Local charge nonequilibrium and anomalous energy dependence of normalized moments in narrow rapidity windows

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From the study of even and odd multiplicity distributions for hadron-hadron collision in different rapidity windows, we propose a simple picture for charge correlation with nonzero correlation length and calculate the multiplicity distributions and the normalized moments in different rapidity windows at different energies. The results explain the experimentally observed coincidence and separation of even and odd distributions and also the anomalous energy dependence of normalized moments in narrow rapidity windows. The reason for the separation of even-odd distributions, appearing first at large multiplicities, is shown to be energy conservation. The special role of no-particle events in narrow rapidity windows is pointed out.

I. INTRODUCTION

With the increase of energy in proton-antiproton collision and the improvement of detectors and data-analyzing methods, one can study multiparticle production in a more precise way. The study of charged-multiplicity distributions has been developed from the study of distributions in the full rapidity region to that in different rapidity windows.[1] Very recently, most interest is turned to the properties of charged multiplicity in very narrow rapidity windows.[2-5] There is evidence to believe that the study of charged multiplicity in different rapidity windows, especially in very narrow windows, is very important in the study of multiparticle production.

Recently, two interesting phenomena on charged multiplicity in different rapidity windows in hadron-hadron collisions have been announced.[6,7] One is the anomalous energy dependence of normalized moments of charged multiplicity in narrow rapidity windows. It falls sharply down with the increase of energy (cf. Fig. 2) instead of rising monotonically as in the usual case.[7] This impressive phenomenon is a challenge for phenomenological models. The other one is, at c.m. energy $\sqrt{s}=22$ GeV for intervals $|y|\le y_w$ with $y_w=0.25-2.0$, even and odd multiplicities follow the same distribution. From $y_w=2.5$ onwards, the difference between odd and even multiplicity distributions starts to be visible. The difference appears first at the large-n tail of distribution and grows rapidly with increasing y_w. In the limit of full phase space, only even multiplicities survive. The disappearance of odd multiplicity in the full rapidity region is a well-known fact of charge conservation. But, why is there the same distribution for even and odd multiplicities in narrow rapidity windows? Why does the even-odd difference start to be visible at $y_w=2.5$ for $\sqrt{s}=22$ GeV? And why does it first appear at large-n events? The aim of this paper is to answer these interesting questions, as well as to explain the anomalous energy dependence of normalized multiplicity moments in narrow rapidity windows. We will use as simple a picture as possible, in order to get some idea on the physical origin of these phenomena.

The fact that there are both even and odd charged multiplicities in small rapidity windows tells us that local charge equilibrium in rapidity space breaks down and that the correlation length between positively and negatively charged particles is nonzero. The existence of short-range two-particle correlation in rapidity space has been observed experimentally already in the early 1970s (Ref. 8). This phenomenon can be well explained by the cluster model.[9] It is generally assumed that the main decay mode of the cluster is decaying into two charged particles[10] having a particular rapidity distribution in the center-of-mass frame of the cluster. Using this assumption, the short-range rapidity correlation consistent with the experimental data can be obtained,[11] and at the same time, after supplementing with some other model assumptions, the multiplicity distributions of final-state particles can also be explained.[12]

However, up to now, the rapidity distribution of cluster decay has been taken seriously mainly in connection with short-range correlations. In discussing the multiplicity distributions, both in total phase space and in different rapidity windows,[12] one usually uses an approximation of neglecting the cluster decay width in comparison with the window size (δ-function approximation[13]), and considering the two particles produced by one cluster to lie either both inside or outside the considered window. This approximation corresponds to the assumption of charge equilibrium in the rapidity window under consideration. The limit of applicability of such an assumption is unclear.

In this paper, we will make use of the idea of the cluster model, assuming the following. (1) The clusters are neutral. Every cluster decays into a positively and a negatively charged particle. (2) In the c.m. frame of the cluster, the probability distribution of the two charged particles is isotropic. We will avoid the use of the δ-function approximation, and consider the influence of the finite decay width of a cluster—the nonvanishing correlation length of charge—on the multiplicity distributions in all the rapidity windows in detail.

We will see that, provided the considered rapidity window is not very close to the total rapidity range, it is improper to assume that the pair of particles, produced from the decay of one cluster, both either fall into the window or lie out of the window. It is also necessary to take into account the possibility that only one of them falls into the window. This turns out to be the physical reason of the experimentally observed even-odd multiplicity distributions and the anomalous energy dependence of normalized moments in narrow rapidity windows.

