Single-spin production asymmetries from the hard scattering of pointlike constituents

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When one takes into account the transverse momenta of the constituents in a polarized proton, there exists a kinematic, "trigger-bias," effect in the formulation of the QCD-based hard-scattering model which can lead to single-spin production asymmetries. It seems convenient to represent the coherent spin-orbit forces in a polarized proton by defining an asymmetry in the transverse-momentum distribution of the fundamental constituents. It may then be possible to organize the hard-scattering model so that the kinematic constraints of hard $2\to 2$ scattering provide the leading contribution at large transverse momentum to asymmetries of the type $A_N d\sigma(hp_\uparrow \to \text{jet}+x)$, $A_N d\sigma(hp_\uparrow \to \text{"}\pi\text{"}x)$, where $p_\uparrow$ denotes a transversely polarized proton and "π" represents any spinless meson composed of light quarks. This approach provides testable relationships between different asymmetries.

I. INTRODUCTION

The demonstration of perturbative factorization for quantum chromodynamics[1,2] (QCD) has validated the application of the QCD-based hard-scattering model for calculations involving the production of jets and hadrons at large transverse momentum in high-energy hadron-hadron collisions. The elements of the hard-scattering model include calculable cross sections for the scattering of fundamental constituents and the momentum-space probability densities for these constituents to be found in the hadrons. The latter are not calculable but can be measured in one type of process and applied to others.[3]

The complete content of the hard-scattering model has not yet been thoroughly explored. One reason for this is that, within the framework of the model, there exists the possibility of several types of "higher-twist" effects.[4] For the usual high-p_T jet observables, these higher-twist effects lead to power-suppressed contributions to the measured cross sections which can be neglected. There do exist examples of specific cross sections and special kinematic regions where higher-twist dynamics can predominate.[5] However, even in these cases where the effects can be observed, there are questions concerning the universality of the mechanisms invoked because the proofs of perturbative factorization are known to break down at the level of power-suppressed contributions.[6]

In spite of the need for caution there are powerful arguments for pursuing the consequences of the QCD-based parton model beyond the level of leading twist.[7] Even if the mechanisms involved at this level do not prove to be universal, the elucidation of the underlying dynamics can provide important physical insight and the "breakdown" of the approach can be informative. Spin observables, in particular, have historically provided powerful constraints on hadronic dynamics and may also turn out to be instructive in pinning down the QCD parton model. For example, the polarization of hyperons produced in high-energy collisions ($pp\to\Lambda_\uparrow X$) has been discussed in specialized models which combine hard scattering at the constituent level with particular coherent mechanisms.[8] These explicit models are compatible with the validity of the QCD hard-scattering approach and also provide a reasonable phenomenological framework for the analysis of the experimental data. The interpretation of the models involves interesting questions in the space-time structure of hard scattering.

This paper will discuss another type of single-spin observable which plays an important role in the design of experiments with polarized proton beams or polarized targets. The relevant observable is the asymmetry

$$A_N d\sigma(pp_\uparrow \to \text{jet}+x), \quad A_N d\sigma(pp_\uparrow \to \pi X) \tag{1.1}$$

(where π denotes a spin-0 meson), for the production of a jet or spinless hadron at large transverse momentum normal to the plane formed by the polarized proton's momentum and its spin. The observation of such asymmetries is frequently quoted as a puzzle or challenge for theory.[9] One frequently encounters the allegation that the QCD-hard-scattering model "predicts" that this type of single-spin observable should vanish. In fact, it is very hard to convince oneself that the usual formulation of the hard-scattering model makes any prediction at all for these asymmetries. The argument which associates the vanishing of the asymmetry with QCD can be motivated by a specific calculation which associated the result with an underlying transverse single-spin asymmetry at the quark level,[10]

$$A_N(q_\uparrow q\to qX)=\alpha_s \frac{m_q}{\sqrt{s}} f(\theta) , \tag{1.2}$$

and then convolutes this asymmetry with a transverse spin-transfer density from proton to quark. Most theoretical work which has studied the problem has emphasized that the effect represented by (1.2) is not the only type of "higher-twist" dynamics which can lead to the asymmetries (1.1) (Ref. 11). However, other possible mechanisms have not heretofore been explicitly described.

