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Statistical and dynamical aspects of hadronic clusters in high-energy collisions: Tests and conjectures

Chao Wei-qin,* Gao Chong-shou,† Meng Ta-chung, and Pan Ji-cai‡
Institut für Theoretische Physik der Freien Universität Berlin, Berlin, West Germany
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Based on the formalism proposed in a previous paper, simple methods are suggested to test the content of hadronic clusters. The usefulness and the limitation of such methods are also discussed.

I. INTRODUCTION

This is the second of a series of two papers in which we discuss production and decay of hadronic clusters in high-energy collisions. In the first part[1] we introduced a few new concepts and proposed a statistical method, with the help of which useful information on cluster properties can be extracted from multiplicity distributions and multiplicity correlations of hadrons observed in different rapidity windows.

In this part, we discuss further applications of this statistical-based formalism.[1] Model-independent methods are suggested to test the hadronic content of the clusters.[2] It is also pointed out that, while many of the cluster properties can be tested by these statistical methods (in analyzing the experimentally observed multiparticle final states), it is difficult to obtain reliable information on the process of cluster formation. A possible scenario is discussed to describe the dynamics of the formation process.

II. MODEL-INDEPENDENT TESTS

We show in this section that the following methods are helpful in testing the hadronic content of the clusters.

(a) Measurements of multiplicity distributions of negatively charged hadrons in different rapidity intervals.

Experimental data[3] and many theoretical models[4] suggest that most of the clusters produced in high-energy hadron-hadron collisions are charge neutral. They decay into two, four, or six charged hadrons, where clusters which decay via known resonances are also taken into account. If this picture is correct, there should not only exist short-range correlations between positively and negatively charged hardrons, but also this kind of correlation between two positively or two negatively charged ones. Hence measurements of multiplicity distributions of negatively charged hadrons in different rapidity intervals for different collision processes will be very helpful. In these experiments, we shall be able to see whether such correlations indeed exist. In particular, we shall be able to examine the properties of the four-charged-hadron clusters compared to those of two-charged-hadron clusters (their statistical weight, their rapidity dependence, etc.).

According to Eq. (4) in the first part[1] of this study (hereafter simply called Part I) the probability $P_{W^-}(n_{W^-})$ of observing n_{W^-} negatively charged hadrons inside a given rapidity window W can be written as

$$P_{W^-}(n_{W^-})=\sum_N{}' P(N)\left[N!\Big/\prod_{k=0}^{c/2} N_W(k|N)!\right] \times \prod_{k=0}^{c/2} \beta_W(k)^{N_W(k|N)} . \quad (1)$$

Here, $P(N)$ is the multiplicity distribution of the neutral clusters which decay at most into $c/2$ positively and as many negatively charged hadrons; $N_W(k|N)$ is the number of clusters which contribute k negatively charged hadrons to the rapidity window W in such an N-cluster event. Hence, the number of negatively charged hadrons in W is

$$n_{W^-}=\sum_{k=0}^{c/2} kN_W/(k|N) , \quad (2)$$

while the number of clusters can be written as

$$N=\sum_{k=0}^{c/2} N_W(k|N) . \quad (3)$$

The average probability for any one of the $N_W(k|N)$ clusters to exist is denoted by $\beta_W(k)$, and it is taken to be independent of N. The dash on the summation sign in Eq. (1) indicates that the conditions given in Eqs. (2) and (3) should be satisfied.

By analogy to the moments for n_W discussed in Part I, a set of relations between the moments of n_{W^-} and those of N can be derived from Eq. (1). The first three of them are

$$\langle n_{W^-}\rangle=\langle N\rangle \sum_k^{c/2} k\beta_W(k) , \quad (4)$$

$$\langle n_{W^-}(n_{W^-}-1)\rangle=\langle N(N-1)\rangle \left[\sum_k^{c/2} k\beta_W(k)\right]^2+\langle N\rangle \sum_k^{c/2} k(k-1)\beta_W(k) , \quad (5)$$

$$\langle n_{W^-}(n_{W^-}-1)(n_{W^-}-2)\rangle = \langle N(N-1)(N-2)\rangle \left[\sum_k^{c/2} k\beta_W(k)\right]^3 + 3\langle N(N-1)\rangle \left[\sum_k^{c/2} k\beta_W(k)\right]\left[\sum_k^{c/2} k(k-1)\beta_W(k)\right] + \langle N\rangle \sum_k^{c/2} k(k-1)(k-2)\beta_W(k) . \quad (6)$$

Next, we use a method of successive approximation, similar to that discussed in Part I, to determine $\beta_W(1)$ and $\beta_W(2)$ from the measured multiplicity distributions for negatively charged hadrons inside different rapidity windows. At present, the only available data of this kind are those given by the NA22 Collaboration for π^+p and pp collisions at $p_{\rm lab}=200$ GeV/c (Ref. 5). The corresponding results we obtained are shown in Figs. 1 and 2. (For details, see Ref. 6.)