In Sec. II the cluster model we use is described and the even and odd multiplicity distributions are calculated. The connection between the separation of even-odd distributions, appearing first at large multiplicities, and the restriction of energy conservation is discussed. In Sec. III the normalized multiplicity moments in different rapidity windows are given and compared with the experimental data. The special role of no-particle events in the appearance of anomalous behavior in narrow rapidity windows is pointed out. A short conclusion is presented in Sec. IV.

II. EVEN AND ODD MULTIPLICITY DISTRIBUTIONS IN DIFFERENT RAPIDITY WINDOWS

Consider a pair of correlated particles—a "cluster." Assuming that the momentum distribution of the two particles at their c.m. frame is isotropic, we can write the distribution of rapidity correlation length as[14]

$$P_g(y_g)=\frac{1}{\cosh^2(y_g)}\ , \tag{1}$$

where y_g is one-half of the rapidity distance between the two particles.

We can draw some inspiration on the rapidity distributions of cluster centers from the experimentally observed rapidity distributions of final-state particles. It has been observed experimentally[15] that the semi-inclusive rapidity distribution depends on multiplicity. There is a peak at the large rapidity value for the rapidity distribution with low multiplicity. This peak moves toward the small rapidity region as the multiplicity increases. This phenomenon is a natural consequence of the restriction of energy conservation.[16] It tells us that, in events with different numbers of clusters, the cluster centers have different rapidity distributions. For events with fewer clusters, the cluster centers are distributed more widely, while for events with larger number of cluster, the cluster centers are distributed more narrowly.

As an approximation, let the center of cluster distribute with equal probability within the kinematically allowed region with the density

$$P_c(y_c)=\frac{1}{2(y_{\max}^{(N)}-y_g)}\ , \tag{2}$$

where $y_{\max}^{(N)}$ is the maximum rapidity value for a single particle in an N-pair event, $y_{\max}^{(N)}-y_g$ is that value for the center of cluster. Based on the restriction of energy conservation mentioned above, we make a simple assumption that $y_{\max}^{(N)}$ decreases linearly with the increasing of the number N of particle pairs (clusters),

$$y_{\max}^{(N)}=a-kN\ , \tag{3}$$

where k is a proportional constant, $a-k$ $(=y_{\max})$ is the kinematical limit for final-state particles at the considered energy.

Since the correlation length is nonzero, for a certain observation window $|y|\le y_w$, it is unnecessary that a pair of correlated particles both either fall into the window or lie out of the window. It is possible also that only one of them falls into the window. Let q_2, q_1, and q_0 denote the probability of a correlated pair to have two, one, and no particles falling into the window, respectively, $q_2+q_1+q_0=1$. (For the calculation of q_2, q_1, and q_0 see the Appendix.) For an event with N clusters, if N_d of them have two particles falling into the window, N_s and N_z of them have one and no particle falling into the window, respectively, the probability of such an event is taken to be a trinomial distribution. Thus, we can write the conditional probability for an N-pair event to have $n_{\mathrm{ch},w}$ charged particles falling into the window as

$$\begin{aligned}P(n_{\mathrm{ch},w}|N)&\sum_{N_dN_sN_z}\frac{A_N}{N_d!N_s!N_z!}q_2^{N_d}q_1^{N_s}q_0^{N_z}\\&\times\delta(N-N_d-N_s-N_z)\\&\times\delta(n_{\mathrm{ch},w}-N_s-2N_d)\ ,\end{aligned} \tag{4}$$

where A_N is a constant, depending only on N, and is determined by the normalization condition

$$\sum_{n_{\mathrm{ch}},w}P(n_{\mathrm{ch},w}|N)=1\ .$$

Summing Eq. (4) for different number N of pairs, we get the probability for an arbitrary event to have $n_{\mathrm{ch},w}$ charged particles falling into the window

$$P(n_{\mathrm{ch},w})=\sum_N P_{\mathrm{pair}}(N)P(n_{\mathrm{ch},w}|N)\ , \tag{5}$$

where $P_{\mathrm{pair}}(N)$ is the probability for producing N pairs. It can be input from experimental data or from some phenomenological formula which has good fit with the data. For convenience, we adopted the parametrization of the three-fireball model[17]

$$\begin{aligned}P_{\mathrm{pair}}(N)=&\sum_{N_CN_PN_T}\frac{64N_CN_PN_T}{\alpha^2(1-\alpha)^4\langle N\rangle^6}\\&\times\exp\left[\frac{-2}{\langle N\rangle}\left[\frac{N_C}{\alpha}+\frac{2(N_P+N_T)}{1-\alpha}\right]\right]\\&\times\delta(N-N_C-N_P-N_T)\ .\end{aligned} \tag{6}$$