The important theoretical question which appears when confronting transverse spins in QCD is one of organizing the calculation in such a way that the appropriate dynamics are displayed. The proposal we wish to consider here involves the complete neglect of the mechanism of Ref. 10, at least for jets or hadrons involving only light quarks or gluons. Instead we start from the formulation of the hard-scattering model which includes the transverse momentum of the constituents:

$$G_{a/p}(s;\mu^2) \rightarrow G_{a/p}(x,\mathbf{k}_T;\mu^2) \ .$$

This formulation of the QCD-hard-scattering model has been discussed elsewhere.[12] It has been used, for example, to discuss the longitudinal structure function of the proton.[13] The relevance of the transverse momentum for the asymmetry (1.1) can be seen from the venerable Chou-Yang[14] model of the constituent structure of a transversely polarized proton. If we assume a correlation between the spin of the proton and the orbital motion of its constituents, Chou and Yang showed the existence of a nontrivial A_N in elastic scattering. The coherent dynamics which correlates the spin of the proton with the orbital angular momentum of the quarks and gluons can also produce a constituent-level asymmetry in transverse momentum:

$$\begin{aligned}\Delta^N G_{a/p(\uparrow)}(x,\mathbf{k}_T;\mu^2) &= \sum_h [G_{a(h)/p(\uparrow)}(x,\mathbf{k}_T;\mu^2) \\ &\qquad - G_{a(h)/p(\downarrow)}(x,\mathbf{k}_T;\mu^2)] \\ &= \sum_h [G_{a(h)/p(\uparrow)}(x,\mathbf{k}_T;\mu^2) \\ &\qquad - G_{a(h)/p(\uparrow)}(x,-\mathbf{k}_T;\mu^2)] \ , \\ a = q,G,\bar{q}, &\quad h = \text{helicity} = +- \ . \end{aligned} \tag{1.3}$$

It is important to realize that the incoherent scattering of these asymmetrically distributed constituents can lead to the observable asymmetries of (1.1) because of the kinematical dependence of the underlying hard processes on $\mathbf{k}_T$. In this approach the "trigger-bias" of the QCD hard-scattering model translates the orbital motion of the quarks and gluons into observable asymmetries at large p_T. We give a simple illustration of the kinematics in the Appendix which demonstrates how they produce an asymptotic behavior

$$A_N \propto \frac{\langle k_T \rangle}{p_T} \tag{1.4}$$

indicative of a higher-twist effect.

We attempt no proof that this mechanism provides the only "higher-twist" dynamics associated with single-spin asymmetries in QCD. Instead the assumption that other types of coherent effects might vanish here forms the simplifying hypothesis of a prospective model. The model predicts several types of regularities which can be looked for in future experiments. If these regularities are observed, then we have constrained other, more exotic, types of spin-dependent effects. We will discuss these predictions in Sec. III.

Although the asymmetries calculated in this way fall as $1/p_T$ they need not be considered proportional to a quark mass nor are they suppressed by powers of α_s once the spin-dependent effects are absorbed into the distribution (1.3). Simple estimates suggest, therefore, that the magnitude of the asymmetry can be compatible with effects observed in existing experimental data.

II. THE HARD-SCATTERING MODEL AND TRANSVERSE SPIN

The idea that there exists a regime where quantum chromodynamic processes can be calculated perturbatively has led to the formulation of a QCD-based parton model for the production of hadrons at large transverse momentum. For the process $pp \rightarrow \pi X$, the familiar expression for the invariant cross section at large transverse momentum is[15]

$$E_\pi \frac{d^3\sigma}{d^3p_\pi}(pp \rightarrow \pi X) \simeq \frac{1}{\pi} \sum_{ab\rightarrow cd} \int dx_a \int dx_b \int \frac{dx_c}{x_c^2} G_{a/p}(x_a;\mu^2) G_{b/p}(x_b;\mu^2) D_{\pi/c}(x_c;\mu^2) \times \left[\tilde{s}\frac{d\sigma}{d\tilde{t}}(ab \rightarrow cd)\delta(\tilde{s}+\tilde{t}+\tilde{u}) \right] \left[1 + O\left[\frac{\alpha_s}{\pi}\right] \right] , \tag{2.1}$$

where constituent masses are neglected and we have made the kinematic approximation

$$\tilde{s} \simeq x_a x_b s, \quad \tilde{t} \simeq \frac{x_a}{x_c} t, \quad \tilde{u} \simeq \frac{x_b}{x_c} u \ . \tag{2.2}$$

For the single-spin transverse asymmetry

$$A_N = \frac{d\sigma(pp_\uparrow \rightarrow \pi X) - d\sigma(pp_\downarrow \rightarrow \pi X)}{d\sigma(pp_\uparrow \rightarrow \pi X) + d\sigma(pp_\downarrow \rightarrow \pi X)} \ , \tag{2.3}$$

it has been suggested[10,16] that the expression (2.1) can be generalized to give

$$A_N \frac{d^3\sigma}{d^3p_\pi}(pp \to \pi X) \simeq \frac{1}{\pi} \sum_{ab \to cd} \int dx_a \int dx_b \int \frac{dx_c}{x_c^2} \Delta_s^T G_{a_\uparrow/p_\uparrow}(x_a;\mu^2) G_{b/p}(x_b;\mu^2) D_{\pi/c}(x_c;\mu^2) \times \left[\bar{s}\,\hat{a}_{ab} \frac{d\sigma}{dt}(a_\uparrow b \to cd) + \cdots \right], \tag{2.4}$$

