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Quark-meson coupling model for baryon wave functions and properties

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A scalar-isoscalar quark-meson interaction is used to mix fundamental pseudoscalar mesons directly into the wave functions of baryons. The parameters of the model are constrained using properties of the ground-state baryon spectrum, such as charge radii, strong decay widths, and the pion-nucleon coupling constant. Physical wave functions of the ground-state baryons are presented. With the physical wave functions, we calculate nucleon electric form factors and spin-$\frac{3}{2}$ baryon quadrupole moments. The addition of mesons produces effects on baryon properties comparable to effects due to spatial excitations of quarks. Although missing from most calculations, kaons and η's are found to be as important as pions in the baryon spectrum. Including mesons in the physical wave functions of baryons leads to the existence of low-energy resonances consisting primarily of three quarks in the ground state surrounded by a meson field. Most of these resonances decouple from the common production channels, but seven are expected to be observable. An additional $N(\frac{1}{2}^+)$ resonance state is calculated to have a mass close to that of the Roper resonance but a πN width much smaller. A $\Delta(\frac{3}{2}^+)$ candidate state is calculated to have a mass and πN width close to the experimental values for the $\Delta(1600)$.

I. INTRODUCTION

In a multitude of basic forms, nonrelativistic quark models (NRQM's) have been very successful in predicting the overall pattern of ground-state baryon masses[1] and magnetic moments.[2] By including spatial excitations of the quarks, NRQM's have also been able to make reasonable predictions of resonance spectra and strong decay widths.[3-5] The most successful of these models is that of Isgur, Karl (IK), and collaborators[3,4] with refinements by Forsyth and Cutkosky.[5] In all of these calculations, agreement with the data has been achieved without including pionic degrees of freedom.

Relativistic calculations, on the other hand, would imply that the role of the pion is important and good agreement on many baryon properties can be obtained only by including pionic effects. Unfortunately, other properties such as electric form factors and decay rates are not easily calculated in relativistic models. NRQM's, in which it is possible to define analytic wave functions, are often preferred since a wider variety of observables can be calculated within them.

However, calculations of pionic effects within NRQM's have been few and limited.[6-9] The standard approach has been to include a one-pion-exchange potential (OPEP) between quarks, although the actual form of the interaction varies slightly among the calculations. Physical wave functions and observables are then calculated using the full set of quark-quark (qq) interactions to mix the states in the model space. While effects are found to be large, agreement with the data is not always improved. In fact, some qq potentials are found to be better than others for fitting some subset of data, but none of the potentials is best for all the data. As long as there are ambiguities in the qq interactions that should be used, the role of the pion will remain uncertain.

In separate studies by Weber and collaborators,[8] the correct form of OPEP (between quarks) is found to be not completely determined. As in Refs. 6 and 7, pionic effects are large, but *only* if the delta-function part of the OPEP is included. Without the delta function, the fit to the baryon spectrum is slightly better. Based on a variety of theoretical grounds, such as the finite size of the constituent quark, they believe this part should be neglected. They conclude that pionic effects are small, and that the pion plays a minor role in NRQM's.

These questions regarding the role of the pion in nonrelativistic models need to be resolved. This is especially true considering the apparent success of approaches that ignore pionic degrees of freedom. However, given the uncertainty in the measured values of resonance energies and decay rates, and the broad range of predicted values, it remains optimistic to believe that these approaches are wholly sufficient, complete, or uniquely capable of reproducing the observed phenomena. Even though these approaches fit a large number of phenomena, a few anomalies remain. For instance, instead of being a well-defined excited state, the Roper resonance may actually consist of two closely spaced states in the range 1400–1480 MeV (Ref. 10). Also, predicted energies of the first excited state above the $\Delta(1232)$ range from 1650 to 1900 MeV, while recent experiments [listed under $\Delta(1600)$] are in the range 1500–1650 MeV (Ref. 11). In fact, the two most successful studies of the excited baryon spectrum both predict an energy of about 1800 MeV (Refs. 4 and 5), which suggests that the IK and similar models may not have a candidate state for the $\Delta(1600)$. The IK model has certainly demonstrated its ability to reproduce and to be consistent with the excited baryon spectrum of masses and decay widths, but given the uncertainty in the data, there is still room for other effects not included by the IK model.

In the present work, we investigate an alternative approach to baryon structure and the resonance spectrum. Fundamental mesons are admixed directly into nonrelativistic baryon wave functions via a standard SU(3) quark-meson coupling potential. The present approach is the complement to those NRQM's that use spatial excitations to reproduce baryon properties. In particular, we will show that our approach is sufficient to reproduce the negative charge radius of the neutron, as well as the strong decay widths of the $\Delta(1232)$ and $\Sigma(1385)$, without introducing *any* deformation of the three-quark wave functions. Furthermore, it is done in a model-independent way such that neither quark masses nor qq interactions are explicitly specified.

II. DESCRIPTION OF THE MODEL

The present model is characterized by four major features. First, quarks are assumed to form three-quark clusters which are eigenstates of an unspecified quark-quark interaction with parametrized eigenvalues. Second, mesons are assumed to be fundamental; their quark substructure is ignored and they are assumed to interact according to their overall quantum numbers. Third, only one interaction is specified, namely, that which couples quarks and mesons. Fourth, the model space is made up of three-quark states and 3-quark–1-meson states written using harmonic-oscillator spatial wave functions and standardized color-spin-flavor wave functions.

A. The Hamiltonian

A global, strong interaction between quarks, deriving from the nonperturbative exchange of gluons, is assumed to exist, but is left unspecified. The exact forms of the kinetic energy and rest-mass portions of the quark-quark Hamiltonian (H_{quark}) are also unspecified. Therefore, unlike other NRQM's, there are no quark masses in the model, no strong coupling constant, and no strength of the confining interaction. However, it is assumed that the eigenstates of H_{quark} are three-quark, color-singlet hadrons with good spin, isospin, and parity quantum numbers. The spatial wave functions of the eigenstates are unknown.

Each three-quark eigenstate can be associated with one of the observed low-energy baryons. We restrict ourselves to baryons within flavor SU(3) (u,d,s) for which there are two supermultiplets, the nucleon octet and the delta decuplet. There are four multiplets in the nucleon octet $(N,\Lambda^0,\Sigma_{1/2},\Xi_{1/2})$ and four in the delta decuplet $(\Delta,\Sigma_{3/2},\Xi_{3/2},\Omega^-)$. Together, the eight multiplets of baryons will be referred to as *ground-state baryons.*

The three-quark eigenstates are notated $|B,\nu,\mathbf{K}\rangle$, where B stands for the spin (j), isospin (i), and strangeness that distinguish the eight baryons. Similarly, ν stands for the z components of spin (μ_j) and isospin (μ_i), and $\mathbf{K}$ denotes the center-of-mass (c.m.) momentum of the eigenstate.

The eigenvalues of H_{quark} are parametrized using M_B. That is, for a baryon at rest,

$$H_{\text{quark}}|B,\nu,0\rangle = M_B|B,\nu,0\rangle \ . \tag{1}$$

Parametrization suppresses quark degrees of freedom, which eliminates the need to specify any qq interaction or quark masses, and permits isolation of mesonic effects. For instance, in most NRQM's, the one-gluon-exchange potential (OGEP) is responsible for the $N-\Delta$ mass difference. In the present model, the same effect is reproduced simply by choosing different values for M_N and M_Δ. Similarly, the effect of a heavier strange-quark mass can also be incorporated into M_B.

The eigenvalues are assumed to be independent of ν and the states are assumed to obey relativistic kinematics:

$$H_{\text{quark}}|B,\nu,\mathbf{K}\rangle = \omega_B(K)|B,\nu,\mathbf{K}\rangle \ , \tag{2}$$

$$\omega_B(K) = (M_B^2+K^2)^{1/2} \ . \tag{3}$$

We allow coupling only to mesons in the ground-state pseudoscalar octet (π,K,η) and treat them as pointlike and fundamental. One-meson states are represented $|\phi,\mu_\phi,\mathbf{p}\rangle$, where ϕ stands for the isospin and strangeness of the meson, μ_ϕ is its z component of isospin, and $\mathbf{p}$ is its momentum. The mesons obey the Klein-Gordon equation, so the eigenvalue equation for the meson Hamiltonian H_ϕ simply

$$H_\phi|\phi,\mu_\phi,\mathbf{p}\rangle = \omega_\phi(p)|\phi,\mu_\phi,\mathbf{p}\rangle \ , \tag{4}$$

$$\omega_\phi(p) = (m_\phi^2+p^2)^{1/2} \ , \tag{5}$$

where m_ϕ is the observed mass of the meson ϕ. The unperturbed Hamiltonian (H_0) is taken to be $H_{\text{quark}}+H_\phi$.

