Brief Reports

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Theoretical implications of Fermilab Tevatron total and elastic differential-cross-section measurements

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Production of gluon jets becomes a feature of average hadron collisions for multi-TeV energies, leading to a model where interactions are mostly mediated by semihard gluons. We calculate forward scattering in this model as the shadow of the large inelastic cross sections associated with gluon jets. Using Fermilab Tevatron experiment E710 results, we deduce that ρ at 546 GeV is well below the central UA4 value of 0.24, defusing this number as a crucial issue.

I. INTRODUCTION

The E710 experiment[1,2] at the Fermilab Tevatron collider has measured the differential cross section for elastic $\bar{p}p$ scattering at $\sqrt{s}=1.8$ TeV. From this they extract

$$B(0)=\frac{d}{dt}\left[\ln\frac{d\sigma_n}{dt}\right]\Bigg|_{t=0},$$

the forward slope of the nucleon scattering cross section. Using the optical theorem, they also obtain $\sigma_{\text{tot}}(1+\rho^2)^{1/2}$, where ρ is the ratio of the real to the imaginary part of the forward strong-interaction scattering amplitude f_N. This quantity essentially determines σ_{tot}, since the correction due to $(1+\rho^2)^{1/2}$ is expected to be about 1%. The new experimental results are shown in Figs. 1–3, along with previous measurements. The fitted curves shown are described below. In Fig. 4, a prediction is made for the differential elastic-scattering cross section $d\sigma/dt$ vs $|t|$, at $\sqrt{s}=1.8$ TeV.

The unexpectedly large ρ value of 0.24 ± 0.04, measured[3] by the UA4 Collaboration at $\sqrt{s}=546$ GeV, and shown in Fig. 2, has suggested the possibility[4] of an anomalously rapid increase of σ_{tot} at higher energies. The new measurement of σ_{tot} at $\sqrt{s}=1.8$ TeV allows us to make a quantitative evaluation of this conjecture.

We analyze in this Brief Report the data shown in Figs. 1–3 in conjunction with a parametrization of a QCD-inspired eikonal description of the data. The results are similar to simple $\ln^2 s$ analytic considerations.[5] We conclude that $\rho\sim0.14$ at $\sqrt{s}=546$ GeV. This is well below the central value of the UA4 measurement. Our QCD model implies a $\ln^2 s$ behavior in the presently accessible energy region and predicts a cross section of ~125 mb at Superconducting Super Collider energies.

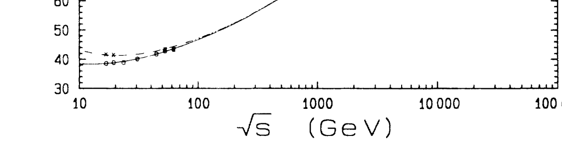

FIG. 1. Total cross sections calculated and measured for $\bar{p}p$ (dashed curve and crosses) and pp (full curve and squares). The parameters in the calculation are $\epsilon=0.05$, $\mu_{qq}=0.89$ GeV, $\mu_{gg}=0.73$ GeV, $\mu_{\text{odd}}=0.53$ GeV, $m_0=0.60$ GeV, $a=41.8$ mb, $b=220.6$ mb, $a'=-3.56$ mb, $b'=1.22$ mb, $a''=-57.68$ mb, $\sigma_{gg}\equiv9\pi\alpha_s^2/\delta^2=0.0634$ mb.

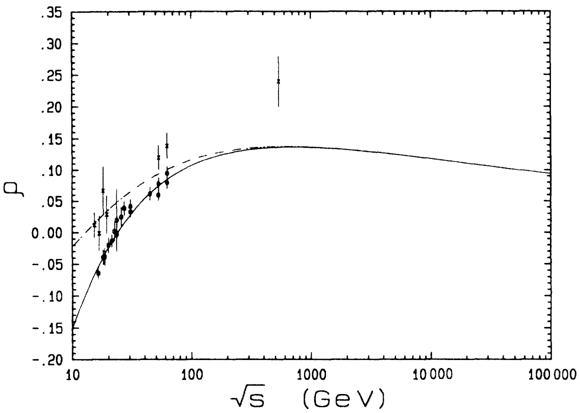

FIG. 2. The calculated ratio $\rho = \text{Re}f(0)/\text{Im}f(0)$, using the same parameters as in Fig. 1, compared with experiments for $\bar{p}p$ (dashed curve and crosses) and pp.