It should be mentioned in this connection that the average probability $\alpha_W(l)$ for having a cluster which contributes exactly l charged (that is either positively or negatively charged) hadrons can be obtained by the method given in Part I. An attempt has been made to determine $\alpha_W(1)$ and $\alpha_W(2)$ from the corresponding set of π^+p and pp data.[5] No solution has been found under the condition that all high $\alpha_W(l)$'s [that is, $\alpha_W(3)$, $\alpha_W(4)$, etc.] can be neglected. Since the error bars in the presently available data are still too large to allow a reliable estimate of $\alpha_W(3)$ and $\alpha_W(4)$, we decide to postpone this calculation until more precise data are available.

This result, taken together with the fact that $\beta_W(1)$ and $\beta_W(2)$ are significantly different from zero (except for extremely small rapidity windows) which we have seen in Figs. 1 and 2, leads to the following conclusion: In contrast with e^+e^--annihilation processes,[7] the π^+p and pp multiplicity distribution data[5] for different rapidity windows strongly suggest that in such processes there is a considerably large percentage of neutral clusters which decay into *more than two charged hadrons.*

(b) Measurements of multiplicity distributions of charged hadrons in different rapidity intervals under the condition that the total number of charged hadrons in the full rapidity space is fixed.

We recall that the application of the general formula for $P_W(n_W)$ given in Eq. (4) of Part I, as well as that for $P_{W^+}(n_{W^+})$ given in Eq. (1) in this part of the paper are limited by the following condition: The average probabilities $\alpha_W(l)$ and $\beta_W(k)$ are taken to be independent of the cluster multiplicity of the emitting system. This is an approximation which needs to be improved when we wish to extract more precise information about the cluster properties from the experimental data. Such an improvement can be achieved in the following way. We recall that experimental data[7] for multiplicity distributions in different rapidity windows irrespective of the number of charged hadrons observed in the full rapidity space, and measurements[8] of rapidity distributions, *not* confined to given rapidity windows, in different ranges of multiplicities (for example, $n\le 10$, $11\le n\le 20$, $21\le n\le 30, \ldots$) exist for various reactions at different incident energies. In order to learn more about hadronic clusters, it would be helpful to measure multiplicity distributions in different rapidity windows for fixed numbers of charged hadrons observed in the full rapidity space (for example, in the same n intervals as those for rapidity distributions mentioned above); and use this kind of multiplicity distribution data to carry out the analyses mentioned in Part I as well as that discussed in (a) of this section.

The results of such analyses are expected to yield more accurate information about the clusters for the following

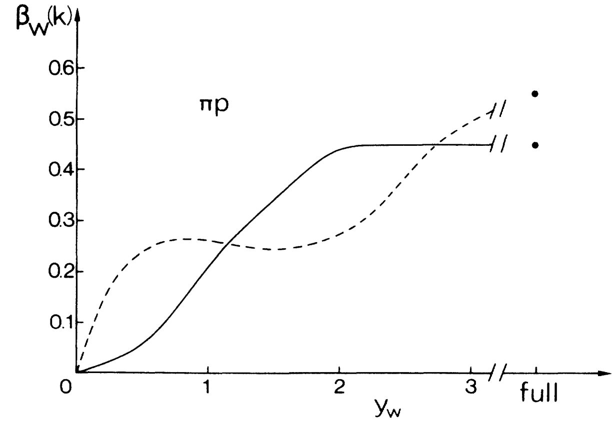

FIG. 1. The probabilities $\beta_W(1)$ (dashed curve) and $\beta_W(2)$ (solid curve) calculated by using the measured (Ref. 5) multiplicity distributions for negatively charged hadrons in different rapidity windows for π^+p collisions at $p_{\rm lab}=200$ GeV/c. The input is $\beta_{W\max}(1)=0.55$, $\beta_{W\max}(2)=0.45$.

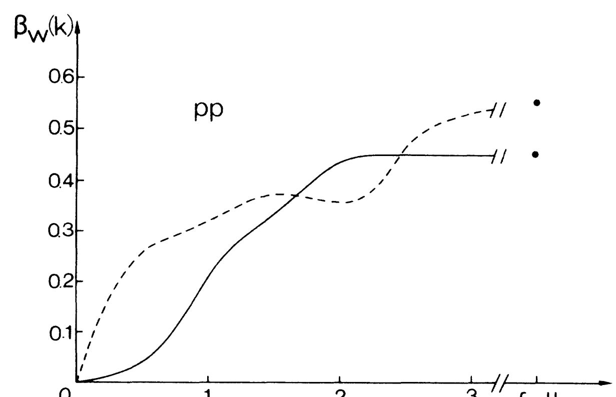

FIG. 2. The same as Fig. 1 for pp collisions at the same energy (Ref. 5).

