad (9)$$

where $H_{S,1}^{(2)}$ and $H_{S,2}^{(2)}$ are the Hamiltonians of the single mesons 1 and 2, respectively, and $H_{S,12}^{(2)}$ is the interaction between them and usually can be ignored at this energy scale. If we assume the real part of the matrix elements of two particles is suppressed, only the first term in (8) remains. For $H_S^{(2)}$, all $|1,l\rangle$ constitute a complete set, so Eq. (8) converges into

$$-M_{fi}=\sum_l\left\langle f\left|H_W\frac{1}{H_S^{(1)}-E_f}\right|1,l\right\rangle\langle 1,l|H_W|i\rangle$$
$$=\left\langle f\left|H_W\frac{1}{H_S^{(1)}-E_f}H_W\right|i\right\rangle\ . \qquad (10)$$

For a one-particle intermediate state which has $u\bar{u}$ flavor, there is no particle with a mass equal to m_K, so that there is no singularity. By the way, in the case of multiparticles, E_n can be equal to m_K and then give an absorptive part which corresponds to $\Delta\Gamma$[5]. In nuclear physics, the conventional treatment in Eq. (8) is to set (E_n-E_f) as an average value and pull it out from the sum; then by completeness, $\sum_n|n\rangle\langle n|$ converges to unity. Ho and Ching suggested the insertion of $1/(E_n-E_f)$ into the brackets and turning E_n into the Hamiltonian operator of the strong-interaction H_S. Since the states $|n,l\rangle$ are eigenstates of H_S $(H_S=H_S^{(1)}+H_S^{(2)}+\cdots=\sum_i H_S^{(i)})$ with eigenvalues E_n's the substitution is obvious. For one-particle $H^{(1)}$, $\sum_l|1,l\rangle\langle 1,l|=1$ can be dropped out because other parts in the formulation do not dependent on l at all.

Defining

$$\frac{1}{H_S-E_f}H_W|i\rangle\equiv|\varphi\rangle\ , \qquad (11)$$

we obtain

$$(H_S-E_f)|\varphi\rangle=H_W|i\rangle \qquad (12)$$

and

$$-M_{fi}=\langle\bar{K}^0|H_W|\varphi\rangle\ , \qquad (13)$$

where

$$H_W=\frac{G_F}{\sqrt{2}}\sin\theta_C\cos\theta_C[\bar{s}\gamma_\mu(1-\gamma_5)u][\bar{u}\,\gamma^\mu(1-\gamma_5)d]$$

and θ_C is the Cabibbo angle. $|i\rangle=|K^0\rangle$ is understood. So we convert the problem of summing over all intermediate states in Eq. (8) to solving Eq. (12). Equation (12) is not an eigenequation, because $E_f=m_K$ is fixed. From H_W, it is easy to see $|\varphi\rangle$ has a $u\bar{u}$ flavor as we expect.

To solve this equation, we need a concrete form for H_S. H_S is the strong-interaction operator and has a set of eigenvalues E_l. The main purpose of this paper is to demonstrate the usage of this method, so that we employ a simple potential model. Fortunately, many studies of potential models have been done[20] for not only finding the spectra, but also the dynamical quantities; for instance, Cea, Colangelo, Cosmai, and Nardulli and Krasmann calculated the decay constants of heavy and light mesons in a potential model.[21,22] As Godfrey and Isgur claimed, mesons from π to Υ can be described by a unified quark model with QCD.[23] Here we would take the potential and parameters given in Ref. 23:

$$H_S=H_0+V_{ij} \qquad (14)$$

and

$$H_0=\sum_{i=1}^{2}\left[m_i+\frac{P^2}{2m_i}\right]\ ,$$
$$V_{ij}=H_{ij}^{\rm conf}+H_{ij}^{\rm hyp}+H_{ij}^{\rm so}+H_A\ , \qquad (15)$$

where

$$V_{ij}^{\rm conf}=-\left[\frac{3}{4}c+\frac{3}{4}br-\frac{\alpha_s(r)}{r}\right](\mathbf{F}_i\cdot\mathbf{F}_j) \qquad (16)$$

includes the spin-independent linear confinement and Coulomb-type interactions. The $H_{ij}^{\rm hyp}$ and $H_{ij}^{\rm so}$ are the color-hyperfine and spin-orbit interactions, respectively, and H_A is the annihilation interaction. Generally speaking, the relativistic corrections are not very small; however, for simplicity we first seek for the solution $|\varphi\rangle$ only from $H_0+H_{ij}^{\rm conf}$, while other terms from relativistic effects would be treated as perturbations which modify our resultant $|\varphi\rangle$ in a reasonable scale.