From our analysis we conclude that no dramatic energy dependence above 1.8 TeV should be inferred, in contrast with conjectures based on the UA4 measurement of ρ.

II. GLUON INTERACTIONS AND DIFFRACTION

We previously[6,7] proposed a model for very-high-energy interactions mediated by gluons. We present a more general version here taking quarks into account. This gives the flexibility to parametrize more adequately the low-energy ($\sqrt{s} \gtrsim 15$ GeV) data. In this model, hadrons interact via semihard collisions of their quark and gluon constituents. The probability of collision at impact parameter b is assumed to be

$$P_{ij}(b,s) = W_{ij}(b)\sigma_{ij}^{\text{QCD}}(s) \qquad (1)$$

with

$$\sigma_{ij}^{\text{QCD}}(s) = \int d\left[\frac{\hat{s}}{s}\right] F_{ij}\left[x_1x_2 = \frac{\hat{s}}{s}\right]\sigma_{ij}(\hat{s}) \; . \qquad (2)$$

Here i,j refer to colliding quarks or gluons. The factor $W_{ij}(b)$ describes the impact parameter dependence of the interaction probability; the integral in (2) describes its dependence on the longitudinal fractional momenta x_1, x_2 of the partons i,j mediating the collision. We next describe these factors in more detail.

$W_{ij}(b)$ represents the overlap at impact parameter b of the surface area occupied by quarks or gluons inside the Lorentz-contracted high-energy hadrons: i.e.,

$$W_{ij} = \int d^2b' D_i(b') D_j(|\mathbf{b}-\mathbf{b}'|) \; . \qquad (3)$$

We will assume that $D(b)$, the density function, is the Fourier transform of a dipole form factor $(1-t/\mu^2)^{-2}$. This form is usually associated with the charge structure of the proton. For $i=j$,

$$W_{ij}(b) = \frac{\mu_{ii}^2}{96\pi}(\mu_{ii}b)^3 K_3(\mu_{ii}b) \; . \qquad (4)$$

The second factor in Eq. (1) computes the interaction cross section of partons i,j for $\hat{s} > m_0^2$. Here $\hat{s} = \tau s = x_1 x_2 s$ is the subprocess c.m. energy squared. In Eq. (2),

$$F_{ij} = \int dx_1 dx_2 f_i(x_1) f_j(x_2) \delta(x_1x_2 - \tau) \qquad (5)$$

Here f_i are the usual structure functions and

$$\sigma_{ij}(\hat{s}) = K_{ij}\frac{9\pi\alpha_s^2}{\delta^2}\theta(\hat{s}-m_0^2) \; , \qquad (6)$$

where K_{ij} is a color factor equal to 1, $2(\frac{4}{9})$, and $(\frac{4}{9})^2$ for gg, qg, and $q\bar{q}$, respectively. Except for possible logarithmic variation of α_s, which we will neglect, the energy dependence of the cross section is totally determined by m_0. This is the critical energy scale defining the onset of the semihard collisions which dictate the interaction of hadrons. The quantity δ is, like m_0, of the order 1 GeV and defines the gluon momentum-transfer scale.

We first examine the gg term in Eq. (3) in detail since it dominates at high energy and, by itself, determines the asymptotic behavior of the total cross section. The physics is most transparent when working with the parametrization of the gluon structure function

$$f(x) \sim \frac{(1-x)^5}{x^J} \; . \qquad (7)$$

The crucial quantity is J, which controls the evolution of the gluon structure at small x. In Regge language J is the intercept of the Pomeron. The factor in Eq. (1) determining the energy dependence is

$$\int_{m_0^2/s}^{1} d\tau F_{gg}(\tau) \; ,$$

which is evaluated by substituting Eqs. (5) and (6) into Eq. (2). One obtains, for large enough s,

$$\int_{m_0^2/s}^{1} d\tau F_{gg}(\tau) \sim \int_{m_0^2/s}^{1} d\tau \frac{-\ln\tau}{\tau^J} \sim \left[\frac{s}{m_0^2}\right]^{J-1} . \qquad (8)$$

Note that F_{gg} counts the number of gluons in the colliding hadrons. The number increases rapidly at