where

$$\Delta_s^T G_{a_\uparrow/p_\uparrow}(x_a;\mu^2) = G_{a_\uparrow/p_\uparrow}(x_a;\mu^2) - G_{a_\downarrow/p_\uparrow}(x_a;\mu^2) \tag{2.5}$$

is the transverse-polarization density of the quark or gluon in the spinning proton and

$$\hat{a}_{ab} = \frac{d\sigma(a_\uparrow b \to cd) - d\sigma(a_\downarrow b \to cd)}{d\sigma(a_\uparrow b \to cd) + d\sigma(a_\downarrow b \to cd)} \tag{2.6}$$

is the perturbatively calculated transverse spin asymmetry at the parton level. There is a problem with this suggestion which appears immediately when one tries to implement it with a specific calculation. The asymmetry (2.6) for a $2 \to 2$ process with pointlike particles, arises as a result of the interference between a nonflip and a single-flip helicity amplitude. Since Born amplitudes in perturbation theory are real, a calculation at the one-loop level is necessary to generate an imaginary part and provide the interference.

Note that Eq. (2.4) contains a counterintuitive mixture of coherent and incoherent dynamics where *all* interference effects are associated with the hard scattering. In addition, quark helicity flip is vanishing at all orders of perturbation theory in the limit $m_q = 0$. This suggests that, for hadrons containing only light quarks, an approach based on (2.4) will never give substantial asymmetries. It is, in fact, premature to conclude anything about the calculation of single-spin asymmetries within the framework of the hard-scattering model at this point because we have simply made too many assumptions in writing (2.4). The inadequacies of this basic approach have long been recognized.[9,11] After further theoretical analysis, it has been suggested that a formulation similar to (2.4) might be valid with the stipulation that perturbative asymmetries be calculated using quark mass parameters given by constituent-quark masses of the order of typical hadronic masses.[17,18] While this general strategy for fixing up (2.4) may be possible, it still unnecessarily associates the coherent part of the dynamics with the hard scattering of the constituents and leads to a confusing space-time structure. We would like to attempt here an alternate way of organizing the calculation which is more consistent with the original formulation of the hard-scattering model and can be valid when $m_q \to 0$. This alternate organization displays dynamics which must be present and may, in fact, be dominant. The alternate starting point is a generalization of (2.1) which includes the transverse momentum of the constituents:

$$E_\pi \frac{d^3\sigma}{d^3p_\pi}(pp_\uparrow \to \pi X) \simeq \frac{1}{\pi} \sum_{\substack{ab \to cd \\ h_a, h_b}} \int d^2k_T^a dx_a \int d^2k_T^b dx_b \int d^2k_T^c \frac{dx_c}{x_c^2} G_{a(h_a)/p_\uparrow}(x_a, \mathbf{k}_{T_a};\mu^2) \times G_{b(h_b)/p}(x_b, \mathbf{k}_{Tb};\mu^2) D_{\pi/c}(x_c, \mathbf{k}_{Tc};\mu^2) \times \left[\tilde{s} \frac{d\sigma}{d\tilde{t}}(a(h_a)b(h_b) \to cd)\,\delta(\tilde{s}+\tilde{t}+\tilde{u}) \right] \left[1 + O\left[\frac{\alpha_s}{\pi}\right] \right]. \tag{2.7}$$

In writing (2.7) we have explicitly included the sum over the helicities of a and b. The constituent transverse momenta are assumed to be much smaller than the p_T of the detected hadron but we shall see they cannot be completely neglected. The reason why (2.7) is more accurate than (2.1) for the description of spin effects is that the Mandelstam invariants of the perturbative $2 \to 2$ subprocess can depend on the relative orientation of $\mathbf{k}_T^a$ and the direction of $\mathbf{P}_c$. The form of this dependence is demonstrated for a simple configuration in the Appendix. Therefore, even if we assume that the distributions in (2.7) do not depend on helicity, the spin information from the polarized proton which shows up in $G_{a(h)/p_\uparrow}(x, \mathbf{k}_T;\mu^2)$ can be transmitted to the cross section at large transverse momentum. It is convenient to parametrize the spin information directly in terms of a transverse-momentum asymmetry. For example, if we assume that the polarized proton has its momentum in the $+z$ direction and transverse spin along the unit vector $\hat{\mathbf{s}}_T$, we can define the normal direction by a unit vector

$$\hat{\mathbf{n}} = \hat{\mathbf{z}} + \hat{\mathbf{s}}_T \tag{2.8}$$

and write

$$\mathbf{k}_T^a = k_{TN}^a \hat{\mathbf{n}} + k_{TS}^a \hat{\mathbf{s}}_T \ . \tag{2.9}$$