The perturbing Hamiltonian (H_{int}) is a scalar-isoscalar interaction which couples quarks and pseudoscalar mesons, but it is not one of the many forms of OPEP between quarks (as in Refs. 6–9). Instead, the form is partially reduced from the standard γ_5 coupling and reflects the particle symmetries of the quarks and mesons. The form is taken to be

$$H_{\text{int}} = \frac{G_{qq\phi}}{m_\pi}\sum_n \boldsymbol{\sigma}_n\cdot\boldsymbol{\nabla}_n \vec{\lambda}_n\cdot\vec{\phi}(\mathbf{r}_n) \ , \tag{6}$$

where $G_{qq\phi}$ is a dimensionless coupling constant, m_π sets the energy scale (so that $G_{qq\phi}$ is unitless), and the sum is over all quarks. The $\{\lambda_n^a; a=1,\ldots,8\}$ are the eight generators of SU(3) acting on the nth quark, and $\{\phi_a; a=1,\ldots,8\}$ are combinations of the field operators of the eight pseudoscalar mesons $(\pi^\pm,\pi^0,K^\pm,K^0,\overline{K}^0,\eta)$. As in most previous calculations, the quark-meson coupling constant is assumed to be independent of quark and meson flavor.[12] The use of m_π in the definition of H_{int} is not meant to suggest that the pion is more important than the kaon or η. Any energy may be used in place of m_π and, except for a different *value* for $G_{qq\phi}$, the results will remain entirely unchanged. That is, all other parameters, all wave-function coefficients, and all observables would have the same values, since it is the ratio $G_{qq\phi}/m_\pi$ which determines these quantities.

B. The model space

In general, the spatial wave functions of the three-quark (q^3) eigenstates depend on the exact form of H_{quark}. Since H_{quark} is not known, the wave functions are assumed to be harmonic-oscillator (HO) S waves of the same length parameter α. So, for a baryon with c.m. momentum $\mathbf{K}$, the projection of a state vector onto spatial coordinates is

$$\langle \rho,\lambda,\mathbf{r}_c|B,\nu,\mathbf{K}\rangle=\frac{\alpha^3}{\pi^{3/2}}\exp[-\tfrac{1}{2}\alpha^2(\rho^2+\lambda^2)] \times\frac{\exp(i\mathbf{K}\cdot\mathbf{r}_c)}{(2\pi)^{3/2}}|B,\nu\rangle \,, \tag{7}$$

where ρ and λ are the standard Jacobi coordinates for the internal degrees of freedom of the three quarks, $\mathbf{r}_c$ is the c.m. of the three quarks, and $|B,\nu\rangle$ is a standard, totally symmetric, spin-isospin wave vector. Spatial excitations certainly exist and will deform the spatial wave function, but it is not known exactly how important these excitations are. Furthermore, we wish to investigate the extent to which mesonic excitations can reproduce baryon properties; therefore, *spatial excitations and deformations are ignored.*

Three-quark basis states and 1-meson states are combined in relative HO wave functions to form a basis of 3-quark–1-meson ($q^3\phi$) states. The notation for a $q^3\phi$ state is

$$|(C\phi)^{kl}_{B,\nu,\mathbf{K}}\rangle \,,$$

where C is the overall spin (s), isospin (t) and strangeness of the three quarks, and ϕ is again one of the spinless mesons. The symbols k and l are the usual quantum numbers of the HO wave function in the relative coordinate between C and ϕ. The symbols B, ν, and $\mathbf{K}$ are the overall quantum numbers of the basis state.

The projection of a $q^3\phi$ wave vector onto spatial coordinates requires uncoupling the spin and isospin quantum numbers using Clebsch-Gordan coefficients. The result is

$$\begin{aligned}\langle \mathbf{R},\mathbf{R}_{\text{c.m.}}|(C\phi)^{kl}_{B,\nu,\mathbf{K}}\rangle &=u_{k,l}(\beta R)\frac{\exp(i\mathbf{K}\cdot\mathbf{R}_{\text{c.m.}})}{(2\pi)^{3/2}}\\ &\times\sum_{\mu_t\mu_\phi}(t\mu_t\phi\mu_\phi|i\mu_i)\\ &\times\sum_{\mu_s\mu_l}(s\mu_s l\mu_l|j\mu_j)Y_l^{\mu_l}(\Omega_R)|C,\mu_s,\mu_t;\phi,\mu_\phi\rangle \,,\end{aligned} \tag{8}$$

where $\mathbf{R}$ is the relative coordinate between the meson ϕ and the c.m. of the three-quark eigenstate C, β is the length parameter in $\mathbf{R}$, and $\mathbf{R}_{\text{c.m.}}$ is the c.m. of the basis state. The function $u_{k,l}(x)$ is a standard HO wave function:

$$u_{k,l}(x)=N_{kl}\exp(-\tfrac{1}{2}x^2)x^l y_{k,l}(x) \,, \tag{9}$$

where $y_{k,l}(x)$ is a polynomial of order k. For $k=0$, the normalization factor N_{kl} is given by the expression

$$N^2_{0l}(\beta)=\frac{2^{2+l}\beta^{2l+3}}{(2l+1)!!\sqrt{\pi}} \,. \tag{10}$$

Although $q^3\phi$ states will couple to 3-quark–2-meson ($q^3\phi^2$) states, the model space is truncated at 3-quark–1-meson states. This is considered justifiable for the ground-state baryons because the $q^3\phi^2$ states are much higher in energy than $q^3\phi$ states (which are themselves expected to comprise only about 20% of the physical wave functions) and $q^3\phi^2$ states cannot couple directly to q^3 states. However, 3-quark–2-meson states should be important when studying excited states, but as we are only interested in rough estimates of the masses and decay widths, this simplification is considered acceptable for the excited states as well.

C. Matrix elements

There are only three types of Hamiltonian matrix elements in the entire calculation: matrix elements of H_0 between q^3 states; the same between $q^3\phi$ states; and matrix elements of H_{int} connecting q^3 states to $q^3\phi$ states. All matrix elements are taken in the $\mathbf{K}=0$ limit.

Between q^3 states, matrix elements of H_0 are trivial because the states are assumed to be eigenstates of H_{quark}:

$$\lim_{\mathbf{K}\to 0}\langle B',\nu',\mathbf{K}'|H_0|B,\nu,\mathbf{K}\rangle=\delta_{BB'}\delta_{\nu\nu'}\delta^{(3)}(\mathbf{K}')M_B \,. \tag{11}$$

At zero c.m. momentum, matrix elements of H_0 between $q^3\phi$ states with quantum number $k=0$ are given by

$$\begin{aligned}\lim_{\mathbf{K}\to 0}\langle (C'\phi')^{0l'}_{B',\nu',\mathbf{K}'}|H_0|(C\phi)^{0l}_{B,\nu,\mathbf{K}}\rangle &=\delta_{CC'}\delta_{\phi\phi'}\delta_{ll'}\delta_{BB'}\delta_{\nu\nu'}\delta^{(3)}(\mathbf{K}')N^2_{0l}(1/\beta)\\ &\times\int_0^\infty p^{2l+2}dp\,\exp\left[-\frac{p^2}{\beta^2}\right][\omega_C(p)+\omega_\phi(p)] \,,\end{aligned} \tag{12}$$

where β is the length parameter in the $C-\phi$ relative coordinate $\mathbf{R}$, and ω_C and ω_ϕ are given by Eqs. (3) and (5), respectively.

The perturbing Hamiltonian H_{int} connects q^3 states to $q^3\phi$ states. In the $\mathbf{K}=0$ limit, matrix elements of H_{int} are

$$\begin{aligned}\lim_{\mathbf{K}\to 0}\langle B',\nu',\mathbf{K}'|H_{\text{int}}|(C\phi)^{0l}_{B,\nu,\mathbf{K}}\rangle &=\delta_{l1}\delta_{\nu\nu'}\delta_{BB'}\delta^{(3)}(\mathbf{K}')N_{0l}(1/\beta)\frac{G_{qq\phi}}{2\pi m_\pi\hat{i}\hat{j}\sqrt{3}}(B\|H_{\text{int}}\|C\phi)\\ &\times(-)^{2t+2i'}\int_0^\infty p^4dp\,\omega_\phi^{-1/2}(p)\exp\left[-\left[\frac{1}{2\beta^2}+\frac{1}{6\alpha^2}\right]p^2\right] \,,\end{aligned} \tag{13}$$

where $(B\|H_{\text{int}}\|C\phi)$ is one of the reduced matrix elements shown in Table I. As a direct result of putting the quarks into S-wave states, $l=1$ is required for the HO wave function in the $C-\phi$ relative coordinate.