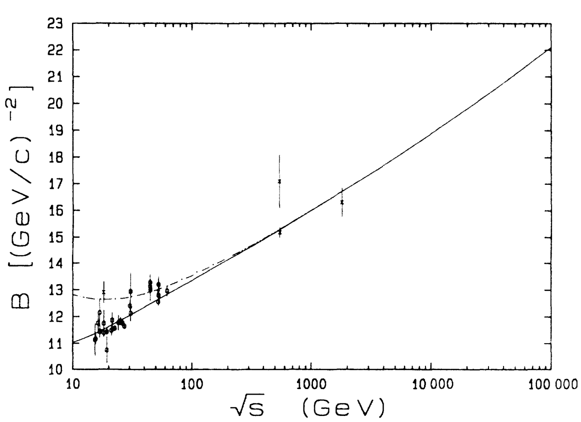

FIG. 3. The forward slope $B(0)$ compared with experiment for $\bar{p}p$ (dashed curve and crosses) and pp.

$x \simeq m_0/\sqrt{s}$ and this is the origin of the rising cross section. More quantitatively,

$$P_{gg}(b,s) \sim W_{gg}(b) s^{J-1} \quad (9)$$

after combining Eqs. (1), (2), and (8). For an interaction probability $P_{gg} \ll 1$ the energy dependence of the amplitude is given by a Pomeron pole-type power law s^{J-1}. When the number of gluons becomes large, P_{gg} exceeds unity for a *critical* impact parameter b_c given by

$$c W_{gg}(\mu_{gg} b_c) s^{J-1} \sim 1 , \quad (10)$$

where c is a constant. For large values of μb, Eqs. (4) and (10) yield the following value of b_c:

$$c'(\mu_{gg} b_c)^{3/2} e^{-\mu_{gg} b_c} s^{J-1} \sim 1 \quad (11)$$

with c' another constant, and, therefore,

$$b_c = \frac{J-1}{\mu_{gg}} \ln \frac{s}{s_0} + O\left[\ln \ln \frac{s}{s_0'}\right] . \quad (12)$$

The large number of gluons turns the nucleon into a black disc of radius b_c. We note the cardinal role of J in determining the energy behavior, since J controls the increase of the number of gluons; see Eq. (8). In summary, the energy dependence of the cross section transforms from s^{J-1} at low energy to the black-disk cross section

$$\sigma_{\text{tot}} = 2\pi \left[\frac{J-1}{\mu_{gg}}\right]^2 \ln^2 \frac{s}{s_0} , \quad (13)$$

as we go to very high energies. We thus reproduce the Froissart bound from QCD arguments as long as $J>1$. The Froissart-bound coefficient of the $\ln^2(s/s_0)$ term, $1/m_\pi^2 = 20$ mb, is here replaced by $[(J-1)/\mu_{gg}]^2 \sim 0.002$ mb. We note that μ_{gg} controls the size of the area occupied by the gluons inside the nucleon.

It is important to point out that the Froissart-type behavior (13) is realized in the collider energy range. The crucial condition is Eq. (9) and the power dependence on s, rather than a logarithmic dependence, which requires $J>1$. From a pragmatic point of view, we can obtain $J = 1+\epsilon$ by matching (7) to experimental determinations of the gluon structure function in the $x \simeq 10^{-3}$, $Q^2 \simeq m_0^2 < 10$ GeV^2 range relevant to this analysis.[7] One concludes that $\epsilon \lesssim 0.1$, consistent with values obtained by fitting total-cross-section data. From a theoretical point of view, the asymptotic behavior of σ_{tot} depends on the value J approaches when $x(=m_0/\sqrt{s}) \to 0$. Also here arguments have been advanced that $\epsilon > 0$; i.e., the Pomeron is "supercritical."[8]

For the purpose of investigating the cross section at lower energies one has to consider the qq and qg terms in Eq. (1). It is instructive to evaluate their energy dependence from (2), (5), and (6) for toy structure functions $f_q \sim x^{-1/2}(1-x)^3$ and $f_g \sim x^{-1}(1-x)^5$. The result has the form

$$P_{qq} = W_{qq}(\mu_{qq} b) \left[\frac{m_0}{\sqrt{s}} \ln \frac{s}{s_0} + \mathcal{P}\left[\frac{m_0}{\sqrt{s}}\right]\right] \quad (14)$$

and

$$P_{qg} = W_{qg}(\mu_{qg} b) \left[a' \ln \frac{s}{s_0} + \mathcal{P}'\left[\frac{m_0}{\sqrt{s}}\right]\right] . \quad (15)$$

