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Inclusive forward-backward asymmetry for light quarks

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The energy-differential forward-backward asymmetry in the reaction $e^+e^- \to M+$ anything, M being a leading, light-quark-constituted meson in a jet, is considered. The measurement of this quantity yields rank-1 and higher-rank fragmentation functions for light quarks hadronizing into light mesons, separately. It is also possible, in the case of a large event sample and good particle identification capabilities at high energies, to obtain information on Z couplings to light quarks.

The forward-backward asymmetry A_{FB}, in the pair creation of elementary fermions $f,\bar{f}$ from e^+e^- annihilation, has received a fair amount of theoretical and experimental attention.[1] When measured on resonance at the Z mass in the forthcoming Z factories, it will provide a clean and powerful precision test of the standard-model couplings of Z to $f,\bar{f}$ as well as an indirect means of searching for new physics.[2] Off-resonance measurements of A_{FB} at lower energies, covering γ-Z interference, exist[3] from e^+e^- storage rings for final-state leptons e, μ, and τ and heavy quarks c and b identified as jets. It has, however, not been possible so far to extend such measurements to the light quarks q_l on account of the difficulty of unambiguous identification of the flavor of the parent of a jet generated by a light quark. This is unfortunate because of the branching ratio of $Z \to q_l\bar{q}_l$ is about 43%. MAC and JADE Collaborations have utilized their hadronic data to measure the combined asymmetry for all quarks.[3] At the energies of the SLAC and DESY storage rings PEP and PETRA, where these measurements are made, this asymmetry arises from γ-Z interference and can be used to test the standard-model predictions for the axial-vector couplings of the quarks.

In this article, we consider the energy-differential forward-backward asymmetry in the 1-particle inclusive reaction $e^+e^- \to M+$anything (Ref. 4). M here is an observed light-quark-constituted meson (pseudoscalar as well as vector) which is the leading particle in a jet. We propose that a measurement of this quantity be used to obtain the fragmentation functions D_q^M, where q is a light quark. This would be an improvement over the existing measurements in e^+e^- annihilations, where only the combined fragmentation function $D^M = \sum_q f_q D_q^M$, f_q being the branching ratio for the specific reaction $e^+e^- \to q\bar{q}$, is known.[5]

The above inclusive forward-backward asymmetry, measured on the Z resonance, can also be used to obtain information on the light-quark couplings to Z. The production of light-quark-constituted mesons is, of course, heavily dependent on the details of the hadronization of light quarks. Nevertheless, if such mesons have high energy (i.e., with momenta greater than, say, 80% of beam momentum), the dependence of the inclusive forward-backward asymmetry on the details of hadronization becomes weak and one can hope to measure the chiral couplings of Z to the corresponding light quarks. While the event fractions for mesons with large fractional momenta are small and the distinction between $\pi^\pm$ and $K^\pm$ at such high energies is hard, a large event sample and a high-grade particle identification capability[6] may overcome these problems. The CERN e^+e^- collider LEP, on the Z resonance, can produce 10^7 Z's over a reasonable period, and this could yield a large enough sample of events.

Designate the angle made by M with the electron beam direction as θ^M and the fraction of beam energy carried by it as z: $0<\theta^M<\pi$ and $0<z<1$. Denote the cross section for the process under consideration as $\sigma^M(q^2)$, q^2 being the total four-momentum squared of the electron-positron pair. The inclusive energy-differential forward-backward asymmetry can be defined as[7]

$$A_{FB}^M(z,q^2) \equiv \left[\frac{d\sigma^M}{dz}\right]^{-1} \times \left[\int_0^1 - \int_{-1}^0\right] d\cos\theta^M \frac{d^2\sigma^M}{dz\, d\cos\theta^M} . \quad (1)$$

We shall make use of the ratio on the right-hand side (RHS) of (1) as well as its numerator and denominator separately.