D. Observables

All observables are calculated using physical-state wave functions. A tilde over the physical state is used to distinguish it from the three-quark basis state with the same quantum numbers. Within our model space, the most general physical wave function is written

$$|\tilde{B},\nu,\mathbf{K}\rangle=(B|\tilde{B})|B,\nu,\mathbf{K}\rangle + \sum_{k,l,C,\phi}((C\phi)_B^{kl}|\tilde{B})|(C\phi)_{B,\nu,\mathbf{K}}^{kl}\rangle \ , \tag{14}$$

where $(B|\tilde{B})$ and $((C\phi)_B^{kl}|\tilde{B})$ are the coefficients of the physical wave function and it is assumed that they do not depend upon the z components ν or the c.m. momentum $\mathbf{K}$. The coefficients are found by diagonalizing the full Hamiltonian (H_0+H_{int}) within each sector (one for each baryon B).

The physical mass of each ground-state baryon is assumed to be the lowest eigenvalue of the diagonalized Hamiltonian within the appropriate sector at $\mathbf{K}=0$. Therefore, given the eigenvectors of the diagonalized Hamiltonian, the physical mass can also be found using

$$m_B=\langle\tilde{B},\nu,0|H_0+H_{\text{int}}|\tilde{B},\nu,0\rangle \ . \tag{15}$$

Eigenvalues that are higher than the ground-state mass are the energies of excited states with the same quantum numbers as the ground state.

Using the physical wave functions, we can calculate observables such as electric form factors, charge radii, quadrupole moments, strong decay widths and the pion-nucleon coupling constant, *without* quark masses or an exact form of the global qq interaction H_{quark}. Since quark masses and H_{quark} are unspecified, the model is somewhat quark-model independent, except that the wave functions are more consistent with nonrelativistic models than with relativistic ones.

Since many observables depend upon the charge of the baryon, we introduce the symbol b to denote a particular charge state of baryon B. That is,

$$b\equiv(B,\mu_i)\equiv(j,i,\mu_i) \ , \tag{16}$$

TABLE I. Reduced matrix elements of the scalar-isoscalar quark-meson interaction. The symbol K is used to represent either a kaon or antikaon, depending upon the context.

B:	N	Λ^0	$\Sigma_{1/2}$	$\Xi_{1/2}$	Δ	$\Sigma_{3/2}$	$\Xi_{3/2}$	Ω^-	B'
				(a) $(B\|H_{\text{int}}\|B'\pi)$					
	10				$\sqrt{128}$				N
			$-\sqrt{24}$			$\sqrt{48}$			Λ^0
		$\sqrt{24}$	8			$\sqrt{32}$			$\Sigma_{1/2}$
				-2			$-\sqrt{32}$		$\Xi_{1/2}$
	$\sqrt{128}$				20				Δ
		$\sqrt{48}$	$-\sqrt{32}$			$\sqrt{160}$			$\Sigma_{3/2}$
				$\sqrt{32}$			$\sqrt{40}$		$\Xi_{3/2}$
									Ω^-
				(b) $(B\|H_{\text{int}}\|B'K)$					
		6	2			$\sqrt{32}$			N
	-6			2			$\sqrt{32}$		Λ^0
	2			10	$\sqrt{128}$		$\sqrt{32}$		$\Sigma_{1/2}$
		2	10			$-\sqrt{32}$		-8	$\Xi_{1/2}$
			$\sqrt{128}$			$-\sqrt{160}$			Δ
	$-\sqrt{32}$			$-\sqrt{32}$	$\sqrt{160}$		$-\sqrt{160}$		$\Sigma_{3/2}$
		$-\sqrt{32}$	$\sqrt{32}$			$-\sqrt{160}$		$\sqrt{80}$	$\Xi_{3/2}$
				-8			$-\sqrt{80}$		Ω^-
				(c) $(B\|H_{\text{int}}\|B'\eta)$					
	2								N
		$-\sqrt{8}$							Λ^0
			$\sqrt{24}$			$\sqrt{48}$			$\Sigma_{1/2}$
				-6			$-\sqrt{32}$		$\Xi_{1/2}$
					$\sqrt{80}$				Δ
			$-\sqrt{48}$						$\Sigma_{3/2}$
				$\sqrt{32}$			$-\sqrt{40}$		$\Xi_{3/2}$
								$-\sqrt{80}$	Ω^-

where μ_i is the z component of isospin.

The electric form factor of baryon b is simply the Fourier transform of the charge density:

$$F_{Eb}(\mathbf{p})=\left\langle \tilde{b},\mu_j,\mathbf{K}' \left| \sum_n \epsilon_n e^{i\mathbf{p}\cdot\mathbf{r}_n} \right| \tilde{b},\mu_j,\mathbf{K} \right\rangle , \tag{17}$$

where $\mathbf{p}$ is the momentum transfer, ϵ_n is the charge of the nth constituent (quark or meson) of the physical wave function, and $\mathbf{r}_n$ is its coordinate.

The mean-squared charge radius and quadrupole moment of baryon b can be found either from the electric form factor or by direct calculation using the physical wave function:

$$\langle r_E^2 \rangle_b=\left\langle \tilde{b},\mu_j,\mathbf{K}' \left| \sum_n \epsilon_n r_n^2 \right| \tilde{b},\mu_j,\mathbf{K} \right\rangle , \tag{18}$$

$$Q_b=\left\langle \tilde{b},\mu_j,\mathbf{K}' \left| \sum_n \epsilon_n (3z_n^2-r_n^2) \right| \tilde{b},\mu_j,\mathbf{K} \right\rangle , \tag{19}$$

where n still refers to all constituents of the physical state $\tilde{b}$.

Partial strong decay widths are well measured for many spin-$\frac{3}{2}$ baryons, and roughly measured for many excited states. The defining equation for the decay width of physical state $\tilde{b}$ into state $\tilde{B}'$ and meson ϕ is

$$\Gamma_{b\to B'\phi}=\lim_{\mathbf{K}\to 0}\frac{2\pi}{2j+1}\sum_{\mu_j\nu'\mu_\phi}\int d^3p\, d^3K'\delta(E-E')|\langle \tilde{b},\mu_j,\mathbf{K}|H_{\text{int}}|\tilde{B}',\nu',\mathbf{K}';\phi,\mu_\phi,\mathbf{p}\rangle|^2 , \tag{20}$$

where E and E' are the energies of the initial and final states, and the sum is over z components of spin and isospin. Unfortunately, this definition of the partial width has two problems. The final-state wave functions are simply products of physical baryons and physical mesons; they are therefore not orthogonal to the initial-state wave functions which contain explicit mesons. Furthermore, the outgoing meson ϕ can come from the bound-state meson in the initial state, instead of being produced by the interaction H_{int}. This form has been chosen for its simplicity, and it is not known how a totally self-consistent definition will affect the final results.

The pion-nucleon coupling constant $G_{NN\pi}$ is used as a measure of the strength of the pion-nucleon vertex function ($V_{NN\pi}$). Written in terms of nucleon operators, the form is

$$V_{NN\pi}(\mathbf{p})=\frac{iG_{NN\pi}}{2m_N}\langle N,\nu,\mathbf{K}|\sigma_N\cdot\mathbf{p}\tau_{N\mu}e^{i\mathbf{p}\cdot\mathbf{x}}|N,\nu',\mathbf{K}'\rangle . \tag{21}$$

We can also write $V_{NN\pi}$ in terms of quark operators using an analogous form that is derived from H_{int}:

$$V_{NN\pi}(\mathbf{p})=\frac{iG_{qq\phi}}{m_\pi}\left\langle \tilde{N},\nu,\mathbf{K} \left| \sum_n \sigma_n\cdot\mathbf{p}\tau_{n\mu}e^{i\mathbf{p}\cdot\mathbf{r}_n} \right| \tilde{N},\nu',\mathbf{K}' \right\rangle , \tag{22}$$

where n refers only to quarks.

These two forms of the vertex function are evaluated in the long-wavelength limit ($\mathbf{p}=\mathbf{0}$) and set equal. Assuming that the physical wave function of the nucleon is written in the form of Eq. (14), the relationship between the two coupling constants is found to be

$$G_{NN\pi}=\frac{m_N}{3m_\pi}\left[10(N|\tilde{N})^2+4\sum_{CC'}((C\pi)_N^{01}|\tilde{N})((C'\pi)_N^{01}|\tilde{N})\left[\frac{2\beta\beta'}{\beta^2+(\beta')^2}\right]^{5/2}(C\|H_{\text{int}}\|C'\pi)\right.$$
$$\left.\times\begin{Bmatrix}\frac{1}{2} & 1 & \frac{1}{2}\\ s' & 1 & s\end{Bmatrix}\begin{Bmatrix}\frac{1}{2} & 1 & \frac{1}{2}\\ t' & 1 & t\end{Bmatrix}+(\text{terms for which } k>0)\right]G_{qq\phi} , \tag{23}$$

where $(C\|H_{\text{int}}\|C'\pi)$ is a reduced matrix element (Table I), (s,t) are the spin and isospin of three-quark cluster C, and (s',t') are those of C'.