For a bound state,[24]

$$|M,\mathbf{P}_{\rm c.m.}\rangle=d_M^\dagger(0)|0\rangle \qquad (17)$$

and, in the nonrelativistic approximation,

$$d_M^\dagger(0)=\sum_{r,s}\int d^3p\, \Phi(\mathbf{p})\varphi(r,s)d^\dagger(-\mathbf{p},s)b^\dagger(\mathbf{p},r)\ , \tag{18}$$

where $d_M^\dagger$ is the creation operator of the meson and $\Phi(\mathbf{p})$ is the momentum distribution function; $\varphi(r,s)$ is the spin function relating the spin indices r and s, and $b^\dagger, d^\dagger$ are creation operators of quark and antiquark, respectively. With normalization $\int d^3p|\Phi(\mathbf{p})|^2=1$, for the K^0 meson,

$$d_M^\dagger(0)=\int \Phi(\mathbf{p})d^3p\tfrac{1}{2}[b_d^\dagger(\mathbf{p},\uparrow)d_{\bar{s}}^\dagger(-\mathbf{p},\downarrow) - b_d^\dagger(\mathbf{p},\downarrow)d_{\bar{s}}^\dagger(-\mathbf{p},\uparrow)]\ . \tag{19}$$

Then one can easily derive

$$H_W|K^0\rangle=(2\pi)^{3/2}\frac{G_F}{\sqrt{2}}\sin\theta_C\cos\theta_C\left[\int\Phi(\mathbf{p})d^3p\right] \times\int\frac{d^3p_u}{(2\pi)^3}\frac{d^3p_{\bar{u}}}{(2\pi)^3}\frac{m_{\bar{s}}m_d m_u m_{\bar{u}}}{E_{\bar{s}}E_dE_uE_{\bar{u}}} \times\sqrt{2}[d_{\bar{u}}^\dagger(\mathbf{p}_{\bar{u}}\uparrow)b_u^\dagger(\mathbf{p}_u,\downarrow) - d_{\bar{u}}^\dagger(\mathbf{p}_{\bar{u}},\downarrow)b_u^\dagger(\mathbf{p}_u,\uparrow)]|0\rangle \tag{20}$$

since we take the nonrelativistic approximation, and there do not exist vector mesons, so there exist only pseudoscalars with flavor $u\bar{u}$. Therefore we can determine the form of the $|\varphi\rangle$ wave function.

We write $|\varphi\rangle$ in the form

$$|\varphi\rangle=(2\pi)^{3/2}\int f(\mathbf{p})d^3p\frac{1}{\sqrt{2}} \times[d_{\bar{u}}^\dagger(-\mathbf{p},\uparrow)b_u^\dagger(\mathbf{p},\downarrow) - d_{\bar{u}}^\dagger(-\mathbf{p},\downarrow)b_u^\dagger(\mathbf{p},\uparrow)]|0\rangle\ , \tag{21}$$

where the Van Royen-Weisskopf convention is employed.

Expanding the wave function and comparing coefficients of the corresponding creation operator on both sides of Eq. (12), we derive

$$(H_S-m_K)f(\mathbf{p})=\sqrt{2}G_F\sin\theta_C\cos\theta_C\int\phi(\mathbf{p})d^3p\ . \tag{22}$$

Coming to the coordinate configuration, it becomes

$$(H_S-m_K)f(\mathbf{r})=\sqrt{2}G_F\sin\theta_C\cos\theta_C\phi(0)\delta(\mathbf{r})\ . \tag{23}$$

It is noted that in our derivations we omit some $(2\pi)^3$ in front of $\delta^3(\mathbf{p})$ and $\delta^3(\mathbf{r})$; careful calculations show that $(2\pi)^3$ finally is canceled and a form of Eq. (23) is given. After some simple manipulations, a compact result is reached