reasons. First, it has been observed a long time ago that high-multiplicity events are associated with events in which the observed charged hadrons are more concentrated in the central rapidity region, while low-multiplicity events are those in which the observed charged hardons are located in the fragmentation regions. Hence, by studying multiplicity distributions in different multiplicity ranges one can obtain information on multiplicity distributions for different kinds of events. That is, by analyzing the data for different groups of events one may in fact be investigating the clusters produced by different emitting systems. Second, this kind of multiplicity distributions is very helpful in finding out the cluster composition (that is, the percentage of each possible kind of clusters). In order to demonstrate this point, we consider the following idealized case. We show, for a given rapidity distribution $\rho_n(y) = (1/\sigma_n)(d\sigma_n/dy)$ for a fixed number (n) of negatively charged hadrons in the full rapidity space, such as the distribution shown in Fig. 3, different cluster composition yields different results for $\beta_W(1)$ and $\beta_W(2)$. In Figs. 4(a)–4(d), we show the results obtained under the assumptions that there are 0%, 30%, 70%, and 100% two-negative-charged-hadron clusters, respectively, while the rest are assumed to be single-negative-charged-hadron clusters. We note that this example shows in particular that by using the statistical method discussed in this paper the cluster composition can be determined, provided that multiplicity distribution for charged and/or negatively charged hadrons in different rapidity windows, under the above-mentioned condition, are measured.

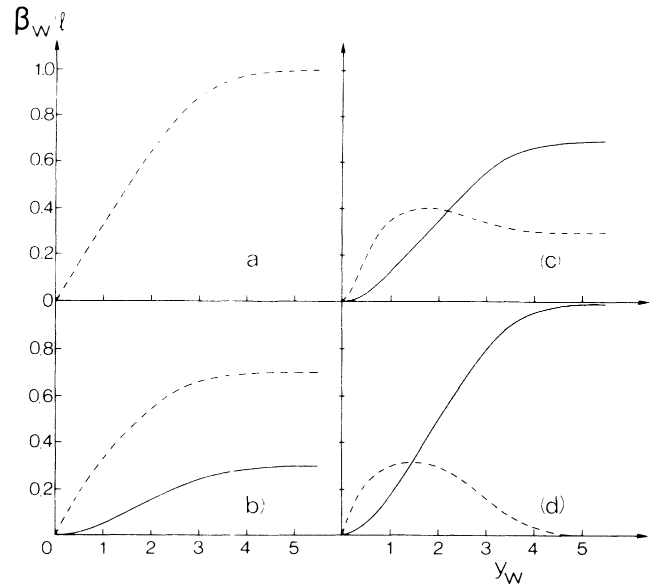

FIG. 4. Calculated $\beta_W(1)$ (dashed curve) and $\beta_W(2)$ (solid curve) for different cluster compositions. (For details, see text.)

III. DYNAMICAL CONJECTURES

It has been known already for a long time that the existing experimental data[3] on short-range rapidity correlations, rapidity gap distributions, multiplicity distributions, forward-backward multiplicity correlations are consistent with the following "cluster picture." The observed hadrons in the central rapidity region of hadron-induced reactions are *decay products of small hadronic clusters.* Such clusters are charge neutral and they decay on the average into 2.2–2.6 hadrons; these properties depend neither on the incident energy nor on the quantum numbers of the incident particles. While some of the above-mentioned properties (for example, charge and size of the clusters) can be tested with the methods discussed in Refs. 1,2, and/or those in the present paper, these statistical methods are less helpful in answering questions concerning the formation stage of the clusters: in particular, "Why do clusters exist in high-energy hadron-induced reaction?" "Why are they charge neutral?" Why are they so small?" "Why are the properties of such clusters independent of the incident energy, and independent of the quantum numbers of the incident particles?"

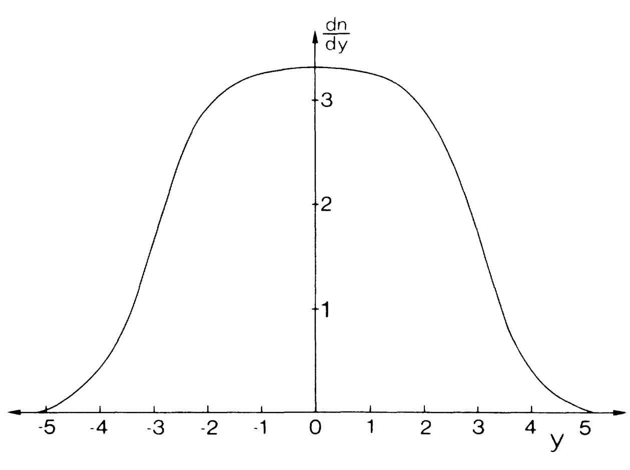

FIG. 3. $\rho_n(y) = (1/\sigma_n)(d\sigma_n/dy)$ for a fixed number (n) of negatively charged hadrons in the full rapidity space.