Equation (6) fits well with experimental data up to $\sqrt{s}=900$ GeV. For the value of parameter α see Ref. 18.

From Eqs. (4)–(6), we calculated the multiplicity distribution in the windows $y_w=2.0,2.5,3.0$ and in full phase space at $\sqrt{s}=22$ GeV. The results are shown in Fig. 1. The values of $y_{\max}^{(N)}$ and $y_{\max}$ (full phase space) used in the calculation are listed in Table I (Ref. 19).

From Fig. 1, we can see that even and odd multiplicities, following the same distribution up to $y_w=2.0$, start to part for $y_w=2.5$ at large-n events. When the window

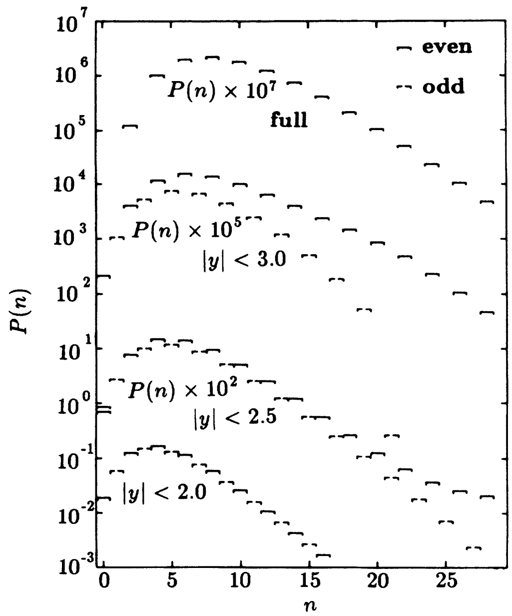

FIG. 1. The calculated odd and even multiplicity distributions in different rapidity windows at $\sqrt{s} = 22$ GeV.

is widened to $y_w = 3.0$, the difference becomes more evident, and finally, at the full rapidity region, there are only even multiplicities left. Thus, we have given a good explanation for multiplicity distributions in different rapidity windows, assuming that there are nonzero correlation lengths between positive and negative particles, instead of local charge equilibrium.

It is the effect of energy conservation that even and odd distributions part first at large-n events. Because of energy conservation, the average rapidity region of charged particles is narrow for large-n events and wide for small-n events. For large-n events, when the width of the observation window is near to that of the average rapidity region, the difference between even and odd multiplicity distributions becomes visible. On the other hand, for small-n events, the average rapidity region may still be much wider than the window, so the even and odd multiplicities follow the same distribution. If we ignored the influence of energy conservation on the rapidity distribution, the split of even-odd multiplicity distributions would start simultaneously for all n events, not first for large-n events.

TABLE I. Kinematically allowed rapidity regions for different number of particle pairs.

$\sqrt{s}$ (GeV)	$y_{\max}^{(N)}$	$y_{\max}$
22	$4.316-0.120N$	4.196
200	$6.348-0.0743N$	6.274
540	$7.259-0.0633N$	7.196
900	$7.740-0.0563N$	7.684

III. THE ANOMALOUS BEHAVIOR OF NORMALIZED MOMENTS IN DIFFERENT RAPIDITY WINDOWS AND ITS ELIMINATION

In Fig. 2 are shown the normalized moments in the windows $y_w = 0.25, 0.5, 1.5, 2.5$ versus c.m. energy. The curves in the figures are obtained using the above-mentioned formulas. It recovers both the general tendency of the slow rise of normalized moments with the increase of energy and the sharp fall of thee moments in small rapidity windows at low energies.