$$M_{ij}=4G_F^2\sin^2\theta_C\cos^2\theta_C|\phi(0)|^2f(0)\ . \tag{24}$$

III. ABOUT $F(\mathbf{r})$ AND THE NUMERICAL RESULTS

From Eq. (23), one can see

$$(H_S-m_K)F(\mathbf{r})=\delta^3(\mathbf{r})\ , \tag{25}$$

where

$$F(\mathbf{r})=f(\mathbf{r})/\sqrt{2}\sin\theta_C\cos\theta_C\phi(0)\ .$$

Therefore, it is easy to associate $[1/(2\mu)]F(\mathbf{r})$ with an effective three-dimensional two-point Green's function in K^0 under an effective potential, or a sort of propagator in a stationary state. First, let us neglect the relativistic correction; Eq. (25) reads

$$\left[\sum_i\left[m_i+\frac{-\Delta^2}{2m_i}\right]+\left[c+br-\frac{4\alpha_s}{3r}\right]-m_K\right]F(\mathbf{r}) =\left[M-\frac{\Delta^2}{2\mu}+\left[c+br-\frac{4\alpha_s}{3r}\right]-m_K\right]F(\mathbf{r}) =\delta^3(\mathbf{r})\ , \tag{26}$$

where the color factor $(\mathbf{F}_i\cdot\mathbf{F}_j)$ is averaged in the meson as $(-\frac{4}{3})$ and M is the total mass (m_1+m_2), μ is the reduced mass as $m_1m_2/(m_1+m_2)$, and in our case m_1 and m_2 are the constituent masses of the u quark which is about 220 MeV. In the center-of-mass frame of K^0, we can drop the c.m. momentum $|\mathbf{P}|$. $F(\mathbf{r})$ is the two-point correlation function, while $\mathbf{r}$ is the relative coordinate between the quark and antiquark.

This question can be numerically solved by using the smearing technique with a smearing function[23]

$$\rho_{ij}(r)=\frac{\sigma_{ij}^3}{\pi^{3/2}}e^{-\sigma_{ij}^2r^2}\ , \tag{27}$$

where

$$\sigma_{ij}^2=\sigma_0^2\left[\frac{1}{2}+\frac{1}{2}\left[\frac{4m_im_j}{(m_i+m_j)^2}\right]^4\right]+s^2\left[\frac{2m_im_j}{m_i+m_j}\right]^2 \tag{28}$$

with $\sigma_0=1.8$ GeV, $s=1.55$, and $m_i=m_j=m_u$.

Numerically, we obtain

$$F(0)=1.1\times10^{-2}\ \text{GeV}^2\ . \tag{29}$$

Because $[1/(2\mu)]F(r)$ can be interpreted as a Green's function in the K meson, one can estimate this value in a way in which the physics picture is clear. We attribute all potentials and mass terms to a boundary condition which confines the propagating within a spherical cavity. A free Green's function is given by Lee.[25] In our case, we modify it due to an effective mass and have a Yukawa-type propagator in the stationary state:

$$\bar{F}(r)\sim\frac{2\mu e^{-r/R}}{4\pi r}\ , \tag{30}$$

where R is an effective radius of the spherical cavity; usually for K^0 it is about $1/m_K$ GeV^{-1}.

To compare this $F(0)$ with that from solving Eq. (26), we also apply the smearing function ρ_{ij} to $\bar{F}(r)$ and obtain the expression

$$F(0)=\frac{2\mu}{4\pi}\left[\frac{\sigma_{ij}}{\pi^{1/2}}-\frac{1}{R}e^{-1/(2R^2\sigma_{ij}^2)}\text{erfc}\left[\frac{1}{2\sigma_{ij}R}\right]\right]\ , \tag{31}$$

where erfc is the complementary error function. The numerical result is

$$F(0)=1.33\times 10^{-2}\ \text{GeV}^2\ . \tag{32}$$

It is noted that the calculated result is not sensitive to the parameter R, i.e., the radius of the cavity of confinement.

This number is very close to that directly obtained from numerically solving Eq. (26) with the parameters given in Ref. 23.

The Van Royen–Weisskopf equation[24] gives

$$\phi(0)=\frac{\sqrt{m_K}f_K}{2}\ , \tag{33}$$

where there is no color factor $\sqrt{3}$ due to the conventional adopted in our calculations; then the expressions of Ref. 23 would read

$$f=\frac{c\mu}{\sqrt{M}},\quad c=52\ \text{MeV}^{1/2}\ . \tag{34}$$

Then we have $|\phi(0)|^2=0.48$ (fm^{-3}) from (32). Substituting all information back to a modified expression of Eq. (24), we have

$$\text{Re}(M_{fi})=4G_F^2\sin^2\theta_C\cos^2\theta_C|\phi(0)|^2F(0)\ ; \tag{35}$$

numerically,

$$\text{Re}(M_{fi})=(0.092\text{–}0.11)\times 10^{-14}\ \text{GeV}\ . \tag{36}$$

That is about 53%–62% of the total $\text{Re}(M_{12})$ $[m_L-m_S=2\,\text{Re}(M_{12})]$. From the Particle Data Group table,[26] $\text{Re}M_{12}=0.175\times 10^{-14}$ GeV.