First, we make a remark concerning the effect of CP symmetry, given an arbitrary particle M. Invariance under CP dictates that $d\sigma^M/d\cos\theta^M = d\sigma^{\bar{M}}/d\cos(\theta^{\bar{M}}+\pi)$ and hence $A_{FB}^M = -A_{FB}^{\bar{M}}$, $\bar{M}$ being the antiparticle of M. Consequently, (1) can be rewritten as

$$A_{FB}^M(z,q^2) = \left[\frac{d\sigma^M}{dz}\right]^{-1} \times \int_0^1 \left[d\cos\theta^M \frac{d^2\sigma^M}{dz\, d\cos\theta^M} - d\cos\theta^{\bar{M}} \frac{d^2\sigma^{\bar{M}}}{dz\, d\cos\theta^{\bar{M}}}\right] . \quad (2)$$

Evidently, $A_{FB}^M(z,q^2)$ vanishes when M is an eigenstate of C. This result, incidentally, holds even for the specific exclusive reaction $e^+e^- \to P+n$, if P is any observed particle and $|n\rangle$ a particular (in general multiparticle) state, all of whose momenta and spins are, respectively, integrated and summed, provided $|n\rangle$ contains self-conjugate particles and/or those accompanied by their antiparticles only. For instance, in the exclusive reaction

$e^+e^- \to \gamma f\bar{f}$ or $\phi f\bar{f}$ (ϕ being the neutral Higgs boson), $A_{FB}^{\gamma,\mathrm{excl}}(z,q^2)=A_{FB}^{\phi,\mathrm{excl}}(z,q^2)=0$; in contrast, the reaction $e^+e^- \to K^-\pi^+K^0$ has a nonvanishing $A_{FB}^{K^0,\mathrm{excl}}(z,q^2)$.

Next, specialize to the situation where M is the leading particle in a jet. Consider the process in the QCD parton picture.[8] Let σ_p be the cross section for the production of a parton of type p in e^+e^- annihilation. The probability distribution for the parton p to produce the meson M with a fraction z of its momentum is given by the fragmentation function $D_p^M(z)$. To zeroth order in QCD (naive parton model with perfect scaling) only $q\bar{q}$ pairs are produced and $D_p^M(z)$ are functions of z only. To first order in α_s, the QCD fine-structure constant, one must consider gluon-loop effects to $e^+e^- \to q\bar{q}$ as well as soft gluon bremsstrahlung by final-state quarks. (We confine ourselves only to events with two hard jets). Perfect scaling is thus violated and the functions $D_p^M(z)$ become q^2 dependent. This q^2 dependence is given by[5,8]

$$D_q^M(z,q^2)=D_q^M(z,q_0^2)+\frac{\alpha_S(q^2)}{2\pi}\ln\frac{q^2}{q_0^2}\left\{\int_z^1\frac{dx}{x}\left[P_{qq}\left[\frac{z}{x}\right]D_q^M(x,q_0^2)+P_{gq}\left[\frac{z}{x}\right]D_g^M(x,q_0^2)\right]\right\} . \tag{3}$$

In Eq. (3) P_{qq} and P_{gq} are Altarelli-Parisi splitting functions[8] and q_0 is a reference momentum, where data on fragmentation functions exist. Here we take $q_0=30$ GeV. One can then write

$$\frac{d^2\sigma^M}{dz\,d\cos\theta}=\sum_p\frac{d\sigma_p}{d\cos\theta}D_p^M(z,q^2) , \tag{4}$$

$$\frac{d^2\sigma^{\bar{M}}}{dz\,d\cos\theta}=\sum_p\frac{d\sigma_p}{d\cos\theta}D_p^{\bar{M}}(z,q^2) , \tag{5}$$

where θ is the angle made by the thrust axis of the jet containing M ($\bar{M}$) to the electron beam. The difference between this angle and θ^M ($\theta^{\bar{M}}$), the angle made by M ($\bar{M}$) with the electron beam, is typically negligible for leading particles.