Some of these observables will be used as model constraints, while the rest will be used as predictions of the model.

E. Constraints on model parameters

There are four types of model parameters: α, the HO length parameter for the quark degrees of freedom within the three-quark eigenstates of H_{quark}; β, the HO length parameter for the relative coordinate between the three-quark cluster and the meson in the $q^3\phi$ basis states; M_B, the eigenvalues of H_{quark}; and $G_{qq\phi}$, the strength of the quark-meson coupling interaction H_{int}.

In general, the length parameter α of the internal quark degrees of freedom can be different for each q^3 eigenstate, but for simplicity we use the same α throughout. The length parameter β can also be different for each choice of three-quark cluster C, meson ϕ, HO quantum number k, orbital angular momentum l, and overall spin and isospin B. For simplicity, we restrict ourselves to a different β for each B, and notate them β_B.

Each observable used as a constraint is chosen because its calculated value is primarily determined by a single model parameter. For example, the proton's electric charge radius is 95–99 % determined by the size of the three-quark core, which is given by the length parameter α. On the other hand, the neutron's electric charge radius is zero without mesonic excitations or deformations. Thus, the spatial extent of the $C-\phi$ wave function completely determines $\langle r_E^2 \rangle_{\text{neutron}}$, which is found from the length parameter β_N. The β's of the nucleon octet are chosen to be equal since there are no firm constraints for the Λ, $\Sigma_{1/2}$, and $\Xi_{1/2}$ sectors.

Strong decay widths are most sensitive to the length parameter β of the decaying baryon. In all multiplets, the width of more than one charge state has been measured, so the best-measured width in each multiplet is used as the constraint. The parameters β_Δ and $\beta_{\Sigma_{3/2}}$ are therefore set using the measured strong decay widths of Δ^{++} and $\Sigma_{3/2}^+$. Unfortunately, the $\Xi_{3/2}$ strong decay width is not very sensitive to $\beta_{\Xi_{3/2}}$, so a value is chosen which fits best into the pattern of β's among the states of the delta decuplet.

The Ω^- is stable against strong decay, so the parameter β_Ω is hard to constrain. However, the $\Omega'(2253)$ is a recently discovered resonance of unknown spin and parity which decays predominantly to $\Xi_{3/2}K$ (Ref. 13). In our model, the first excited state of Ω^- is dominated by the basis state with the same constituents, and has the same dominant decay mode. We therefore tentatively associate the observed resonance with our first-excited state, and set β_Ω using the measured resonance energy as the constraint.

The parametrized masses M_B of the three-quark eigenstates are set using the observed masses of the ground-state baryons. Since the observed masses are slightly dependent on the z components of isospin, the physical mass m_B is taken to be the weighted average of the measured values within each multiplet.

The quark-meson coupling constant $G_{qq\phi}$ is constrained by the pion-nucleon coupling constant using Eq. (23). To leading order, $G_{qq\phi}$ has the value

$$G_{qq\phi}^{(0)} = \frac{3m_\pi}{10m_N} G_{NN\pi} \approx -0.6 \; . \tag{24}$$

The addition of mesons will renormalize $G_{qq\phi}$ slightly.

Given the chosen constraints on the parameters, we have reduced the problem to a system of 17 equations, some of which are nonlinear, and 18 unknowns. The only unconstrained parameter is $\beta_{\Xi_{3/2}}$. However, if we consider the $\Xi_{3/2}$ sector separately, and treat $\beta_{\Xi_{3/2}}$ and $M_{\Xi_{3/2}}$ independently of the other 16 parameters, the choice of $\beta_{\Xi_{3/2}}$ does not significantly affect the solution to the remaining system of 16 equations and 16 unknowns. Results are presented in the following section.

III. RESULTS

The basis has been truncated at $q^3\phi$ states under the assumption that contributions are negligible or unimportant from states with higher numbers of mesons. In this truncated basis, H_{int} will mix only $l=1$ states into the physical wave functions [see Eq. (13)]. The model space is also truncated at states with HO quantum number $k=0$ (in the $C-\phi$ relative coordinate) since states with $k>0$ will be considerably higher in energy. As a consequence of both truncations, only rough estimates of the properties of excited baryons are possible.

A. Determination of model parameters

The constraints used to set the parameters of the model are shown along with the model values in Table II. Also

TABLE II. Results. The final values of the length parameters β_B and masses of the three-quark cores M_B are given in MeV and used to calculate all observables. The values of the other parameters are $\alpha=258.8$ MeV and $G_{qq\phi}=-0.628$. The quantity m_B is the physical mass of baryon B in MeV. Underlined values are used as constraints. Where appropriate, measurement errors are shown in parentheses.

Baryon	β_B	M_B	m_B	Observable	Calculated value	Measured value	Observable	Calculated value	Measured value
N	176.3	1031.1	$\underline{938.9}$	$\langle r_E^2 \rangle_{\text{proton}}$	$\underline{0.720}$	0.72(0.02) fm^2	$\Gamma(\Delta^{++})$	$\underline{112.0}$	112(1) MeV
							$\Gamma(\Delta^{+})$	113.8	
Λ^0	176.3	1189.9	$\underline{1115.6}$	$\langle r_E^2 \rangle_{\text{neutron}}$	$\underline{-0.120}$	$-0.12(0.01)$ fm^2	$\Gamma(\Delta^{0})$	115.8	120(2) MeV
							$\Gamma(\Delta^{-})$	114.8	
$\Sigma_{1/2}$	176.3	1265.1	$\underline{1193.4}$						
				$G_{NN\pi}$	$-\underline{13.4}$	-13.4	$\Gamma(\Sigma_{3/2}^{+})$	$\underline{35.0}$	35(1) MeV
$\Xi_{1/2}$	176.3	1374.6	$\underline{1318.1}$				$\Gamma(\Sigma_{3/2}^{0})$	35.8	36(5) MeV
							$\Gamma(\Sigma_{3/2}^{-})$	36.7	39(2) MeV
Δ	252.7	1406.6	$\underline{1232.0}$	Resonance					
				energy of			$\Gamma(\Xi_{3/2}^{0})$	10.2	9.1(0.5) MeV
$\Sigma_{3/2}$	303.1	1569.4	$\underline{1386.9}$	$\Omega'(2253)$	$\underline{2253.0}$	2253(13) MeV	$\Gamma(\Xi_{3/2}^{-})$	11.4	10.1(1.9) MeV
$\Xi_{3/2}$	330.0	1708.4	$\underline{1533.4}$	Full			$B(\Sigma_{3/2}^{+})$[a]	86.0	
				width of			$B(\Sigma_{3/2}^{0})$	87.5	
Ω^-	342.3	1837.6	$\underline{1672.4}$	$\Omega'(2253)$	140	81(38) MeV	$B(\Sigma_{3/2}^{-})$	86.4	
							Average	86.6	88(2)%

[a]Branching ratio: $\Lambda\pi$/total.

included in Table II are the calculated and measured values of some other observables. Data are taken from the Particle Data Group.[11]

The values of all model parameters can be found in columns 2 and 3 of Table II or its caption. While there remains some freedom in the choice of the parameters for the Ξ and Ω sectors, the sets of parameters for the N, Δ, Λ, and Σ sectors are *uniquely well determined.*

As shown in column 2 of Table II, β is found to be about 175 MeV for the nucleon octet, and between roughly 250 and 340 MeV for the delta decuplet. For $\beta \approx 175$ MeV, the meson distribution peaks at roughly twice the distance from the baryon c.m. that the quark distribution peaks. The main reason that our calculation results in a small value for β_N (and therefore a spread-out meson distribution) is that the meson field is assumed to be the source of the entire electric charge radius of the neutron. If spatial excitations were to be included, reproducing the neutron electric charge radius would require a larger β_N. For $\beta \approx 340$ MeV, the peak of the meson distribution lies just outside the peak of the quark distribution. This result is consistent with cloudy bag model (CBM) calculations,[14,15] for which the pion distribution peaks at the surface of the bag.

The model values of β for the Δ and $\Sigma_{3/2}$ sectors are well determined since a 2% shift in β would remove the good agreement between the calculated strong decay widths and the data. As mentioned earlier, the value of β for the $\Xi_{3/2}$ is not well determined since the $\Xi_{3/2}$ strong decay width is rather insensitive to $\beta_{\Xi_{3/2}}$. This insensitivity of the strong decay width to β is a direct consequence of the small role that pions play in the physical Ξ wave functions. While values for $\beta_{\Xi_{3/2}}$ closer to 400 MeV would yield slightly better widths, a value of 330 MeV has been used since it fit better into the pattern of β's. Although a measurement of the kaonic decay width of $\Xi_{3/2}$ is unlikely, it would place a strong constraint on $\beta_{\Xi_{3/2}}$.