IV. DISCUSSION AND CONCLUSION

From the above calculations, we obtain long-distance effects in K^0-$\bar{K}^0$ mixing about 53%–62% of the total $\text{Re}(M_{12})$. Donoghue, Golowich, and Holstein[27] obtained the long-distance effect parameter D as

$$D=\begin{cases}+0.64(\pm 0.17) & \text{if } B>0\ ,\\ 1.33(\pm 0.17) & \text{if } B<0\ ,\end{cases} \tag{37}$$

compared with our result of about 53%–62%, reasonably consistent with their value.

In this work, we only consider one-particle intermediate states as the source of long-range effects; however, multiparticle-intermediate states also contribute through loops. We can give a rough estimation of the infinite series. If $H^{(n)}=\sum_{m=1}^{n}H_m^{(n)}+\sum_{m<l}H_{ml}+$ (three-body interactions), we can omit the interaction between two and three mesons in the intermediate states; then $H^{(n)}=\sum_{m=1}^{n}H_m^{(n)}$ and we can expect that a reasonable factorization can give $F^{(n)}(0)\propto[F^{(1)}(0)]^n$. Then the series becomes

$$\begin{aligned}F(0)&=c_1F^{(1)}(0)+c_2F^{(2)}(0)+\cdots+c_nF^{(n)}(0)+\cdots\\ &\simeq c_1F^{(1)}(0)+c_2[F^{(1)}(0)]^2\\ &\quad+\cdots+c_n[F^{(1)}(0)]^n+\cdots\ .\end{aligned} \tag{38}$$

By a dimension analysis, $c_2\sim 1/m_K^2$, $c_n\sim(1/m_K^2)^{n-1}$, etc. From our value, $F(0)/m_K^2\sim 0.053$, and it means that the series (39) converges fast enough, and the contribution from the two-particle intermediate state is only 6% of that of one particle. In our future paper, we will give the estimation in more detail.

From field theory there is

$$\begin{aligned}M_{ij}+\frac{i}{2}\Gamma_{ij}&=\sum_{\lambda}\frac{\langle i|H_{W,\Delta s=1}|\lambda\rangle\langle\lambda|H_{W,\Delta s=1}|f\rangle}{m_K-m_\lambda}\\ &=P\sum_{\lambda}\frac{\langle i|H_{W,\Delta s=1}|\lambda\rangle\langle\lambda|H_{W,\Delta s=1}|f\rangle}{m_K=m_\lambda}\\ &\quad+2\pi i\sum_{\lambda}\delta(m_K-m_\lambda)\langle i|H_{W,\Delta s=1}|\lambda\rangle\\ &\quad\times\langle\lambda|H_{W,\Delta s=1}|f\rangle\ .\end{aligned} \tag{39}$$

The second term represents resonances with the effective mass $m_\lambda=m_K$. For one-particle intermediate states, there is no such physical resonance on the particle table, so that to the imaginary part, i.e., Γ_{ij}, at least two-particle intermediate states are necessary. This is easily obtained from the expressions given in Ref. 27 by calculating the absorptive part of $\Sigma(s)$.

Even though we apply the relativistic correction, in the main calculation process we employ the nonrelativistic approximation, which may introduce some errors,[28] and furthermore there are some potential model parameters obtained from fitting the data so that the accuracy of the calculations may be influenced. Numerically, the D parameter takes a value of about 53%–62%, which is reasonably consistent with data. The advantage of this method is that one can evade the troublesome summing over all individual intermediate states and give results directly. This method can be generalized to other areas where an infinite series is involved.

Our conclusion is that this method is applicable in K^0-$\bar{K}^0$ mixing long-distance-effects evaluation and the resultant D value taken 53%–62%, which coincides with data reasonably.

ACKNOWLEDGMENTS

The authors would like to thank Professor Z. Y. Zhu for helpful discussions on the quark-hadron degrees of freedom, and we benefited greatly from the fruitful suggestions of Professor C. H. Chang. This work was supported in part by the National Natural Science Foundation, China.