In this section we discuss a possible scenario for the production and decay of clusters.[9] This scenario is based on a set of dynamical conjectures. The purpose of this attempt is to offer a simple and natural answer to the above-mentioned questions. The basic assumption is the following. The hadronic clusters are nothing else but color-singlet gluon clusters.[10] Such gluon clusters are in general extremely short lived. That is, they are metastable objects, the decay widths of which are in general much broader than those of ordinary resonances. There are mainly two types of them, the C_ϵ type and the C_ω type, and each type is characterized by its specific decay mode. Type-C_ϵ clusters decay into two mesons while type-C_ω clusters decay either into baryon-antibaryon pairs or into three mesons.

It is the following facts and open questions which have led us to this assumption.

(i) Deep-inelastic lepton-proton collision experiments[11] indicate that the gluons in a fast moving proton carries about half of the total momentum. What kind of role do the gluons in hadrons play in high-energy *hadron-hadron* collisions? *How* do the gluons in the colliding hadrons interact with one another, especially in ordinary soft/gentle process?

(ii) A major difference between photons in QED and gluons in QCD is that the gluons are not only responsible for the confining (color) force, but they also carry themselves (color) quanta. If this is indeed the case, why is it so *difficult* to find in nature color-singlet objects consisting of *gluons only*? In particular, if the colliding hadrons are indeed made out of few valence quarks and a large number of gluons (which carry on the average approximately as much momentum as do the valence quarks), why do we not see many such objects in high-energy hadron-hadron collision processes?

(iii) Elementary perturbative QCD calculations show that,[12] especially at large scattering angles (that is, in the central rapidity region), the contribution to hadron-hadron cross sections from gluon-gluon subprocesses is much larger than from gluon-quark and that from quark-quark subprocesses. Nonperturbative considerations[13] also show significant domination of gluon-induced effects. What are the reasons which *force* us to *believe* that gluon-gluon processes do *not* give a significant contribution to the central rapidity region in high-energy hadron-hadron collisions?

(iv) Monte Carlo lattice calculations[14] show that there can be confinement in gluon plasma without quarks. Such calculations also indicate the existence of glueballs.[15]

(v) Based on the experimental fact mentioned in (i), a quark-gluon picture for nondiffractive hadron-hadron collisions was proposed many years ago by Pokorski and Van Hove.[16] In their picture, the valence quarks of one hadron penetrate the other hadron and become the constituents of the corresponding leading particles, while the remaining energy-momentum is arrested in some collision volume by virtue of the much stronger interactions between the gluons.

(vi) The quark-gluon picture of Pokorski and Van Hove[16] is very useful in describing gentle (soft) hadron-hadron collision processes. For example, Pokorski, Van Hove, and their collaborators[16] have used this picture to describe particle production in fragmentation regions. Carruthers and Ming Doung-Van[17] and Shuryak[13] have used this picture in connection with the Fermi-Landau model to calculate the energy dependence of the average multiplicity and the gluon mean free path in excited quark-gluon matter. It has been shown[18] that the statistical model proposed by the Berlin group to describe multiplicity distributions, rapidity distributions, long-range multiplicity correlations, and short-range rapidity correlation in such processes, can also be intepreted in terms of this quark-gluon picture.

The above-mentioned facts and questions strongly suggest that gluon-gluon interaction should play a dominating role in high-energy nondiffractive hadron-hadron collisions.

Now, we recall that in QCD, gluons are color octets; and when two gluons interact with each other, the only two-gluon states in which the gluons attract each other are color singlet and color octet. Hence, it seems natural to assume that the proposed C_ϵ-type gluon clusters are due to the collision of two gluons with exactly the opposite color, and that a C_ϵ-type gluon cluster is due to the collision of a gluon and a member of a two-gluon color octet C_8 mentioned above. The formation and the decay of such gluon clusters are illustrated in Fig. 5. Here, it is envisaged that once a color-singlet gluon cluster is formed, it remains a color singlet when it interacts with other color objects. That is, the formation and decay of C_ϵ- and C_ω-type gluon clusters are assumed to be irreversible processes. We note that it follows (for details see the Appendix) from this assumption and the well-known properties of interactions between different color multiplets in the QCD framework that the chance for color-singlet gluon clusters larger than those of type C_ω to exist is neglegibly small. That is to say, in a sufficiently good approximation, the color-singlet gluon clusters produced in the central rapidity region of high-energy nondiffractive hadron-hadron collisions are either of type C_ϵ or of type C_ω.