The model values of M_B can be most easily interpreted by considering the mass shifts due to meson couplings ($\delta m_B \equiv m_B - M_B$). These mass shifts are shown in Table III along with values obtained in previous calculations.[7,16,17] Within the nucleon octet, our mass shifts, which range from 56 MeV for the $\Xi_{1/2}$ to 92 MeV for the nucleon, are comparable to previous calculations, all of which considered only pionic effects. Within the delta decuplet, our model predicts mass shifts of around 175 MeV for all four multiplets, which are much larger than other estimates. These differences are due primarily to nonpionic effects rather than model dependence. One clear indication of the importance of kaons and η's is the mass shift of the Ω^-, which cannot couple to the pion. To further demonstrate that kaons and η's have large effects elsewhere in the baryon spectrum, we have redone our calculation using only pionic couplings. The resulting mass shifts are shown in column 3 of Table III.

Comparing columns 2 and 3 of Table III, it is clear that nonpionic effects are large, especially in the delta decuplet. With nonpionic effects removed, the mass shifts (column 3) are in rough agreement with previous models for the nucleon octet, but still slightly higher than other models for the delta decuplet. Most notably, the mass shift of the delta (δm_Δ) is larger than that of the nucleon (δm_N) in our approach, but smaller than δm_N in

TABLE III. Comparison of mass shifts due to meson couplings. Key to table: (a) Nogami and Ohtsuka (Ref. 7), potential I, $\Lambda = 2m_\pi$; (b) Nogami and Ohtsuka (Ref. 7), potential I, $\Lambda = 2.5m_\pi$; (c) Mulders and Thomas (Ref. 16); (d) Barik and Dash (Ref. 17). The results using two sets of parameters within Ref. 7 are chosen as typical of NRQM's, and the results of Refs. 16 and 17 are chosen as typical of relativistic calculations. All four sets of previous results are calculated using only pionic excitations. All shifts are in MeV.

	Present model		Previous models (π only)			
Baryon	π,K,η	π only	(a)	(b)	(c)	(d)
N	-92	-86	-50	-108	-199	-160
Λ^0	-74	-52	-27	-60	-127	-101
$\Sigma_{1/2}$	-72	-42	-33	-56	-71	-56
$\Xi_{1/2}$	-56	-14	-5	-12	-32	-25
Δ	-175	-139	-48	-108	-98	-92
$\Sigma_{3/2}$	-183	-100	-43	-79	-60	-56
$\Xi_{3/2}$	-175	-50	-38	-52	-28	-25
Ω^-	-165	0	0	0	0	0

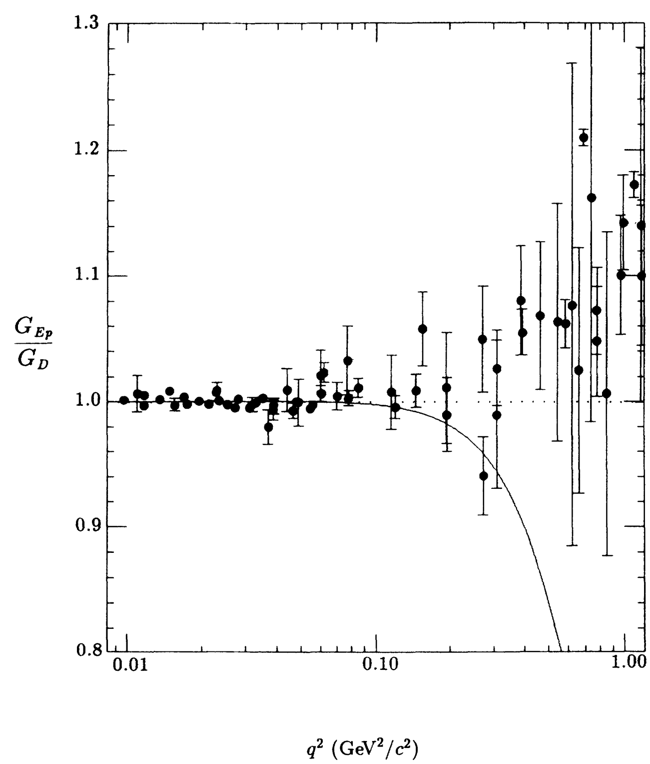

FIG. 1. Proton electric form factor. Data are taken from Refs. 19–21. The calculated form factor is shown as a solid curve. The data and calculation are divided by a dipole form factor G_D with $\langle r^2 \rangle = 0.72$ fm^2.

others. The importance of kaons and η's, as well as the relative mass shifts of the nucleon and delta, are more closely examined in Sec. III D.

B. Wave functions and observables of the ground-state baryons

All observables are calculated from the physical wave functions. The components of the model space, as well as the dominant contributions of the ground-state physical wave functions, are shown in Table IV. In the nucleon octet, the three-quark content of baryons (as opposed to the 3-quark–1-meson content) ranges from about 86% to 92%, so the truncation of the basis appears justified for the octet. However, for baryons in the decuplet, the three-quark content is only 70–80%, so it may be that 3-quark–2-meson states and/or $k>0$ states are important for the delta decuplet.

As examples of other calculations that can be done with the physical wave functions, the electric form factors of the proton and neutron are shown in Figs. 1 and 2. A monopole form factor has been used for the charged mesons, to agree with elastic scattering data on pions.[18] Data from Refs. 19–24 are shown for comparison.

Good agreement at low q^2 is not surprising since the slope of each plot at $q^2=0$ is used as a model constraint. Although both sets of data are well reproduced out to $q^2 \approx 0.3(\mathrm{GeV}/c)^2$, these results are not significantly better than those of previous calculations.

Another example of a typical calculation is the baryon quadrupole moment (Q_b), which is nonzero for the states of the delta decuplet. Although none of the Q_b has been measured, some have been estimated within different models.[25] Our estimates, plus those of other authors, are shown in Table V.

We would like to stress that the estimates of columns 2 and 4 have been made without quark masses and without deformations of the three-quark eigenstates. The deformation necessary to yield nonzero quadrupole moments is provided entirely by the meson distribution of the

TABLE IV. Composition of the physical ground-state baryons. The notation for a three-quark basis state is (B), where B refers to the overall quantum numbers of the state. The notation for 3-quark–1-meson basis states is $(C\phi;B)$ where C represents the overall quantum numbers of the three quarks, ϕ is the meson, and B represents the overall quantum numbers of the basis state. The spatial wave function of the three quarks is an S-wave harmonic oscillator with length parameter α, which can be found in Table II. The spatial wave function in the relative $C-\phi$ coordinate is a harmonic oscillator with $k=0, l=1$, and length parameter β_B, as given in Table II. The symbol K is used for both kaons and antikaons.