We can define the normal asymmetry

$$\Delta^N G_{a/p_\uparrow}(x,\mathbf{k}_T;\mu^2)=\sum_h [G_{a(h)/p_\uparrow}(x,k^a_{TN}\hat{\mathbf{n}}+k^a_{TS}\hat{\mathbf{s}}_T;\mu^2)-G_{a(h)/p_\downarrow}(x,k^a_{TN}\hat{\mathbf{n}}+k^a_{TS}\hat{\mathbf{s}}_T;\mu^2)]$$
$$=\sum_h [G_{a(h)/p_\uparrow}(x,k^a_{TN}\hat{\mathbf{n}}+k^a_{TS}\hat{\mathbf{s}}_T;\mu^2)-G_{a(h)/p_\uparrow}(x,-k^a_{TN}\hat{\mathbf{n}}-k^a_{TS}\hat{\mathbf{s}}_T;\mu^2)] , \quad (2.10)$$

here we have used the fact that both $\hat{\mathbf{n}}$ and $\hat{\mathbf{s}}_T$ change sign under a spin flip. From the parity and time-reversal invariance of the strong interactions, $G_{a/p_\uparrow}(x,k_{TN},k_{TS};\mu^2)$ must be even under $k_{TS}\leftrightarrow -k_{TS}$ but there can be a nontrivial asymmetry in the $\hat{\mathbf{n}}$ direction. Thus strong interaction dynamics allow

$$\Delta^N G_{a/p_\uparrow}(x,k_{TN},k_{TS};\mu^2)\neq 0 . \quad (2.11)$$

The existence of this asymmetry distribution can be seen as a reflection of the "soft" asymmetries observed in peripheral processes such as $pp_\uparrow \to \pi X$ at low p_T translated to the level of quarks and gluons. The physics inherent in the possible existence of the distribution can also be seen more directly. Given spin-orbit forces, it is possible to formulate a semiclassical model which leads to a nontrivial $\Delta^N G_{a/p_\uparrow}$. In Ref. 14 Chou and Yang showed how rotating constituents in a polarized proton can lead to a nontrivial asymmetry in elastic scattering. The interpretation of this picture in quark-parton language can lead to (2.11). The distributions (2.10) are not completely arbitrary. The fact that the proton itself has no momentum in the $\hat{\mathbf{n}}$ direction leads to the sum rule

$$\int dx \sum_a \Delta^N G_{a/p_\uparrow}(x,k_{TN},k_{TS};\mu^2)=0 , \quad (2.12)$$

where the sum is over flavors and the integral over the longitudinal-momentum fraction. However, since spin-orbit forces can lead to a correlation between x and k_{TN}, this sum rule does not imply that the integrand vanishes. Once we parametrize the transverse spin information into (2.11), we see that there can exist a limit in which it contains all the coherent dynamics and, therefore,

$$A_N\left[E\frac{d^3\sigma}{d^3p}(pp_\uparrow\to mX)\right]\simeq\sum_{ab\to cd}\int d^2\mathbf{k}^a_T dx_a\int d^2\mathbf{k}^b_T dx_b\int d^2k_{TC}\frac{dx_c}{x_c^2}\Delta^N G_{a/p_\uparrow}(x_a,k^a_{TN},k^a_{TS};\mu^2)$$
$$\times G_{b/p}(x_b,k^{b2}_T;\mu^2)D_{m/c}(x_c k^{c2}_T;\mu^2)$$
$$\times\left\{\bar{s}\frac{d\sigma}{d\bar{t}}(ab\to cd)\delta(\bar{s}+\bar{t}+\bar{u})\left[1+O\left[\frac{\alpha_s}{\pi}\right]\right]\right\} . \quad (2.13)$$

This reformulation of the hard-scattering model is our basic result. In contrast with the "attempt" to generalize from (2.1) to (2.4), this equation corresponds to a probabilistic formula in the original spirit of the parton model where the unknown soft nonperturbative dynamics have been absorbed into the specification of the density $\Delta^N G_{a/p_\uparrow}$. The hard subprocess merely serves to transmit the asymmetry to large transverse momentum by the dependence of the kinematical invariants in the scattering on k^a_{TN}. This means the asymmetry effect is "higher twist" in the language discussed earlier. To see this we assume that all intrinsic transverse momenta are limited in (2.13):

$$|k^a_T|,|k^b_T|,|k^c_T|<\mu \ll p_T . \quad (2.14)$$

In doing the integrations in (2.13) we also assume that $\langle k^a_{TN}\rangle=\epsilon\mu$ where