Let M have the quark content $q_v\bar{q}\,'_v$. Consider the difference between (4) and (5). Its RHS receives contributions only from quarks q_v,q'_v and antiquarks $\bar{q}_v,\bar{q}\,'_v$; those from other sea quarks cancel out:

$$\int_0^1\left[d\cos\theta^m\frac{d^2\sigma^M}{dz\,d\cos\theta^M}-d\cos\theta^{\bar{M}}\frac{d^2\sigma^{\bar{M}}}{dz\,d\cos\theta^{\bar{M}}}\right]=\left[\int_0^1-\int_{-1}^0\right]d\cos\theta\left[\frac{d\sigma_{q_v}}{d\cos\theta}[D_{q_v}^M(z,q^2)-D_{\bar{q}_v}^M(z,q^2)]+\frac{d\sigma_{q'_v}}{d\cos\theta}[D_{q'_v}^M(z,q^2)-D_{\bar{q}\,'_v}^M(z,q^2)]\right] . \tag{6}$$

In contrast with the above, the sum of (4) and (5) receives in its RHS, contributions not only from $q_v,q'_v,\bar{q}_v,\bar{q}\,'_v$ but also from all other sea quarks and antiquarks $q_s,\bar{q}_s$. Thus the denominator of (2) becomes

$$\frac{d\sigma^M}{dz}=\sum_q\int_{-1}^1 d\cos\theta\frac{d\sigma_q}{d\cos\theta}[D_q^M(z,q^2)+D_{\bar{q}}^M(z,q^2)] , \tag{7}$$

where $\sum_q$ runs over valence quarks q_v and q'_v and sea quarks q_s.

If q_v,q'_v form a doublet of a good flavor-SU(2) symmetry (e.g., isospin) and M and $\bar{M}$ are the two terminal members of the corresponding triplet (e.g., $\pi^\pm$ or $\rho^\pm$), then, by isospin and C invariance, we have

$$D_{q_v}^M=D_{\bar{q}\,'_v}^M \quad\text{and}\quad D_{q'_v}^M=D_{\bar{q}_v}^M . \tag{8}$$

On substituting these relations in (6) and (7), they take the simpler forms

$$\int_0^{-1}\left[d\cos\theta^M\frac{d^2\sigma^M}{dz\,d\cos\theta^M}-d\cos\theta^{\bar{M}}\frac{d^2\sigma^{\bar{M}}}{dz\,d\cos\theta^{\bar{M}}}\right]=\left[\int_0^1-\int_{-1}^0\right]d\cos\theta\left[\frac{d\sigma_{q_v}}{d\cos\theta}-\frac{d\sigma_{q'_v}}{d\cos\theta}\right][D_{q_v}^M(z,q^2)-D_{q'_v}^M(z,q^2)] , \tag{9}$$

and

$$\frac{d\sigma^M}{dz}=\int_{-1}^1 d\cos\theta\left[\left[\frac{d\sigma_{q_v}}{d\cos\theta}+\frac{d\sigma_{q'_v}}{d\cos\theta}\right][D_{q_v}^M(z,q^2)+D_{q'_v}^M(z,q^2)]+\frac{d\sigma_{q_s}}{d\cos\theta}2D_{q_s}^M(z,q^2)\right] . \tag{10}$$

I. MEASUREMENTS OF THE FRAGMENTATION FUNCTIONS

The fragmentation functions $D_q^M(z)$ cannot be derived from first principles of QCD. They involve the processes of quarks combining into mesons which are in the domain of soft physics and are not calculable. There are various phenomenological models which are designed to explain the data.[5] Most of these models use the cascade picture of fragmentation.[9,10]

The meson M has quark content $q_v, \bar{q}\,'_v$. If a q_v (or a $\bar{q}\,'_v$) is produced in e^+e^- annihilation, it can directly fragment into M by picking up a $\bar{q}\,'_v$ (or a q_v) at the first step itself. Therefore, $D_{q_v}^M$ and $D_{\bar{q}\,'_v}^M$ are rank-1 fragmentation functions.[9,10] Any other quark (or antiquark) must first fragment into some meson M' such that a q_v (or a $\bar{q}\,'_v$) is left behind, which can in turn fragment into M. So all D_p^{M}'s, where p is not q_v or $\bar{q}\,'_v$, are higher-rank fragmentation functions. One can measure these fragmentation functions using (6) and (7), by making some reasonable assumptions as discussed below. In the cases where isospin relations (8) hold, one can use the simpler Eqs. (9) and (10).

