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Baryon magnetic moments with confined quarks

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Within a rather general framework for confined quarks in a baryon, we derive an approximate expression for the effective quark masses and quark magnetic moments. Use of the additive quark model with the spin-flavor SU(6) wave functions may be justified for the relativistic quarks and lead to a good agreement between theory and experiment for the baryon magnetic moments.

The naive quark model is remarkable in qualitatively explaining the observed magnetic moments of baryons composed of u, d, and s quarks.[1] However, recent accurate measurements[2] of $\mu(\Xi^0)$ and $\mu(\Sigma^+)$ indicate that the model predictions are far from satisfactory. Moreover, with $m_u < m_d$, which is required by the hadron electromagnetic mass differences, the model can hardly explain the small but important discrepancy between the famous spin-flavor SU(6) ratio $\mu(n)/\mu(p) = -\frac{2}{3}$ and its observed value. Despite many theoretical attempts to improve the model[3-6] some discrepancies between theory and experiment seem to persist. The difficulties encountered in simple quark models are possibly due to many effects such as the neglect of orbital angular momentum, quark-antiquark pairs in the baryon wave functions, quark-quark correlations, meson currents, etc. Among others, the quark confinement is probably one of the most important effects, and here we will concentrate on this effect. In fact there is no strict justification for the naive assumption that the quarks behave in response to the external electromagnetic field as if they are free Dirac particles and have magnetic moments $e_q/2m_q$ (m_q and e_q being the quark mass and electric charge, respectively). This is because, when quarks are probed by the external electromagnetic field, i.e., by the soft photon carrying small momentum transfer, the quarks are not free but confined in a baryon. Therefore it is necessary to study how the confined quarks respond to the external electromagnetic field. In the MIT bag model[6] as well as its improved version, which treat the confinement by essentially introducing a vacuum volume energy, the relativistically confined quarks and their magnetic moments are studied. However, within the widely used quark potential models (e.g., Ref. 4) in which the confinement is implemented by a confining interquark potential, we still need convincing discussions on baryon magnetic moments for relativistically confined quarks. In this paper, within a rather general framework for the confined quarks, we will derive an approximate expression for the effective quark masses and magnetic moments, clarify the applicability of the SU(6) formula for relativistically confined quarks, and calculate the baryon magnetic moments.

At present we are still unable to write a fundamental equation of motion for the quarks confined in a baryon from first principles of the strong-interaction theory—QCD. Instead, we will assume that a baryon, composed of three quarks q_1, q_2, and q_3, satisfies the equation

$$(H_1 + H_2 + H_3 + \beta_1\beta_2\beta_3 V)\psi = M_B\psi \ , \tag{1}$$

where $H_i = \boldsymbol{\alpha}_i \cdot \mathbf{p}_i + \beta_i m_i$ ($i=1,2,3$) is the free Dirac Hamiltonian for the ith quark, m_i the quark mass, and V is the confining potential, which is postulated to transform as a Lorentz scalar. Here the relativistic wave function ψ has 64 components, four each for quarks q_1, q_2, and q_3, and the Dirac matrices $\boldsymbol{\alpha}_i$ and β_i operate on the spinor components of ψ for quarks q_i. M_B is the baryon mass eigenvalue (in the baryon rest frame where $\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3=0$). We further assume that when the ith quark with electric charge e_i is probed in a given external electromagnetic field $A_\mu = (A_0, \mathbf{A})$ the Hamiltonian is obtained by the replacement $H_i \to \boldsymbol{\alpha}_i \cdot [\mathbf{p}_i - e_i \mathbf{A}(x_i)] + e_i A_0(x_i) + \beta_i m_i$.

We now examine the effect of the vector potential $\mathbf{A}$ on, say, quark q_1 (in the following we will ignore the scalar potential A_0). Let $\psi = \binom{\phi}{\chi}$, where ϕ and χ are, respectively, the upper and lower components for quark q_1 (each having 32 components). Before introducing the external potential $\mathbf{A}$, Eq. (1) can be written as two coupled equations:

$$(M_B - H_2 - H_3)\phi = (\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)\chi + (m_1 + \beta_2\beta_3 V)\phi \ , \tag{2}$$

$$(M_B - H_2 - H_3)\chi = (\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)\phi - (m_1 + \beta_2\beta_3 V)\chi \ . \tag{3}$$