$$\epsilon=\epsilon(x_T) \quad (2.15)$$

(with $x_T=2p_T/\sqrt{s}$) is a small parameter which characterizes the underlying asymmetry in $\Delta^N G_{a/p_\uparrow}$. If we define

$$x_T^{\pm}=\frac{2p_T\pm 2\epsilon\mu}{\sqrt{s}} \quad (2.16)$$

and insert a pointlike cross section depending on the $2\to 2$ invariants then scaling arguments applied to (2.13) lead to an approximation

$$A_N(p_\pi=p_T\hat{\mathbf{n}})\simeq\frac{f(x_T^-)[(p_T-\epsilon\mu)^2+\mu^2]^{-2}-f(x_T^+)[(p_T+\epsilon\mu)^2+\mu^2]^2}{f(x_T^-)[(p_T-\epsilon\mu)^2+\mu^2]^{-2}+f(x_T^+)[(p_T+\epsilon\mu)^2+\mu^2]^2} , \quad (2.17)$$

where $f(x_T)/p_T^4$ is the spin-averaged distribution in the scaling limit. Finally, if we parametrize

$$f(x_T)\simeq Ce^{-Bx_T} , \quad (2.18)$$

then expanding (2.7) for small $\epsilon\mu/p_T$ gives

$$A_N\simeq\frac{2B\epsilon(x_T)\mu}{\sqrt{s}}+\frac{4\epsilon(x_T)\mu p_T}{p_T^2+\mu^2}$$
$$\simeq(Bx_T+4)\frac{\epsilon(x_T)\mu}{p_T} . \quad (2.19)$$

This expression provides a reasonable form to use for guidance in the region $p_T \gg \mu$ where the spin-averaged p_T distribution has approximate scaling form. In the absence of further information on $\Delta^N G_{a/p_\uparrow}(x, k_{TN}, k_{TS}; \mu^2)$ we can make some reasonable assumptions to estimate the magnitude of the asymmetry. For example, we can assume that μ is a mass parameter representing the typical scale of a transverse-momentum distribution

$$\mu^2 \simeq m_\rho^2 \simeq 0.5\ \text{GeV}^2 \tag{2.20}$$

and that $B \simeq 10$ in correspondence with spin-averaged data. If we also neglect any kinematic variation of ϵ and set $\epsilon \simeq 0.05$ this gives the rough approximation

$$A_N \simeq \frac{0.35(x_T + 0.4)}{p_T/(1\ \text{GeV}/c)}\ , \tag{2.21}$$

so that if (2.17) is valid an asymmetry of a few percent can persist out to transverse momenta of order 10 GeV/c.

The fundamental assumption underlying (2.13) is that all of the spin information can be absorbed into $\Delta^N G_{a/p_\uparrow}(x, k_{TN}, k_{TS}; \mu^2)$ and that further non-spin-dependent corrections are proportional to μ^2/p_T^2. Once these assumptions are made, (2.13) can be seen to have the same general factorization properties as the hard-scattering model for an unpolarized scattering process. Of course, we need to worry about the validity of these assumptions, their stability under changes of the factorization scale, and whether they can be formulated consistently. Nonetheless, we have made some progress. Superficially, (2.13) is a lot more transparent in its physical content than (2.4) where one is confronted directly with quark mass parameters and an uncertain mixture of coherent and incoherent effects. It remains to be seen whether (2.13) is a reasonable translation of parton-model concepts into processes involving transverse spin. It is known, for example, that there are violations of the Block-Nordziek theorem concerning cancellations between real and virtual graphs which appear in QCD at the level of (m_q^2/Q^2) (Ref. 6). This has led to the speculation that it is inherently misleading to try to extend parton model concepts to deal with "higher-twist" processes. Beyond the leading-twist level there is not necessarily a *simple* parton description of dynamics. The operator basis can be chosen in several different ways[13] which, in parton language, involve different types of parton correlation functions. The trick is to have a physical picture which can provide a guide to economically and consistently incorporating the appropriate dynamics for the process measured.

It may be necessary to have more experimental information to aid in the organization of the calculation but we feel that (2.13) is a better "starting point" than other attempts to describe large-p_T asymmetries. The approach here can be related to the model description of the longitudinal structure function for deep-inelastic scattering in terms of the k_T^2-dependent parton distributions[13]

$$F_L(x;Q^2) \simeq \sum_a e_a^2 \frac{4}{Q_2} \int d^2k_T^a k_T^2 G_{a/p}(x, k_T^2; Q^2)$$
$$\simeq \sum_a e_a^2 \frac{4\langle k_T^{a2}\rangle}{Q^2}\ . \tag{2.22}$$

Although the model which leads to (2.22) is wrong, it has many of the features of the interacting theory. It can also be argued that the asymmetry inherent in (2.4) has been absorbed into the definition of $\Delta^N G_{a/p_\uparrow}(x, k_{TN}, k_{TS}; \mu^2)$ by assuming that the dominant contribution of the dynamics at the quark level involves soft momenta. One must be aware that the extra complication of introducing transverse momenta as well as allowing *arbitrary* spin-related dynamics has restricted the predictive power of (2.13). We will attempt to deal briefly with the regularities inherent in (2.13) in the next section.