For the case when M is π^+ or ρ^+, we have $q_v = u$ and $q'_v = d$. The isospin relations (8) hold and we can use (9) and (10). These equations involve one rank-1 fragmentation function $D_u^{\pi^+}$ and two higher-rank fragmentation functions $D_{d,s}^{\pi^+}$. In the cascade picture of fragmentation, all the higher-rank fragmentation functions into a particular meson are assumed to be equal. (This is since both q_v and $\bar{q}\,'_v$ must be produced from the vacuum in the case of higher-rank fragmentation while only one of them is so produced for rank-1 fragmentation.[11]) Now the number of independent fragmentation functions is reduced to two. The differential cross sections for quark production in (9) and (10) are easily calculable in the standard model. By considering the sum and difference of the numbers of charged pions (or ρ's) in the forward and backward hemispheres one can directly measure the fragmentation functions of light quarks into such charged mesons.

The situation is a little more complicated for the case when M is K^+. Here also all the higher-rank fragmentation functions are equal. But the two rank-1 fragmentation functions $D_u^{K^+}$ and $D_{\bar{s}}^{K^+}$ are not equal because SU(3) is broken.[9] However, they can be related to each other and the higher-rank fragmentation function if one makes an assumption about the relative probability of the production of $u\bar{u}$ and $s\bar{s}$ pairs from the vacuum. If this probability ratio is taken to be 0.4:0.2, then the relationship between the fragmentation functions becomes[12]

$$D_u^{K^+} = \frac{1}{2}\left[D_{\bar{s}}^{K^+} + D_s^{K^+} \right] . \tag{11}$$

Substituting this in (6) and (7), one can obtain the fragmentation functions of light quarks into charged kaons.

To measure the fragmentation functions in the way proposed, one must isolate the mesons coming from the decay of resonances and consider only those that are products of direct fragmentation. This can be done fairly easily for the leading particles in a jet.[13] The above procedure is not suitable for measuring fragmentation functions for nonleading mesons, since it is difficult to isolate the directly fragmented mesons from those emerging out of resonance decays.

The European Muon Collaboration[12] (EMC) has measured the fragmentation functions of light quarks into charged pions and kaons by a procedure which is somewhat similar to that discussed here (i.e., by taking the sum and difference of the numbers of mesons in the forward and backward hemispheres). But they had to assume the quark luminosity functions to calculate the quark production cross sections in the process of determining the fragmentation functions. In e^+e^- annihilations these cross sections are given directly by the standard model. So the fragmentation functions determined in this way are less parametrization dependent.

II. MEASUREMENT OF CHIRAL COUPLINGS OF Z TO LIGHT QUARKS

The chiral couplings of Z to u and d quarks have been obtained from deep-inelastic scattering of ν and $\bar{\nu}$ on isoscalar nuclear targets.[14] Measurement of the ratios of neutral-current to charged-current cross sections

$$R_\nu \equiv \frac{\sigma_{\nu N}^{\rm NC}}{\sigma_{\nu N}^{\rm CC}} = (g_{Lu}^2 + g_{Ld}^2) + \tfrac{1}{3}(g_{Ru}^2 + g_{Rd}^2) , \tag{12}$$

and

$$R_{\bar{\nu}} \equiv \frac{\sigma_{\bar{\nu} N}^{\rm NC}}{\sigma_{\bar{\nu} N}^{\rm CC}} = (g_{Lu}^2 + g_{Ld}^2) + 3(g_{Ru}^2 + g_{Rd}^2) , \tag{13}$$

where the g's refer to the chiral gauge couplings, allows us to determine $(g_{Lu}^2 + g_{Ld}^2)$ and $(g_{Ru}^2 + g_{Rd}^2)$. Our method will obtain the combination $[(g_{Lu}^2 - g_{Ru}^2) - (g_{Ld}^2 - g_{Rd}^2)][(g_{Lu}^2 + g_{Ru}^2) + (g_{Ld}^2 + g_{Rd}^2)]^{-1}$.