Formally, using Eq. (3) we can express χ in terms of ϕ and then eliminate χ in Eq. (2):

$$\chi = (M_B + m_1 - H_2 - H_3 + \beta_2\beta_3 V)^{-1}(\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)\phi \ , \tag{4}$$

$$\begin{aligned}(M_B - m_1 - H_2 - H_3 - \beta_2\beta_3 V)\phi &= (\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)(M_B + m_1 - H_2 - H_3 + \beta_2\beta_3 V)^{-1} \\ &\quad \times (\boldsymbol{\sigma}_1 \cdot \mathbf{p}_1)\phi \ .\end{aligned} \tag{5}$$

When quark q_1 is probed in the external field the momentum $\mathbf{p}_1$ in Eq. (5) will be replaced by $\mathbf{p}_1 - e_1\mathbf{A}(x_1)$ and we then have

$$(M_B-m_1-H_2-H_3-\beta_2\beta_3V)\phi=[\sigma_1\cdot(\mathbf{p}_1-e_1\mathbf{A})](M_B+m_1-H_2-H_3+\beta_2\beta_3V)^{-1}[\sigma_1\cdot(\mathbf{p}_1-e_1\mathbf{A})]\phi$$
$$=(M_B+m_1-H_2-H_3+\beta_2\beta_3V)^{-1}[\sigma_1\cdot(\mathbf{p}_1-e_1\mathbf{A})]^2\phi+\cdots$$
$$=(M_B+m_1-H_2-H_3+\beta_2\beta_3V)^{-1}[(\mathbf{p}_1-e_1\mathbf{A})^2-e_1\sigma_1\cdot\mathbf{H}]\phi+\cdots, \quad (6)$$

where the operator $\sigma_1\cdot\mathbf{p}_1=-i\sigma_1\cdot\nabla_1$ commutes with H_2 and H_3 but not V, and the ellipsis denotes terms containing the gradient of the confining potential $\nabla_1 V$ but not the magnetic field $\mathbf{H}$. Equation (6) indicates that the interaction of the confined quark (q_1) with an external magnetic field $\mathbf{H}$ will contribute to the baryon mass an additional term $-\boldsymbol{\mu}_1\cdot\mathbf{H}$, as if the confined quark has an effective mass (operator) m_1^B and a magnetic moment (operator) $\boldsymbol{\mu}_1$

$$m_1^B=\tfrac{1}{2}(M_B+m_1-H_2-H_3+\beta_2\beta_3V),\quad \boldsymbol{\mu}_1=(e_1/2m_1^B)\sigma_1 . \quad (7)$$

This $-\boldsymbol{\mu}_1\cdot\mathbf{H}$ term can be treated as perturbation and the baryon mass shift will be obtained by taking the expectation value over the wave function ϕ, $\Delta M_B^{(1)}=-\overline{\boldsymbol{\mu}_1}\cdot\mathbf{H}$ where $\overline{\boldsymbol{\mu}_1}=\langle\phi|\boldsymbol{\mu}_1|\phi\rangle/\langle\phi|\phi\rangle$ and ϕ is the solution of Eq. (5). Furthermore, it can be seen that the introduction of $\mathbf{p}_2\to\mathbf{p}_2-e_2A(x_2)$ and $\mathbf{p}_3\to\mathbf{p}_3-e_3\mathbf{A}(x_3)$ in Eq. (6) will not affect this interaction term of quark q_1 with $\mathbf{H}$. The baryon mass shift due to interactions of all three quarks in the baryon with the external magnetic field $\mathbf{H}$ will be given by $\Delta M_B^{(1)}+\Delta M_B^{(2)}+\Delta M_B^{(3)}=-(\overline{\boldsymbol{\mu}_1}+\overline{\boldsymbol{\mu}_2}+\overline{\boldsymbol{\mu}_3})\cdot\mathbf{H}$, where $\boldsymbol{\mu}_{2,3}$ and $m_{2,3}^B$ are defined in the same way as for $\boldsymbol{\mu}_1$ and m_1^B in Eq. (7). This indicates that the baryon magnetic moment is given by the vector addition of the quark magnetic moments. But each expectation value $\overline{\boldsymbol{\mu}_i}$ should be taken over its own corresponding wave function. Namely, ϕ corresponds to $\boldsymbol{\mu}_1$ but not $\boldsymbol{\mu}_2$ and $\boldsymbol{\mu}_3$. Note that the term on the right-hand side of Eq. (5) can be converted into two terms one being $(1/2m_1^B)\mathbf{p}_1^2\phi$ and the other containing the spin-orbit operator

$$\sigma_1\cdot\nabla_1V\times\mathbf{p}_1=\frac{\partial V}{\partial r_{12}}\sigma_1\cdot\frac{\mathbf{r}_{12}}{r_{12}}\times\mathbf{p}_1+\frac{\partial V}{\partial r_{13}}\sigma_1\cdot\frac{\mathbf{r}_{13}}{r_{13}}\times\mathbf{p}_1$$
$$(\mathbf{r}_{12}\equiv\mathbf{r}_1-\mathbf{r}_2,\mathbf{r}_{13}\equiv\mathbf{r}_1-\mathbf{r}_3) .$$