The main reason that normalized moments rise anomalously at low energies in small rapidity windows is that the average multiplicities $\langle n_{\mathrm{ch},w} \rangle$'s in these windows are less than unity. According to definition, the normalized moment is $C_{iw} = \langle n^i_{\mathrm{ch},w} \rangle / \langle n_{\mathrm{ch},w} \rangle^i$. From Table II one can see that, if we use the *moments* $\langle n^i \rangle$ instead of *normalized moments,* there will not be any anomaly in small windows.

As we know, the advantage of using normalized moment C_{iw} instead of moment $\langle n^i \rangle$ (Ref. 20) is that, in the usual case, when the average multiplicity is larger than unity, the normalized moment increases slowly with the increasing of c.m. energy, while moment $M_i = \langle n^i \rangle$ itself increases abnormally, almost in direct proportion to the ith power of average multiplicity $\langle n \rangle$. It is why one usually regards the normalized moment as a better characteristic quantity for multiplicity distribution. However, this is no longer true when the average multiplicity $\langle n \rangle$ becomes very small, especially when it is less than unity. In order to show clearly what happens, consider a simple inequality

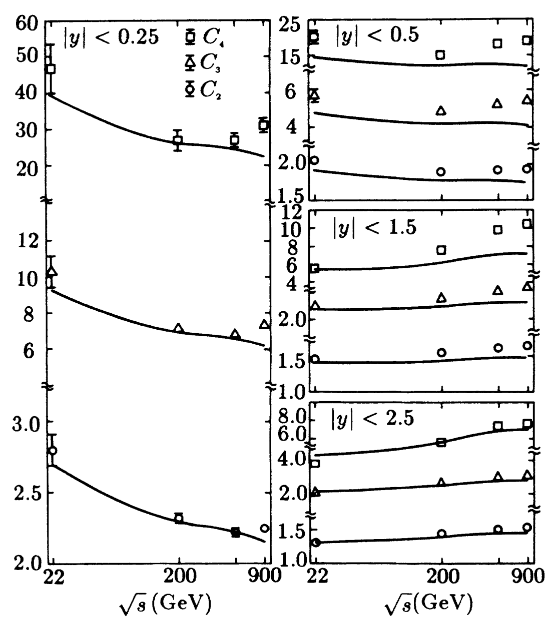

FIG. 2. The normalized multiplicity moments in different rapidity windows. The curves are results of our calculation. Data are taken from Ref. 6.

TABLE II. Multiplicity moments $M_i = \langle n^i \rangle$ in two rapidity windows.

y_w / $\sqrt{s}$ (GeV)	0.25 M_2	0.25 M_3	0.25 M_4	0.5 M_2	0.5 M_3	0.5 M_4
22	1.044	2.271	6.126	2.987	9.409	36.427
200	3.242	11.000	46.500	10.639	58.374	395.044
540	3.868	14.503	68.264	12.916	79.952	616.259
900	3.930	14.924	70.916	13.168	82.723	643.140

$$C_{iw} = \frac{\sum_{n=0}^{\infty} n^i P_w(n)}{\left[\sum_{n=0}^{\infty} n P_w(n)\right]^i} = \frac{\sum_{n=1}^{\infty} n^i P_w(n)}{\left[\sum_{n=1}^{\infty} n P_w(n)\right]^i} \geq \frac{1}{\langle n \rangle^{i-1}} . \tag{7}$$

This inequality gives the lower bound of the normalized moment by the replacement of n^i by n in the numerator. This is, of course, a very crude estimate and is not good enough to explain all the phenomena. However, we can see already from this inequality that, when the average multiplicity $\langle n \rangle$ in window is much lower than unity, the normalized moment C_{iw} $(i \geq 2)$ will take very large values. It is clear from the inequality (7) that the reason for this anomalous growth of normalized moment in the narrow window at low energy is due to the large probability of no particle falling into the window, i.e., large probability for $n=0$. This gives us a hint that, in order to get useful information in narrow rapidity windows, we should give a special consideration for $n=0$ events. Note that the same conclusion is drawn in Ref. 4, starting from a different argument.