Consider the energy-differential inclusive forward-backward asymmetry for the case when M is a charged pion, for which $q_v = u$ and $q'_v = d$. (Exactly the same considerations go through when M is a charged ρ.) Our aim here is to find a situation where the strong-interaction corrections decouple from the weak-interaction quantities so that a combination of the chiral couplings of Z to u and d quarks gets directly related to experimentally measured quantities. One can obtain the relevant expression for $A_{FB}^{\pi^+}(z,q^2)$ by substituting (9) and (10) in (2). The expression for the $d\sigma(e^+e^- \to q\bar{q})/d\cos\theta$ for all q^2 is given in Ref. 3. Though our method is applicable to all q^2, let us specialize to the case $q^2 = M_Z^2$. The reason for this is the resonance production of Z which leads to a tremendous increase in the number of events. Also, on resonance the γ-Z interference term drops out and $d\sigma/d\cos\theta$ as well as $A_{FB}^{\pi^+}$ take particularly simple forms. First, we have

$$\left[\int_0^1 - \int_{-1}^0 \right] \frac{d\sigma(e^+e^- \to q\bar{q})}{d\cos\theta} d\cos\theta = \frac{\pi\alpha_Z^2}{4\Gamma_Z^2}(g_{Le}^2 - g_{Re}^2)(g_{Lq}^2 - g_{Rq}^2) \tag{14}$$

$$\int_{-1}^{1}\frac{d\sigma(e^+e^-\to q\bar{q})}{d\cos\theta}d\cos\theta=\frac{\pi\alpha_Z^2}{3\Gamma_Z^2}(g_{Le}^2+g_{Re}^2)(g_{Lq}^2+g_{Rq}^2)\,, \tag{15}$$

where Γ_Z is the Z width and $\alpha_Z=\alpha/\sin^2\theta_W\cos^2\theta_W$. It follows that

$$A_{FB}^{\pi^+}(z,M_Z^2)=\frac{3}{4}\left[\frac{g_{Le}^2-g_{Re}^2}{g_{Le}^2+g_{Re}^2}\right][(g_{Lu}^2-g_{Ru}^2)-(g_{Ld}^2-g_{Rd}^2)][D_u^{\pi^+}(z,M_Z^2)-D_d^{\pi^+}(z,M_Z^2)]$$
$$\times\{[(g_{Lu}^2+g_{Ru}^2)+(g_{Ld}^2+g_{Rd}^2)][D_u^{\pi^+}(z,M_Z^2)+D_d^{\pi^+}(z,M_Z^2)]+(g_{Ls}^2+g_{Rs}^2)2D_s^{\pi^+}(z,M_Z^2)\}^{-1}$$
$$=\frac{3}{4}\left[\frac{g_{Le}^2-g_{Re}^2}{g_{Le}^2+g_{Re}^2}\right]\frac{(g_{Lu}^2-g_{Ru}^2)-(g_{Ld}^2-g_{Rd}^2)}{(g_{Lu}^2+g_{Ru}^2)+(g_{Ld}^2+g_{Rd}^2)}F(z,q^2)\,, \tag{16}$$

where $F(z,q^2)$ contains all the information involving the fragmentation functions. As mentioned before, $D_u^{\pi^+}$ is a rank-1 fragmentation function and $D_{d,s}^{\pi^+}$ are higher-rank ones. Both theoretical expectations[9,10] and experimental data[12] indicate that the ratio of higher-rank fragmentation function to the rank-1 fragmentation function becomes small at high z, i.e., $D_{d,s}^{\pi^+}(z)/D_u^{\pi^+}(z)\to 0$ and $F(z,q^2)\to 1$ as $z\to 1$. Thus we can define