This is again due to the fact that $\sigma_1\cdot\mathbf{p}_1$ commutes with H_2,H_3 but not V. For the ground-state (S-wave) baryons the spin-orbit forces make no contributions; therefore, Eq. (5) actually becomes

$$[M_B-m_1-H_2-H_3-\beta_2\beta_3V-(1/2m_1^B)\mathbf{p}_1^2]\phi=0 . \quad (8)$$

The operators in Eq. (8) do not contain the Pauli matrix σ_1; hence, ϕ can be written as the product of a spatial wave function and a two-component Pauli spinor (note that here the "spatial" and the spinor only refer to quark q_1). Accordingly, the expectation value of $\overline{\boldsymbol{\mu}_1}$ taken over ϕ will be factorized into two parts: $e_1/2m_1^B$ (concerning the spatial wave function of q_1 only) and $\overline{\sigma}_1$ (concerning the Pauli spinor of q_1 only). This may justify the use of the spin-flavor SU(6) wave functions. Namely, even for the relativistic quarks we can still use the **56** representation for the ground-state (S-wave) $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons to calculate the baryon magnetic moments which are the vector addition of the quark magnetic moments, as have been shown above.

The relativistic effects and complexity are contained in the expectation values of the effective quark masses, e.g., [see Eq. (7)], $\overline{m_1^B}=\frac{1}{2}(M_B+m_1-\overline{H}_2-\overline{H}_3+\overline{\beta_2\beta_3V})$. By using the expectation value taken over ϕ for Eq. (8), $M_B-m_1-\overline{H}_2-\overline{H}_3-\overline{\beta_2\beta_3V}-\overline{(1/2m_1^B)\mathbf{p}_1^2}=0$, we can eliminate $\overline{\beta_2\beta_3V}$ and get

$$\overline{m_1^B}=M_B-\overline{H}_2-\overline{H}_3-\overline{(1/4m_1^B)\mathbf{p}_1^2}\approx M_B-\overline{H}_2-\overline{H}_3 , \quad (9)$$

where the last step is due to $\overline{(1/4m_1^B)\mathbf{p}_1^2}\ll\overline{H}_2+\overline{H}_3$ [which holds not only in the nonrelativistic limit but also for relativistic quarks with, say, $\overline{\mathbf{p}_i^2}\sim(m_i^B)^2\sim(300-500\ \mathrm{MeV})^2$]. For relativistic quarks the expectation value $\overline{H}_i=\overline{\alpha_i\cdot\mathbf{p}_i+\beta_i m_i}$ receives contributions from both the intrinsic quark mass (e.g., the current-quark mass) and the kinetic energy. It is certainly difficult to find the true value of $\overline{H}_i$ in terms of the fundamental parameters in Eq. (1). However, if the kinetic energy in $\overline{H}_i$ is not sensitive to baryon species then $\overline{H}_i$ may be approximately treated as a fixed mass parameter for quark q_i, in all baryons composed of u, d, s quarks. We may write $\overline{H}_i=m_i'$ and regard m_i' as the constituent mass of quark q_i, Eq. (9) then becomes $\overline{m_1^B}\approx M_B-m_2'-m_3'$. Ideally, only in the nonrelativistic limit (with a very smooth confining potential) $\mathbf{p}_i^2\to0,\overline{H}_2\to m_2,\overline{H}_3\to m_3$; we can get from Eq. (9) an exact relation

$$m_1^B=M_B-m_2-m_3 . \quad (10)$$

For convenience in the following we will use the form of Eq. (10) but remembering where the parameter m_i may also stand for $m_i'=\overline{H}_i$ in relativistic cases.