	Physical state		Main components					
Baryon	$I(J^\pi)$	Mass	Composition	& basis	Other components			
N	$\frac{1}{2}(\frac{1}{2}^+)$	939	0.926	(N)	$(N\eta;N)$	$(\Lambda K;N)$		
			0.312	$(N\pi;N)$		$(\Sigma_{1/2}K;N)$		
			0.191	$(\Delta\pi;N)$		$(\Sigma_{3/2}K;N)$		
Λ^0	$0(\frac{1}{2}^+)$	1116	0.945	(Λ)		$(\Lambda\eta;\Lambda)$	$(\Xi_{1/2}K;\Lambda)$	
			0.197	$(\Sigma_{1/2}\pi;\Lambda)$			$(\Xi_{3/2}K;\Lambda)$	
			0.172	$(\Sigma_{3/2}\pi;\Lambda)$				
			−0.171	$(NK;\Lambda)$				
$\Sigma_{1/2}$	$1(\frac{1}{2}^+)$	1193	0.938	$(\Sigma_{1/2})$	$(NK;\Sigma_{1/2})$	$(\Sigma_{1/2}\eta;\Sigma_{1/2})$	$(\Xi_{1/2}K;\Sigma_{1/2})$	
			0.220	$(\Sigma_{1/2}\pi;\Sigma_{1/2})$	$(\Delta K;\Sigma_{1/2})$	$(\Sigma_{3/2}\pi;\Sigma_{1/2})$	$(\Xi_{3/2}K;\Sigma_{1/2})$	
			−0.165	$(\Lambda\pi;\Sigma_{1/2})$		$(\Sigma_{3/2}\eta;\Sigma_{1/2})$		
$\Xi_{1/2}$	$\frac{1}{2}(\frac{1}{2}^+)$	1318	0.959	$(\Xi_{1/2})$		$(\Lambda K;\Xi_{1/2})$	$(\Xi_{1/2}\pi;\Xi_{1/2})$	$(\Omega K;\Xi_{1/2})$
			−0.196	$(\Xi_{1/2}K;\Xi_{1/2})$		$(\Sigma_{3/2}K;\Xi_{1/2})$	$(\Xi_{1/2}\eta;\Xi_{1/2})$	
			0.110	$(\Xi_{3/2}\pi;\Xi_{1/2})$			$(\Xi_{3/2}\eta;\Xi_{1/2})$	
Δ	$\frac{3}{2}(\frac{3}{2}^+)$	1232	0.852	(Δ)	$(\Delta\eta;\Delta)$	$(\Sigma_{1/2}K;\Delta)$		
			0.390	$(N\pi;\Delta)$		$(\Sigma_{3/2}K;\Delta)$		
			0.303	$(\Delta\pi;\Delta)$				
$\Sigma_{3/2}$	$1(\frac{3}{2}^+)$	1387	0.877	$(\Sigma_{3/2})$	$(NK;\Sigma_{3/2})$	$(\Sigma_{1/2}\pi;\Sigma_{3/2})$	$(\Sigma_{1/2}K;\Sigma_{3/2})$	
			0.245	$(\Lambda\pi;\Sigma_{3/2})$	$(\Delta K;\Sigma_{3/2})$	$(\Sigma_{1/2}\eta;\Sigma_{3/2})$	$(\Xi_{3/2}K;\Sigma_{3/2})$	
			0.228	$(\Sigma_{3/2}\pi;\Sigma_{3/2})$				
$\Xi_{3/2}$	$\frac{1}{2}(\frac{3}{2}^+)$	1533	0.891	$(\Xi_{3/2})$		$(\Lambda K;\Xi_{3/2})$	$(\Xi_{1/2}\eta;\Xi_{3/2})$	$(\Omega K;\Xi_{3/2})$
			0.224	$(\Sigma_{3/2}K;\Xi_{3/2})$		$(\Sigma_{1/2}K;\Xi_{3/2})$	$(\Xi_{3/2}\pi;\Xi_{3/2})$	
			−0.215	$(\Xi_{1/2}\pi;\Xi_{3/2})$			$(\Xi_{3/2}\eta;\Xi_{3/2})$	
Ω^-	$0(\frac{3}{2}^+)$	1672	0.899	(Ω)				
			0.324	$(\Xi_{1/2}K;\Omega)$				
			−0.227	$(\Xi_{3/2}K;\Omega)$				
			−0.186	$(\Omega\eta;\Omega)$				

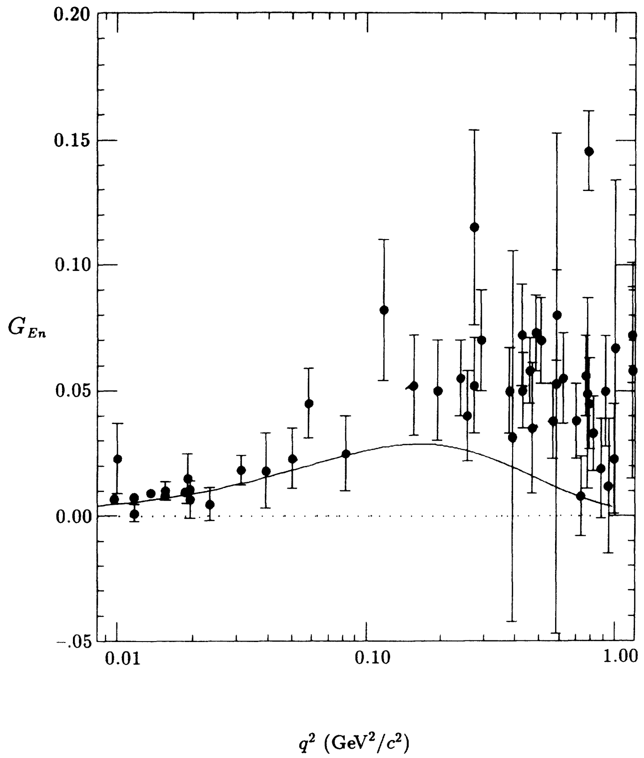

FIG. 2. Neutron electric form factor. Data are taken from Refs. 21–24. The calculated form factor is shown as a solid curve.

TABLE V. Quadrupole moments of the spin-$\frac{3}{2}$ ground-state baryons. Model parameters can be found in Table II, and wave functions can be found in Table IV. For comparison, the results of other theoretical calculations (Ref. 25) are shown.

	$Q_b(\times 10^{-2}e\ \mathrm{fm}^2)$		$\frac{Q_b}{\langle r_E^2 \rangle_b}(\times 10^{-2}e)$	
Baryon	Present model	Previous models	Present model	Isgur, Karl, and Koniuk
Δ^{++}	-6.0	-12.6[a]	-4.3	-14
Δ^{+}	-2.1	-6.3[a]	-3.1	-7
Δ^{0}	1.8	0.0[a]	-27	0
Δ^{-}	5.7	6.3[a]	-7.1	-7
$\Sigma^{+}_{3/2}$	-2.2		-3.1	
$\Sigma^{0}_{3/2}$	-0.01		2.7	
$\Sigma^{-}_{3/2}$	2.0		-3.0	
$\Xi^{0}_{3/2}$	-0.59		-32	
$\Xi^{-}_{3/2}$	1.0		-1.5	
Ω^{-}	0.57	5.2[a]	-0.92	
		3.1[b]		
		1.8[c]		
		0.4[d]		

[a]Krivoruchenko.
[b]Isgur, Karl, and Koniuk.
[c]Gershtein and Zinov'ev.
[d]Richard.

physical wave functions. The present approach yields equivalent though slightly smaller results than most of the other approaches.

We can also calculate transition quadrupole moments ($Q_{bb'}$) as the expectation value of the quadrupole operator [see Eq. (19)] between different baryons. In our model, the nucleon to delta transition quadrupole moments are calculated to be $Q_{p\Delta^+}=Q_{n\Delta^0}=-0.034e\ \mathrm{fm}^2$, indicating that the nucleon is deformed.

The calculated value of $Q_{N\Delta}$ can be used to find a value of the ratio of the electric quadrupole ($E2$) to magnetic dipole ($M1$) transition amplitudes for the process $\gamma+N\rightarrow\Delta(1232)$. In the long-wavelength limit, this ratio can be approximated as[26]

$$R=\frac{E2}{M1}\approx\frac{1}{12}\frac{qQ_{N\Delta}}{\mu_{N\Delta}},$$

where q is the on-shell photon momentum (0.259 GeV/c) and $\mu_{N\Delta}$ is the nucleon to delta transition magnetic moment [$3.76\pm0.19\mu_N=2.0\pm0.1e\hbar c\ \mathrm{GeV}^{-1}$ (Ref. 11)]. Using the model value of $Q_{N\Delta}$, this expression yields a value of $(0.94\pm0.05)\%$ for R, compared to $(1.5\pm0.2)\%$ (Ref. 27). Although the agreement is reasonable, one must remember that the long-wavelength limit is not a good approximation for this transition and that the transition magnetic moment is not really an experimental number, but an extraction made ignoring background contributions and unitarity effects.[28]

C. Excited states

The masses of resonances or excited states are identified as the larger eigenvalues of the diagonalized Hamiltonian within each sector. Since a quark-meson coupling interaction is used instead of OPEP, excited states are dominated by $q^3\phi$ basis states, and are therefore completely different from the excited states in other quark models. In Tables VI and VII, we display the excited spectrum predicted by our model for the strangeness $S=0$ and 1 sectors. The lowest spin-$\frac{3}{2}$ states are missing from the tables because although they are resonances, from a quark-model viewpoint, they are actually ground states.

The most common production mechanism for $S=0$ resonances is pion-nucleon scattering, although electron scattering is often used to study the $\Delta(1232)$. The πN widths therefore provide a good measure of which of the predicted excited states should be observed in the data. Similarly, kaon-nucleon scattering is usually used to create $S=1$ resonances, and the $\bar{K}N$ widths will indicate which $S=1$ states are observable. Approximate ranges of the data are also provided for comparison.

In the $N-\Delta$ sector (Table VI), most of the predicted states have very small πN widths. Therefore, they decouple from the primary production channel and will not be observed. However, the predicted P_{11} state at 1413 MeV, and the P_{33} state at 1556 MeV have large enough partial widths that they should be observed. The reason most states have a small πN width is that only a small fraction of the physical wave function is composed of a nucleon

TABLE VI. Nonstrange excited baryon spectrum with ground-state quantum numbers. Masses and partial widths are shown in MeV. The main components of the physical wave functions are also shown. The notation for three-quark and 3-quark–1-meson basis states can be found in the caption of Table IV.