$\mathcal{P}$ and $\mathcal{P}'$ are polynomials in $m_0/\sqrt{s}$. The analysis we presented to evaluate the high-energy behavior of the gg term is inapplicable to the qq and qg terms which are expected to describe much of the cross section below collider energies. We will parametrize Eqs. (14) and (15) with a constant, a term $s^{-1/2}$, and a term $\ln s$, as suggested by standard Regge arguments. All we will attempt is to decipher the high-energy behavior of the forward amplitude and will limit ourselves to parametrizing the low-energy data.

The experimental observables are calculated from $P(b,s)$ and

$$\sigma_{\text{tot}} = 4\pi \, \text{Im} f_N , \quad (16)$$

$$\frac{d\sigma}{dt} = \pi |f_N|^2 , \quad (17)$$

with

$$f_N = i \int b \, db \, J_0(b\sqrt{-t})(1 - e^{-P(b,s)/2}) . \quad (18)$$

In this notation the usual eikonal function is given by

$$2\chi(b,s) = W(b)\sigma(s) = P(b,s) . \quad (19)$$

In Eq. (18), $P = P_{gg} + P_{qg} + P_{qq}$. We will compute P_{gg} exactly using (1), (6), and (7). Guided by Eqs. (14) and (15), we parametrize the terms involving quarks, needed to describe the lower-energy data, as

$$P_{qq} = W(\mu_{qq} b) \left[a + b \frac{m_0}{\sqrt{s}}\right] , \quad (20)$$

$$P_{qg} = W(\sqrt{\mu_{qq}\mu_{gg}}\, b) \left[a' + b' \ln \frac{s}{m_0^2}\right] . \quad (21)$$

To the extent that the coefficients in Eqs. (20) and (21) are treated as free parameters, the low-energy parametrization can also accommodate elastic and diffractive contributions. We ensure the correct analyticity properties of our model amplitudes by substituting $s \to s e^{-i\pi/2}$

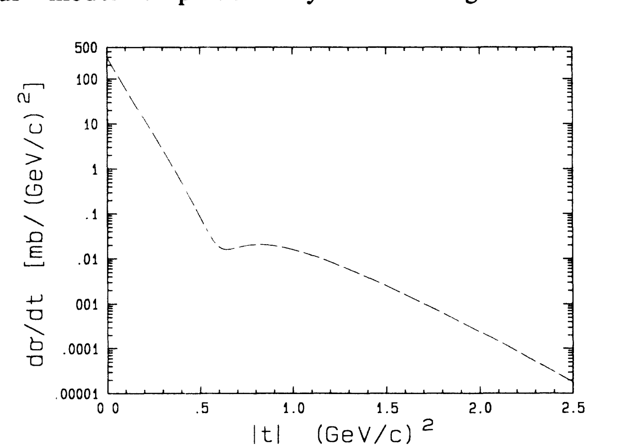

FIG. 4. A prediction for the differential elastic-scattering cross section $d\sigma/dt$ vs $|t|$, at $\sqrt{s} = 1.8$ TeV. Note the flatness in the small $|t|$ slope. The upward curvature at lower energies has vanished as the proton becomes blacker. A downward curvature develops at higher energies (Refs. 5 and 6).

throughout Eqs. (1)–(21). We also introduce an *ad hoc* crossing-odd amplitude parametrizing the difference between the pp and $p\bar{p}$ scattering amplitudes as

$$P_{\text{odd}} = W(\mu_{\text{odd}} b) a'' \frac{m_0}{\sqrt{s}} e^{-i\pi/4} . \tag{22}$$

We show in Figs. 1–3 our calculations for σ_{tot}, $\rho(0)$, and $B(0)$ for $\epsilon=0.05$ compared with pp and $\bar{p}p$ measurements. The other fit parameters are given in the Fig. 1 caption. Figure 4 gives a prediction for $d\sigma_n/dt$ at 1.8 TeV.

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