It is clear that the proposed scenario is only an example which can give answers to the questions discussed at the beginning of this section. However, this scenario not only offers a simple, natural explanation for the experimental fact that there is local charge compensation, and that there is a short-range rapidity correlation among the observed hadrons, but it also has a number of other consequences which can be checked experimentally. Some examples are given below.

First, it suggests a simple solution for the puzzle concerning the double role of the gluons in QCD. According to this model, there should be plenty of gluon clusters in hadron-induced multiparticle production processes. While the decay widths of these metastable objects are in general broader than those of ordinary resonances, some of the gluon-clusters may have widths which are sufficiently narrow to be identified as resonances with reasonable lifetime. It is extremely interesting to see that a large number of such objects have indeed been observed:[19,20] for example, the new 0^{++} resonance observed by Tanimori *et al.*, the three 0^{++} resonances found by Au *et al.*, etc. In fact, this seems to give a simple and natural explanation for "the chaotic situation"[20] that there are too many candidates for the glueballs.

Second, the proposed model can be used to obtain a new lower limit for the meson-hadron collisions. To be more precise, it follows from SU(3)-color symmetry and

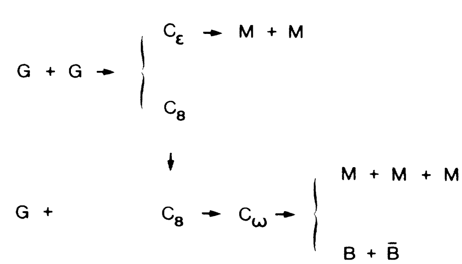

FIG. 5. Formation and decay of gluon clusters. Here, G, M, B, and $\bar{B}$ are gluon, meson, baryon, and antibaryon, respectively, while C_8 stands for color octet. (This is the only color multiplet that can be formed; see text for further details.) C_ϵ and C_ω are the color-singlet gluon clusters which decay into hadrons. Note that all processes are irreversible.

the decay of C_ϵ and C_ω that the meson-baryon ration M/B can be written as (the proof is given in the Appendix)

$$\frac{M}{B}=\frac{10}{3}r+2\ , \tag{7}$$

where r is the average production rate for C_ϵ-type to that for C_ω-type clusters. By taking into account the color symmetry and by considering gluons instead of quark pairs in a combination calculation analog to that of Anisovich and Shekhter[21] we obtain, from Eq. (7) (see the Appendix for details),

$$\frac{M}{B}=12.3\ . \tag{8}$$

We note that this lower limit is more than twice as large as that given in the literature.[21] It is consistent with the data[22] at presently available energies and can be readily checked in future (Fermilab Tevatron, Superconducting Super Collider, etc.) experiments.

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APPENDIX

We recall that the Hamiltonian density which describes the color interaction between the quarks and gluons can be written as

$$H_{\rm int}\sim\sum_{j=k}\mathbf{F}_j\cdot\mathbf{F}_k\ , \tag{A1}$$

where the $\mathbf{F}_j$'s are the generators of the color SU(3). This form is based on an analogy in nuclear physics concerning the isospin states in two-nucleon systems, and it is valid in the ladder approximation. Since the gluons are hypothesized to exist in color octet (**8**), the interaction energy $E_{\rm int}$ between two gluons (denoted by 1 and 2) is the expectation value of

$$H_{\rm int}=V_0\sum_{j=1}^{8}\mathbf{F}_j(1)\cdot\mathbf{F}_j(2)\ , \tag{A2}$$

where V_0 is a positive constant. Equation (A2) can be written as

$$H_{\rm int}=\frac{V_0}{2}[C(1+2)-C(1)-C(2)]\ , \tag{A3}$$

where

$$C(\alpha)=\sum_{j=1}^{8}[F_j(\alpha)]^2,\quad \alpha=1,2\ . \tag{A4}$$

is the Casimir operator of the αth gluon, and

$$C(1+2)=\sum_{j=1}^{8}[F_j(1)+F_j(2)]^2 \tag{A5}$$

is the Casimir operator of the compound system.

Two-gluon systems are associated with the SU(3) representation $\mathbf{8}\times\mathbf{8}$ and hence belong to one of the irreducible representations $\mathbf{1}+\mathbf{8}+\mathbf{8}+\mathbf{8}+\mathbf{10}+\mathbf{10}^*+\mathbf{27}$. It can be easily seen, by inserting the corresponding eigenvalues of the Casimir operator on the right-hand side of Eq. (A3), that attractive interactions occur only in the singlet (**1**) and in the octet (**8**). Hence, two-gluon systems with relatively stable binding are only possible in **1** and in **8**. These are the C_ϵ and the C_ω gluon clusters, respectively. We note that the C_ϵ clusters are more stable than the C_8 clusters. This is not only because the color singlets are color neutral (like a hadron), but also because the energy level of C_ϵ is indeed lower than that of C_8.