$$\mathcal{A}_{FB}^{\pi^+}(M_Z^2)\equiv\lim_{z\to 1}A_{FB}^{\pi^+}(z,M_Z^2)$$
$$=\frac{3}{4}\left[\frac{g_{Le}^2-g_{Re}^2}{g_{Le}^2+g_{Re}^2}\right]\frac{(g_{Lu}^2-g_{Ru}^2)-(g_{Ld}^2-g_{Rd}^2)}{(g_{Lu}^2+g_{Ru}^2)+(g_{Ld}^2+g_{Rd}^2)}\,. \tag{17}$$

The RHS of (17) contains a product of two ratios of functions of Z couplings: one involving the electron couplings and a second involving those u and d quarks. The first can be measured from the forward-backward asymmetry (on the Z resonance) in the reaction $e^+e^-\to\mu^+\mu^-$, assuming e-μ universality. Then (17) can be used to obtain $[(g_{Lu}^2-g_{Ru}^2)-(g_{Ld}^2-g_{Rd}^2)]\,[(g_{Lu}^2+g_{Ru}^2)+(g_{Ld}^2+g_{Rd}^2)]^{-1}$. It is remarkable that the inclusive forward-backward asymmetry for a meson is independent of strong interactions, in the limit $z\to 1$, yielding a combination of weak couplings.

One can also consider a generalized version of (17), for any value of q^2. To write out this expression, define the complex variable $\xi\equiv[\alpha_Z/\alpha(1-M_Z^2/q^2+iM_Z\Gamma_Z/q^2)]$. Then the inclusive forward-backward asymmetry, in the limit $z\to 1$, is given by

$$\mathcal{A}_{FB}^{\pi^+}(q^2)=[-2Q_f\mathrm{Re}\xi(g_{Le}-g_{Re})(g_{Lf}-g_{Rf})$$
$$+|\xi|^2(g_{Le}^2-g_{Re}^2)(g_{Lf}^2-g_{Rf}^2)]_{f=u-d}$$
$$\times[4Q_f^2-2Q_f\mathrm{Re}\xi(g_{Le}+g_{Re})(g_{Lf}+g_{Rf})$$
$$+|\xi|^2(g_{Le}^2+g_{Re}^2)(g_{Lf}^2+g_{Rf}^2)]_{f=u+d}^{-1}\,, \tag{18}$$

where Q_f is the electromagnetic charge of the fermion f.

In the standard model, (17) can be written as

$$\mathcal{A}_{FB}^{\pi^+}=-\frac{3}{4}\frac{1-4\sin^2\theta_W}{1-4\sin^2\theta_W+8\sin^4\theta_W}$$
$$\times\frac{\sin^2\theta_W}{\frac{3}{2}-3\sin^2\theta_W+\frac{10}{3}\sin^4\theta_W}\,, \tag{19}$$

where θ_W is the Weinberg angle. With the presently known value of $\sin^2\theta_W=0.23$, $\mathcal{A}_{FB}^{\pi^+}\simeq 0.026$ in the standard model. Any significant deviation, observed from this value, will indicate new physics beyond the standard model. If the electron beam is totally longitudinally polarized, the factor involving the electron couplings in the RHS of (17) becomes ∓ 1 depending on whether the helicity of the electron is positive or negative. This evidently enhances the magnitude of the asymmetry. For e^- and e^+ beams with partial longitudinal polarizations P^- and P^+, respectively, in the electron beam direction, (17) is modified to[15]

$$\mathcal{A}_{FB}^{\pi^+}(M_Z^2)$$
$$=\frac{3}{4}\frac{(g_{Lu}^2-g_{Ru}^2)-(g_{Ld}^2-g_{Rd}^2)}{(g_{Lu}^2+g_{Ru}^2)+(g_{Ld}^2+g_{Rd}^2)}$$
$$\times\frac{(1+P^+)(1-P^-)g_{Le}^2-(1-P^+)(1+P^-)g_{\mathrm{Re}}^2}{(1+P^+)(1-P^-)g_{Le}^2+(1-P^+)(1+P^-)g_{\mathrm{Re}}^2}\,. \tag{20}$$