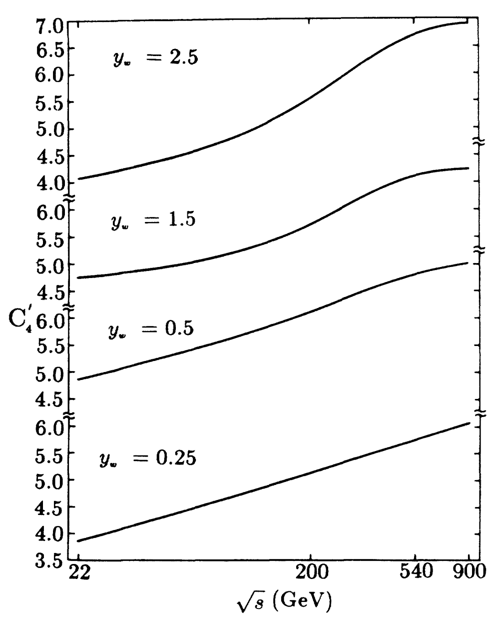

FIG. 3. The renormalized multiplicity moments defined in Eq. (8).

The right way for dealing with the $n=0$ events depends on the nature of the physical problem. Here we give a demonstration by simply omitting the $n=0$ events in multiplicity distribution. Consider $\mathcal{N}$ nondiffractive collision events. Let $\mathcal{N}_0$ denote the number of events in which there is no particle falling into the considered window. After deducting the $n=0$ events, the multiplicity distribution becomes

$$P'_w(n) = \frac{1}{a} P_w(n), \quad a = 1 - \frac{\mathcal{N}_0}{\mathcal{N}} . \tag{8}$$

Using this distribution, we can redefine the normalized moments as

$$C'_{iw} = \frac{\sum_{n=1}^{\infty} n_i P'_w(n)}{\left[\sum_{n=1}^{\infty} n P'_w(n)\right]^i} = a^{i-1} C_{iw} . \tag{9}$$

In Fig. 3 are shown the redefined normalized moments C'_{4w} versus c.m. energies for four rapidity windows. There is no longer any anomaly in small rapidity windows. All the moments change smoothly with energy and window size.

IV. CONCLUSION

In this paper, we proposed a simple physical picture to describe the even-odd multiplicity distributions in different rapidity windows and the energy dependence of its normalized moments. Our main assumption is that, the correlation length of charged particles is nonzero (i.e., no local charge equilibrium in rapidity space), and the probability for a correlated charged-particle pair to have two, one, and no particles falling into the observation window obeys a trinomial distribution. For the convenience of calculation, we have also made use of the isotropic distribution for the momenta of correlated particles in their c.m. systems i.e., Eq. (1), and equal probability distribution for the pair center, i.e., Eq. (2), but the results of calculation do not depend qualitatively on both of them. We have succeeded in explaining both the coincidence and the separation of even-odd multiplicity distributions as well as the anomalous energy dependence of normalized charged-multiplicity moments in narrow windows. Our result shows that there is no strict local charge equilibrium in rapidity space in the final states of high-energy hadron-hadron collisions. It also tells us that, it is necessary to give a special consideration for $n=0$ events in narrow rapidity windows.

TABLE III. $P_{ci}(y_g)$ in different segments of y_g.

$0 < y_w \le \frac{y_{\max}^{(N)}}{3}$

y_g	0 to y_w	y_w to $(y_{\max}^{(N)} - y_w)/2$	$(y_{\max}^{(N)} - y_w)/2$ to $(y_{\max}^{(N)} + y_w)/2$	$(y_{\max}^{(N)} + y_w)/2$ to $y_{\max}^{(N)}$
P_{c2}	$\frac{y_w - y_g}{y_{\max}^{(N)} - y_g}$	0	0	0
P_{c1}	$\frac{2y_g}{y_{\max}^{(N)} - y_g}$	$\frac{2y_w}{y_{\max}^{(N)} - y_g}$	$\frac{y_{\max}^{(N)} - 2y_g + y_w}{y_{\max}^{(N)} - y_g}$	0
P_{c0}	$\frac{y_{\max}^{(N)} - 2y_g - y_w}{y_{\max}^{(N)} - y_g}$	$\frac{y_{\max}^{(N)} - 2y_g - y_w}{y_{\max}^{(N)} - y_g}$	$\frac{y_g - y_w}{y_{\max}^{(N)} - y_g}$	1