We now consider the interaction between a C_8 cluster and a gluon. Since both C_8 clusters and gluons are members of SU(3) color octets, a discussion similar to that given above leads us to the conclusion that compound systems formed by the C_8 cluster and a gluon can only have relatively stable bindings when the three-gluon system belongs to the singlet (**1**) or the octet (**8**). These are called the C_ω and the C_8' clusters. For reasons similar to those in the C_ϵ and C_8 cases, C_ω clusters are more stable than C_8' clusters.

Furthermore, since the interactions between three gluons (denoted by 1, 2, and 3) are given by

$$H_{\rm int}=V_0\left[\sum_{i=1}^{8}F_i(1)F_i(2)+\sum_{j=1}^{8}F_j(2)F_j(3)+\sum_{k=1}^{8}F_k(3)F_k(1)\right], \tag{A6}$$

we have

$$H_{\rm int}=\frac{V_0}{2}[C(1+2+3)-C(1)-C(2)-C(3)]\ . \tag{A7}$$

Here we note that, because of the fact that $C(1)=C(2)=C(3)$, the interaction is determined by the multiplet associated by the compound system only. That is, when the compound system is a given multiplet (an octet, say), the energy level is fixed—independent of the possible states of any two of the three gluons in the system. In other words, the energy level of a C_8' cluster (formed by a C_8 cluster and a gluon) is the same as that of a "compound" built by a C_ϵ and a gluon. Now, since the latter "compound" is simply a color-singlet object and an extra gluon which may readily leave the "compound," the C_8' clusters are also unstable, and may also easily decay into a C_ϵ clusters and a gluon through slight perturbations. This is why C_ϵ and C_ω gluon clusters are the ones which manifest themselves in multiparticle production processes, while clusters such as C'_8 and larger ones are negligible in practice.

The decay modes of the C_ϵ and C_ω clusters are

$$C_\epsilon \to M+M \ , \qquad \text{(A8)}$$

$$C_\omega \to \begin{cases} M+M+M \ , \\ B+\bar{B} \ . \end{cases} \qquad \text{(A9)}$$

The SU(3)-color symmetry, together with the fact that the irreducible representations of $3\times 3^*$ and that of $3\times 3\times 3$ are $1+8$ and $1+8+8+10$, respectively, requires that the color weight for each meson is $\frac{1}{9}$, and that for each baryon is $\frac{1}{27}$. Since there are two possible ways to form a meson by counting quarks and antiquarks originating from different gluons, the chance of having three mesons to that of having a $B\bar{B}$ pair from C_ω is $(\frac{1}{9})^2\times 2{:}\frac{1}{27}$ which is 2:3; thus the corresponding normalized color weights are $\frac{2}{5}$ and $\frac{3}{5}$.

Let us denote the average production rate of C_ϵ and that of C_ω gluon clusters by C_ϵ and C_ω, and the average rate for mesons and that for baryons by M and B, respectively. Then, we have

$$M=2C_\epsilon+3\times\frac{2}{5}C_\omega \ , \qquad \text{(A10)}$$

$$B=\frac{3}{5}C_\omega \ , \qquad \text{(A11)}$$

from which the relationship given in Eq. (1) in the text follows.

While the formation and decay processes of gluon clusters due to gluon-gluon collisions in real life are certainly more complicated, we think the following idealized case may not be too far off, at least in the high-energy limit where the interacting gluons are moving fast and the number of such gluons are large ($\gg 1$). In this case we assume that the interaction time is so short that every gluon can only take part in one collision *actively*. That is, during this period every gluon that enters the interaction region can only hit once, either on another gluon or to an N-gluon system in which the probability for finding a gluon in single gluon, a C_ϵ, a C_ω, or a C_8 state is $\alpha_G(N)$, $\alpha_\epsilon(n)$, $\alpha_\omega(n)$, or $\alpha_8(N)$, respectively. According to the formation rules mentioned in the text, we have

$$(N+1)\alpha_G(N+1)=N\alpha_G(N)+\frac{15}{32}\alpha_G(N)+\frac{61}{64}\alpha_8(N)+\alpha_\epsilon(N)+\alpha_\omega(N) \ , \qquad \text{(A12)}$$

$$(N+1)\alpha_8(N+1)=N\alpha_8(N)+\frac{1}{2}\alpha_G(n)-\frac{64+63}{64\times 32}\alpha_8(N) \ , \qquad \text{(A13)}$$

$$(N+1)\alpha_\epsilon(N+1)=N\alpha_\epsilon(N)+\frac{1}{32}\alpha_G(N)+\frac{63}{64\times 32}\alpha_8(N) \ , \qquad \text{(A14)}$$

$$(N+1)\alpha_\omega(N+1)=N\alpha_\omega(N)+\frac{3}{64}\alpha_8(N) \ . \qquad \text{(A15)}$$