Partial wave	Mass Data	Mass Model	πN Width Data	πN Width Model	Main components		
P_{11}	1400–1480	1413	60–250	11	$0.95(N\pi;N)$	$-0.28(N)$	$-0.14(\Delta\pi;N)$
	1680–1740	1682	10–25	1	$-0.998(N\eta;N)$		
		1766		2	$0.87(\Delta\pi;N)$	$-0.46(\Lambda K;N)$	$-0.16(N)$
		1802		1	$0.89(\Lambda K;N)$	$0.42(\Delta\pi;N)$	$-0.17(N)$
		1863		0	$0.997(\Sigma_{1/2}K;N)$		
	2000–2200	2165	10–45	0	$-0.998(\Sigma_{3/2}K;N)$		
P_{33}	1500–1650	1556	30–75	23	$-0.91(N\pi;\Delta)$	$0.32(\Delta)$	$0.23(\Delta\pi;\Delta)$
	1860–2000	1926	15–85	0	$-0.76(\Delta\pi;\Delta)$	$-0.61(\Sigma_{1/2}K;\Delta)$	$0.20(\Delta)$
		1999		3	$0.77(\Sigma_{1/2}K;\Delta)$	$-0.51(\Delta\pi;\Delta)$	$0.30(\Delta)$
		2148		1	$-0.97(\Delta\eta;\Delta)$	$0.14(\Sigma_{3/2}K;\Delta)$	$0.13(\Delta)$
		2273		1	$-0.97(\Sigma_{3/2}K;\Delta)$	$0.18(\Delta)$	

and a pion. As shown in Table VI, those states with small widths are dominated by other combinations of three-quark clusters and mesons. Furthermore, both observable states are dominated by the $(N\pi)$ combination.

The Roper resonance has a four-star rating from the Particle Data Group,[11] but remains controversial nonetheless.[10] The data suggest there could actually be two separate states within the nominal range of 1400–1480 MeV, while the IK and similar models predict only one state in this energy range. Our model suggests that if, in fact, there are two states, the other could be composed of mesonic excitations. However, our calculated width is much smaller than either of the observed widths, so to agree with the data the two resonance states would be strong mixtures of the excited states predicted by the two approaches.

TABLE VII. Singly strange excited baryon spectrum with ground-state quantum numbers. Masses and partial widths are shown in MeV. The main components of the physical wave functions are also shown. The notation for three-quark and 3-quark–1-meson basis states can be found in the caption of Table IV.

Partial wave	Mass Data	Mass Model	$\bar{K}N$ Width Data	$\bar{K}N$ Width Model	Main components		
P_{01}	1550–1650	1609	20–80	14	$0.91(\Sigma_{1/2}\pi;\Lambda)$	$0.39(NK;\Lambda)$	$-0.11(\Lambda)$
		1657		21	$0.90(NK;\Lambda)$	$-0.35(\Sigma_{1/2}\pi;\Lambda)$	$0.21(\Lambda)$
	1730–1860	1838	35–45	0	$-0.99(\Lambda\eta;\Lambda)$	$-0.11(\Sigma_{3/2}\pi;\Lambda)$	
		1925		13	$-0.96(\Sigma_{3/2}\pi;\Lambda)$	$0.20(\Lambda)$	$-0.13(\Xi_{1/2}K;\Lambda)$
		1972		1	$-0.99(\Xi_{1/2}K;\Lambda)$	$0.12(\Sigma_{3/2}\pi;\Lambda)$	
		2306		1	$-0.996(\Xi_{3/2}K;\Lambda)$		
P_{11}		1531		0	$0.98(\Lambda\pi;\Sigma_{1/2})$	$0.17(\Sigma_{1/2}\pi;\Sigma_{1/2})$	$0.12(\Sigma_{1/2})$
	1630–1690	1618	5–25	3	$-0.94(\Sigma_{1/2}\pi;\Sigma_{1/2})$	$-0.22(NK;\Sigma_{1/2})$	$0.21(\Sigma_{1/2})$
		1637		34	$0.97(NK;\Sigma_{1/2})$	$-0.20(\Sigma_{1/2}\pi;\Sigma_{1/2})$	
	1750–1850		≈ 5				
	1850–2000	1900	45–75	3	$0.96(\Sigma_{3/2}\pi;\Sigma_{1/2})$	$0.26(\Sigma_{1/2}\eta;\Sigma_{1/2})$	
		1912		0	$0.96(\Sigma_{1/2}\eta;\Sigma_{1/2})$	$-0.24(\Sigma_{3/2}\pi;\Sigma_{1/2})$	$0.10(\Xi_{1/2}K;\Sigma_{1/2})$
		1977		0	$0.93(\Xi_{1/2}K;\Sigma_{1/2})$	$-0.35(\Delta K;\Sigma_{1/2})$	
		2021		0	$-0.92(\Delta K;\Sigma_{1/2})$	$-0.32(\Xi_{1/2}K;\Sigma_{1/2})$	$-0.18(\Sigma_{1/2})$
		2213		0	$0.995(\Sigma_{3/2}\eta;\Sigma_{1/2})$		
		2301		0	$0.997(\Xi_{3/2}K;\Sigma_{1/2})$		
P_{13}	1700–1900	1775	≈ 45	2	$0.93(\Lambda\pi;\Sigma_{3/2})$	$0.23(NK;\Sigma_{3/2})$	$-0.22(\Sigma_{1/2}\pi;\Sigma_{3/2})$
		1822		21	$-0.85(NK;\Sigma_{3/2})$	$-0.52(\Sigma_{1/2}\pi;\Sigma_{3/2})$	
		1852		3	$-0.81(\Sigma_{1/2}\pi;\Sigma_{3/2})$	$0.45(NK;\Sigma_{3/2})$	$-0.24(\Lambda\pi;\Sigma_{3/2})$
	2050–2200	2078	unknown	0	$-0.97(\Sigma_{1/2}\eta;\Sigma_{3/2})$	$0.23(\Sigma_{3/2}\pi;\Sigma_{3/2})$	
		2133		0	$-0.76(\Xi_{1/2}K;\Sigma_{3/2})$	$0.60(\Sigma_{3/2}\pi;\Sigma_{3/2})$	$-0.23(\Delta K;\Sigma_{3/2})$
		2148		0	$0.62(\Delta K;\Sigma_{3/2})$	$-0.60(\Xi_{1/2}K;\Sigma_{3/2})$	$-0.50(\Sigma_{3/2}\pi;\Sigma_{3/2})$
		2242		1	$-0.71(\Delta K;\Sigma_{3/2})$	$-0.50(\Sigma_{3/2}\pi;\Sigma_{3/2})$	$0.32(\Sigma_{1/2})$
		2494		0	$0.96(\Xi_{3/2}K;\Sigma_{3/2})$	$0.22(\Sigma_{1/2})$	$-0.11(\Sigma_{3/2}\pi;\Sigma_{3/2})$

The $\Delta(1600)$ is a P_{33} resonance with a two-star rating.[11] The data covers a wide range, but is most consistent with a mass near 1550 MeV and a πN width of about 50 MeV. Quark models using spatial excitations predict masses in the range of 1650–1900 MeV (Refs. 4, 5, and 29), but the two most reliable estimates are both very close to 1800 MeV (Refs. 4 and 5). Spatial excitation models therefore have no viable candidate for the $\Delta(1600)$. Although the calculated value of the πN width (23 MeV) in our approach is slightly outside the expected range (30–75 MeV), we conjecture that this observed state could still be a mesonic excitation of quarks.

For the $S=1$ sector, it is very difficult to make any comparisons between theory and experiment because it is impossible to count the actual number of observed states in the data. For instance, the comments from the Particle Data Group for the $\Lambda(1600)$ and the $\Lambda(1800)$ suggest that there could be two states within each energy range. The existence of the $\Sigma(1770)$ is based on a single listed experiment, yet confirming evidence appears to be listed under $\Sigma(1880)$. In general, the measured masses and widths vary so greatly that almost nothing can be said definitively. The IK model predicts that there are three states in the P_{01} channel that are observable. They also predict three states in the P_{11} and three states in the P_{13} channels. Our model predicts three additional P_{01} states: at about 1610, 1660, and 1925 MeV. We also predict one additional P_{11} state at around 1640 MeV, and a P_{13} state at 1820 MeV. All of these additional states couple strongly to the $\bar{K}N$ channel. The rest of the predicted states should not be seen. As in the case of the $N-\Delta$ sector, $\bar{K}N$ widths are largest when the excited state is dominated by the (NK) combination and smaller otherwise (see Table VII). In both the strangeness-0 and strangeness-1 sectors, the existence of observable model states does not contradict either the results of spatial excitation models or the data.

D. The importance of nonpionic degrees of freedom

It is clear from Table III that our results for the baryon mass shifts due to H_{int} (column 2) are very different from previous calculations. It is not obvious, however, where to attribute the differences. It is typical for most calculations to choose the same pion distribution for all states. Therefore, to reduce model dependence and make a proper comparison with other calculations, we have repeated our calculation using the same value of β for all the states in the model space. We have chosen a value of 300 MeV as a compromise between those values of β which reproduce the Δ and $\Sigma_{3/2}$ widths. Results are shown in Table VIII. Also shown are results using the same set of parameters, but including fewer species of mesons in the basis.