III. DISCUSSION

The form of (2.13) is very much like the formulation of the hard-scattering model for the spin-averaged cross section (2.1) except that the distribution $\Delta^N G_{a/p_\uparrow}(x_a, k_{TS}^a, k_{TN}^a; \mu^2)$ can be either positive or negative. The spin-dependent information involved in the calculation is completely absorbed into this arbitrary distribution, which is "measured" by the experimental asymmetry. Without knowing $\Delta^N G_{a/p_\uparrow}(x, \mathbf{k}_T; \mu^2)$, the predictive power of (2.13) is therefore restricted to predicting regularities between the asymmetries of different processes.

One type of regularity which appears involves the weak dependence of the asymmetry in (2.13) on $G_{b/h}(x, \mathbf{k}_T; \mu^2)$ and $D_{\pi/c}(x, \mathbf{k}_T; \mu^2)$. All of the coherent dynamics in (2.13) are concentrated into the asymmetry $\Delta^N G_{a/p_\uparrow}(x, k_{TS}^a, k_{TN}^a; \mu^2)$ and the parton distribution functions of the other hadrons appear in both the spin-averaged and spin-subtracted cross sections. These functions thus merely serve to sample $\Delta^N G_{a/p_\uparrow}(x, \mathbf{k}_T; \mu^2)$ and $G_{a/p}(x, \mathbf{k}_T; \mu^2)$ in different kinematic regions to give $A_N d\sigma$ and $d\sigma$, respectively. If we make certain smoothness assumptions their effects approximately cancel in the ratios of asymmetries. This leads to the rough expectation

$$\frac{A_N(h_1 p_\uparrow \to \pi X)}{A_N(h_2 p_\uparrow \to \pi X)} \simeq 1\ (\text{central region})\ , \tag{3.1}$$

of universality. If coherent processes involving the quark masses can be neglected even for strange quarks this can be extended to give

$$\frac{A_N(h p_\uparrow \to \pi X)}{A_N(h p_\uparrow \to K X)} \simeq 1\ (\text{central region})\ . \tag{3.2}$$

There can, of course, be nontrivial flavor dependence if the underlying asymmetry $\Delta^N G_{q/p_\uparrow}$ depends on the flavor of the quark involved in the hard scattering. We would have, for example, in a kinematic region dominated by "valence" $qq \to qq$ scattering, a ratio