We emphasize that Eq. (10) [or Eq. (9) in general] is derived based on the assumption that the confining potential in Eq. (1) is a Lorentz scalar. If a Lorentz-vector confining potential were used the expression for the effective mass m_1^B would be very different (even in the nonrelativistic limit). An important point is that the baryon mass M_B in Eq. (10) should not be regarded as the physical baryon mass m_B but the baryon mass before taking account of the hyperfine splittings. This is simply because the eigenvalue M_B in Eq. (1) only receives contribution of the scalar (spin-independent) confining energy. As nowadays widely expected, the spin-dependent mass splittings for hadrons are probably due to the gluon exchange between quarks.[4,6] If using[4] the color hyperfine splitting term $\delta H = C\sum_{i<j} \mathbf{s}_i\cdot\mathbf{s}_j/m_i m_j$ ($\mathbf{s}_i$ and m_i being the spin and mass of the ith quark, and C being flavor independent) to calculate the mass splittings for the ground-state $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons, we will get the following M_B from the observed physical masses m_B:

$$M_N = m_N + \tfrac{1}{2}(m_\Delta - m_N),\quad M_\Lambda = m_\Lambda + \tfrac{1}{2}(m_\Delta - m_N),$$
$$M_{\Sigma^0} = M_\Lambda,\quad M_\Xi \approx m_\Xi + \tfrac{2}{3}(m_{\Xi^*} - m_\Xi). \tag{11}$$

With (in MeV) $m_\Delta - m_N = 294$, $m_{\Xi^*} - m_\Xi = 217$, we get $M_p = 1085.3$, $M_n = 1086.6$, $M_\Lambda = M_{\Sigma^0} = 1262$, $M_{\Sigma^+} = 1258$, $M_{\Sigma^-} = 1266$, $M_{\Xi^0} = 1460$, $M_{\Xi^-} = 1466$. These baryon masses are those in Eq. (1) and will be used in Eq. (10).

Using

$$\mu_1 = \overline{\left[\frac{e_1}{2m_1^B}\right]} \approx \frac{e_1}{2\overline{m}_1^B}$$

with $\overline{m_1^B} = M_B - m_2 - m_3$ [Eqs. (9) and (10)] for each probed quark (q_1) in a baryon, then adding the magnetic moments of three probed quarks over the baryon wave function given by the spin-flavor SU(6) group (see the formula in Table I), we calculate the baryon magnetic moments, which are shown in Table I. The quark mass parameters m_i in Eq. (10) are chosen to be (in MeV)

$$m_u = 364,\quad m_d = 387,\quad m_s = 529. \tag{12}$$

In fact we use observed $\mu(p)$ and $\mu(n)$ as inputs to determine m_u and m_d, and then use $\mu(\Xi^0)$ as the input to determine m_s. The obtained $\mu(\Lambda)$ and $\mu(\Sigma^+)$ are in agreement with data.[2] In the calculation of $\mu(\Omega^-)$ we use

$$M_{\Omega^-} = m_{\Omega^-} - \tfrac{1}{2}(m_\Delta - m_N)(m_u/m_s)^2 = 1614\ \text{MeV},$$

where $m_u/m_s = 0.63$, a value obtained from fitting the $\Sigma - \Lambda$ mass difference with the hyperfine splitting formula used above.

Moreover, $\mu(p)$ and $\mu(n)$ are fitted with $m_u < m_d$, but our $m_d - m_u$ is considerably larger than that of ~ 4 MeV, as determined from the hadron mass differences. This can be partially explained by observing that our masses in (12) should actually be the values of $m'_{2,3} = \overline{H}_{2,3} + \overline{(1/8m_1^B)\mathbf{p}_1^2}$ when two spectator quarks q_2 and q_3 are the same kind [e.g., $m'_u = \overline{H}_u + (1/8m_d^p)\mathbf{p}_d^2$, $m'_d = \overline{H}_d + (1/8m_u^n)\mathbf{p}_u^2$, see Eq. (9)] and that the obtained spectator mass difference $m'_d - m'_u$ from (12) is actually enlarged by the kinetic energy difference of corresponding probed quarks, in addition to the constituent quark mass difference $\overline{H}_d - \overline{H}_u$.