Column 4 (π only) agrees very well with the results of other authors shown in Table III, especially calculations (b) and (d). Because of the extremely light mass of the pion, most calculations assume that pionic effects dom-

TABLE VIII. Results using a single value of the length parameter β. The parametrized masses M_B of the three-quark eigenstates are set as before, but $\alpha=237.8$ MeV, $\beta=300$ MeV, and $G_{qq\phi}=-0.622$ are used throughout. The quantity δm_B is the energy shift of baryon B which is due to meson couplings (m_B-M_B). The parameters M_B can be found by subtracting the energy shifts from the physical masses. Underlined values have been used as constraints. Where appropriate, measured values are shown with errors in parentheses.

Parameter/ observable	Baryon	Fit/calculated values using $\alpha=237.8$ MeV, $\beta=300$ MeV, and $G_{qq\phi}=-0.622$: No mesons	π only	π,K only	π,K,η	Measured value
δm_B (MeV)	N	0	−140	−159	−160	
	Λ^0	0	−90	−140	−144	
	$\Sigma_{1/2}$	0	−63	−123	−137	
	$\Xi_{1/2}$	0	−24	−103	−125	
	Δ	0	−116	−139	−146	
	$\Sigma_{3/2}$	0	−78	−129	−135	
	$\Xi_{3/2}$	0	−40	−111	−124	
	Ω^-	0	0	−86	−114	
$\langle r_E^2 \rangle_b$ (fm^2)	p	0.688	0.716	0.719	0.720	0.72(0.02)
	n	0.000	−0.026	−0.022	−0.023	−0.12(0.01)
$G_{NN\pi}$		−13.5	−13.4	−13.4	−13.4	−13.4
$\Gamma_{b\to B'\pi}$ (MeV)	Δ^{++}	56.0	100.0	103.0	103.0	112(1)
	$\Sigma_{3/2}^{+}$	27.5	40.4	42.4	42.9	35(1)
	$\Xi_{3/2}^{0}$	10.8	11.2	12.1	13.1	9.1(0.5)
ABR[a](%)	$\Sigma_{3/2}$	85.2	87.5	87.3	87.0	88(2)

[a] Average branching ratio.

inate, and do not include contributions from kaons and etas. However, our calculations indicate that mass shifts are not saturated by pionic effects, as shown by comparing columns 4, 5, and 6. As the strangeness of the baryon increases, the contributions from the kaon and η also increase, and partially compensate for the decreasing contributions from the pions. In fact, pionic effects are dominant for the *nucleon only.* The reason is that it usually requires about the same energy to create a pionic excitation as a kaonic excitation. For instance, the $\Sigma_{1/2}$ three-quark state can couple to $\Lambda\pi$, $\Sigma_{1/2}\pi$, and $\Sigma_{3/2}\pi$, as well as NK and ΔK. The η becomes important when the masses of the three-quark states are so large that the relative mass difference between the pion and the η is not very large. In general, we find that kaons and η's are as important as pions in the baryon spectrum.

As mentioned earlier, Table III indicates that, with or without kaons and η's, δm_Δ is larger than δm_N. However, when we use a single value of β, our mass shifts agree qualitatively with all previous work, which have found that δm_Δ is less than δm_N. We conjecture that others have not observed the possibility that $\delta m_\Delta > \delta m_N$ since the spatial distributions of the meson fields were not allowed to be different for different baryons.

IV. DISCUSSION

We have developed an approach to modeling the baryon spectrum which has sufficient freedom to reproduce the mean-squared charge radii of the proton and neutron, as well as strong decay widths in the Δ and $\Sigma_{3/2}$ sectors. The model presupposes the existence of explicit mesons in the physical wave functions of the baryons, and as a consequence, predicts the existence of low-lying resonant excitations made up predominantly of three quarks and a pseudoscalar meson.

The present model has demonstrated that mesonic couplings have measurable effects on the masses and charge distributions of the ground-state baryons. Mesonic couplings lead to quadrupole moments comparable to those predicted by other models. We have also shown that pionic couplings are dominant for the nucleon only, and that the three pseudoscalar mesons are equally important for the baryon spectrum.

The present model does not reproduce the observed excited spectrum of masses and decay widths. In fact, only a very few of the predicted states couple strongly to the production channels most commonly used. Thus, other effects such as spatial excitations are clearly necessary to reproduce most of the excited spectrum. However, the present approach is not inconsistent with the IK and similar models. Instead, our model resolves two possible controversies among excited states with ground-state quantum numbers: We find a second state in the mass range of the Roper resonance, and a candidate state in the region of the $\Delta(1600)$.

Our model also predicts five states in the $\Lambda-\Sigma$ sector that should be observable in $\bar{K}N$ scattering experiments. Including those predicted by other models, there are now six observable $\Lambda(\frac{1}{2}^+)$ states in the range 1550–1950 MeV. There are also four $\Sigma(\frac{1}{2}^+)$ states between 1600 and 2000 MeV, and four $\Sigma(\frac{3}{2}^+)$ states between 1700 and 2100 MeV. Given the acknowledged variety of experimental results,[11] and the number of predicted states, further experiments are certainly necessary to distinguish the excited baryon spectrum.

Although many studies of nonrelativistic quark models calculate the effects of mesonic couplings, in fact, mesons have not been explicitly included. For instance, in Ref. 7 OPEP is used to estimate energy shifts and wave-function perturbations, but charge radii are calculated with corrections due to the *implicit* presence of pions. By "implicit" we mean that fundamental mesons are not included as part of the physical wave function. In nuclear physics, these are usually called meson-exchange currents. This hybrid approach of using OPEP for some calculations and implicit mesons for others is justified in nuclear physics because much less energy is required to excite a nucleus than to create a pion.

In contrast with most nuclear calculations, in our model, mesonic excitations have about the same energy as spatial excitations of quarks. Studies of multiquark hadrons ($q^m\bar{q}^n; m+n>3$) have found similar results for $q^4\bar{q}$ states in the bag model.[30,31] Adding a single $q\bar{q}$ excitation, however, may not necessarily be equivalent to adding a meson. For instance, Bernard *et al.*[32] have shown that, when the pion is treated as a Goldstone boson, the pion wave function can be written with large contributions from many-quark–many-antiquark excitations. By extension, we might expect the structure of the rest of the pseudoscalar mesons to be complicated as well. By treating the mesons as fundamental, it is possible that higher-order combinations of $q\bar{q}$ pairs in the meson wave function are taken into account.

Since the energies of mesonic and spatial excitations are roughly the same, it may not be sufficient to calculate effects *as though* mesons were present. We believe it is necessary to include meson degrees of freedom explicitly. In an approach that combines mesonic and spatial excitations, the strength and form of the quark-meson coupling interaction will remain largely unchanged, since the strength is determined by the pion-nucleon coupling constant and the form is determined from particle symmetries. The freedom to adjust parameters will come from the quark-quark interaction, which has been left unspecified in our approach, and is uncertain in other approaches.

V. CONCLUSIONS

An approach to baryon wave functions and properties has been developed which isolates in a model-independent way the effects of a standard, scalar-isoscalar, quark-meson coupling interaction. The model successfully correlates a wide variety of observables, but its predictive power has not yet been fully tested, since most of the observables calculated are used as model constraints. Agreement with the data has been achieved without including any deformed three-quark states in the model space. Instead, deformation of the physical states is provided by explicit 3-quark–1-meson states mixed into the physical wave functions.

Explicit mesons implies the existence of low-energy resonances composed primarily of three quarks and a meson. Most of the predicted states are not observable because they do not couple strongly to the experimental scattering channels. In fact, the present approach fails to reproduce most of the excited baryon spectrum. However, there are a number of strangeness-0 and strangeness-1 resonance states that should be observable in πN and $\bar{K}N$ reactions, but are not predicted by other models. None of the observable states is inconsistent with the data or the IK model.

In contrast with previous studies, we find that kaons and η's are as important in the strange baryon sector as pions are in the nonstrange sector. Furthermore, mesonic couplings are found to be an important part of ground-state baryon properties, and provide an excellent complement to spatial excitations.

Therefore, we conclude that a complete study of the baryon spectrum should include *all* ground-state pseudoscalar mesons, and that they can be added *explicitly* to the three-quark basis, with mesonic and spatial excitations being treated equally. We also conclude that more precise measurements would be helpful to fully understand the excited baryon spectrum, especially in the region of the Roper resonance as well as the $\Delta(\frac{3}{2}^+)$, $\Lambda(\frac{1}{2}^+)$, $\Sigma(\frac{1}{2}^+)$, and $\Sigma(\frac{3}{2}^+)$ sectors.

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