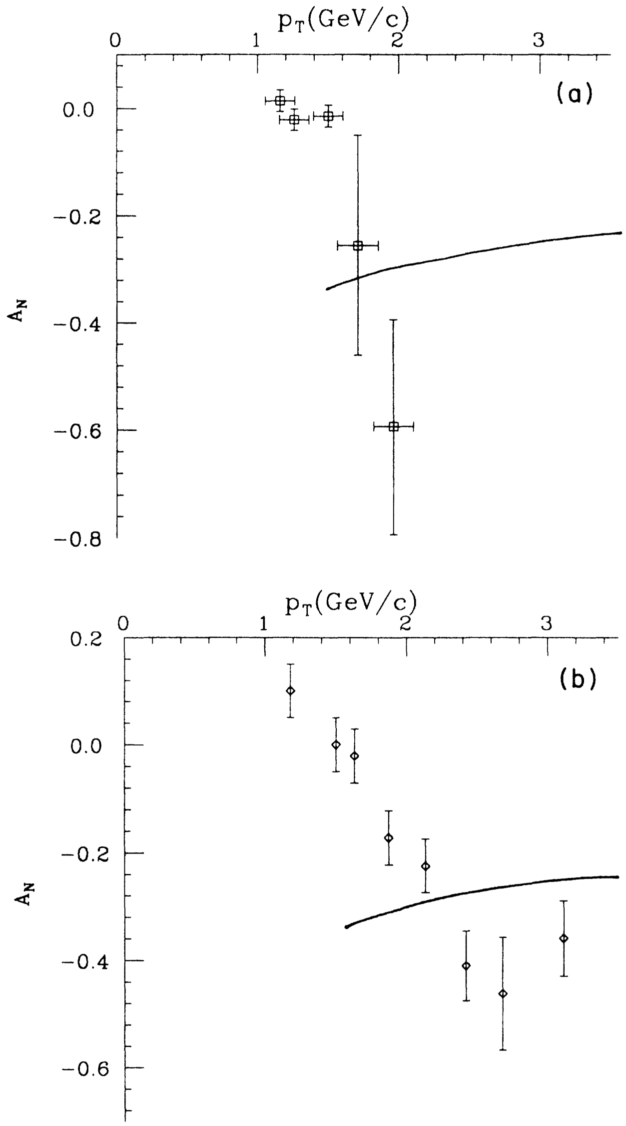

FIG. 1. (a) Data from Ref. 19 on $pp_\uparrow \rightarrow \pi^0 X$ at $p_{\rm lab}=24$ GeV/c, $x_F \in (0,0.1)$. (b) Data from Ref. 20 on $\pi^- p_\uparrow \rightarrow \pi^0 X$ at $p_{\rm lab}=40$ GeV/c, $x_F=0.0$ The curve is from Eq. (2.19) with $\epsilon=0.1$.

$$\left.\frac{A_N(hp_\uparrow \rightarrow \pi^+ X)}{A_n(hp_\uparrow \rightarrow \pi^- X)}\right|_{\text{large } x_T} \simeq \frac{\langle \Delta^N G_{u/p_\uparrow} \rangle}{\langle \Delta^N G_{d/p_\uparrow} \rangle} . \tag{3.3}$$

Even when such ratios are not unity the form of (2.13) suggests that they should depend only weakly on angles.

For the full range of kinematics, we should have the isospin invariant

$$\frac{2A_N(hp_\uparrow \rightarrow \pi^0 X)}{A_n(hp_\uparrow \rightarrow \pi^+ X) + A_N(hp_\uparrow \rightarrow \pi^- X)} = 1 . \tag{3.4}$$

It is interesting to confront the simplest version of these ideas with existing data. An experiment from CERN on $pp_\uparrow \rightarrow \pi^0 X$ at 24 GeV/c (Ref. 19) and an experiment from Serpukhov on $\pi^- p \rightarrow \pi^0 X$ at 40 GeV/c (Ref. 20) are shown in Figs. 1(a) and 1(b). Although these data are not in a region where the QCD cross section (2.1) can be considered to give a good fit to the spin-averaged distribution we have gone ahead and used (2.19) in its most naive form to estimate the size of $\langle \epsilon \rangle$ needed to characterize the experimental results. Curves are shown for $\epsilon=0.1$. Although we do not necessarily have a good fit to the data, this simple exercise provides a starting point for predicting asymmetries at higher transverse momentum. For comparison, this same value of ϵ is used in (2.19) to estimate the asymmetry for $pp_\uparrow \rightarrow \pi^0 X$ at small x_F for $\sqrt{s}=20$ GeV and $p_T=2$–6 GeV/c. The curve is shown in Fig. 2. This experiment should be done in the near future.

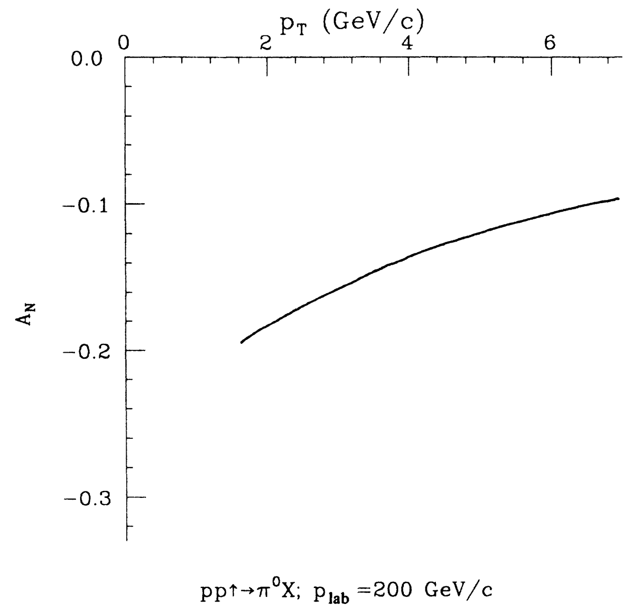

FIG. 2. The estimate (2.19) with $\epsilon=0.1$ is applied to $pp_\uparrow \rightarrow \pi^0 X$ at $p_{\rm lab}=200$ GeV/c.

There is room for theoretical work to explore the connection between (2.13) and the generalization of (2.4) advocated in Refs. 17 and 18.