In fact, our m'_u and m'_d are determined by using

$$\mu(p) = \frac{8}{9}\frac{1}{2(M_p - m'_u - m'_d)} + \frac{1}{9}\frac{1}{2(M_p - m'_u - m'_u)}, \tag{13}$$

$$\mu(n) = -\frac{4}{9}\frac{1}{2(M_n - m'_u - m'_d)} - \frac{2}{9}\frac{1}{2(M_n - m'_d - m'_d)}. \tag{14}$$

With observed values of $\mu(p)$ and $\mu(n)$ we find $m_d^p = M_p - m'_u - m'_u = 357$ MeV and $m_u^n = M_n - m'_d - m'_d = 312$ MeV. If taking, say, $\overline{\mathbf{p}_u^2} \approx \overline{\mathbf{p}_d^2} \approx (350\ \text{MeV})^2$, we will get $(1/8m_d^p)\overline{\mathbf{p}_d^2} \approx 43$ MeV and $(1/8m_u^n)\overline{\mathbf{p}_u^2} \approx 49$ MeV, and therefore $m'_d - m'_u = \overline{H}_d - \overline{H}_u + 6$ MeV. This may indicate that $m'_d - m'_u$ should indeed be considerably larger than the constituent-quark mass difference. However, our value of $m'_d - m'_u = 23$ MeV is still too large and this may be due to the crudeness of our approximate expressions for quark masses and magnetic moments. In any case, with $m_u < m_d$ the ratio of $\mu(n)/\mu(p)$ is improved. This is in the right direction and is not a trivial result.

The derived effective quark mass formula Eq. (10) coin-

TABLE I. Comparison of experimental (Ref. 2) baryon magnetic moments (in nuclear magnetons) with three theoretical results: the naive quark model (NQM), the SO model (Ref. 5), and our approach. Inputs are indicated by asterisks.

$\mu(B)$	Formula	NQM	SO	Ours	Experiment
$\mu(p)$	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_d$	2.79*	2.79*	2.79*	2.793
$\mu(n)$	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_u$	−1.91*	−1.91*	−1.91*	−1.913
$\mu(\Lambda)$	μ_s	−0.61*	−0.61	−0.61	−0.613±0.004
$\mu(\Sigma^+)$	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_s$	2.67	2.06	2.48	2.42 ±0.05
$\mu(\Sigma^-)$	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_s$	−1.09	−0.79	−0.98	−1.157±0.025
$\mu(\Xi^0)$	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_u$	−1.43	−1.25*	−1.25*	−1.250±0.014
$\mu(\Xi^-)$	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_d$	−0.49	−0.50	−0.50	−0.69 ±0.04
$\mu(\Sigma\Lambda)$	$\frac{\sqrt{3}}{3}\mu_d - \frac{\sqrt{3}}{3}\mu_u$	−1.63	−1.38	−1.53	−1.61 ±0.08
$\mu(\Omega^-)$	$3\mu_s$	−1.83		−1.69	

cides with a purely empirical formula proposed by Sogami and Oh'yamaguchi[5] (SO) but there is an important difference that SO assume M_B to be the physical baryon mass whereas in our derivation M_B should apparently be the baryon mass when switching off the hyperfine splittings. This difference leads to significant changes in the values of calculated magnetic moments. As can be seen in Table I, SO succeeded in fitting $\mu(p)$ and $\mu(n)$ with $m_u < m_d$, also $\mu(\Lambda)$ and $\mu(\Xi^0)$ (using $m_u = 291$ MeV, $m_d = 314$ MeV, $m_s = 456$ MeV) but gave a rather awful result for $\mu(\Sigma^+)$. Our results turned out to be much better for $\mu(\Sigma^+)$, $\mu(\Sigma^-)$, and $\mu(\Sigma\Lambda)$.

We emphasize that the obtained mass parameters in (12) are actually the values of m_i' ($\approx \bar{H}_i$) for u,d,s quarks. With these values we now check the approximations made in our calculations. We are not trying to specify the quark mass (m_i) values in Eq. (1) but just considering two special cases. First, as noted previously, all the approximations made in Eqs. (9) and (10) would become exact in the nonrelativistic limit where $\mathbf{p}_i^2 \ll m_i^2$ and $m_i' \approx \bar{H}_i \approx m_i$. Unfortunately, with $m_u' \approx m_d' \approx 350$ MeV obtained in (12), the nonrelativistic quark momenta are required to be $\mathbf{p}_{u,d}^2 \ll (350\ \text{MeV})^2$, which is in contradiction with the fact that the baryon, e.g., the nucleon, has a radius of about one fermi. Therefore the nonrelativistic case does not seem to be realistic in the physical world and the use of quark magnetic moment being $\mu_q = e_q/2m_q$ is not truly justified for nonrelativistic quarks. Second, for relativistic quarks we consider an extreme case in which the quark masses in Eq. (1) are taken to be the current quark masses which are much smaller than the confinement energy scale, e.g., $m_1 \approx m_2 \approx m_3 \approx 0$ for the nucleons. In this case in Eq. (9) $\bar{H}_2 \approx |\mathbf{p}_2|$, $\bar{H}_3 \approx |\mathbf{p}_3|$, and $|\mathbf{p}_1| \approx |\mathbf{p}_2| \approx |\mathbf{p}_3|$. With the parameters of m_i' ($\approx \bar{H}_i$) given in (12) we then have $m_1^B \approx 350$ MeV $\approx |\mathbf{p}_1|$. Therefore the inequality $(1/4m_1^B)\mathbf{p}_1^2 \ll \bar{H}_2 + \bar{H}_3$ in Eq. (9) is indeed valid. Furthermore, using the expansion [see Eq. (7)]