Since N is usually a large number, the corresponding probabilities for the $(N+1)$-gluon system $\alpha_i(N+1)$, $(i=G,\epsilon,\omega,8)$ can be considered to be the same as $\alpha_i(N)$. Hence the equations which correspond to Eq. (2) in the paper of Anisovich and Shekhter[21] are

$$\frac{17}{32}\alpha_G=\frac{63}{64}\alpha_8+\alpha_\epsilon+\alpha_\omega \ , \qquad \text{(A16)}$$

$$\left[1+\frac{64+63}{64\times 32}\right]\alpha_8=\frac{1}{2}\alpha_G \ , \qquad \text{(A17)}$$

$$\alpha_\epsilon=\frac{1}{32}\alpha_G+\frac{63}{32\times 64}\alpha_8 \ , \qquad \text{(A18)}$$

$$\alpha_\omega=\frac{3}{64}\alpha_8 \ . \qquad \text{(A19)}$$

These equations give

$$r=\frac{\alpha_\epsilon/2}{\alpha_\omega/3}=3.1 \ . \qquad \text{(A20)}$$

It should be mentioned that another idealized case in which the interaction time is long enough to allow infintely many gluon collisions has also been studied. The corresponding value for r is 3.7. Details on this, and on related problems, will be discussed elsewhere.

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*Now at Institute of High-Energy Physics, Academia Sinica, Beijing, China.

†Now at Peking University, Beijing, China.

‡Permanent address: Hua-Zhong Normal University, Wuhan, China.

[1]Chao Wei-qin, Gao Chong-shou, Meng Ta-chung, and Pan Ji-cai, Phys. Rev. D **36**, 2702 (1987).

[2]The idea of using model-independent methods to test the hadronic content of a cluster has also been dicussed, in a different context, by J. Dias de Deus, Phys. Lett. B **185**, 189 (1987).

[3]See, for example, the experimental papers given in Ref. 1.

[4]See, for example, A. Bialas, in *Proceedings of the IVth International Symposium on Multiparticle Hadron Dynamics,* Pavia, Italy, 1973, edited by F. Duimio *et al.* (Istituto Nazionale di Fisica Nucleare La Gollardica Pavese, Pavia, Italy, 1973), p. 93; L. Fao, Phys. Rep. **22**, 1 (1975); C. Quigg, P. Pirilä, and G. H. Thomas, Phys. Rev. Lett. **34**, 290 (1975); J. Whitmore, Phys. Rep. **27**, 187 (1976); G. Giacomelli and M. Jacob, Phys. Rep. **55**, 1 (1979); K. Böckmann and B. Eckart, in *Multiparticle Dynamics 1984,* proceedings of the XVth International Symposium, Lund, Sweden, 1984, edited by G. Gustafson and C. Peterson (World Scientific, Singapore, 1984); G. Ekspong, in *Multiparticle Dynamcs 1985,* proceeding of the XVI International Symposium, Kiryat Anavim, Israel, 1985, edited by J. Grunhaus (World Scientific, Singapore, 1985); L. K. Mangotra, I. Otterlund, and E. Stenlund, Z. Phys. C **31**, 199 (1986); W. Bell *et al.*, *ibid.* **32**, 335 (1986); L. van Hove and A. Giovannini, *ibid.* **30**, 391 (1986); C. C. Shih, Phys. Rev. D **34**, 2710 (1986); **34**, 2720 (1986); S. Barshay and E. Eich, Phys. Lett. B **178**, 431 (1986), and the references given therein.

[5]NA22 Collaboration, M. Admus *et al.*, Phys. Lett. B **177**, 239 (1986); W. Kittel and F. Meijers (private communication).

[6]The moments of n_{W^-} and their experimental error bars on the left-hand side of Eqs. (4)–(6), which are expressed as $\sigma(\langle n_{W^-}\rangle)$, $\sigma(\langle n_{W^-}(n_{W^-}-1)\rangle)$, and $\sigma(\langle n_{W^-}(n_{W^-}-1)(n_{W^-}-2)\rangle)$, are calculated from the negative-binomial fits of π^+p and pp data given by the NA22 Collaboration in Ref. 5. The corresponding values on the right-hand side [expressed as $\langle n_{W^-}\rangle_R$, $\langle n_{W^-}(n_{W^-}-1)\rangle_R$, and $\langle n_{W^-}(n_{W^-}-1)(n_{W^-}-2)\rangle_R$] are calculated for different inputs of $\beta_{W\max}(1)$ and $\beta_{W\max}(2)$ [the values of $\beta_W(1)$ and $\beta_W(2)$ for full phase space] using the method of successive approximation described in Part I. The results shows in Figs. 1 and 2 for $\beta_{W\max}(1)=0.55$, $\beta_{W\max}(2)=0.45$ correspond to the minimum value of the quantity.