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APPENDIX: 2→2 KINEMATICS WITH A TRANSVERSE SHIFT

We will use a simplified approach to the kinematics of the hard-scattering QCD model to demonstrate how the information from the $\Delta^N G(x, \mathbf{k}_T; \mu^2)$ is transmitted to the observable asymmetry at large transverse momentum. We will consider the process $ab \rightarrow cd$ with all "partons" massless and on mass shell. The four-momenta will be parametrized:

$$P_a=\left[x_aP_0\left[1+\frac{\epsilon^2(x_a)\mu^2}{x_a^2P_0^2}\right]^{1/2},\epsilon(x_a)\mu,0,x_aP_0\right]\ \text{(proton polarized)}\ ,$$

$$P_b=[x_bP_0,0,0-x_bP_0]\ \text{(other parton)},\quad P_c=\left[\frac{P_T}{\sin\theta_c},P_T,0,P_T\cot\theta_c\right]\ \text{(detected jet)}\ , \tag{A1}$$

$$P_d=\left[\frac{P_T-\epsilon(x_a)\mu}{\sin\theta_d},\epsilon(x_a)\mu-P_T,0,[P_T-\epsilon(x_a)\mu]\cot\theta_d\right]\ \text{(recoil jet)},\quad P_0>P_T\gg\epsilon(x_a)\mu\ ,$$

where the spin of proton a is oriented along the $+\hat{y}$ axis and there is a transverse shift of P_a along the x axis. Momentum and energy conservation give the constraints

$$(x_a-x_b)P_0=P_T\cot\theta_c+[P_T-\epsilon(x_a)\mu]\cot\theta_d\ . \tag{A2a}$$

$$(x_a+x_b)P_0+\frac{1}{2}\frac{\epsilon^2\mu^2}{x_aP_0}+\cdots=\frac{P_T}{\sin\theta_c}+\frac{P_T-\epsilon(x_a)\mu}{\sin\theta_d}\ . \tag{A2b}$$

We will confine attention to the kinematic region where it is safe to drop the second term on the left-hand side (LHS) of (A2b). Defining

$$x_T=P_T/P_0,\quad \xi_T=\frac{\epsilon(x_a)\mu}{P_0} \tag{A3}$$

with $\xi_T\ll x_T$ we introduce rapidity variables $e^{y_d}=\tan\frac{1}{2}\theta_d$ and $e^{y_c}=\tan\frac{1}{2}\theta_c$. The constraints (A2a) and (A2b) give

$$x_a\simeq\tfrac{1}{2}x_Te^{-y_c}+\tfrac{1}{2}(x_T-\xi_T)e^{-y_d}\ , \tag{A4a}$$

$$x_d\simeq\tfrac{1}{2}x_Te^{+y_c}+\tfrac{1}{2}(x_T-\xi_T)e^{+y_d}\ . \tag{A4b}$$

Equations (A4a) and (A4b) can be solved recursively (for example, by Newton's method) when the x_a dependence of $\epsilon(x_a)$ and ξ_T is included.

The kinematics of the $2\to2$ process $ab\to cd$ are described in terms of the Mandelstam invariants

$$\tilde{s}\simeq4x_ax_bP_0^2\simeq2P_0^2x_T(x_T-\xi_T)(1+\cosh\Delta y)\ , \tag{A5}$$

with $\Delta y=y_c-y_d$,

$$-\tilde{t}=P_0^2[x_T^2-2x_T\xi_T+x_T(x_T-\xi_T)e^{\Delta y}]\ , \tag{A6}$$

$$-\tilde{u}=P_0^2[x_T^2+x_T(x_T-\xi_T)e^{-\Delta y}]\ , \tag{A7}$$

so that $\tilde{s}+\tilde{t}+\tilde{u}=0$ in agreement with the mass-shell constraint for the $2\to2$ process. The c.m. system scattering angle is

$$\cos\theta^*=\frac{\tilde{t}-\tilde{u}}{\tilde{s}}=\frac{x_T\xi_T-x_T(x_T-\xi_T)\sinh\Delta y}{x_T(x_T-\xi_T)(1+\cosh\Delta y)}\ . \tag{A8}$$

If we now identify the partons with quarks and use perturbative QCD with the lowest-order cross section

$$\frac{d\sigma}{d\tilde{t}}=\frac{\alpha_s^2}{\tilde{s}^2}\frac{2}{9}\frac{\tilde{s}^2+\tilde{u}^2}{\tilde{t}^2}\ . \tag{A9}$$

This gives, for $y_d=y_c=0$, the scattering asymmetry

$$\frac{d\sigma}{d\tilde{t}}(+\epsilon)-\frac{d\sigma}{d\tilde{t}}(-\epsilon)=\frac{4}{9}\frac{\alpha_s^2}{P_T^4}\left[\frac{\epsilon\mu}{P_T}\right],\quad P_T\gg\mu\ , \tag{A10}$$

showing the general structure of the model in (2.13). The integrations over other transverse momenta in the hard-scattering preserve the higher-twist form (A10). Equation (2.13) therefore leads to a nontrivial higher-twist asymmetry at large transverse momentum from the incoherent scattering of pointlike constituents.

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