$\frac{y_{\max}^{(N)}}{3} \le y_w \le y_{\max}^{(N)}$

y_g	0 to $(y_{\max}^{(N)} - y_w)/2$	$(y_{\max}^{(N)} - y_w)/2$ to y_w	y_w to $(y_{\max}^{(N)} + y_w)/2$	$(y_{\max}^{(N)} + y_w)/2$ to $y_{\max}^{(N)}$
P_{c2}	$\frac{y_w - y_g}{y_{\max}^{(N)} - y_g}$	$\frac{y_w - y_g}{y_{\max}^{(N)} - y_g}$	0	0
P_{c1}	$\frac{2y_g}{y_{\max}^{(N)} - y_g}$	$\frac{y_{\max}^{(N)} - y_w}{y_{\max}^{(N)} - y_g}$	$\frac{y_{\max}^{(N)} - 2y_g + y_w}{y_{\max}^{(N)} - y_g}$	0
P_{c0}	$\frac{y_{\max}^{(N)} - 2y_g - y_w}{y_{\max}^{(N)} - y_g}$	0	$\frac{y_g - y_w}{y_{\max}^{(N)} - y_g}$	1

$y_w \ge y_{\max}^{(N)}$

y_g	0 to $y_{\max}^{(N)}$
P_{c2}	1
P_{c1}	0
P_{c0}	0

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APPENDIX: THE PROBABILITY FOR A CORRELATED PAIR TO HAVE TWO, ONE, AND NO PARTICLES FALLING INTO THE WINDOW

For a correlated particle pair (cluster) with correlation length y_g, let $P_{ci}(y_g)$ $(i=2,1,0)$ denote the probability to have i particles falling into the window. Integrating with respect to y_g, we get the probability q_i for an arbitrary correlated pair to have i particles falling into the window

$$q_i = \int_0^{y_{\max}^{(N)}} P_g(y_g) P_{ci}(y_g) dy_g \ , \tag{A1}$$

where, $P_g(y_g)$ is given by Eq. (1).

According to the assumption, the centers of the correlated pair are distributed with equal probability between $-(y_{\max}^{(N)} - y_g)$ and $(y_{\max}^{(N)} - y_g)$, the probability density being $1/2(y_{\max}^{(N)} - y_g)$. So $P_{ci}(y_g)$ is determined by the width of the rapidity region for the center of correlated pairs having i particles falling into the window.

First, consider the case of $y_g < y_w$. When the center of the correlated pair is between $\pm(y_w - y_g)$, both of the two correlated particles will fall into the window. When the center of the correlated pair is between $y_w - y_g$ and $y_w + y_g$ or $-(y_w - y_g)$ and $-(y_w + y_g)$, one of the correlated particles will fall into the window. When the center of the correlated pair is at the left of $-(y_w + y_g)$ or right of $y_w + y_g$, there is no correlated particle falling into the window.

Then, let $y_g > y_w$. In this case, it is impossible for the two correlated particles both falling into the window. When the center of the correlated pair is between $y_g - y_w$ and $y_g + y_w$ or $-(y_g - y_w)$ and $-(y_g + y_w)$, there is only one particle falling into the window. Outside these regions, there is no particle falling into the window.

The integration with respect to y_g in Eq. (A1) is divided into several segments. The boundary points of these segments are y_w and $(y_{\max}^{(N)} \pm y_w)/2$. The cases of y_g being larger or smaller than $(y_{\max}^{(N)} \pm y_w)/2$ correspond to $y_g \mp y_w$ bigger or smaller than $y_{\max}^{(N)} - y_g$. From simple geometrical consideration, we get $P_{ci}(y_g)$ in different regimes of y_g, as shown in Table III. Substituting into Eq. (A1), we get q_i.

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[19]The values of $y^N_{\max}$ are given by Eq. (3). The kinematical limit $a-k=y_{\max}=\ln(\sqrt{s}/m_\perp)$ gives a natural condition for the determination of the two constants a and k. In order to get another condition, we make use of the experimental hint that even and odd multiplicity distributions start to part at large multiplicity tail $[n\approx 4\langle n\rangle]$ for a rapidity window approximately half of the width of total rapidity range (cf. Fig. 1). This means that $y^{(N)}_{\max}\approx y_{\max}/2$ for $n=4\langle n\rangle$. Note that this condition is consistent with the experimentally observed rapidity distributions in different multiplicity intervals, see Ref. 15. With these two conditions in hand we get the values of a and k in Eq. (3).

[20]For simplicity, in the following we omitted the subscripts ch,w.