$$\frac{1}{2m_1^B} \equiv \frac{1}{M_B+m_1}\left[1-\frac{H_2+H_3-\beta_2\beta_3 V}{M_B+m_1}\right]^{-1}$$
$$= \frac{1}{M_B+m_1}\left[1+\frac{H_2+H_3-\beta_2\beta_3 V}{M_B+m_1} + \left[\frac{H_2+H_3-\beta_2\beta_3 V}{M_B+m_1}\right]^2 + \cdots\right] \tag{15}$$

with inequalities such as

$$[(\bar{H}_2)^2-\overline{(H_2)^2}]/(M_B+m_1)^2 \approx [(\overline{|\mathbf{p}_2|})^2-\overline{\mathbf{p}_2^2}]/M_B^2 \ll 1\ ,$$

etc., we may justify the approximate equality $\overline{(1/2m_1^B)} \approx 1/2m_1^B$, which has been used in the calculations for quark magnetic moments. We then expect our approximations to be generally good for confined quarks with relativistic motion.

In conclusion, within a rather general framework for the quarks relativistically confined in a baryon and probed in an external magnetic field, we find an approximate expression for their effective masses and magnetic moments, i.e., $m_1^B \approx M_B - m_2' - m_3'$ and $\mu_1 \approx e_1/2m_1^B$ for the quark q_1, where M_B is the baryon mass without hyperfine splittings and where $m_i' \approx \bar{H}_i \equiv \overline{\boldsymbol{\alpha}_i\cdot\mathbf{p}_i + \beta_i m_i}$ are expected to be approximately independent of baryon species and may be regarded as the constituent quark mass parameters. With this expression we may justify the SU(6) quark model formulation for relativistically confined quarks and obtained a fairly good fit for the baryon magnetic moments.

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[1]G. Morpurgo, Ann. Phys. (N.Y.) **2**, 95 (1965); W. Thirring, Acta Phys. Austriaca Suppl. **2**, 205 (1965); J. Franklin, Phys. Rev. **172**, 1807 (1968).

[2]Particle Data Group, G. P. Yost *et al.*, Phys. Lett. B **204**, 1 (1988), and references therein.

[3]For a recent review, see J. Franklin, in *High Energy Spin Physics,* proceedings of the Eighth International Symposium, edited by K. J. Heller (AIP Conf. Proc. No. 187) (AIP, New York, 1989), p. 384; see also J. L. Rosner, in *High Energy Physics—1980,* proceedings of XXth International Conference, Madison, Wisconsin, edited by L. Durand and L. Pondrom (AIP Conf. Proc. No. 68) (AIP, New York, 1981), p. 540; N. Isgur and G. Karl, Phys. Rev. D **21**, 3175 (1980); H. J. Lipkin, Phys. Lett. **89B**, 358 (1980); Phys. Rev. D **24**, 1437 (1981); Y. Tomozawa, *ibid.* **25**, 795 (1982); M. Bohm *et al.*, *ibid.* **25**, 223 (1982); M. Gupta and N. Kaur, *ibid.* **28**, 534 (1983); D. B. Lichtenberg *et al.*, Z. Phys. C **17**, 57 (1983); G. E. Brown, M. Rho, and V. Vento, Phys. Lett. **97B**, 423 (1980); J. Franklin, Phys. Rev. D **29**, 2648 (1984); **30**, 1542 (1984).

[4]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D **12**, 147 (1975).

[5]I. S. Sogami and N. Oh'yamaguchi, Phys. Rev. Lett. **54**, 2295 (1985).

[6]T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D **12**, 2060 (1975).