One of the difficulties in verifying (19) or in measuring the chiral Z couplings of light quarks from (17) is achieving the limit $z\to 1$. This is since both theory[9,10] and experiment[5,12] indicate that all fragmentation functions go to zero in this limit. The situation here is analogous to that of the ratio of the neutron to proton structure functions in the limit of the scaling variable x approaching unity. This ratio, being equal to that of the squared electromagnetic charges of the d and u quarks,

$$\lim_{x\to 1}\frac{F_2^{en}(x)}{F_2^{ep}(x)}=\frac{Q_d^2}{Q_u^2}=\frac{1}{4}\,, \tag{21}$$

a benchmark of the quark parton model, is nonzero even though both $F_2^{en}(x)$ and $F_2^{ep}(x)$ vanish when $x\to 1$. A prolonged effort has been made over the years to experimentally verify this relation by a limiting procedure based on the accumulation of larger and larger data samples. The present results[16] indicate that the limiting value of the ratio in (21) is indeed close to $\frac{1}{4}$.

An alternative but more model-dependent procedure is to try to extract the chiral couplings in (17) from experiment by use of fragmentation models. Here all the pions with z greater than some z_0 are used to measure the forward-backward asymmetry. That is, both the numerator and the denominator of $F(z,q^2)$ are integrated from Z_0 to 1. Thus we define a correction factor

$$C=\frac{\int_{z_0}^{1} dz[D_u^{\pi^+}(z,M_Z^2)-D_d^{\pi^+}(z,M_Z^2)]}{\int_{z_0}^{1} dz[D_u^{\pi^+}(z,M_Z^2)+D_d^{\pi^+}(z,M_Z^2)+1.12D_s^{\pi^+}(z,M_Z^2)]}, \tag{22}$$

1.12 in the denominator being $2(g_{Ls}^2+g_{Rs}^2)(g_{Lu}^2+g_{Ru}^2+g_{Ld}^2+g_{Rd}^2)^{-1}$ in the standard model. The factor C can be evaluated by using a theoretical fragmentation model.

Two of the most widely used models of the fragmentation of quarks into mesons are the Field-Feynman[9] and Lund Monte Carlo[10] models. Both these models are based on the cascade picture, which makes the following two assumptions. In the cascade, each step of the fragmentation is similar to the previous step and the momentum distribution of a meson is taken to be just a function of the ratio of the meson momentum to the momentum of the quark which produced it. Let $f(1-\eta)$ be the probability for a quark to produce a meson with fractional momentum $\eta=p_M/p_q$ [or, equivalently, the probability to leave behind a momentum fraction $(1-\eta)$ after the meson production]. Then the probability $F(z)$ of finding final particles with a fraction $z=p_M/p_{\text{beam}}$ of the beam momentum is given by[9]

$$F(z)=f(1-z)+\int_z^1 \frac{d\eta}{\eta} f(\eta) F\left[\frac{z}{\eta}\right] . \tag{23}$$

The final fragmentation functions must also take into account the probabilities of production of various flavors and spins of the mesons. This introduces two parameters into the fragmentation picture. The relative probabilities of $q\bar{q}$ pairs being produced from vacuum are taken to be $\gamma:\gamma:1-2\gamma$ for $u\bar{u}, d\bar{d}, s\bar{s}$ pairs. The fraction of pseudoscalar mesons to pseudoscalar and vector mesons produced is denoted by the fraction $P/(P+V)$ (Ref. 5). Thus the fragmentation functions of light quarks into π^+'s turn out to be[9]

$$\begin{aligned} D_u^{\pi^+}(z)&=\frac{P}{P+V}\{\gamma f(1-z)+\gamma^2[F(z)-f(1-z)]\} , \\ D_d^{\pi^+}(z)&=D_s^{\pi^+}(z)=\frac{P}{P+V}\gamma^2[F(z)-f(1-z)] . \end{aligned} \tag{24}$$

This picture could easily be extended to take into account gluon fragmentation.[17] The QCD evolution of the fragmentation functions was given in (3).