$$\left[\frac{\langle n_{W^-}\rangle-\langle n_{W^-}\rangle_R}{\sigma(n_{W^-})}\right]^2+\left[\frac{\langle n_{W^-}(n_{W^-}-1)\rangle-\langle n_{W^-}(n_{W^-}-1)\rangle_R}{\sigma(\langle n_{W^-}(n_{W^-}-1)\rangle)}\right]^2+\left[\frac{\langle n_{W^-}(n_{W^-}-1)(n_{W^-}-2)\rangle-\langle n_{W^-}(n_{W^-}-1)(n_{W^-}-2)\rangle_R}{\sigma(\langle n_{W^-}(n_{W^-}-1)(n_{W^-}-2)\rangle)}\right]^2.$$

[7]See the papers by M. Derrick in Ref. 3.

[8]J. G. Rushbrooke, in *Multiparticle Dynamics 1983,* proceedings of the XIVth International Symposium, Lake Tahoe, California, 1983, edited by J. F. Gunion and D. M. Yoger (World Scientific, Singapore, 1984).

[9]An earlier attempt and a part of the result discussed in Sec. II was reported as a contributed paper by Chao Wei-qin, Gao Chong-shou, and Meng Ta-chung; see *Proceedings of the 23rd International Conference on High-Energy Physics,* Berkeley, California, 1986, edited by S. C. Loken (World, Scientific, Singapore, 1986).

[10]The term "gluon cluster" has also been used in connection with hard collisions, using Monte Carlo techniques, by a number of authors. See, for example, G. Marchesini and B. R. Webber, Nucl. Phys **B238**, 1 (1984); B. R. Webber, *ibid.* **B238**, 492 (1984); R. D. Field and S. Wolfram, *ibid.* **B213**, 65 (1983); R. D. Field, E. G. Fox, and R. L. Kelly, Phys. Lett. **119B**, 439 (1982), and papers cited therein.

[11]See, for example, C. H. Llewellyn Smith, Phys. Rep. C **3**, 261 (1974), and papers cited therein.

[12]See, for example, B. L. Combridge, J. Kripfganz, and J. Ranft, Phys. Lett. **70B**, 234 (1977). In order to see the relative importance of the different contributions at large scattering angles, consider the case $t=u=s/2$ in Table I of that paper.

[13]See, for example, E. V. Shuryak, Phys. Rep. **61**, 71 (1980), and the references given therein.

[14]See, for example, M. Creutz, L. Jacobs, and C. Rebbi, Phys. Rep. **95**, 201 (1983), and papers cited therein.

[15]B. Berg and A. Billoire, Phys. Lett. **113B**, 65 (1982); **114B**, 324 (1982); K. Ishikawa, M. Teper, and G. Schierholz, *ibid.* **116B**, 429 (1982).

[16]S. Pokorski and L. Van Hove, Acta Phys. Pol. B **5**, 229 (1974); Nucl. Phys. **B86**, 243 (1975); L. Van Hove, Acta. Phys. Pol. B **7**, 339 (1976), and papers cited therein.

[17]P. Carruthers and Ming Duong-Van, Phys. Rev. D **28**, 130 (1983).

[18]Liu Lian-sou and Meng Ta-chung, Phys. Rev. D **27**, 2640 (1983); Cai Xu, Chao Wei-qin, Huang Chao-shang, and Meng Ta-chung, Phys. Rev. D **33**, 1287 (1986).

[19]See, for example, T. Tanimori *et al.*, Phys. Rev. Lett. **55**, 1835 (1985); K. L. Au, D. Morgan, and M. R. Pennington, Phys. Lett. **167B**, 229 (1986); D. Morgan, in *Strong Interactions and Gauge Theories,* proceedings of the 21st Rencontre de Moriond, Les Arcs, France, 1986, edited by J. Tran Thanh Van (Editions Frontieres, Gif-sur-Yvette, 1986), p. 455, and the papers cited therein.

[20]S. Cooper, in *Proceedings of the 23rd International Conference on High-Energy Physics* (Ref. 9), p. 67.

[21]V. V. Anisovich and V. M. Shekhter, Nucl. Phys. **B55**, 455 (1973); see also V. V. Anisovich, M. N. Kobrinsky, and P. E. Vokovitski, Z. Phys. C **19**, 221 (1983).

[22]UA5 Collaboration, K. Alpgard *et al.*, Phys. Lett. **121B**, 209 (1983), and the papers cited therein.