[1]L. L. Chau, Phys. Rep. **95**, 1 (1983).
[2]M. K. Gaillard and B. W. Lee, Phys. Rev. D **10**, 897 (1974).
[3]T. Inami and C. Lim, Prog. Theor. Phys. **65**, 297 (1981); J. Hagelin, Nucl. Phys. **B193**, 123 (1981).
[4]J. Donoghue, E. Golowich, and B. Holstein, Phys. Lett. **119B**, 412 (1982).
[5]J. Donoghue, Phys. Rev. D **30**, 1499 (1984); A. Le Yaouanc, L. Oliver, O. Péne, and J. C. Raynal, Phys. Lett. B **182**, 383 (1986); S. Godfrey, Phys. Rev. D **33**, 1391 (1986).
[6]R. Marshak, Riazuddin, and C. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969).
[7]P. Colic, B. Guberina, D. Tadic, and J. Trampetic, Nucl. Phys. **B221**, 141 (1983); B. McWilliams and O. Shanker, Phys. Rev. D **22**, 2853 (1980).
[8]K. Chetyrkin, A. Kataev, A. Krasulin, and A. Pivovarov, Phys. Lett. B **174**, 104 (1986).
[9]M. Leurer, Nucl. Phys. **B266**, 147 (1986).
[10]T. Inami and C. Lim, Prog. Theor. Phys. **65**, 297 (1981).
[11]W. C. Kuo, G. Lu, B.-L. Young, and Z. Zhou, Phys. Lett. B **214**, 253 (1988).
[12]L. Wolfenstein, Nucl. Phys. **B160**, 501 (1979); C. Hill, Phys. Lett. **97B**, 574 (1980).
[13]C. Itzykson, M. Jacob, and G. Mahoux, Nuovo Cimento Suppl. **5**, 978 (1967); T. Truong, Phys. Rev. Lett. **17**, 1102 (1966); R. Rockmore and T. Yao, *ibid.* **18**, 501 (1967); K. Kang and D. Land, *ibid.* **18**, 503 (1967); D. Greenberg, Nuovo Cimento **56**, 597 (1968).
[14]N. Billić and B. Guberina, Phys. Lett. **136B**, 440 (1984); I. Bigi and A. Sanda, *ibid.* **148B**, 205 (1984); G. Nardulli and G. Preparata, *ibid.* **104B**, 399 (1981); G. Nardulli, G. Preparata, and D. Rotondi, Phys. Rev. D **27**, 557 (1983); P. Cea, G. Nardulli, and G. Preparata, Phys. Lett. **148B**, 477 (1984); P. Cea and G. Nardulli, *ibid.* **152B**, 251 (1985).
[15]T. H. Ho and C. R. Ching, Academia Sinica Institute of Theoretical Physics Reports Nos. AS-ITP-88-021, AS-ITP-88-004, AS-ITP-88-035, 1988 (unpublished).
[16]L. Wolfenstein, Phys. Lett. **164B**, 170 (1985); J. Donoghue, E. Golowich, and B. Holstein, Phys. Rev. D **33**, 179 (1986).
[17]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[18]A. Buras, Max-Planck-Institute Report No. MPI-PAE-/PTH/1/88, 1988 (unpublished).
[19]F. Gilman and M. Wise, Phys. Rev. D **27**, 1128 (1983); M. Wise, SLAC Report No. 227, 1980 (unpublished).
[20]For example, A. Martin, Phys. Lett. **93B**, 338 (1980); **100B**, 511 (1981); P. Moxhay and J. Rosner, Phys. Rev. D **28**, 1132 (1983).
[21]P. Cea, P. Colangelo, L. Cosmai, and G. Nardulli, Phys. Lett. B **206**, 691 (1988).
[22]H. Krasmann, Phys. Lett. **96B**, 397 (1980).
[23]D. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[24]R. Van Royen and V. Weisskopf, Nuovo Cimento **50A**, 617 (1967).
[25]T. D. Lee, Phys. Rev. D **19**, 1802 (1979).
[26]Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. **170B**, 1 (1986).
[27]J. Donoghue, E. Golowich, and R. Holstein, Phys. Lett. **135B**, 481 (1984).
[28]As we mention in the text, because of the nonrelativistic approximation some errors are unavoidable. A more detailed calculation on the relativistic Bethe-Salpeter equation is under preparation and will be published elsewhere.