In Table I we have listed the values of C for two forms of the function $f(1-z)$ that are widely used:[5]

$$f1(-z)=1-a+3a(1-z)^2 \quad \text{with } a=0.77 ,$$

$$f(1-z)=(1+d)(1-z)^d \quad \text{with } d=0.7 .$$

We have taken[5,9,12] $\gamma=0.4$ and[5,9,13] $P/(P+V)=\frac{1}{2}$. The values of C from the two fragmentation models agree[18] to within 7%.

Finally, let us consider briefly the number of events which would measure $\mathcal{A}_{FB}^{\pi^+}$ at LEP. Assume a sample of events with a 10^6 Z bosons. The branching ratio $B(Z\to u\bar{u}+d\bar{d})$ being nearly 34%, the number of positively charged pions with $z\geq 0.8$ is given by

$$\begin{aligned} N^{\pi^+}(z\geq 0.08)&=10^6\times 0.34\times(\text{denominator of } C) \\ &=10^6\times 0.34\times 0.017=5780 . \end{aligned} \tag{25}$$

The statistical uncertainty in N^{π^+} is $\sqrt{N^{\pi^+}}\simeq 76$. On the other hand, the difference in N^{π^+} between the contributions from the forward and backward hemispheres is the number of events leading to the inclusive asymmetry. This is given bv

$$\mathcal{A}_{FB}^{\pi^+} C N^{\pi^+}(z\geq 0.8)\simeq 120 . \tag{26}$$

Therefore, with a million Z's, the inclusive asymmetry would be only a 1.5σ effect. In contrast, if the initial sample contains $10^7 Z$'s, then N^{π^+} and the number of events leading to the inclusive asymmetry are increased by a factor of 10, whereas the uncertainty increases only by a factor 3. Now the asymmetry is a much more significant (5σ) effect. In case the electron beam is longitudinally polarized (even partially), $\mathcal{A}_{FB}^{\pi^+}$ and hence the signal for the inclusive asymmetry is increased tremendously by virtue of (20). Then it is likely to become a precision measurement. From Table I we note that the correction factor C, defined in (22), can be determined only up to about an accuracy of 10% due to the theoretical uncertainty in $D_q^{\pi^+}(z)$. The uncertainty in C can be reduced by integrating over z from 0.9 to 1, instead of from 0.8 to 1. Though this reduces the data sample and increases the relative statistical error to the point of overcoming the signal for a million Z's and an unpolarized beam, such a procedure is likely to yield dividends if data are accumulated for long periods to collect many more Z's and using a longitudinally polarized beam.

To conclude, in this article we have pointed out a direct, parametrization-independent way of measuring fragmentation functions of light quarks into various mesons. For π's and ρ's the method is quite clean and makes only a mild assumption about higher-rank fragmentation functions. For K's one needs to make an additional assumption on the relative production of $s\bar{s}$ and $u\bar{u}$ pairs from the vacuum. We have also highlighted a new way of extracting information on the chiral couplings of the Z boson to liqht quarks, using the inclusive forward-backward asymmetry in meson production. This asymmetry, in the limit when the meson energy approaches the beam energy, becomes independent of strong interactions. In that sense it is somewhat on par with the famous $\lim_{x\to 1} F_2^{en}(x)/F_2^{ep}(x)$ ratio and calls for a similar

TABLE I. The values of the correction factor C for various fragmentation functions.

$f(1-z)$	Numerator of C	Denominator of C	C
$0.23+2.31(1-z)^2$	0.014	0.017	0.82
$1.7(1-z)^{0.7}$	0.015	0.017	0.88

experimental effort as has been directed to the latter. The combination of light quark chiral couplings probed by our method is different from those studied in deep-inelastic ν and $\bar{\nu}$ scattering off isoscalar nuclear targets.

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